Convolutional Neural Networks

Introduction à l'apprentissage automatique – GIF-4101 / GIF-7005

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Week 10

UNIVERSITÉ LAVAL

10.1 Convolution and image processing

Convolution

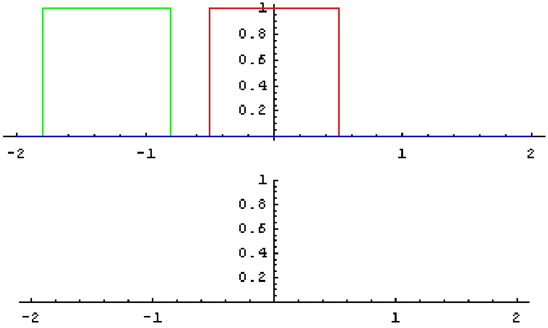
- Convolution: product of two functions on the same domain

$$f(x) * g(x) \equiv \int_{t=-\infty}^{\infty} f(x-t)\, g(x)\, \mathrm{d}t$$

- Discrete formulation

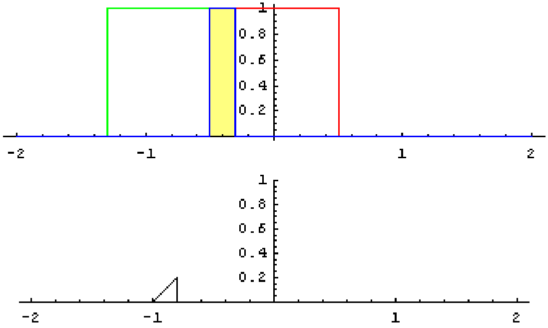
$$f(x) * g(x) \equiv \sum_{t=-\infty}^{\infty} f(x-t)\, g(x)$$

Convolution example



By Lautaro Carmona, CC-SA 4.0, `https://commons.wikimedia.org/wiki/File:Convolucion_Funcion_Pi.gif`.

Convolution example



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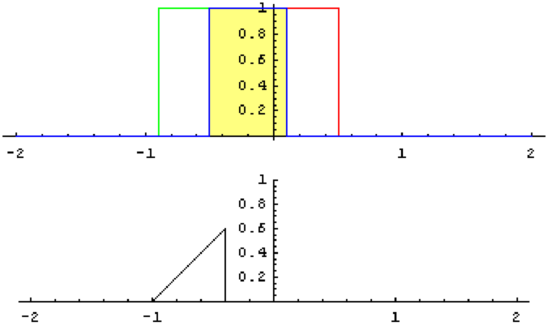
Convolution example



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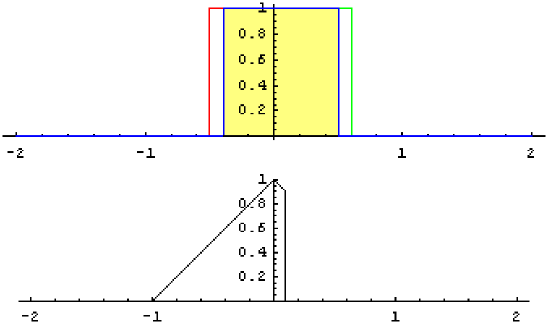
Convolution example



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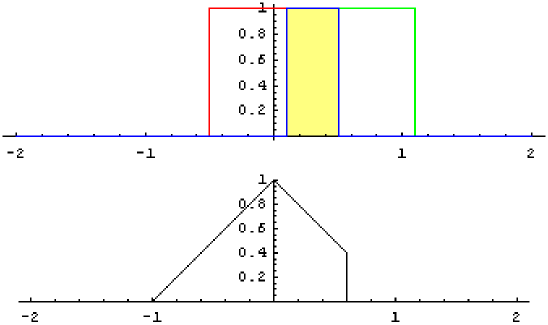
Convolution example



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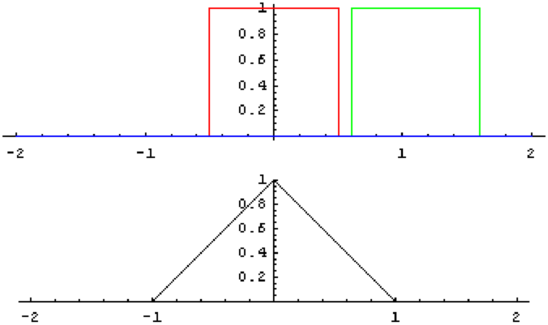
Convolution example



By Lautaro Carmona, CC-SA 4.0, `https://commons.wikimedia.org/wiki/File:Convolucion_Funcion_Pi.gif`.

