Quantum_tea3_coppenhagen2

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Quantum_...

Overview

1. **Copenhagen Interpretation**
2. **Stern-Gerlach Experiment**

The Postulates of Quantum Mechanics

- Ψ) Any state of a quantum system at time t_0 is characterized by a vector (ket) $|\psi(t_0)\rangle$ that belongs to the Hilbert space. This vector in Hilbert space describes completely the physical state of the system. Everything that can be said about the system is contained in $|\psi(t_0)\rangle$
- Ψ) Every measurable physical quantity is described by a self-adjoint operator (called observable) acting in the Hilbert space of the system
- Ψ) The only possible result of a measurement of a physical quantity is one of the eigenvalues of the self-adjoint operator associated with it

The Postulates of Quantum Mechanics

- Ψ) The probability of finding one of the eigenvalues (for example, a_n) associated with the observed quantity is given by (in the discrete case):

$$\mathcal{P}(a_n) = \sum_{i=1}^{g_n} |\langle u_n^i|\psi\rangle|^2 = \langle\psi|P_n|\psi\rangle$$

where P_n is the projector over the eigensubspace of the Hilbert space with eigenvalue a_n. $|u_n^i\rangle$ is one of the eigenstates with this eigenvalue with degeneracy g_n.

- Ψ) After a measurement generating the eigenvalue a_n, the state of the system collapses to

$$|\psi'\rangle = \frac{P_n|\psi\rangle}{\sqrt{\langle\psi|P_n|\psi\rangle}}$$

The Postulates of Quantum Mechanics

- Ψ) The evolution of the state vector while no experiments are carried out is governed by the Schrödinger equation:

$$i\hbar \frac{d|\psi(t)\rangle}{dt} = H|\psi(t)\rangle$$

where $H(t)$ is the Hamiltonian operator of the system.
In the Heisenberg description, the observable A varies with time using the equation

$$\frac{dA}{dt} = \frac{i}{\hbar}[H, A]$$

while the quantum state remains constant.

Let us use these postulates to describe the double-slit experiment

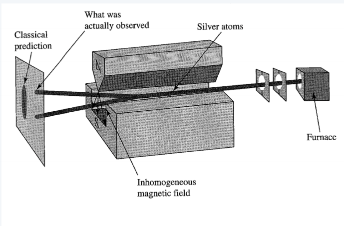


Figure: Modern Quantum Mechanics by Sakurai and Napolitano

In 1922 Stern and Gerlach measured the possible values of the magnetic dipole moment for silver atoms by sending a beam of these atoms through a nonuniform magnetic field.

- Ψ) A beam of neutral atoms is formed by evaporating silver from an oven. The beam is collimated by a diaphragm, and it enters a magnet.
- Ψ) The field that increases in intensity in the z direction is defined which is also the direction of the magnetic field itself in the region of the beam.
- Ψ) As the atoms are neutral overall, the only net force acting on them is the force $\vec{F} = \nabla(\vec{\mu}.\vec{B})$, which is proportional to μ_z.
- Ψ) Since the force acting on each atom of the beam is proportional to its value of μ_z, each atom is deflected in passing through the magnetic field by an amount proportional to μ_z.
- Ψ) Thus the beam is analyzed into components according to the various values of μ_z. The deflected atoms strike a metallic plate, upon which they condense and leave a visible trace.

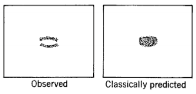
- Ψ) If the orbital magnetic moment vector of the atom has a magnitude μ, then in classical physics the z-component μ_z of this quantity can have any value from $-\mu$ to μ. The reason is that classically the atom can have any orientation relative to the z axis, and so this will also be true of its orbital angular momentum and its magnetic dipole moment.

The predictions of quantum mechanics, can be summarized by

$$\mu_z \simeq -\frac{e\hbar}{2m} m_l,$$

where $m_l = -l, -l+1, ...0, ..., +l-1, +l$.

- Ψ) Thus the classical prediction is that the deflected beam would be spread into a continuous band, corresponding to a continuous distribution of values of μ_z from one atom to the next.
- Ψ) The quantum mechanical prediction is that the deflected beam would be split into several discrete components. Furthermore, quantum mechanics predicts that this should happen for all orientations of the analyzing magnet.



