RSS-Based Source Localization When Path-Loss Model Parameters are Unknown

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Abstract—**In this letter we present a novel method for source node localization using received signal strength in situations where path-loss model parameters are unknown. First, a ratio approach is used to eliminate the transmitting power uncertainty, then, combined with a search method for path-loss exponent under certain constraints, linear least squares is utilized to determine the location of the source node. Simulations are used to demonstrate the feasibility and suitability of the proposed method. The simulation results indicate that the method presented outperforms other off-the-shelf source node localization algorithms intended for use when path-loss model parameters are unknown.**

Index Terms—**Source localization, received signal strength, path-loss model.**

I. Introduction

SOURCE node localization is the focus of much research in communications engineering due to its widespread adoption in applications such as radar, sonar, telecommunications, mobile communications, and wireless sensor networks [1]. Commonly used measurement methods used for localizing source nodes are: time-of-arrival (TOA) [2], time-difference-of-arrival (TDOA) [3], angle-of-arrival (AOA) [4], acoustic energy [5], and received signal strength (RSS) [6]. Compared with the other methods, because the RSS measurement method does not require time synchronization or use of an antenna array, it makes it an effective and cost-saving method for source localization in terms of both software and hardware.

RSS-based source localization can be divided into two categories—physical localization and symbolic localization—according to whether the coordinates of receivers are needed or not, respectively. For symbolic localization, only RSS measurements are used to get close to the source node. For instance, [7] relies on the computation of a local gradient in the neighborhood of the moving RSS measurements to find the location of a Wi-Fi-equipped device. [8] rotates a wireless receiver around the user's body to produce a directional analysis that accurately predicts the direction of the source node.

Physical source localization refers to obtaining the position of the source node based on RSS measurements of receivers and their known coordinates. Common algorithms for RSS-based physical source localization are: centroid [9], gradient [10], and trilateration [11]. The centroid algorithm takes the average of all the receivers within the transmission range of the source node as its location; it is simple to implement but low in localization accuracy. The gradient algorithm first uses the local signal strength distribution to estimate the direction of the access point, and then acquires its location by combining multiple directional estimates. Although gradient algorithm can overcome sampling bias and reduce non-uniform signal propagation effect, it requires extensive RSS measurements. Trilateration converts the RSS measurements to distances between the source node and receivers using a path-loss model, and then combines these distances to obtain the location of the source node. This algorithm requires the path-loss model parameters, such as source transmitting power and path-loss exponent, which usually are assumed to be known *a priori* through a calibration phase.

However, in anonymous environments, it is impractical to know this path-loss model information in advance. To address this limitation, [12] considers path-loss exponent as an unknown parameter and estimates it simultaneously with the unknown location coordinates of the target node; and [13] utilizes differential RSS measurements to eliminate the transmitting power uncertainty. While both approaches compensate for where one or other of the path-loss model parameters are unknown, neither method can be employed unless at least one model parameter is known.

Reference [14] linearly approximates the exponential relationship between RSS measurement and distance, making it possible to estimate the location of the source node when both transmitting power and path-loss exponent are unknown. But the localization error is relatively large when the geometric conditions are poor.

In this letter, we propose a novel RSS-based physical source localization algorithm which suits anonymous environments where the path-loss model parameters are unknown. No calibration phase is needed, because it firstly eliminates the source transmitting power uncertainty by using a ratio approach, then, combined with a search method in path-loss exponent space, linear least squares is utilized to determine the location of the source node. Simulations are used to demonstrate the feasibility and suitability of the proposed method, and comparisons with other algorithms which can be used when path-loss model parameters are unknown, such as centroid and linearization, show its advantages.

II. Method Description

In the proposed method the coordinates of the source to be estimated are defined as (x, y), and the known coordinates of

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the ith receiver as $(x_i, y_i)(1 \le i \le m)$, where the number of receivers, $m \ge 3$. The first step is to convert the measured RSS at the ith receiver R_i to a distance d_i between the source node and the ith receiver.