Convolution and density estimation

- Off-center Dirac distribution

$$\delta(x-t) = \begin{cases} \infty & \text{if } x = t \\ 0 & \text{otherwise} \end{cases}, \quad \int_{x=-\infty}^{\infty} \delta(x-t)\,\mathrm{d}x = 1.$$

  - Convolution on off-center Diracs

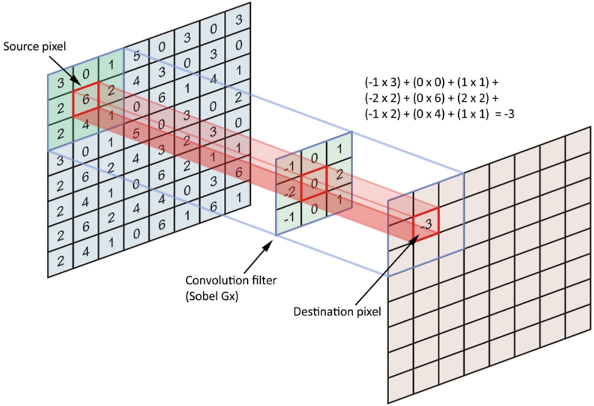
$$f(x) * \delta(x-u) = f(x-u)$$

- Kernel density estimation: kernel convolution with several Diracs centrered on the data

$$\hat{p}(x) = \frac{1}{Nh}\sum_{t=1}^{N} K\left(\frac{x-x^t}{h}\right) = \frac{1}{Nh}\sum_{t=1}^{N} K\left(\frac{x}{h}\right) * \delta(x-x^t)$$

Image processing

- 2D convolution is a building blocks for image processing

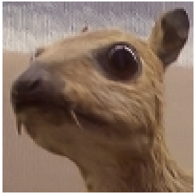


Source: `https://thigiacmaytinh.com/wp-content/uploads/2018/05/kernel.png`, accessed November 13, 2018.

Examples of filters

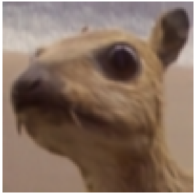
Identity (3×3):

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



Gaussian blur:

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

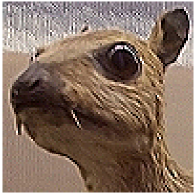


Edge detection:

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Sharpen:

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

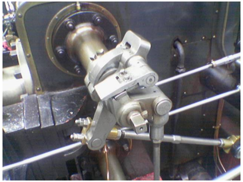


Sobel operator

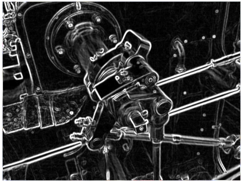
- Classic filter for edge detection
  - Compute local gradients of image intensity
  - Uses two convolutions to obtain the vertical gradient $\mathbf{G}_x$ and the horizontal gradient $\mathbf{G}_y$ of an image $\mathbf{A}$, the result is an image $\mathbf{G} = \sqrt{\mathbf{G}_x^2 + \mathbf{G}_y^2}$

$$\mathbf{G}_x = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix} * \mathbf{A}, \quad \mathbf{G}_y = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} * \mathbf{A}$$

Original image:



Application of Sobel:

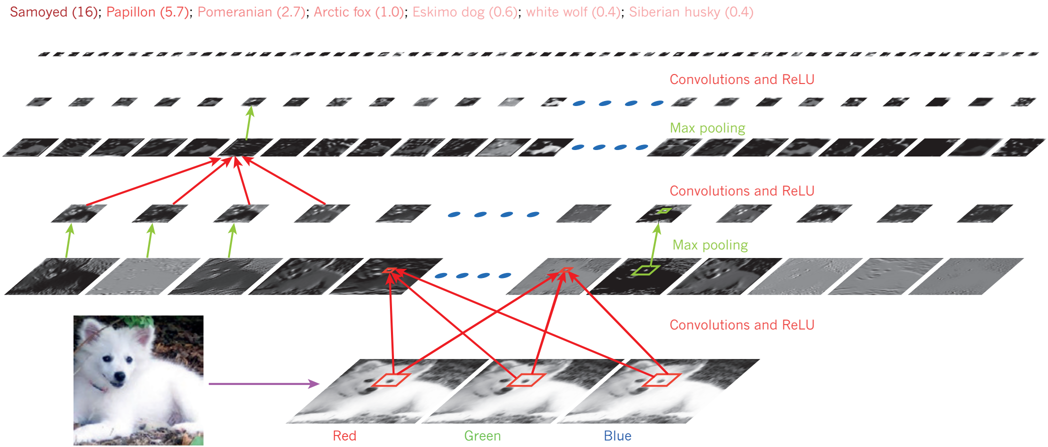


10.2 Convolutional neural networks

Convolutional neural networks

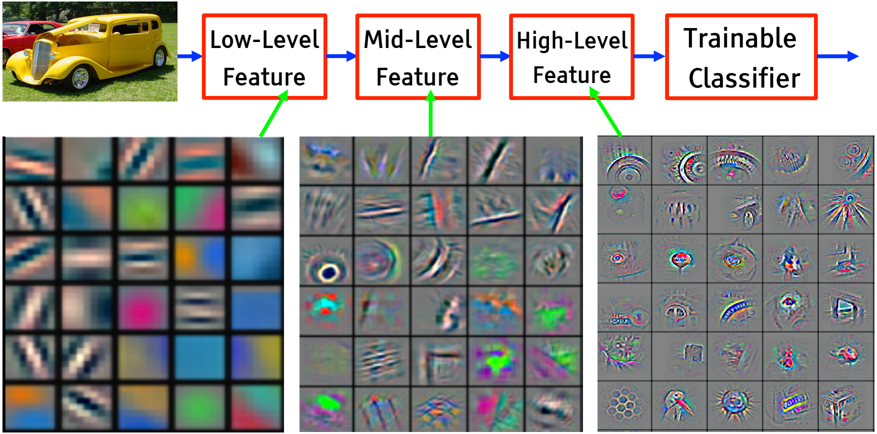
- Idea: Define neural networks with convolution operations
  - Learning the numerical values of convoluted filters
  - Define a network exploiting elements of the data structure
    - Sound or speech: temporal data (1D convolutions)
    - Image: spatial data (2D convolutions)
    - Video: spatio-temporal data (3D convolutions)
  - Sequence of convolution stages, filtering output of previous layer
  - Allows for more compact modelling than fully connected networks and translation invariant
- Some components of a convolution network
  - Layer of convoluted filters on the different channels
  - Pooling: maximum (max pool) or average (avg pool) value in a certain convoluted window
  - Transfer functions: ReLU, etc.
  - Near output, fully connected layers (like with multi-layer perceptron)

Convolution network



From *Y. LeCun, Y. Bengio and G. Hinton, Deep Learning, Nature, vol. 521, 28 mai 2015.* Accessed online November 6, 2020 at `https://www.nature.com/articles/nature14539`.

Filters composition

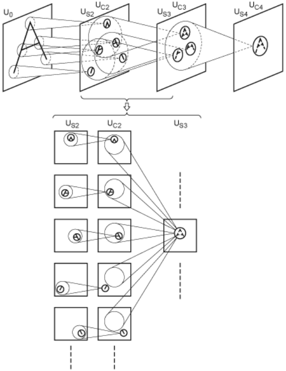


From *G. Hinton, Y. Bengio and Y. LeCun, Deep Learning NIPS'15 Tutorial, 2015.* Accessed online on November 6, 2020 at `https://nips.cc/Conferences/2015/Schedule?showEvent=4891`.

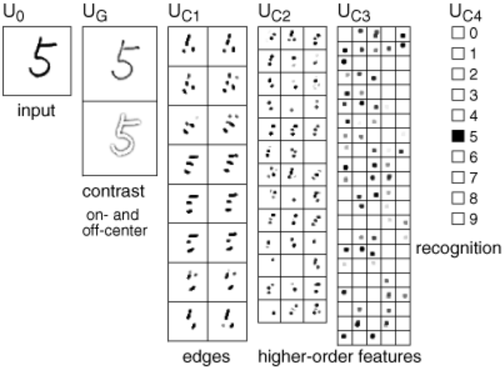
10.3 Examples of convolution networks

Neocognitron

- Neocognitron: the ancestor of convolutional networks
    - Proposed by Kunihiko Fukushima in the 1980s
    - Inspiration for LeCun in the development of convolutional networks



