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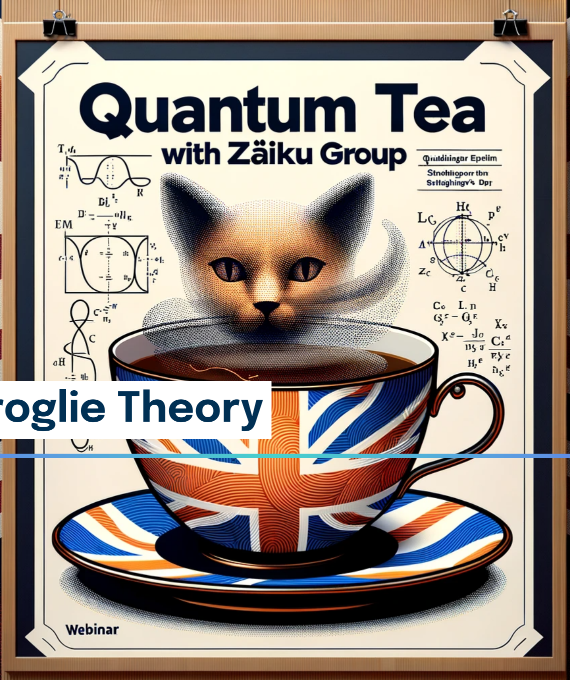
Quantum Formalism

Overview

1. **Bohm-de Broglie Theory**



Bohm-de Broglie Theory

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- Ψ) In 1927 de Broglie suggested a quantum theory where the position $(\vec{x}_a)$ and trajectory of a quantum particle a should have reality independently of the observation
- Ψ) However, it would be necessary a new mechanic to explain these trajectories, something that was not present in the Heisenberg/Born Mechanics
- Ψ) Since the weird quantum effects can appear in situations where the quantum particles are free of interaction (like in the double-slit experiment)
- Ψ) De Broglie suggested a change in the Newtonian laws proposing that every particle, free or not, has a velocity determined by a wave (called wave pilot) from the expression

$$\frac{d\vec{x}_a}{dt} = \frac{\vec{j}_a(\vec{x}_1 ... \vec{x}_N, t)}{|\psi(\vec{x}_1 ... \vec{x}_N, t)|^2}$$

- Ψ) To find the expression for the pilot wave, we just need to solve the usual Schrödinger equation. Thus, a particle interacts with other particles, and the background by $\psi(\vec{x}_1 ... \vec{x}_N, t)$
- Ψ) Later on, de Broglie would give up on his theory mainly because he was unable to explain the process of quantum measure

New Physical Interpretation of Schrödinger's Equation

Ψ) David Bohm in 1952 returned with de Broglie ideas in the paper below

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A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I

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The usual interpretation of the quantum theory is self-consistent, but it involves an assumption that cannot be tested experimentally, *viz.*, that the most complete possible specification of an individual system is in terms of a wave function that determines only probable results of actual measurement processes. The only way of investigating the truth of this assumption is by trying to find some other interpretation of the quantum theory in terms of at present "hidden" variables, which in principle determine the precise behavior of an individual system, but which are in practice averaged over in measurements of the types that can now be carried out. In this paper and in a subsequent paper, an interpretation of the quantum theory in terms of just such "hidden" variables is suggested. It is shown that as long as the mathematical theory retains its present general form, this suggested interpretation leads to precisely the same results for all physical processes as does the usual interpretation. Nevertheless, the suggested interpretation provides a broader conceptual framework than the usual interpretation, because it makes possible a precise and continuous description of all processes, even at the quantum level. This broader conceptual framework allows more general mathematical formulations of the theory than those allowed by the usual interpretation. Now, the usual mathematical formulation seems to lead to insoluble difficulties when it is extrapolated into the domain of distances of the order of 10^{-13} cm or less. It is therefore entirely possible that the interpretation suggested here may be needed for the resolution of these difficulties. In any case, the mere possibility of such an interpretation proves that it is not necessary for us to give up a precise, rational, and objective description of individual systems at a quantum level of accuracy.

Ψ) Following Bohm, initially, to simplify, we will not take into account the spin of the particles, this will be considered in the future.

