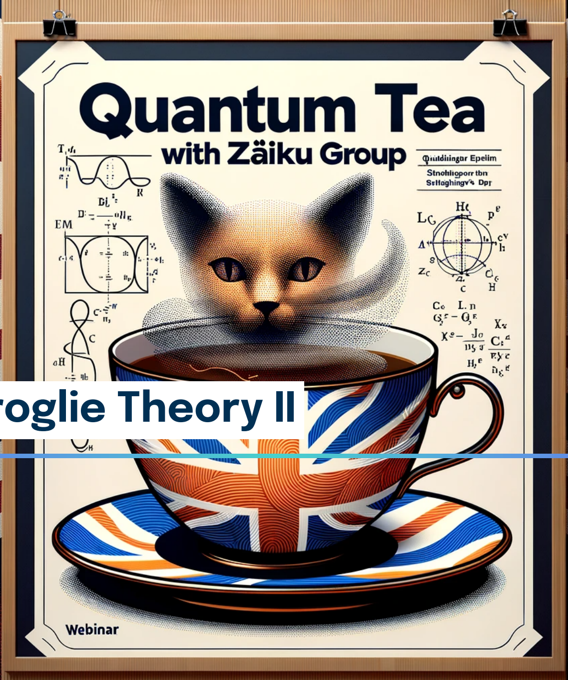
Carlos Bessa

Zaiku Group Ltd.
Quantum Formalism

Overview

1. **Bohm-de Broglie Theory II**



Bohm-de Broglie Theory II

The Measurement Problem

- Ψ) Questions involving the measure of a quantum observable and the frontier between classic and quantum are controversial subjects in the quantum theory
- Ψ) The collapse postulate says that the only possible result of the measurement of a physical quantity is one of the eigenvalues of the corresponding observable. However, some people say that this is just a way to avoid a deeper discussion about the theme
- Ψ) Bohr says basically that since our minds work classically, the concepts that we understand and can communicate are classic $\longrightarrow$ Thus a scientific theory should be formulated in classical terms
- Ψ) The measurement apparatus is classical, and the measure is just the interaction between the apparatus (classical) and the quantum object
- Ψ) In this context, Bohr said that the corpuscular and wave nature of quantum objects are complementary attributes – The Complementary Principle

The Measurement Problem

- Ψ) Thus, in the double-slit experiment if we determine from which slit each photon passed through, we identify their particle attribute and destroy the interference figure
- Ψ) In this view, the wave behavior for the macroscopic objects is not seen since their masses are too big. Consequently, the de Broglie wavelength is small: $\lambda = h/p = h/mv$.
- Ψ) On the opposite side, the quantization of fields (waves) with big λ has small ν and $E_{photon} = h\nu$
- Ψ) However, this interpretation is contested by many people

If everything is formed by atoms, so in what moment does it need to see the characteristic of the "quantum world"?

The Measurement Problem

- Ψ) Schrödinger and Wigner considered fundamental questions related to the classical limit – Schrödinger cat, Wigner friend – showing some ambiguities about the superposition of macroscopic objects
- Ψ) It was von Neumann who tried to describe the quantum process of a measure with a postulate about the collapse of the wave function

- Consider $\hat{S}$, an observable that will be measured, and $|s_i\rangle$ the respective eigenstates

$$\hat{S}|s_i\rangle = s_i|s_i\rangle \tag{1}$$

the eigenvalues are discrete and $i = 1, 2, ...N$, and N is the total number of orthogonal eigenstates

The Measurement Problem

- Consider $|x\rangle$ the eigenstate of the position operator $\hat{X}$ linked with the center of mass of the measurement apparatus

$$\hat{X}|x\rangle = x|x\rangle. \tag{2}$$

The interaction between the system and the apparatus is given in a finite time interval and in a time interval where both do not interact, the system is said to be isolated

- Thus, we can write the state of both systems in this way

$$|\psi\rangle = |\varphi_S\rangle \otimes |\varphi_A\rangle$$

each one belonging to their own Hilbert space. The interaction (or measure) will introduce a correlation between both states

The Measurement Problem

- Note that, if the initial state of the system is an eigenvector from $\hat{S}$, the measurement will not change, but will modify the state of the measurement apparatus
- This modification will be something proportional to the eigenvalue that corresponds to this eigenstate
- Thus, the evolution operator

$$U_I = e^{-i\lambda \hat{S} \otimes \hat{p}/\hbar}$$

  that corresponds to this interaction must change the initial state (the state before the measure) to a final state (the state after the measure). Here, $\lambda = \int_{-\epsilon}^{\epsilon} g(t)dt$, and $g(t)$ is non-null in the interval $-\epsilon < t < \epsilon$, $\hat{p}$ is the momentum operator and it is related with the apparatus position by $[\hat{X}, \hat{p}] = i\hbar$
- In this context, the interaction Hamiltonian is

