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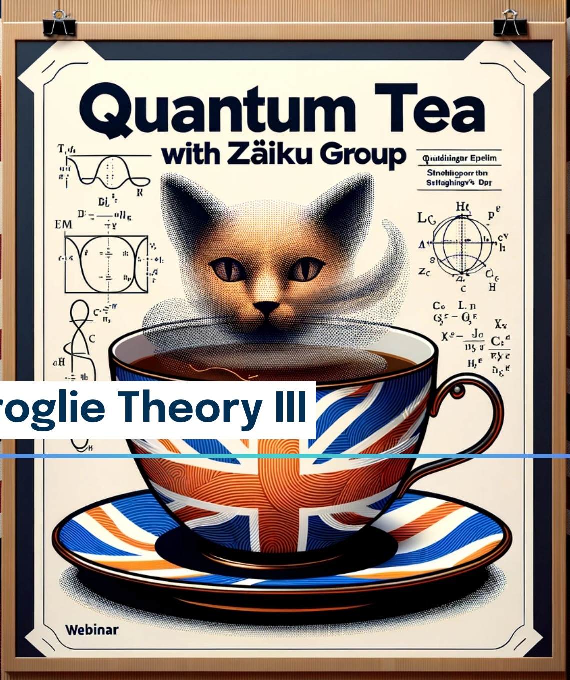
Quantum Formalism

Overview

1. Bohm-de Broglie Theory III



Bohm-de Broglie Theory III

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Quantum Mechanic's Interpretation in terms of Hidden Variables

- Ψ) Today we will first review what we have discussed in previous sections about the Bohm-de Broglie (BdB) proposal for a new interpretation of quantum mechanics (QM) in terms of hidden variables
- Ψ) Then we will discuss how this proposal interpreted the existence of the spin of the electrons in QM
- Ψ) As emphasized by David Bohm in several references, these proposals are preliminary, and the main goal is that they might be considered as starting points in the development of a more detailed theory
- Ψ) The main points can be emphasized in the following postulates

Quantum Mechanic's Interpretation in terms of Hidden Variables

1) The wave function

$$\psi = Re^{iS/\hbar}$$

   is assumed as representing a real and objective field, where R and $S \in \mathcal{R}$, and are the amplitude and phase of the field, respectively
2) It is supposed that, together with the field, there is a particle represented mathematically by well-defined coordinates that vary in a definite way
3) It is assumed that the velocity of the particle is given by

$$\vec{v} = \nabla S/m, \tag{1}$$

   where m is the mass of the particle

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4) It is supposed that the particle is influenced by a classical potential $V(\vec{x})$ plus a quantum potential

$$U_q = \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} \tag{2}$$

5) And finally, it is assumed that the field ψ is in a state of fast random oscillation, in such a way that, the ψ values are a type of average in a certain characteristic time τ. Such interval must be long when compared with the quantum mechanical processes. The fluctuations of the field ψ could have originated from a much deeper sub-quantum mechanical level

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- Ψ) Taking into account these postulates, let's consider that the field ψ oscillates and Eq. (1) implies that the fluctuations will be communicated to the particle's movement through the quantum potential U_q, Eq. (2)
- Ψ) In this case, the particle will not follow a regular trajectory. Instead, it will have an irregular path
- Ψ) In this path, the particle will possess a velocity given by averaging Eq. (1) in terms of the field fluctuations that occur during the interval τ
- Ψ) Thus, it is possible to show that these random motions will guide us to consider that the field ψ gives rise to a probability density

Quantum Mechanic's Interpretation in terms of Hidden Variables

$$dP = |\psi|^2 dV \tag{3}$$

where dV is the volume element where the particles are taking their random movements.

- Ψ) Thus, ψ can be interpreted as determining the motion of the particles from Eq. (1), the quantum potential from Eq. (2), and the probability density from Eq. (3).
- Ψ) It is also possible to show that such a theory has the same results previewed by the non-relativistic QM orthodox interpretation
- Ψ) However, this is done differently, in terms of the existence of a deeper knowledge about Nature's law

Example: Double Slit Experiment again – Copenhagen $\times$ BdB

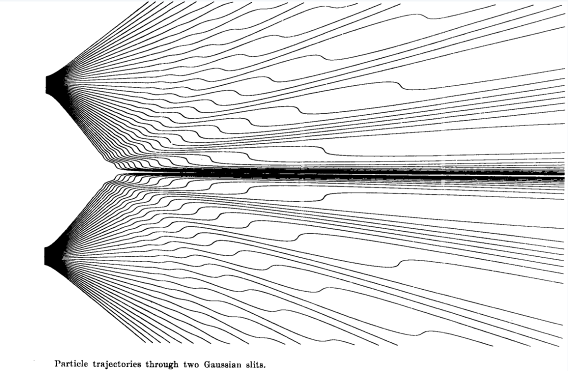

Figure: Double Slit Experiment

- Ψ) In BdB, the particle follows a defined path but with a random trajectory, and this type of behavior is ψ-dependent
- Ψ) The places where the particles are detected by the apparatus are determined by several factors:
  - the particle's initial position,
  - the initial form of its ψ-field,
  - the changes in ψ due to the slit's influence
  - the random changes in ψ from sub-atomic origins
- Ψ) In a statistical analysis of all cases, the fluctuations of the ψ-field would reproduce the same usual interpretation of QM

Example 2: Spin

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A relativistic, causal account of a spin measurement

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Abstract

An objective account of the action of a Stern–Gerlach apparatus on spin-$\frac{1}{2}$ particles is given, using the Dirac equation. This generalizes earlier work on a causal interpretation of the Pauli equation to the relativistic domain, leading to a more natural choice for the current in the model.

