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Lecture14...

Quantum Hardware – Optical Models

Class IX

Carlos Bessa *

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* Quantum Formalism, Zaiku Group Ltd.

Based on the book, Quantum Computation and Quantum Information, by M. Nielsen and I. Chuang

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Introduction to non-linear optics

In the next couple of lectures, we will discuss a little bit about non-linear optical materials and their application to quantum computer models.

Ψ) Non-linear optics is the study of phenomena that occur as a consequence of the modification of the optical properties of a material system by the presence of light

Ψ) Typically, only laser light is sufficiently intense to modify the optical properties of a material system

Ψ) Non-linear optical phenomena are "non-linear" in the sense that they may occur when the response of a material system to an optical field depends in a non-linear manner on the strength of the optical field

Ψ) To describe more precisely what we mean by an optical nonlinearity, let us consider how the dipole moment per unit volume, or polarization $P(t)$, of a material system depends on the strength $E(t)$ of an applied optical field

Ψ) In the case of linear optics, the induced polarization depends linearly on the electric field strength in a manner that can often be described by

$$P(t) = \epsilon_0 \chi^{(1)} E(t),$$

where $\chi^{(1)}$ is the linear susceptibility and ϵ_0 is the permittivity of free space.

Ψ) In nonlinear optics, the optical response can often be described by generalizing the equation above by expressing the polarization $P(t)$ as a power series in the field strength $E(t)$ as

$$P(t) = \epsilon_0 \left[\chi^{(1)} E(t) + \chi^{(2)} E^2(t) + \chi^{(3)} E^3(t) + ... \right],$$

where $\chi^{(2)}, \chi^{(3)}$ are the second-and third-order non-linear optical susceptibilities, respectively

Ψ) For simplicity, we will take $P(t)$ and $E(t)$ to be scalar quantities, however, due to the vector nature of $\vec{P}(t)$ and $\vec{E}(t)$, $\chi^{(1)}$ becomes a second-rank tensor, and so on

$$P_i = \epsilon_0 \left[\chi^{(1)}_{ij} E_j + \chi^{(2)}_{ijk} E_j E_k + \chi^{(3)}_{ijkl} E_j E_k E_l + ... \right]$$

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- Ψ) In writing $P(t)$ in the scalar form, we have assumed that the polarization at time t depends only on the instantaneous value of the electric field strength
- Ψ) The assumption that the medium responds instantaneously also implies that the medium must be lossless and dispersionless
- Ψ) In general, the non-linear susceptibilities depend on the frequencies of the applied field, but under our present assumption of instantaneous response, we take them to be constants

3

6

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WHITEBOARD

4

REFERENCE: NON-LINEAR OPTICS
3rd EDITION
ROBERT W. BOYD
CHAPTER 4

- THE REFRACTIVE INDEX OF MANY MATERIAL CAN BE DESCRIBED BY THE RELATION

$$n = n_0 + \bar{n}_2 \langle \tilde{E}^2 \rangle_t \quad , \quad \langle \ \rangle_t \rightarrow \text{AVERAGING IN TIME}$$

$n_0 \rightarrow$ WEAK-FIELD REFRACTIVE INDEX

$\bar{n}_2 \rightarrow$ NEW OPTICAL CONSTANT (SOMETIMES CALLED SECOND ORDER INDEX OF REFRACTION)

$\bar{n}_2$ IT GIVES THE RATE AT WHICH THE REFRACTIVE INDEX INCREASES WITH INCREASING OPTICAL INTENSITY

- FOR THE ELECTRIC FIELD, WE COULD REPRESENT IT BY

$$\tilde{E}(t) = E(\omega) e^{-i\omega t} + E^*(\omega) e^{i\omega t}$$

$$\tilde{E}^2 = 2|E|^2 + E^2 e^{-2i\omega t} + E^{*2} e^{2i\omega t}$$

REMEMBER: $\langle f(x) \rangle_x = \dfrac{1}{b-a} \displaystyle\int_a^b f(x)\, dx$

