Lecture10_QHardware_class5

sexta-feira, 12 de janeiro de 2024 14:07

Lecture10...

Quantum Hardware – Optical Models

Class V

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January 12, 2024

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Based on the book, Quantum Computation and Quantum Information, by M. Nielsen and I. Chuang

Optical Photon Quantum Computer - Coherent States - Part II

Let us start today by reviewing some of the formalisms we studied in the previous lectures

- Ψ) Lase light is a coherent electromagnetic wave, the closest possible analog to a classical electromagnetic wave
- Ψ) In the last lecture, we discussed quantum states that described ideal laser light – coherent states
- Ψ) Since laser light has a well-defined amplitude, we have defined coherent states as the eigenstates of the amplitude operator (annihilation operator) $\longleftrightarrow a|\alpha\rangle = \alpha|\alpha\rangle$
- Ψ) Because the annihilation operator a is not Hermitian, the eigenvalues of a are complex
- Ψ) We note that the annihilation operator acts on the state $|\alpha\rangle$ with no annihilation since this state is its eigenvalue
- Ψ) This suggests that there might exist other types of states where the annihilation operator can act with no annihilation, and sometimes even create!

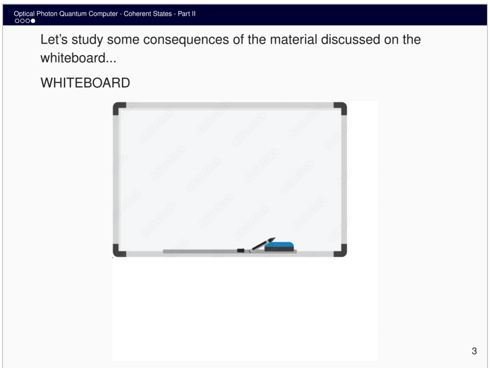
- Ψ) In fact, it was shown that the operator a acts as an "annihilator" in sub-Poissonian states as the $|n\rangle$-states, and as a "creator" in super-Poissonians states (like in thermal states, phase states, etc) and do not create or annihilate in Poissonian states, like the coherent states
- Ψ) To study coherent states more carefully, we introduced the displacement operator $\longleftrightarrow D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$, which is unitary.
- Ψ) This operator displaces the amplitude a by the complex number α

$$D^\dagger(\alpha) a D(\alpha) = a + \alpha$$

- Ψ) In terms of the $|n\rangle$ states (sometimes named Fock states), the state $|\alpha\rangle$ can be written as

$$|\alpha\rangle = e^{\frac{-|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

- Ψ) In the SHO, n refers to the energy level, while in the EM field, n refers to the number of photons, which are associated with the energy level as well



$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \qquad (7.16) \rightarrow \text{NIELSEN CHUANG}$$

COHERENT STATES PROPERTIES

a) NOTE THAT

$$N|\alpha\rangle = a^\dagger\underbrace{(a|\alpha\rangle)}_{\hat{I}} = a^\dagger \alpha |\alpha\rangle = \alpha \underbrace{a^\dagger|\alpha\rangle}_{\alpha^*} = |\alpha|^2 |\alpha\rangle$$

$$\langle\alpha|N|\alpha\rangle = |\alpha|^2 \langle\alpha|\alpha\rangle = |\alpha|^2$$

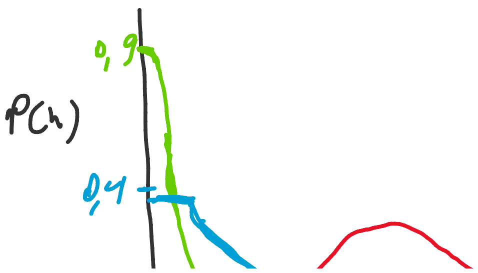
$$\langle\alpha|n|\alpha\rangle = |\alpha|^2$$

b) THE PROBABILITY OF FINDING n PHOTONS IN THE STATE $|\alpha\rangle$ IS GIVEN BY

$$p(n) = \langle\alpha|n\rangle\langle n|\alpha\rangle = |\langle n|\alpha\rangle|^2$$

$$P(n) = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \underbrace{\langle n|n\rangle}_{\delta_{nn}} = e^{-|\alpha|^2} \frac{\alpha^n}{n!} = \frac{e^{-\langle n\rangle} \alpha^n}{n!}$$