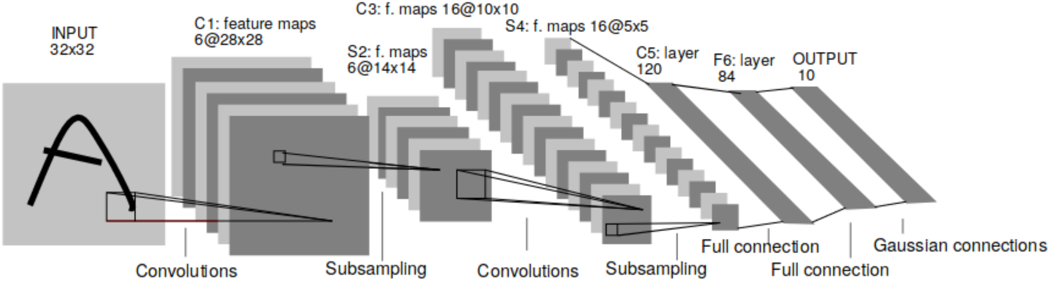
Source: `http://www.scholarpedia.org/article/File:ScholarFig2.gif`, accessed November 13, 2018.



Source: `http://www.scholarpedia.org/article/File:ScholarFig4.gif`, accessed November 13, 2018.

LeNet5

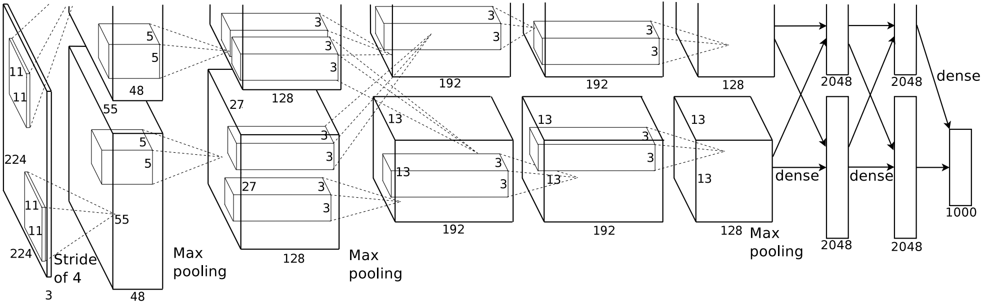
- LeNet5: classical convolutional network, proposed in the 1990s
  - 3 convolution layers, 2 average pooling layers, 2 fully connected layers
  - 60k parameters (from 10M to 100M with modern networks)



From *Y. LeCun, L. Bottou, Y. Bengio et P. Haffner, Gradient-based learning applied to document recognition, Proceedings of the IEEE, 86(11), 1998.* Accessed online on November 6, 2020 at `http://yann.lecun.com/exdb/publis/pdf/lecun-98.pdf`.

AlexNet

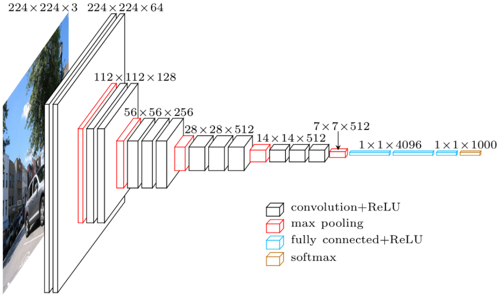
- AlexNet: network for object recognition
  - Winner of the ImageNet 2012 contest
  - Implemented for GPU Computing
  - Often used as a basic model for representation transfer
  - 8 convolution layers, some max pooling layers, 3 fully connected layers



From *A. Krizhevsky, I. Sutskever, et G. Hinton, Imagenet classification with deep convolutional neural networks. NIPS, 2012.* Accessed online November 6, 2020 at `https://papers.nips.cc/paper/2012/file/c399862d3b9d6b76c8436e924a68c45b-Paper.pdf`.

VGG

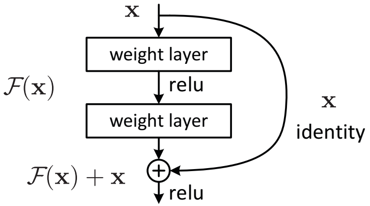
- VGGNet: greater depth with simplified topology
  - Winner of the ImageNet 2013 contest
  - Depth is critical for good performance
  - Similar to AlexNet, but with only 3×3 convolutions, 2×2 max pooling, 3 layers fully connected and 16 layers in total (VGG-16)



Source: `https://heuritech.files.wordpress.com/2016/02/vgg16.png`, accessed November 13, 2018.