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$$\langle \tilde{E}^2 \rangle_t = \frac{1}{\frac{\pi}{\omega}} \int_0^{\pi/\omega} dt \left(\underset{\underset{\pi/\omega}{\parallel}}{2|E|^2} + E^2 \underset{\underset{0}{\parallel}}{e^{-2i\omega t}} + E^{*2} \underset{\underset{0}{\parallel}}{e^{2i\omega t}} \right)$$

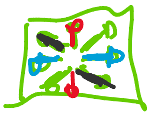
$$\langle \tilde{E}^2 \rangle_t = \frac{1}{\pi/\omega} \cdot \pi/\omega \, 2|E|^2$$

$$= 2|E|^2$$

So, $n = n_0 + 2\bar{n}_2 |E(\omega)|^2$ (1)

- THIS CHANGE IN THE REFRACTIVE INDEX DESCRIBED BY Eq. (1) IS SOMETIMES CALLED THE OPTICAL <u>KERR EFFECT</u>

- THE INTERACTION OF A BEAM OF LIGHT WITH NON-LINEAR OPTICAL MEDIUM CAN ALSO BE DESCRIBED IN TERMS OF THE NON-LINEAR POLARIZATION

- THE TOTAL POLARIZATION TO CENTROSYMMETRIC MATERIALS IS DESCRIBED BY  → CENTROSYMMETRIC MATERIAL $\chi^{(2)} = 0$

$$P(\omega) = \epsilon_0 \chi^{(1)} E(\omega) + 3\epsilon_0 \chi^{(3)} |E(\omega)|^2 E(\omega) + \dots$$

$$= \epsilon_0 \chi_{eff} E(\omega)$$

WHERE $\chi_{eff} = \chi^{(1)} + 3\chi^{(3)} |E(\omega)|^2$ (2)

- IN ORDER TO RELATE THE NON-LINEAR SUSCEPTIBILITY $\chi^{(3)}$ TO THE NONLINEAR REFRACTIVE INDEX n_2, WE NOTE THAT

$$n^2 = 1 + \chi_{eff}$$ (3)

USING Eq. (1) IN THE LHS OF Eq. (3) AND Eq. (2) IN THE RHS

$$\left(n_0 + 2\bar{n}_2 |E(\omega')|^2\right)^2 = 1 + \chi^{(1)} + 3\chi^{(3)} |E(\omega')|^2$$

EXPANDING TO TERMS UP TO $|E|^2$

$$\underbrace{n_0^2} + \underbrace{4 n_0 \bar{n}_2} |E(\omega')|^2 + \cdots = \underbrace{1 + \chi^{(1)}} + \underline{3\chi^{(3)}} |E(\omega')|^2$$

$$n_0^2 = 1 + \chi^{(1)} \Rightarrow n_0 = \sqrt{1+\chi^{(1)}}$$

$$4 n_0 \bar{n}_2 = 3\chi^{(3)} \Rightarrow \bar{n}_2 = \frac{3\chi^{(3)}}{4 n_0}, \quad n_0 = \sqrt{1+\chi^{(1)}}$$

$$\bar{n}_2 = \frac{3\chi^{(3)}}{4\sqrt{1+\chi^{(1)}}}$$

— AN ALTERNATIVE WAY OF WRITTING THE INTENSITY DEPENDENT REFRACTIVE INDEX IS BY MEANS OF THE EQUATION

$$n = n_0 + n_2 I$$

— TO FIND I AND n_2, REMEMBER THAT

$$I = v \langle u \rangle_t, \quad \langle u \rangle_t \rightarrow \text{TIME AVERAGED ENERGY DENSITY}$$

