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Quantum Formalism

Overview

1. **Bohm-de Broglie Theory IV**

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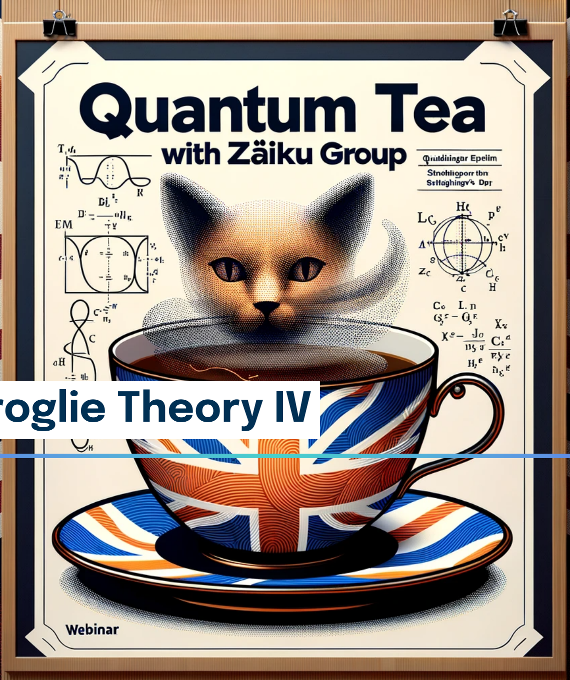

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Non-Relativistic theory

–SUMMARY–

Ψ) The theory is based on defined trajectories, where the wave function

$$\psi(\vec{x},t) = R(\vec{x},t)e^{iS(\vec{x},t)/\hbar}$$

is assumed as representing a real and objective field, where R and $S \in \mathcal{R}$, and are the amplitude and phase of the ψ-field, respectively

Ψ) It obeys Schrödinger equation

$$\frac{\partial\psi(\vec{x},t)}{\partial t} = \frac{\hbar}{2m}\nabla^2\psi(\vec{x},t) + V(\vec{x},t)$$

, and we have two fundamental equations

Non-Relativistic theory

Ψ) A CONTINUITY EQUATION

$$\frac{\partial R^2}{\partial t} + \nabla . \left(R^2 \frac{\nabla S}{m} \right) = 0$$

Ψ) A HAMILTON-JACOBI EQUATION

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + U_q = 0$$

where $U_q = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$ is the quantum potential, and the term $\nabla S/m$ is interpreted as a velocity and them $\vec{p} = \nabla S$ is the momentum, and R^2 is the probability density

Non-Relativistic theory

- Ψ) Using these equations Newton's law can be re-interpreted by

$$m\frac{d^2\vec{x}}{dt^2} = -\nabla V - \nabla U_q$$

- Ψ) In this sense, we have a trajectory-based theory where the wave function plays a probabilistic and a dynamical role
  - it guides the evolution of the particles in time, a pilot wave
  - it gives the probabilities of the particle's initial positions. These positions are the hidden variables in the theory since we can not observe them experimentally
- Ψ) In the limit $U_q \to 0$ we archive the classical limit

Relativistic theory – Bosons

The quantum relativistic theory is formulated consistently in the Quantum Field theory (QFT)

Ψ) For Bosons, it was shown that it is more consistent to attribute an objective reality to the scalar fields $[\phi(\vec{x}, t)]$, and if we use the Schrödinger representation involving functionals derivatives of the fields $(\delta/\delta\phi)$ we obtain

$$i\hbar \frac{\partial \psi(\phi, t)}{\partial t} = \int d^3x \frac{1}{2} \left[-\hbar^2 \frac{\delta^2}{\delta\phi^2} + (\nabla\phi)^2 + V(\phi) \right] \psi(\phi, t) \tag{1}$$

Relativistic Theory - Bosons

$\Psi)$ Writing again $\psi = Re^{iS/\hbar}$ we get

$$\frac{\partial S}{\partial t} + \int d^3x \frac{1}{2}\left[-\left(\frac{\delta S}{\delta \phi}\right)^2 + (\nabla\phi)^2 + V(\phi)\right] + U_q = 0 \tag{2}$$

$$\frac{\partial R^2}{\partial t} + \int d^3x \frac{\delta}{\delta \phi}\left(R^2 \frac{\delta S}{\delta \phi}\right) = 0 \tag{3}$$

where now $U_q \to U_q(\phi, t)$ is the quantum potential given by

$$U_q(\phi, t) = -\frac{\hbar^2}{2R}\int d^3x \frac{\delta^2 R}{\delta \phi^2} \tag{4}$$

Relativistic Theory - Bosons

- Ψ) The evolution equation for the field ϕ is

$$\frac{\partial \phi(\vec{x}, t)}{\partial t} = \lim_{\phi(\vec{x})=\phi(\vec{x},t)} \frac{\delta S[\phi(\vec{x}), t)]}{\delta \phi(\vec{x})} \tag{5}$$

- Ψ) Note that S is now the solution from Eq. (2), which corresponds to the Hamilton-Jacobi equation for a scalar field ϕ
- Ψ) This equation suffers the influence of the classical potential $V(\phi)$ and the quantum potential $U_q(\phi, t)$
- Ψ) If we take a second functional derivative in Eq. (2) and use Eq. (5) we obtain

$$-\frac{\partial^2 \phi}{\partial t^2} + \nabla^2 \phi = -\frac{\delta V}{\delta \phi} - \frac{\delta U_q}{\delta \phi} \tag{6}$$

when $U_q \to 0$ we obtain the usual equation.

Relativistic Theory - Bosons

Ψ) Note that U_q is given in terms of high-order functional derivatives, evaluated at the same point

Ψ) Its expression must pass through an appropriate regularization process to be well-defined

Ψ) Suppose that the probability density of the field respect the Born's rule, $P = |\psi(\phi(\vec{x}), t = 0)|^2$. Thus Eq. (6) says that the probability will be $P = |\psi(\phi(\vec{x}), t)|^2$ to any given time.

Ψ) So, the statistical provisions of the theory will be the same as the usual QFT

Relativistic Theory – Problems

- Ψ) In the case of Fermions, there are three different approaches, all of which have their theoretical (still) unsolved problems. We will discuss these approaches in other opportunities in the future.
- Ψ) A common issue to all of these approaches (including the bosonic case) has the same statistical previsions of the usual theory
- Ψ) However, the non-locality of the quantum potential destroys the relativistic invariance of the theory in the sub-atomic level
- Ψ) In this way, the fundamental group of symmetry in the BdB theory is not the Poincaré group anymore.
- Ψ) This is the symmetry group just for the statistical provisions of the theory

Non-Relativistic theory - Problems

$\Psi)$ One could argue that the notion of a quantum potential $U_q = -\hbar^2/2m(\nabla^2 R/R)$ is not physically satisfactory since it does not possess a visible source (like the electromagnetic field).

$\Psi)$ Another criticism is related to the interpretation of the oscillating field ψ and the random coordinates of the particles in the sense that the observable result on a large scale is the same as the orthodox theory. In this case, there are no pieces of experimental evidence about the hidden variables.

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