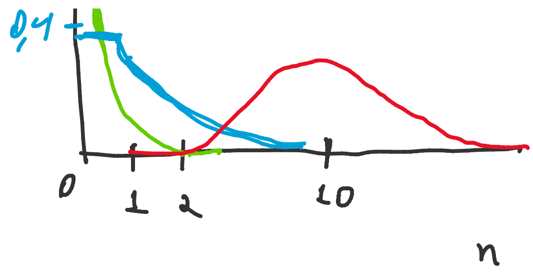
WHICH IS A POISSON DISTRIBUTION



$|\alpha|^2 = 0.1$

$|\alpha|^2 = 1$

$|\alpha|^2 = 10$



$|\alpha|^2 = 10$

- WE CAN SEE THAT, FOR $|\alpha|^2 \leq 1$, $P(n)$ IS MAXIMUM AT $n = 0$
- FOR $|\alpha|^2 > 1$, $P(n)$ HAS A PEAK CLOSE TO $n = |\alpha|^2$

c) AS DISCUSSED EARLIER, THE COHERENT STATE IS A MINIMUM UNCERTAINTY STATE

- TO DEMONSTRATE THIS STATEMENT, LET'S USE THE COORDINATE REPRESENTATION OF THE OSCILLATOR NUMBER STATE $|n\rangle$

$$\psi_n(r) = \langle r | n \rangle$$

- IT FOLLOW FROM LECTURE 7, PAGE 3

$$\begin{cases} a = \dfrac{1}{\sqrt{2m\omega\hbar}}(m\omega r + ip) \\ a^+ = \dfrac{1}{\sqrt{2m\omega\hbar}}(m\omega r - ip) \end{cases}$$

USING THE FACT THAT $p = -i\hbar \dfrac{\partial}{\partial r}$ (LECTURE 6, PAGE 13) AND FROM LECTURE 8, PAGE 4

$$\left(m\omega r + \hbar\frac{d}{dr}\right)\psi_0(r) = 0$$

$$\psi_0(r) = \frac{1}{\pi^{1/4}\sqrt{r_0}} e^{-r^2/2r_0^2}, \quad r_0 = \sqrt{\frac{\hbar}{m\omega}}$$

AND THE GENERAL $\psi_n(r)$

$$\psi_n(r) = \frac{1}{\sqrt{n!}} \frac{1}{r_0^n}\left(r - r_0^2\frac{d}{dr}\right)^n \psi_0(r) \quad \text{(LECTURE, PAGE 5)}$$

– IT CAN BE VERIFIED THAT THESE WAVE FUNCTIONS SATISFY THE ORTHONORMALITY CONDITION

$$\int_{-\infty}^{+\infty} \psi_n^*(x)\, \psi_m(x) = \delta_{nm} \qquad \text{(HOMEWORK)}$$

$$\psi_n(x) = \frac{1}{(2^n n!)} H_n\left(\frac{x}{x_0}\right) \psi_0(x) \quad , \quad H_n \to \text{HERMITE POLYNOMIALS}$$

– THE EXPECTATION VALUES IN THE MOMENTUM AND POSITION ARE:

$$\begin{cases} (\Delta p)^2 = \langle p^2 \rangle - \langle p \rangle^2 \\ (\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2 \end{cases}$$

– LET'S USE EQUATIONS FROM LECTURE 8, PAGE 3

$$\langle n | x | n \rangle = \sqrt{\frac{\hbar}{2m\omega}} \left(\sqrt{n}\, \overset{=0}{\delta_{n,n-1}} + \sqrt{n+1}\, \delta_{n,n+1} \right) = 0$$

$$\langle n | p | n \rangle = i \sqrt{\frac{m\hbar\omega}{2}} \left(\sqrt{n+1}\, \overset{=0}{\delta_{n,n+1}} - \sqrt{n}\, \overset{=0}{\delta_{n,n-1}} \right) = 0$$

– FROM LECTURE 8, PAGE 3

$$x^2 = \left(\frac{\hbar}{2m\omega}\right) \left(a^2 + a^{+2} + a^+ a + a a^+\right) \qquad \begin{cases} a|n\rangle = \sqrt{n}\,|n-1\rangle \\ a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle \end{cases}$$

- $\langle n | a^2 | n \rangle = 0$, • $\langle n | a^+ a | n \rangle = n \langle n | n \rangle = n$
- $\langle n | a^{+2} | n \rangle = 0$, • $\langle n | a a^+ | n \rangle = (n+1) \langle n | n \rangle = n$

$$\langle n | x^2 | n \rangle = \frac{\hbar}{2m\omega} (2n+1) = \frac{\hbar}{m\omega} (n + 1/2)$$

– THE SAME FOR $\langle n | p^2 | n \rangle$

$$\langle n | p^2 | n \rangle = \hbar\omega m \,(n + 1/2)$$

$$\langle n | p^2 | n \rangle = \hbar \omega m (n + 1/2)$$

– THE UNCERTAINTIES IN THE GENERALIZED MOMENTUM AND COORDINATE VARIABLES ARE GIVEN BY

$$(\Delta p)^2 = \langle p^2 \rangle - \overset{=0}{\langle p \rangle} = \hbar \omega m (n + 1/2)$$

$$(\Delta x)^2 = \langle x^2 \rangle - \overset{=0}{\langle x \rangle} = \frac{\hbar}{m\omega} (n + 1/2)$$

