Lecture15_QHardware_class10

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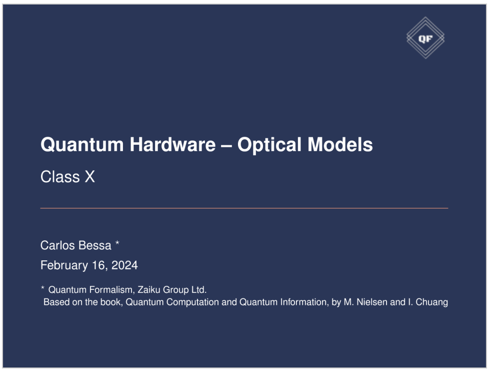
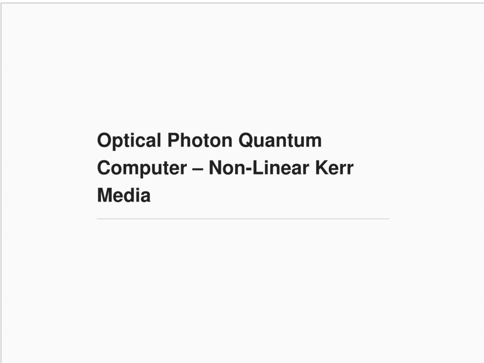
Lecture15...





NON-LINEAR KERR MEDIA

- THE MOST IMPORTANT EFFECT OF A KERR MEDIA IS THE CROSS PHASE MODULATION

- IT PROVIDES BETWEEN TWO MODES OF LIGHT THAT IS CLASSICALLY DESCRIBED BY THE n_2 TERM IN EQ. (7.14)

(7.15) $\Rightarrow$ $n(I) = n_0 + n_2 I$,

$n_2 \sim \chi^{(3)}$

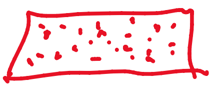
$I \sim E^2(\omega)$

CENTROSYMMETRIC MATERIALS

0

$P = \epsilon_0(\chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3 + \dots)$

- IT IS AN INTERACTION BETWEEN PHOTONS MEDIATED BY ATOMS IN THE KERR MEDIUM



- QUANTUM MECHANICALLY, THIS EFFECT IS DESCRIBED BY

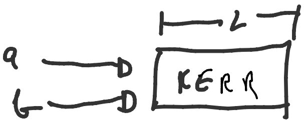
- QUANTUM MECHANICALLY, THIS EFFECT IS DESCRIBED BY THE HAMILTONIAN

$$H_{xpm} = -\chi\, a^{+} a\, b^{+} b$$

$xpm \to$ CROSS PHASE MODULATION

$a, b \to$ TWO MODES PROPAGATING THROUGH THE MEDIUM

$\chi = \hbar \bar{\kappa}$, $\bar{\kappa} \to$ COUPLING CONSTANT



- FOR A CRYSTAL OF SIZE L WE OBTAIN THE UNITARY TRANSFORM

$$K = e^{-iH_{xpm} L}$$

- BY COMBINING THE KERR MEDIA WITH BEAM SPLITTERS WE CAN CONSTRUCT A CNOT GATE

- LET'S SEE HOW TO DO IT

- $K|00\rangle = |00\rangle$

$$K|00\rangle = e^{i\chi L\, a^{+}a\, b^{+}b}|00\rangle = \left(1 + i\chi L\, a^{+}a\, b^{+}b + \dots\right)|00\rangle$$

$\to 0_a 0_b$

$$= |00\rangle + i\chi L \cdot 0 = |00\rangle$$

- $$K|01\rangle = e^{i\chi L a^{+}a\, b^{+}b}|01\rangle = \left(1 + i\chi L\, a^{+}a\, b^{+}b + \dots\right)|01\rangle$$

$$= |01\rangle + i\chi L\, 0$$
$$= |01\rangle$$

- $K|10\rangle = |10\rangle$

- $K|11\rangle = e^{i\chi L}|11\rangle$

$$e^{i\chi L\, a^{+}a\, b^{+}b}|11\rangle = \left(1 + i\chi L\, a^{+}a\, b^{+}b + \dots\right)|11\rangle$$

• $K|11\rangle = c \; |11\rangle$

$$e^{i\chi L\, a^\dagger a\, b^\dagger b}|11\rangle = (1 + i\chi L\, a^\dagger a\, b^\dagger b + \dots)|11\rangle$$

$$= |11\rangle + i\chi L|11\rangle + \dots = (1 + i\chi L + \dots)|11\rangle$$

$$K|11\rangle = e^{i\chi L}|11\rangle$$

$$\chi L = \pi \Rightarrow K|11\rangle = -|11\rangle$$

– NOW CONSIDER TWO DUAL-RAIL STATES, THAT IS, FOUR MODES OF LIGHT

– THESE MODES LIVES IN A SPACE SPANNED BY THE FOLLOWING BASIS

• $|e_{00}\rangle = |1\,0\,0\,1\rangle = \underbrace{|1\rangle_{b_2} \otimes |0\rangle_{a_2}}_{|0\rangle_L} \otimes \underbrace{|0\rangle_{a_1} \otimes |1\rangle_{b_1}}_{|0\rangle_L}$

$a_1 \to$, $b_1 \to$, $a_2 \to$, $b_2 \to$ Kerr

• $|e_{01}\rangle = |1010\rangle = \underbrace{|10\rangle_2}_{|0\rangle_L} \otimes \underbrace{|10\rangle_1}_{|1\rangle_L}$