ResNet

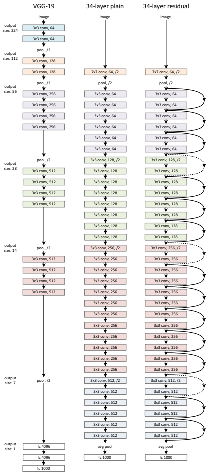
- Residual networks: allowing direct connections between non-adjacent layers (*skip links*)



From *K. He, X. Zhang, S. Ren, and J. Sun, Deep residual learning for image recognition. CVPR, 2016.* Accessed online November 6, 2020 at `https://arxiv.org/abs/1512.03385`.

- Allows for much deeper and more efficient networks
  - Winner of ImageNet 2015 competition (3.57 % top 5 error)
  - Facilitates signal optimization and propagation across the network
  - Residual block must do better than a treatment directly on the previous block

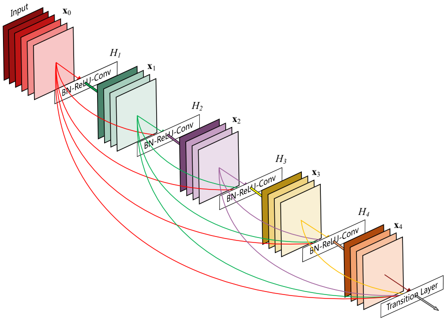
ResNet



From *K. He, X. Zhang, S. Ren, et J. Sun, Deep residual learning for image recognition. CVPR, 2016.* Accessed online November 6, 2020 at `https://arxiv.org/abs/1512.03385`.

DenseNet

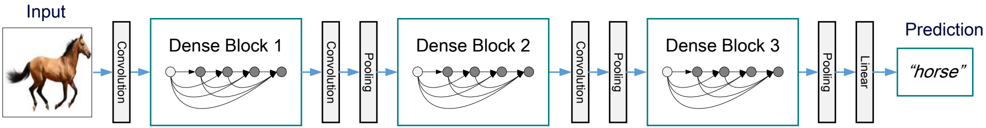
- Observation: convolution networks can be deeper and get better performance with close connections throughout the network at its input.
- DenseNet: connect each layer to all of the above layers
  - Network with L layers will have $L(L+1)/2$ direct connections between layers



From *G. Huang, Z. Liu, L. Van Der Maaten et K.Q. Weinberger, Densely Connected Convolutional Networks. CVPR, 2017.* Accessed online on November 6, 2020 at `https://arxiv.org/abs/1608.06993`.

DenseNet

- In practice, we create dense blocks separated by convolution and pooling layers



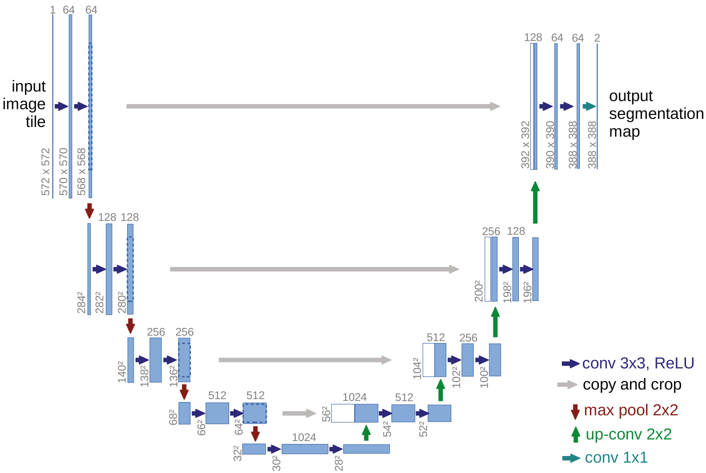
From *G. Huang, Z. Liu, L. Van Der Maaten et K.Q. Weinberger, Densely Connected Convolutional Networks. CVPR, 2017.* Accessed online on November 6, 2020 at `https://arxiv.org/abs/1608.06993`.

- Each layer in a dense block can be relatively narrow, i.e. can contain few neurons.

U-Net

- Networks presented so far first proposed and tested for object recognition (classification)
  - Other possible tasks in vision: detection, tracking, etc.
- Segmentation: identify coherent regions of the image
  - Separate the different regions
  - Give a label to each region
- U-Net: network proposed for biomedical imaging
  - Fully convolutional network, gives an output image
  - Compression of information in a network environment, similar to an auto-encoder
  - Skip links allow to preserve spatial structure

U-Net



From *O. Ronneberger, P. Fischer, et T. Brox, U-net: Convolutional networks for biomedical image segmentation. MICCAI, 2015.* Accessed online on November 6, 2020 at `https://arxiv.org/abs/1505.04597`.