Ψ) The Schrödinger equation for N particles is

$$i\hbar\frac{\partial\psi(\vec{x}_1...\vec{x}_N,t)}{\partial t}=\left[-\sum_{a=1}^{N}\frac{\hbar^2}{2m_a}\nabla_a^2+V(\vec{x}_1...\vec{x}_N)\right]\psi(\vec{x}_1...\vec{x}_N,t) \tag{1}$$

where ∇_a^2 is the Laplacian operator relative to the particle a, $V(\vec{x}_1...\vec{x}_N)$ is the classical particle's interaction potential , and Ψ is a complex function that can be expressed as $\psi=Re^{iS/\hbar}$, where $R,S\in\mathcal{R}$. It is convenient to write $P(\vec{x}_1...\vec{x}_N)=R^2(\vec{x}_1...\vec{x}_N)$,

$$\frac{\partial S}{\partial t}+\sum_{a=1}^{N}\frac{(\nabla_a S).\nabla_a S}{2m_a}+V-\sum_{a=1}^{N}\frac{\hbar^2}{2m_a}\left[\frac{\nabla_a^2 P}{P}-\frac{1}{2}\frac{(\nabla P)^2}{P^2}\right]=0 \tag{2}$$

$$\frac{\partial P}{\partial t}+\sum_{a=1}^{N}\nabla_a.\left(P\frac{\nabla_a S}{m_a}\right)=0 \tag{3}$$

In the first equation, we note the addition of an extra potential $U_q = -\sum_{a=1}^{N} \frac{\hbar^2}{2m_a}\left[\frac{\nabla_a^2 P}{P} - \frac{1}{2}\frac{(\nabla P)^2}{P^2}\right] = -\sum_{a=1}^{N} \frac{\hbar^2}{2m_a}\frac{\nabla_a^2 R}{R}$ and

- $\frac{\nabla S(\vec{x}_a)}{m_a} = \vec{v}(\vec{x}_a) \longrightarrow$ velocity vector for any particle passing from $\vec{x}$
- $\vec{j}_a = P_a \frac{\nabla S_a}{m_a} \longrightarrow$ mean current of particles in this ensemble
- This satisfies the conservation equation, $\frac{\partial P}{\partial t} + \nabla_a.(P\vec{v}_a) = 0$
- And the momentum $\vec{p}_a = m_a \frac{\vec{x}_a}{dt} = \nabla_a S$
- The equation of motion of a particle a acted by the classical potential $V(\vec{x})$ and the "quantum mechanical" potential U_q is now

$$m_a \frac{d^2\vec{x}_a}{dt^2} = -\nabla_a \left[V(\vec{x}_a) - \frac{\hbar^2}{2m_a}\frac{\nabla_a^2 R}{R}\right]$$

$\Psi)$ One can see from these equations that the trajectory of a particle a will be different from the classical trajectory if $U_q \neq 0$

$\Psi)$ The classical limit in the Bohm-de Broglie theory is obtained when U_q is negligible

- Summarizing the ideas above, we can say that, from Eq. (3) for a set of systems of N particles whose probability densities of these particles being (as opposed to being found, as in the Copenhagen interpretation) in positions $x_1, ..., x_N$ at time t is given by $P(x_1, ..., x_N)$.
- In other words, if we do not know the initial positions of the N particles that constitute each member of a set of systems of N quantum particles, but if the initial positions in the set are given by $P_i = P(t = 0)$, then, at the generic instant t, Eq. (3) guarantees that the distribution of positions implies the Born rule $P(t) = |\psi|^2$, and all statistical results of the theory will be the same as those obtained in the Copenhagen interpretation.