Theoretical and practical tests indicate in [15] that measured RSS decreases logarithmically as the distance increases, and that RSS values can be modeled by the following equation:

$$R_i = R_0 - 10n\log_{10}(d_i/d_0) + X_i \tag{1}$$

where R_i is the measured RSS at the ith receiver; R_0 is the received signal strength in dBm at a reference distance d_0 from the source node (usually $d_0 = 1$ m); n is the path-loss exponent of the environment; and X_i represents the shadow noise modeled as an uncorrelated zero-mean Gaussian variable with standard deviation σ dB.

As R_0 is a source transmitting power-related constant, and path-loss exponent n is environment-dependent and usually can be assumed constant in a particular environment for a certain period of time, then R_i can be modeled as a Gaussian random variable. Therefore the probability-density function of R_i conditioned on the distance d_i between the source node and the receiver is expressed as [16]:

$$f(R_i/d_i) = \frac{1}{\sigma}\exp\left[-\frac{\left(R_i - (R_0 - 10n\log_{10}(d_i/d_0))\right)^2}{2\sigma^2}\right]. \tag{2}$$

Thus we can calculate the maximum likelihood estimator of d_i using:

$$\hat{d}_i = d_0 10^{(R_0 - R_i)/10n}. \tag{3}$$

However, in an anonymous environment, parameters R_0 and n are usually unknown in advance, therefore R_i cannot be directly converted to distance $\hat{d}_i$ from the source to the receiver. To address this, (3) is modified as follows:

$$\hat{d}_i = d_0\left(10^{\frac{R_0}{10}}/10^{\frac{R_i}{10}}\right)^{1/n}. \tag{4}$$

Simplifying as follows, $10^{R_0/10} = P_0$ and $10^{R_i/10} = P_i$, and then using the first receiver as the reference, a ratio approach, that is divide distances of other receivers by the distance of the first reference receiver, is used to eliminate the uncertainty of the source transmitting power:

$$\hat{d}_1/\hat{d}_i = (P_i/P_1)^{1/n}. \tag{5}$$

As $\hat{d}_i \approx \sqrt{(x_i - x)^2 + (y_i - y)^2}$, we rearrange (5):

$$\left(P_1^{\frac{2}{n}} - P_i^{\frac{2}{n}}\right)\left(x^2 + y^2\right) + 2\left(P_i^{\frac{2}{n}} x_i - P_1^{\frac{2}{n}} x_1\right)x + 2\left(P_i^{\frac{2}{n}} y_i - P_1^{\frac{2}{n}} y_1\right)y = P_i^{\frac{2}{n}}\left(x_i^2 + y_i^2\right) - P_1^{\frac{2}{n}}\left(x_1^2 + y_1^2\right). \tag{6}$$

By introducing a variable $S = x^2 + y^2$ and substituting it in (6), we can stack all equations from (6) in a matrix form:

$$\mathbf{A}\boldsymbol{\theta} = \mathbf{b} \tag{7}$$

where:

$$\mathbf{A} = \begin{bmatrix} 2\left(P_2^{\frac{2}{n}} x_2 - P_1^{\frac{2}{n}} x_1\right) & 2\left(P_2^{\frac{2}{n}} y_2 - P_1^{\frac{2}{n}} y_1\right) & P_1^{\frac{2}{n}} - P_2^{\frac{2}{n}} \\ 2\left(P_3^{\frac{2}{n}} x_3 - P_1^{\frac{2}{n}} x_1\right) & 2\left(P_3^{\frac{2}{n}} y_3 - P_1^{\frac{2}{n}} y_1\right) & P_1^{\frac{2}{n}} - P_3^{\frac{2}{n}} \\ \cdots & \cdots & \cdots \\ 2\left(P_m^{\frac{2}{n}} x_m - P_1^{\frac{2}{n}} x_1\right) & 2\left(P_m^{\frac{2}{n}} y_m - P_1^{\frac{2}{n}} y_1\right) & P_1^{\frac{2}{n}} - P_m^{\frac{2}{n}} \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} P_2^{\frac{2}{n}}\left(x_2^2 + y_2^2\right) - P_1^{\frac{2}{n}}\left(x_1^2 + y_1^2\right) \\ P_3^{\frac{2}{n}}\left(x_3^2 + y_3^2\right) - P_1^{\frac{2}{n}}\left(x_1^2 + y_1^2\right) \\ \cdots \\ P_m^{\frac{2}{n}}\left(x_m^2 + y_m^2\right) - P_1^{\frac{2}{n}}\left(x_1^2 + y_1^2\right) \end{bmatrix} \quad \boldsymbol{\theta} = \begin{bmatrix} x \\ y \\ S \end{bmatrix}.$$