– THE UNCERTAINTY PRODUCT IS

$$\Delta p \Delta x = \sqrt{\hbar \omega m} (n + 1/2)^{1/2} \sqrt{\frac{\hbar}{m\omega}} (n + 1/2)^{1/2}$$

$$\Delta p \Delta x = \hbar (n + 1/2)$$

– FOR $n=0$, WE HAVE $\Delta p \Delta x = \frac{\hbar}{2}$

– THIS IS THE MOST CLASSICAL OF THE QUANTUM STATES

– ADVANTAGE: EASIER TO CARRY OUT EXPERIMENTALLY

d) THE SET OF ALL COHERENT STATES IS A COMPLETE SET

– TO SHOW THIS LET US CONSIDER THE INTEGRAL

$$\int |\alpha\rangle\langle\alpha| \, d^2\alpha = \int d^2\alpha \left(e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \right) \times$$

$$\left(e^{-|\alpha|^2/2} \sum_{n'=0}^{\infty} \frac{(\alpha^*)^{n'}}{\sqrt{n'!}} \langle n'| \right)$$

$n'=0 \quad \sqrt{n'!}$

$$= \sum_n \sum_{n'} \frac{|n\rangle\langle n'|}{\sqrt{n!\,n'!}} \underbrace{\int d^2\alpha\, e^{-|\alpha|^2} (\alpha^*)^{n'} \alpha^n}_{INT}$$

$$INT = \pi\, n!\, \delta_{nn'} \qquad \Big\{ \quad \alpha = |\alpha| e^{i\theta}$$

$$INT = \int_0^\infty d|\alpha|\, e^{-|\alpha|^2} |\alpha|^{n'+n+1} \int_0^{2\pi} d\theta\, e^{i(n-n')\theta}$$

$$\int_{-\infty}^{+\infty} |\alpha\rangle\langle\alpha|\, d^2\alpha = \sum_n \sum_{n'} \frac{|n\rangle\langle n'|}{\sqrt{n!\,n'!}} \pi\, n!\, \delta_{nn'}$$

$$= \pi \sum_n \frac{|n\rangle\langle n|}{n!} n! = \pi \overbrace{\sum_n |n\rangle\langle n|}^{\hat{I}}$$

$$= \pi \hat{1}$$

$$\frac{1}{\pi} \int_{-\infty}^{+\infty} |\alpha\rangle\langle\alpha|\, d^2\alpha = \hat{1}$$

- THIS IS THE COMPLETE RELATION FOR THE COHERENT STATES

- THE BASE IS CONTINUOS AND SOMETIMES CALLED OVERCOMPLETE

- THIS IS IN OPPOSITION TO THE FOCK STATES

$$\sum_n |n\rangle\langle n| = \hat{1} \qquad \text{(DISCRETE AND COMPLETE)}$$

AND $\{|x\rangle\}$, AND $\{|p\rangle\}$

$$\int |x\rangle\langle x|\, dx = \hat{1} \quad \text{(CONTINUOS AND COMPLETE)}$$

$$\int |p\rangle\langle p|\, dp = \hat{1} \quad (\;\text{"} \quad \text{"} \quad \text{"}\;)$$

e) THE COHERENT STATES CORRESPONDING TO DIFFERENT EIGENVALUES α AND α' ARE NOT ORTHOGONAL

$$\langle \alpha | \alpha' \rangle = e^{-\frac{|\alpha|^2}{2} - \frac{|\alpha'|^2}{2}} \sum_n \sum_n \frac{(\alpha' \alpha^*)^n}{n!} \delta_{nn}$$

$$= e^{-\frac{|\alpha|^2}{2} - \frac{|\alpha'|^2}{2}} \sum_n \frac{(\alpha' \alpha^*)^n}{n!}$$

$$= e^{-\frac{|\alpha|^2}{2} - \frac{|\alpha'|^2}{2}} e^{\alpha' \alpha^*} = e^{-\frac{|\alpha|^2}{2} - \frac{|\alpha'|^2}{2} + \alpha' \alpha^*} \neq \delta(\alpha - \alpha')$$

- $|\langle \alpha | \alpha' \rangle|^2 = e^{-|\alpha - \alpha'|^2} \longrightarrow 0$, ONLY WHEN α AND α' DIFFERS SIGNIFICANTLY

REFERENCE :

QUANTUM OPTICS
M. O. SCULLY AND M. S. ZUBAIRY (CAP. 2)
PAGE 50 – 55