- Thus, the wave function has a double role in the theory: the first is dynamic, as it guides the particles through the relationship $\vec{v} = \vec{j}/P$, and the second is statistical by providing the distribution of initial positions for a statistical set of systems
- This duplicity of attributions does not occur in the Copenhagen interpretation, where the wave function does not play any dynamic role.
- The trajectories obtained from Eq. (3) are called Bohmians trajectories and we highlight here that the particle can not go through the points where $R^2 = |\psi(t)|^2 = 0$, since the probability of the particle being in that place is null
- From Eq.

$$\frac{d^2\vec{x}_a(t)}{dt^2} = -\nabla_a V - \nabla_a U_q$$

for a given classical potential (V) it could exist several quantum potential (U_q) depending of the type of solution of Eq. (1) would be used to evaluate the momentum equation $\vec{p}_a = \nabla_a S$

- This is a consequence of the superposition principle and the solution of the Schorödinger equation depends on the boundary conditions of the system (like walls, slits, etc)
- Thus, the phase of the wave function and the quantum potential could be complicated, since they depend on the parameters associated with the experiment
- It is from these parameters that one could note the influence from the experiments in the quantum system (as postulated by Bohr) but in BdB this is an explicit influence that could be testable

Example: Double-Slit Experiment

- The phase of a superposition of wave functions can be quite complicated, even if the phases of its components are simple.
- In the case of a double slit, the global phase depends on parameters related to the experiment, such as the width and separation between the slits.
- In this way, these parameters, which are not linked to any fundamental interaction, can influence, through the phase of the wave function, the movement of the particle, even in regions where there is no mechanical contact between it and any of the slits.
- We will now discuss one example considered in Ref. [3] where the trajectories and the quantum potential are shown in Figs. 1 and 2 respectively

- The initial wave function was a superposition of two Gaussian functions each one centered in each slit
- The authors used this function to evaluate the Schrödinger equation and the solution found was used in Eq. (3)
- Thus, they could find the Bohmian trajectories whose initial positions were present in the two slits

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Quantum Interference and the Quantum Potential.

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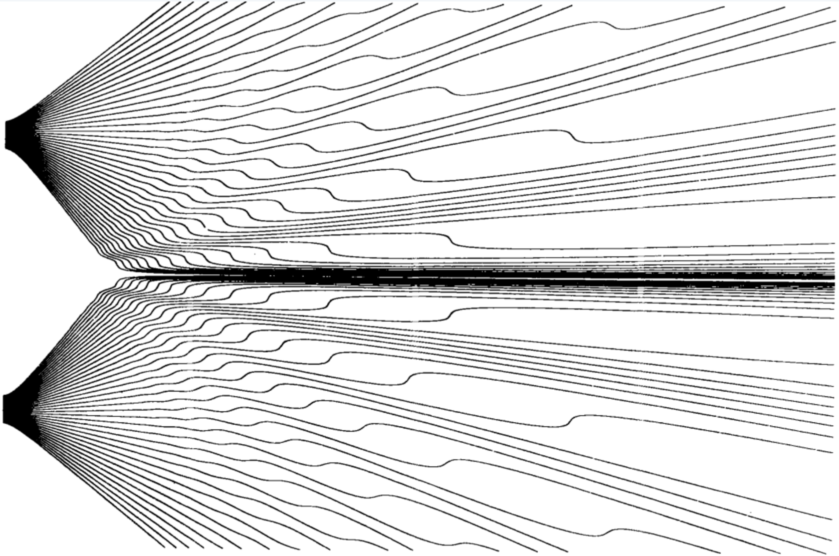
(ricevuto il 27 Dicembre 1978)

Summary. — We re-examine the notion of the quantum potential introduced by de Broglie and Bohm and calculate its explicit form in the case of the two-slit interference experiment. We also calculate the ensemble of particle trajectories through the two slits. The results show clearly how the quantum potential produces the bunching of trajectories that is required to obtain the usual fringe intensity pattern. Hence we are able to account for the interference fringes while retaining the notion of a well-defined particle trajectory. The wider implications of the quantum potential particularly in regard to the quantum interconnectedness are discussed.



Particle trajectories through two Gaussian slits.

- From a physical point of view, the particle enters through one of the slits, but the wave function passes through both and informs the particle about the other slit through Eq. (3)
- The information that is continuously passed to the particle contains data on the size and separation of the slits, the possible existence of detectors, and everything that is relevant to its movement.
- Note the jumps made by the particles in the region where they are free. It can be seen that its distribution in the detector describes an interference figure.
- As the wave function is null at the center line, we can say with certainty, contrary to the usual interpretation, that the particles detected in the upper part came from the upper slit, and analogously to the lower one, since they cannot cross the central line

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