In these equations the source transmitting power-related parameter R_0 has been eliminated through the ratio approach. However, the path-loss exponent n remains unknown, making $\mathbf{A}$ and $\mathbf{b}$ undeterminable, and preventing estimation of the source node. To address this, a search technique for the path-loss exponent n is proposed.

Usually the path-loss exponent values vary in different environments, and can not be obtained in advance for an anonymous environment. Fortunately, there is a constraint to the path-loss exponent—it always lies between a minimum $N_{\min}$ and a maximum $N_{\max}$—with typical values between two and four [17]. This constraint can be used to search for the optimum path-loss exponent n_{opt}.

For a given path-loss exponent n_j under the constraint $N_{\min} \le n_j \le N_{\max}$, distance ratios can be calculated using the RSS measurements R_i according to (5). Together with the assumed n_j, by calculating $\mathbf{A}_{n_j}$ and $\mathbf{b}_{n_j}$, and the corresponding linear least squares estimator $\hat{\boldsymbol{\theta}}_{n_j} = [\hat{x}_{n_j}\ \hat{y}_{n_j}\ \hat{S}_{n_j}]^T$, a second distance ratios can be calculated based on the receivers' coordinates (x_i, y_i). The n_j that produces the closest match between the two distance ratios is the optimum path-loss exponent n_{opt}.

Without considering the dependency between x, y and S we can firstly get the linear least squares estimator $\hat{\boldsymbol{\theta}}_{n_j}$ as follows:

$$\hat{\boldsymbol{\theta}}_{n_j} = (\mathbf{A}_{n_j}^{\mathbf{T}}\mathbf{A}_{n_j})^{-\mathbf{1}}\mathbf{A}_{n_j}^{\mathbf{T}}\mathbf{b}_{n_j}. \tag{8}$$

Then, with $(\hat{x}_{n_j}, \hat{y}_{n_j})$ and (x_i, y_i), the distance d_{i,n_j} between the source and the ith receiver can be calculated as:

$$d_{i,n_j} = \sqrt{\left(\hat{x}_{n_j} - x_i\right)^2 + \left(\hat{y}_{n_j} - y_i\right)^2}. \tag{9}$$

When the distance ratios from (5) most match the distance ratios from (9), we obtain the optimum path-loss exponent n_{opt}:

$$n_{\text{opt}} = \underset{N_{\min} \le n_j \le N_{\max}}{\arg\min} \sum_{i=2}^{m}\left[(d_{1,n_j}/d_{i,n_j}) - (P_i/P_1)^{1/n_j}\right]^2. \tag{10}$$

When we obtain the optimum path-loss exponent n_{opt}, the corresponding linear least squares estimator $\hat{\boldsymbol{\theta}}_{n_{\text{opt}}}$ represents the location of the source node. Without a loss of generality the path-loss exponent constraint can be set as $N_{\min} = 1$, $N_{\max} = 5$, and the search step $step = 0.05$ empirically. This is a trade-off between localization accuracy and search time. The reason why an iteration method [1] is not used to determine the path-loss exponent is that an initial value chosen at random sometimes may converge to a local minimum and a consequential large estimation error.

