Lecture13_QHardware_class8

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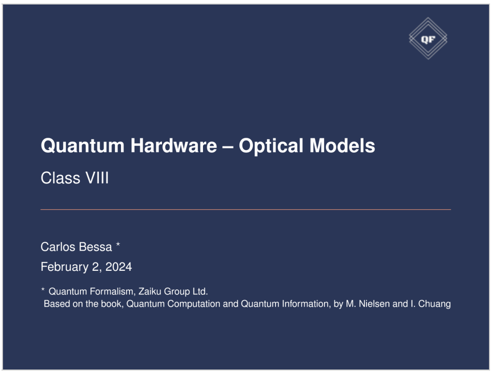
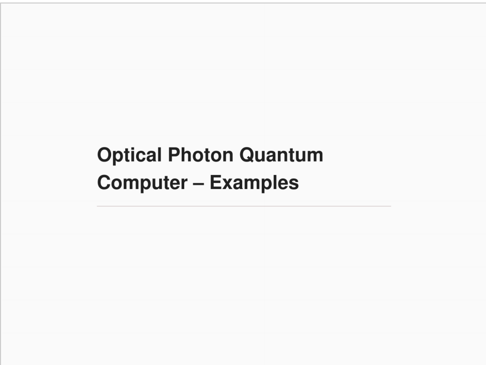
Lecture13...



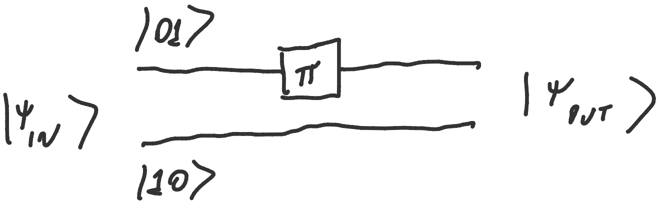


- PHASE-SHIFTERS

$$
\begin{cases}
H_p = (n_0 - n) Z \\
P = e^{-i H_p L / c_0} \\
P = \begin{pmatrix} e^{-i\Delta} & 0 \\ 0 & e^{i\Delta} \end{pmatrix}
\end{cases}
\quad , \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
$$

$\vdash L \dashv$, $c_0 \rightarrow$ SPEED OF LIGHT IN VACUUM

EXERCISE 7.7



$$|\psi_{out}\rangle_{TOP} = P|01\rangle \quad , \qquad P|0\rangle = |0\rangle$$

$$P|1\rangle = e^{i\Delta}|1\rangle$$

$$= P|0\rangle \otimes |1\rangle$$

$$= e^{i\Delta}|01\rangle$$

$$|\psi_{out}\rangle_{BOTTOM} = |10\rangle$$

$$\Rightarrow \quad |\psi_{out}\rangle = |\psi\rangle_{TOP} + |\psi\rangle_{BOTTOM} = e^{i\Delta}\underbrace{|01\rangle}_{|0_L\rangle} + \underbrace{|10\rangle}_{|1_L\rangle}$$

$$|\psi_{out}\rangle = \overbrace{\begin{pmatrix} e^{i\Delta} & 0 \\ 0 & 1 \end{pmatrix}}^{P} |\psi_{in}\rangle$$

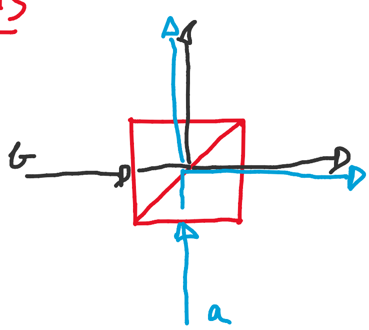
$\Delta = \pi$

$$|\psi_{out}\rangle = \begin{pmatrix} e^{i\pi} & 0 \\ 0 & 1 \end{pmatrix} |\psi_{in}\rangle \qquad (7.23)$$

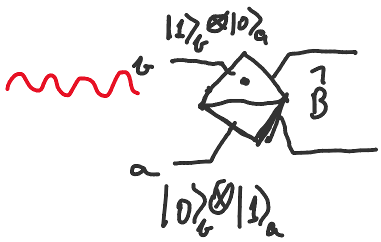
EXERCISE 7.8 (HOMEWORK)

BEAM SPLITTERS

CLASSICAL



QUANTUM

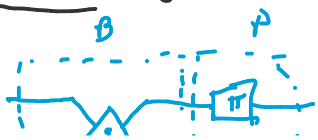


$$b' = \frac{|01\rangle + |10\rangle}{\sqrt{2}} \qquad a' = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

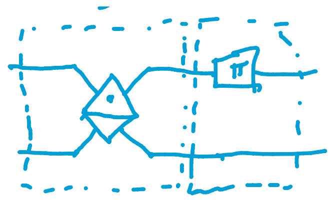
$$\theta = \pi/4$$

$$, \quad \hat{B} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

EXERCISE 7.9 (OPTICAL HADAMARD GATE)



$$B = \begin{pmatrix} \cos\theta & -\sin\theta \end{pmatrix} \qquad (7.35)$$



$$B = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad (7.35)$$

$$P = \begin{pmatrix} e^{i\pi} & 0 \\ 0 & 1 \end{pmatrix} \qquad (7.23)$$

$$\text{CIRCUIT} = B.P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} e^{i\pi} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} e^{i\pi}\cos\theta & -\sin\theta \\ e^{i\pi}\sin\theta & \cos\theta \end{pmatrix}$$