IN VACUUM : $v = c_0 \rightarrow$ VELOCITY OF LIGHT IN VACUUM

IN A MEDIUM : $v = \frac{c_0}{n_0} \rightarrow$ " " " " MEDIUM

— $\langle u \rangle_t$ CAN BE CALCULATED USING POYTING THEOREM

$$\langle u \rangle_t = \frac{1}{2}\langle \vec{D}\cdot\vec{E}\rangle_t + \frac{1}{2}\langle \vec{B}\cdot\vec{H}\rangle_t \simeq \frac{1}{2}\langle \vec{D}\cdot\vec{E}\rangle_t$$

BECAUSE, $|\vec{B}| \ll |\vec{E}|$, $\vec{B} = \mu_0 \vec{H}$, $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

$$\vec{D} = \epsilon_0 \vec{E} + \epsilon_0 \chi^{(1)} \vec{E} \simeq \epsilon_0 \underbrace{(1+\chi^{(1)})}_{n_0^2} \vec{E} = \epsilon_0 n_0^2 \vec{E}$$

$$\langle u \rangle_t = \frac{1}{2}\epsilon_0 n_0^2 \langle E \rangle = \frac{1}{2}\epsilon_0 n^2 |E(\omega)|^2$$

$$\langle u \rangle_t = \frac{1}{2} \epsilon_0 n_0^2 \langle E \rangle_t = \frac{1}{2} \epsilon_0 n_0^2 |E(\omega)|^2$$

$$I = v \langle u \rangle_t = \frac{c_0}{n_0} \frac{1}{2} \epsilon_0 n_0^2 |E|^2 \Rightarrow I = \frac{1}{2} \epsilon_0 n_0 c_0 |E|^2$$

SO,

$$n = n_0 + n_2 \frac{1}{2} \epsilon_0 c_0 n_0 |E|^2$$

FROM EQ. (1) $\Rightarrow n = n_0 + 2 \bar{n}_2 |E|^2$

$$\cancel{n_0} + 2 \bar{n}_2 |E|^2 = \cancel{n_0} + \frac{1}{2} \epsilon_0 c_0 n_0 n_2 |E|^2$$

$$n_2 = \frac{4 \bar{n}_2}{\epsilon_0 c_0 n_0},$$

BUT WE HAVE FIND THAT $\bar{n}_2 = \dfrac{3 \chi^{(3)}}{4\sqrt{1+\chi^{(1)}}} = \dfrac{3 \chi^{(3)}}{4 n_0}$

SO, $n_2 = \dfrac{3 \chi^{(3)}}{\epsilon_0 c_0 n_0^2}$

TO DO QUANTUM COMPUTATION IT IS IMPORTANT FOR US THE FOLLOWING EXPRESSION

$$n(I) = n_0 + n_2 I \qquad (7.19)$$

EXEMPLES: • DOPED GLASSES

$$10^{-14}\ \text{cm}^2/\text{W} < n_2 < 10^{-7}\ \text{cm}^2/\text{W}$$

• SEMICONDUCTORS

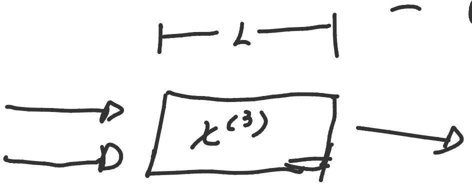
$$10^{-10}\ \text{cm}^2/\text{W} < n_2 < 10^{2}\ \text{cm}^2/\text{W}$$

— EXPERIMENTALLY, THE RELEVANT BEHAVIOUR IS THAT WHEN TWO BEAMS OF LIGHT OF EQUAL INTENSITY ARE NEARLY CO-PROPAGATED THROUGH A KERR MEDIUM

— EACH BEAM WILL EXPERIENCE AN EXTRA PHASE SHIFT OF

$$e^{i(n-n_0)L\omega/c_0} = e^{i(n_0+n_2 I - n_0)L\omega/c_0}$$

$$= e^{i n_2 I L\omega/c_0}$$



$$e^{i n_2 I L \omega/c_0}$$

$$n_2 = \frac{\chi^{(3)}}{\epsilon_0 c_0 (1+\chi^{(1)})^{1/2}}$$