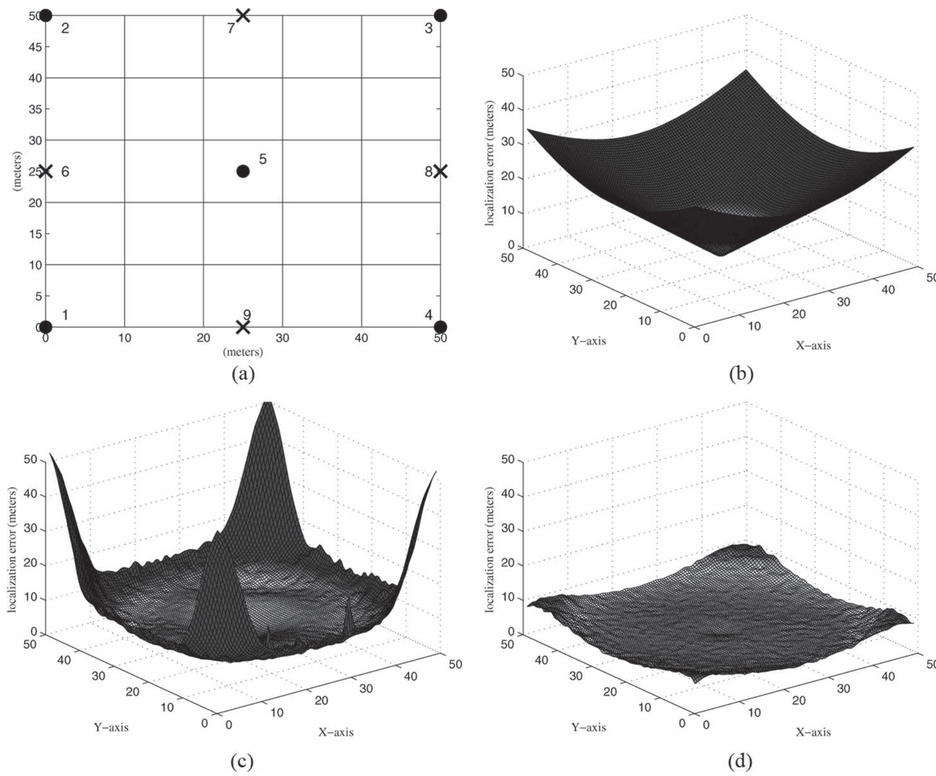

Fig. 1. Performance comparison of different algorithms. (a) Scenario one; (b) centroid; (c) linearization; (d) ratio&search.

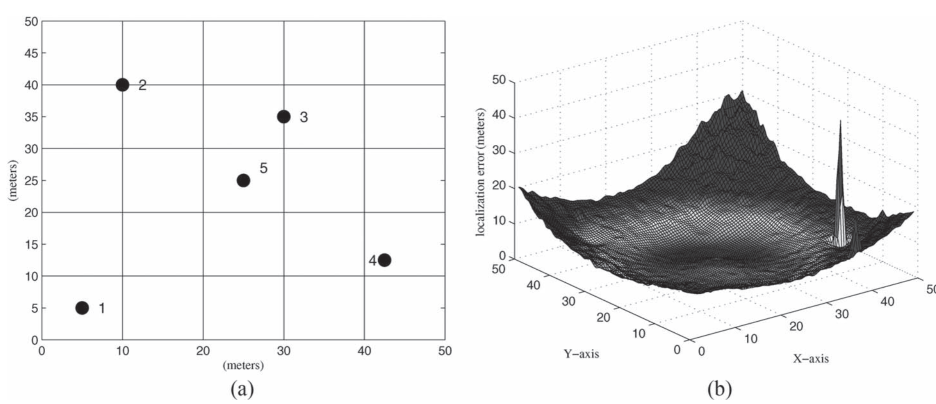

Fig. 2. Performance of the proposed algorithm in different geometries. (a) Scenario two; (b) ratio&search.

III. Numerical Results

Simulations were carried out to evaluate the performance of the proposed algorithm by comparing its performance with that of the centroid and linearization methods, as neither of them needs the path-loss model parameters. In scenario one [see Fig. 1(a)], five receivers were represented by black points on the four corners and in the center of the 50 m $\times$ 50 m square respectively. RSS measurements were generated by the path-loss model with $R_0 = -30$ dBm, $n = 3$ and $\sigma = 2$ dB. The source node localization estimation was conducted in every 1 meter grids and all results were averages of 1000 independent runs.

The performance of the centroid algorithm, which does not need any information about the path-loss model, is shown in Fig. 1(b), its localization accuracy is directly related to the location of the source node and RSS measurement locations. The localization error is large when the distribution of RSS measurement locations is non-uniform.