The deflection pattern recorded on the detecting plate in a Stern-Gerlach measurement of the z component of the magnetic dipole moment of silver atoms. Maximum deflection occurs at the center of the beam because the atoms there pass through the region of maximum field gradient, $\partial B_z/\partial z$. The observed pattern consists of two discrete components due to space quantization. According to the classical prediction a continuous band would be expected.

Figure: See book Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles by R. Eisberg and R. Resnick

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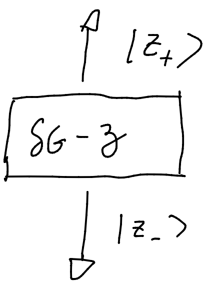
STERN - GERLACH EXPERIMENT

- IN THE CASE OF STERN-GERLACH

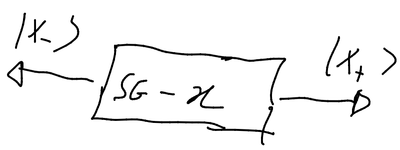
A)



$$\hat{S}_z |z_+\rangle = \frac{\hbar}{2} |z_+\rangle$$

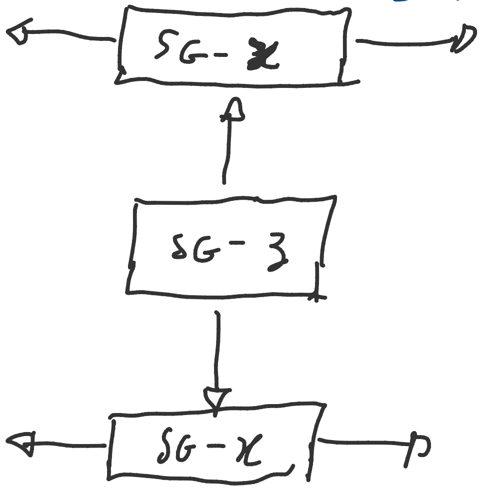
$$\hat{S}_z |z_-\rangle = -\frac{\hbar}{2} |z_-\rangle$$

B)



$$\hat{S}_x |x_-\rangle = -\frac{\hbar}{2} |x_-\rangle \qquad \hat{S}_x |x_+\rangle = \frac{\hbar}{2} |x_+\rangle$$

– SO, LET'S CONSIDER A SEQUENTIAL SG-EXPERIMENT



– THE ATOMS THAT GO UP FROM SG-z ARE (CHARACTERIZED)

$$|z_+\rangle = \frac{1}{\sqrt{2}}\left(|x_+\rangle + |x_-\rangle\right)$$

AND THE ATOMS THAT GO DOWN FROM SG-z

$$|z_-\rangle = \frac{1}{\sqrt{2}}\left(|x_+\rangle - |x_-\rangle\right)$$

SINCE $|x_+\rangle$ AND $|x_-\rangle$ ARE MUTUALLY EXCLUDENT AS WELL $|z_+\rangle$ AND $|z_-\rangle$

$$\langle x_+|x_-\rangle = \langle z_+|z_-\rangle = 0 \quad , \quad \langle x_\pm | x_\pm \rangle = 1$$

• $\langle z_+|z_-\rangle = \frac{1}{\sqrt{2}}\left(\langle x_+| + \langle x_-|\right)\frac{1}{\sqrt{2}}\left(|x_+\rangle - |x_-\rangle\right) = 0$

– ACCORDING TO THE 4th POSTULATE, LET'S EVALUATE THE PROBABILITY FROM AN "UP" ATOM IN THE SG-z GO OUT TO THE "RIGHT" IN THE SG-x

$$P(z_+|x_+) = \langle z_+|\, P(x_+)\, |z_+\rangle = \langle z_+|\underbrace{x_+\rangle\langle x_+|}_{P(x_+)}z_+\rangle$$

$$P(z_+|x_+) = \frac{1}{\sqrt{2}}\left(\langle x_+| + \langle x_-|\right)|x_+\rangle\langle x_+|\frac{1}{\sqrt{2}}\left(|x_+\rangle + |x_-\rangle\right)$$

$$= \frac{1}{2}$$

- PILOT - WAVE
  - → SPECIAL OBSERVERS

- SCHRÖDINGER EQUATION (SPIN)

$$H = H + H_s = \frac{p^2}{2m} + V(x) \pm \vec{\mu}_s \cdot \vec{B}$$

- KLEIN GORDON p → RELATIVISTIC MOMENT

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\psi = 0$$

, $s = 1$ (PHOTONS) BOSONS

- DIRAC EQUATION $s = 1/2$ (ELECTRONS)