$$H_I = -g(t)\hat{S} \otimes \hat{p}$$

The Measurement Problem

$$
\begin{aligned}
U_I|s_i\rangle \otimes |x\rangle &= e^{-i\lambda \hat{S}\otimes \hat{p}/\hbar}|s_i\rangle \otimes |x\rangle \\
U_I|s_i\rangle \otimes |x\rangle &= (\hat{1} - i\lambda \hat{S} \otimes \hat{p}/\hbar + ...)|s_i\rangle \otimes |x\rangle \\
U_I|s_i\rangle \otimes |x\rangle &= (\hat{1} - i\lambda \hat{S}|s_i\rangle \otimes \frac{\hat{p}}{\hbar}|x\rangle \\
U_I|s_i\rangle \otimes |x\rangle &= (\hat{1} - i\lambda s_i \otimes \hat{p}/\hbar + ...)|s_i\rangle \otimes |x\rangle \\
U_I|s_i\rangle \otimes |x\rangle &= |s_i\rangle \otimes e^{-i\lambda s_i \otimes \hat{p}/\hbar}|x\rangle \\
U_I|s_i\rangle \otimes |x\rangle &= |s_i\rangle \otimes |x + \lambda s_i\rangle
\end{aligned}
\tag{3}
$$

where w in the last step we have used the fact that $e^{-i\lambda s_i \otimes \hat{p}/\hbar}$ is a translation operator in $x \implies \tau(dx)|x\rangle = |x + dx\rangle$.

The Measurement Problem

- Now, if we take the following state vector as the initial state

$$|\psi_0\rangle = |\phi_s\rangle \otimes |\phi_A\rangle$$

where $|\phi_s\rangle = \sum_i c_i|s_i\rangle$ and $|\phi_A\rangle = \int f(x)|x\rangle dx$.
The final state is

$$|\psi_F\rangle = U_I|\psi_0\rangle = U_I \sum_i c_i|s_i\rangle \otimes \int f(x)|x\rangle dx$$

.

Using Eq. (3),

$$|\psi_F\rangle = \sum_i c_i|s_i\rangle \otimes |\phi_A(s_i)\rangle \tag{4}$$

where, $|\phi_A\rangle = \int f(x - \lambda s_i)|x\rangle dx$

The Measurement Problem

- Eq. (4) tells us that when a measurement finishes, only one of the several possibilities from the initial systems appears as a single result, which is the measurement result
  Exemple: $|\psi_F\rangle = c_1|s_1\rangle \otimes |\phi_A(s_1)\rangle + c_2|s_2\rangle \otimes |\phi_A(s_2)\rangle + \ldots + c_N|s_N\rangle \otimes |\phi_A(s_N)\rangle$ Let's say we measure $c_1 \Rightarrow \langle s_1|\psi_F\rangle = c_1$.

COPENHAGEN INTERPRETATION

Among all N possibilities, **the wave function collapses** to the result indicated by the measurement apparatus (The collapse postulate)

The Measurement Problem

The problem is that this collapse is never directly observed and this gives rise to different interpretations.

BOHM-DE BROGLIE INTERPRETATION

- The collapse is just apparent. In this case, just one of the possible eigenvalues c_i will be different from zero in the specific position x_i (where the pointer of the apparatus is placed)
- The, other $c_j(j \neq i)$ are not present on this x_i and the wave functions associated with them are called empty waves
- This position (x_i) is unique and depends only on the initial configuration of the total system

The Measurement Problem

- Here there exists an objective reality associated with x_i and the degrees of freedom from the physical system plus the apparatus on their configuration spaces
- However, we do not know the initial configuration, we only know its distribution that obeys the Born rule
- Thus, we can only evaluate the probability of c_i to be selected and the empty waves can not be detected by other apparatus

The Measurement Problem

Example: Let's consider that after a measurement we have:

$$\psi_F(x) = \psi_1(x) + \psi_2(x), \tag{5}$$

x represents all degrees of freedom from the physical system and the detector. Now, consider two extra detectors, one in each region. Thus in Region 1, we place detector $\phi_1(y)$ and in Region 2, the detector $\phi_2(z)$. Before the interaction, the total wave function is

$$\psi_I(x, y, z) = [\psi_1(x) + \psi_2(x)]\phi_1(y)\phi_2(z) \tag{6}$$

After the interaction between ψ_I and the detectors,

The Measurement Problem

$$\psi_F(x, y, z) = \varphi_1(x, y)\phi_2(z) + \varphi_2(x, z)\phi_1(y) \tag{7}$$

where $\varphi_1(x, y)$ and $\varphi_2(x, z)$ are entangled states resulting from the interaction that happened in Region 1 and 2, respectively

- Note that the interaction between $\psi_1(x)\phi_1(y)$ can not affect $\phi_2(z)$
- This is because the locality of each interaction
- So, if the particle is present in Region 1 in Eq. (5)
- The wave function $\psi_2(x)$ is an empty wave, and it will still be empty in Eq. (6), and thus detector 2 will not click

$$\varphi_2 \to 0$$

The Measurement Problem

- In this case, the detector 1 is the only one that clicks

$$\varphi_1 \neq 0$$

- Thus, for the total system only Region 1 is noticeable
- Interaction with the environment during a measurement procedure separates the wave packets in configuration space [Eqs. (6) or (7)], which is where apparent wave function collapse comes from, even though there is no actual collapse
- So, we do not have a collapse but it seems that it has happened one

References

N. Pinto-Neto (2010)
Teorias e Interpretações da Mecânica Quântica
Editora Livraria da Física, São Paulo.

D. Bohm (1952)
A Suggested Interpretation of the Quantum Theory in Terms of “Hidden” Variables, I
Phys. Rev. 85, 166.