The linearization algorithm, approximating the exponential relationship between RSS measurement and distance, does not need the path-loss model parameters and has small localization error in most part of the scenario, but suffers large errors at the corners when the geometric conditions are poor, which is shown in Fig. 1(c).

The performance of the algorithm proposed in this letter is shown in Fig. 1(d). It can be seen that the localization error is relatively small across the scenario and it has better localization accuracy at the corner area compared with the linearization method, which means it is not as susceptible to geometric conditions as the linearization method.

In order to examine the suitability of the proposed algorithm in different geometries, another scenario was chosen. In Fig. 2(a), the black points represented the locations of the receivers and they were placed irregularly. Path-loss model parameters were set the same as in scenario one. Fig. 2(b) shows

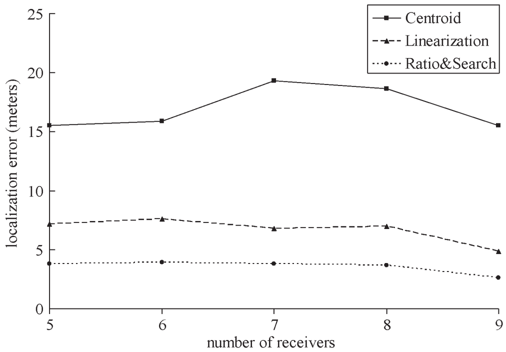

Fig. 3. Performance comparison with different number of receivers.

TABLE I
ESTIMATION ERROR (METERS) FOR DIFFERENT MODEL PARAMETERS

R_0(dBm)	-20	-25	-30	-35	-40
Centroid	15.51	15.51	15.51	15.51	15.51
Linearization	7.28	7.31	7.20	7.31	7.19
Ratio&Search	3.72	3.77	3.77	3.80	3.75

Shadow Noise σ(dB)	1	1.5	2	2.5	3
Centroid	15.51	15.51	15.51	15.51	15.51
Linearization	6.93	7.02	7.20	7.84	8.66
Ratio&Search	1.75	2.72	3.77	5.01	6.18

Path-loss Exponent n	2	2.5	3	3.5	4
Centroid	15.51	15.51	15.51	15.51	15.51
Linearization	8.39	7.70	7.20	7.12	6.94
Ratio&Search	6.04	4.71	3.77	3.18	2.72

the performance of the proposed algorithm in scenario two. Although the localization accuracy is lower than that in the first scenario when the receivers were regularly placed, it still shows relatively small localization error.

More simulations were conducted to evaluate the effects of the number of receivers on the localization accuracy. Scenario one was used and more receivers represented by black crosses were located at the midpoints of the four sides. Source node location (24.5, 9.5) was chosen and other parameters were kept the same as before. The results are shown in Fig. 3. The performance of centroid does not become better with more receivers placed. However, the localization errors of both the linearization algorithm and the proposed algorithm decrease as the number of the receivers increases. And the simulation also shows that the localization error of the proposed algorithm is always lower than that of the linearization algorithm.

The effects of different path-loss model parameters on the proposed algorithm were also evaluated. For each model parameter, other parameters are kept constant as before. The results are shown in Table I. Centroid is robust to different model parameters as its localization accuracy never changes, while the accuracies of both the linearization algorithm and the proposed algorithm show similar trend when a single parameter changes. For both the linearization algorithm and the proposed method, the localization accuracy keeps nearly constant when the transmitting power related parameter R_0 changes, decreases as the shadow noise σ increases and improves as the path-loss exponent n increases. In summary, the simulations indicate that, for different path-loss model parameters, the proposed method results in better localization accuracy compared with centroid and linearization.

IV. CONCLUSION

A RSS-based source localization algorithm that does not require known path-loss model parameters is proposed. This method is proposed as being suitable for anonymous environment and situations where an offline calibration phase is not possible. Initially it eliminates the uncertainty of the source transmitting power using a ratio approach, then combined with a search method in path-loss exponent space, linear least squares is utilized to determine the location of the source node. Simulation results show that the proposed method has better localization accuracy compared with other methods used when path-loss parameters are unknown. A limitation in the proposed method is that the path-loss exponent is assumed to be the same for different measurements, whereas there may be minor differences in these values [16]. This will be the basis for future research work.

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