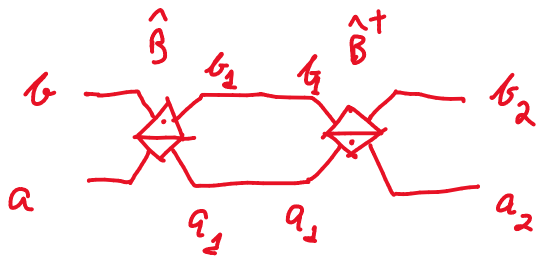
$\theta = \pi/4$

$$\text{CIRCUIT} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\pi} & -1 \\ e^{i\pi} & 1 \end{pmatrix} = \frac{e^{i\pi}}{\sqrt{2}}\begin{pmatrix} 1 & -e^{-i\pi} \\ 1 & e^{-i\pi} \end{pmatrix} = \frac{e^{i\pi}}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= e^{i\pi} H = -H \quad , \quad H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

EXERCISE 7.10 (MACH-ZENDER INTERFEROMETER)

1.



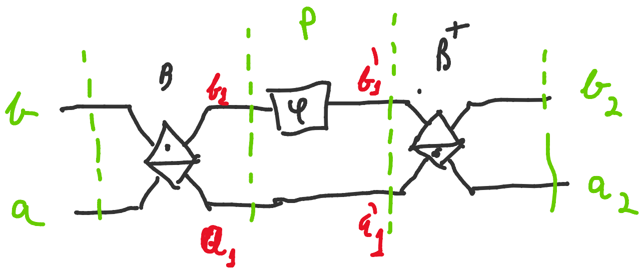
50/50

$\hat{B}$: $b_1 = \dfrac{b-a}{\sqrt{2}}$, $a_1 = \dfrac{b+a}{\sqrt{2}}$; $\hat{B}^{\dagger}$: $b_2 = \dfrac{b_1+a_1}{\sqrt{2}} = \dfrac{b-a+b+a}{2} = b$

$$a_2 = \frac{a_1-b_1}{\sqrt{2}} = \frac{b+a-b+a}{2} = a$$

SO, THE CIRCUIT ABOVE REPRESENTS THE IDENTITY OPERATION

2.



$P|01\rangle = e^{i\delta}|01\rangle$

$P|10\rangle = e^{i\delta}|10\rangle$

$\hat{B}$; $b_1 = \dfrac{b-a}{\sqrt{2}}$, $a_1 = \dfrac{a+b}{\sqrt{2}}$

$\hat{B}$; $b_1 = \frac{b-a}{\sqrt{2}}$, $a_1 = \frac{a+b}{\sqrt{2}}$

$\hat{P}$: $b_1' = e^{i\varphi} b_1$, $a_1' = a_1$

$\hat{B}^{\dagger}$: $b_2 = \frac{a_1' + b_1'}{\sqrt{2}} = \frac{a+b + e^{i\varphi}(b-a)}{2} = \frac{(1+e^{i\varphi})b + (1-e^{i\varphi})a}{2}$

$$b_2 = \frac{e^{i\varphi/2}(e^{-i\varphi/2} + e^{i\varphi/2})b + e^{i\varphi/2}(e^{-i\varphi/2} - e^{i\varphi/2})a}{2}$$

$$b_2 = e^{i\varphi/2}\cos(\varphi/2)\, b - e^{i\varphi/2}\, i \sin(\varphi/2)\, a$$

$$\boxed{b_2 = e^{i\varphi/2}[b\cos(\varphi/2) - i a \sin(\varphi/2)]}$$

$a_2 = \frac{a_1 - b_1}{\sqrt{2}} \Rightarrow \boxed{a_2 = e^{i\varphi/2}[a\cos(\frac{\varphi}{2}) - i b \sin(\frac{\varphi}{2})]}$

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \underbrace{e^{i\varphi/2}\begin{pmatrix} \cos(\varphi/2) & -i\sin(\varphi/2) \\ -i\sin(\varphi/2) & \cos(\varphi/2) \end{pmatrix}}_{\text{circuit}} \begin{pmatrix} a \\ b \end{pmatrix}$$

Eq. (4.4)

$$R_x(\theta) = \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$$

$$\text{circuit} = e^{i\varphi/2} R_x(\varphi)$$

BEAM SPLITTERS : $\hat{B} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = e^{-i\theta Y}$

$\hat{P} = \begin{pmatrix} e^{-i\Delta/2} & 0 \end{pmatrix} = e^{-\frac{\Delta}{2} Z}$

$$\vec{P} = \begin{pmatrix} e^{-i\Delta/2} & 0 \\ 0 & e^{i\Delta/2} \end{pmatrix} = e^{-\frac{\Delta}{2}z}$$