1. Introduction

Dewdney et al. [1,2] have described the action of an inhomogeneous magnetic field on an uncharged spin-$\frac{1}{2}$ particle, as in a Stern–Gerlach type spin measurement. Their aim was to demonstrate that there is a causal interpretation of quantum mechanics which is not only consistent with the equations of quantum mechanics, but which quantitatively reproduces the results of measurements made on ensembles of identically prepared systems. According to such an interpretation we may assign well-defined trajectories and spin vectors to particles and study how these interact with the apparatus during the course of a measurement. For a spin measurement, the conventional collapse of the wavefunction onto eigenstates of the spin operators along the field direction is replaced by a continuous evolution of the particle's spin vector towards either alignment or anti-alignment with the field direction. Dewdney et al. interpret this alignment as the consequence of "quantum torques" which act in addition to the usual spin precession in the magnetic field. These quantum torques depend on the wavefunction and hence are sensitive to the whole experimental context.

In this Letter we provide a fully relativistic account of a similar spin measurement. It is well known that the current employed by Dewdney et al. is inconsistent with that obtained from Dirac theory in the non-relativistic limit, the two differing by a term in the curl of the spin vector [3,4]. However, the qualitative results of Dewdney et al. for the behaviour of the streamlines and the spin vector coincide with our predictions in the non-relativistic case. Our results demonstrate that the deterministic evolution of the wavefunction suffices to account for the discrete outcomes of a quantum spin measurement. We do not need the relativistic analogue of Bohm/de Broglie theory in order to accept the validity of these results. Besides dealing with a well-defined current, a relativistic treatment is necessary as a foundation for future work extending these ideas to correlated

3. A model for the spin measurement

Suppose we wish to measure the z component of the spin of an uncharged spin $\frac{1}{2}$ particle. Let the mass of this particle be m and the magnetic moment μ. We observe the deflection of the particle after passage through an inhomogeneous magnetic field directed along our z coordinate axis, and so infer the component of spin along the field direction. We will take our laboratory system to be the γ_0 system.

Let the initial state of the particle be a positive-energy plane-wave solution of the free-particle Dirac equation. We write this state in the form $\psi = \psi_0 e^{-i\sigma_3 p\cdot x}$, where p is the momentum and the constant spinor ψ_0 satisfies

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by assuming that, at time $t = 0$, the particle receives an impulse from a magnetic field gradient, $F = B' z i \sigma_3 \delta(t)$, where B' is the field gradient along the z direction.

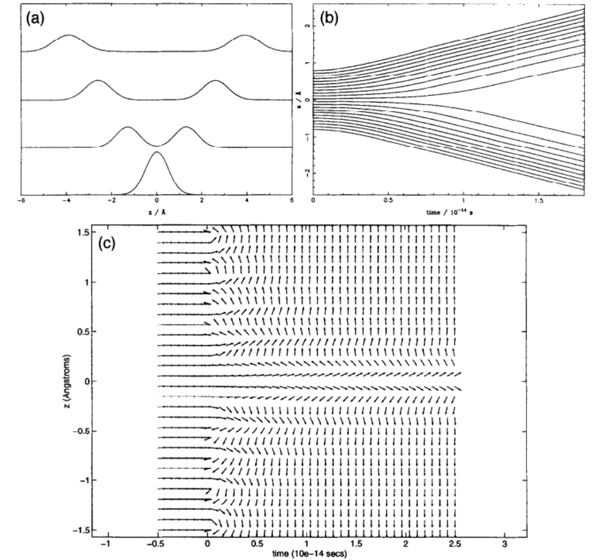

Fig. 1. Splitting of a wavepacket with $\Phi = 1 - i\sigma_2$. (a) Probability density, J_0, at $t = 0, 1.3, 2.6, 3.9 \times 10^{-14}$ s with t increasing up the figure. (b) Streamlines in the (t, z) plane. (c) Evolution of the spin vector projected in the (x, z) plane.

The Equation of motion of the particles with spin $\pm 1/2$ is

$$i\hbar \frac{d\Psi_i(\vec{x},t)}{dt} = \sum_{j=1}^{2} \left[-\frac{\hbar^2}{2m} \hat{I}_{ij} \nabla^2 + \mu \vec{\sigma}_{ij} . \vec{H}(\vec{x}) \right] \Psi_j(\vec{x},t) \tag{4}$$

where μ is the magnetic moment, $\vec{H}$ is the magnetic field, and $\vec{\sigma}_{ij}$ are the components of the three Pauli matrices, $\Psi_i(\vec{x},t)$ is a Pauli spinor parametrized as

$$\begin{aligned} \Psi_1(\vec{x},t) &= Re^{iS/2} \cos(\theta/2) e^{i\phi/2} \\ \Psi_2(\vec{x},t) &= Re^{iS/2} i \sin(\theta/2) e^{i\phi/2} \end{aligned} \tag{5}$$

with

$$\frac{d\vec{x}}{dt} = \frac{\vec{j}(\vec{x},t)}{\sum_{j=1}^{2} |\Psi_j|^2} \tag{6}$$

and $\vec{j} = Im \sum_{j=1}^{2} \Psi^{\star}{}_j(\vec{x},t) \nabla \Psi_j(\vec{x},t)/2m$. The Bohmians were evaluated using Eq. (6)

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