• $|e_{10}\rangle = |\underbrace{01}_{|1\rangle_L}\,\underbrace{01}_{|0\rangle_L}\rangle$

• $|e_{11}\rangle = |\underbrace{01}_{|1\rangle_L}\,\underbrace{10}_{|1\rangle_L}\rangle$

$$\begin{cases} |01\rangle \to |0\rangle_L \\ |10\rangle \to |1\rangle_L \end{cases}$$

$|0\rangle_L = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|1\rangle_L = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

• $|e_{00}\rangle = |00\rangle_L = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

• $|e_{01}\rangle = |01\rangle_L = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

• $|e_{10}\rangle = |10\rangle_L = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

• $|e_{11}\rangle = |11\rangle_L = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

• $K|e_{00}\rangle = |e_{00}\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, • $K|e_{01}\rangle = |e_{01}\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

for $\chi L = \pi$

$\begin{pmatrix} 0 \end{pmatrix}$

for $\chi L = \pi$

• $\mathcal{K}|e_{10}\rangle = |e_{10}\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$, • $\mathcal{K}|e_{11}\rangle = -|e_{11}\rangle = \begin{pmatrix}0\\0\\0\\-1\end{pmatrix}$

$$\mathcal{K} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}, \quad \text{for } \chi L = \pi$$

- This is useful because the CNOT operation can be factored using $\mathcal{K}$

$$\underbrace{\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}}_{CNOT} = \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1&1&0&0\\1&-1&0&0\\0&0&1&1\\0&0&1&-1\end{pmatrix}}_{I\otimes H} \underbrace{\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}}_{\mathcal{K}} \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1&1&0&0\\1&-1&0&0\\0&0&1&1\\0&0&1&-1\end{pmatrix}}_{I\otimes H}$$

↳ Hadamard

↳ beam splitters + phase shifters

- H → Hadamard gate
- $\mathcal{K}$ → the Kerr interaction with $\chi L = \pi$

- An apparatus like this one has been considered for constructing a reversible classical optical logic gate described in Box 7.4

- Summarizing

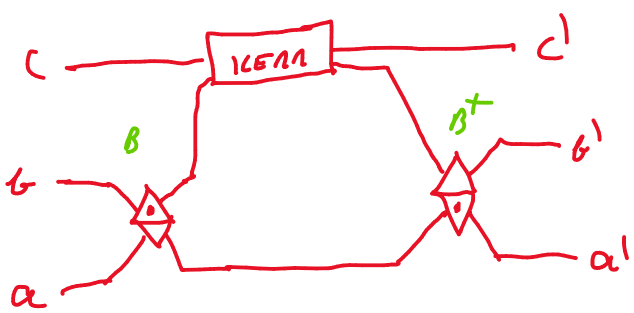
• A CNOT gate can be constructed from Kerr media
• Arbitrary single qubit operations can be realized

USING BEAM SPLITTERS AND PHASE SHIFTERS

- SINGLE PHOTONS CAN BE CREATED USING LASERS AND DETECTED WITH PHOTON DETECTORS
- THUS, IN THEORY, A QUANTUM COMPUTER CAN BE IMPLEMENTED USING THESE OPTICAL COMPONENTS

BOX 7.4 THE QUANTUM FREDKIN GATE

– AN OPTICAL FREDKIN GATE CAN BE BUILT USING TWO BEAM SPLITTERS AND A NON-LINEAR KERR MEDIUM AS SHOWN IN THE DIAGRAM



– THIS PERFORMS THE UNITARY TRANSFORM

$$U = B K B^\dagger \quad , \quad B \to 50/50 \text{ BEAMSPLITTER}$$

$$K \to \text{CROSS PHASE MODULATION}$$

$$\begin{cases} K = e^{i\xi b^\dagger b c^\dagger c} \\ B = e^{\theta(a^\dagger b - a b^\dagger)} \end{cases} \quad , \quad \xi = \chi L$$

– THIS SIMPLIFIES TO GIVE

$$\begin{cases} U = e^{i\xi c^\dagger c \frac{(b^\dagger - a^\dagger)(b-a)}{2}} \qquad (7.47) \\ U = e^{i\frac{\pi}{2} b^\dagger b} e^{\frac{\xi}{2} c^\dagger c (a^\dagger b - b^\dagger a)} e^{-i\frac{\pi}{2} b^\dagger b} e^{i\frac{\xi}{2} a^\dagger a c^\dagger c} e^{i\frac{\xi}{2} b^\dagger b c^\dagger c} \end{cases}$$

$$U = e^{i\frac{\pi}{2} b^\dagger b}\, e^{\frac{\xi}{2} c^\dagger c (a^\dagger b - b^\dagger a)}\, e^{-i\frac{\pi}{2} b^\dagger b}\, e^{i\frac{\xi}{2} a^\dagger a c^\dagger c}\, e^{i\frac{\xi}{2} b^\dagger b c^\dagger c} \tag{7.48}$$

HOMEWORK!

USE THE BCH-FORMULA TO CALCULATE $U = BKB^\dagger$ AS WE DID IN LECTURE 12 PAGE 7

$$U = e^{\lambda G} K e^{-\lambda G} = e^{\lambda G}(1 + i\xi b^\dagger b c^\dagger c + \cdots)\, e^{-\lambda G}$$

$$= 1 + i\xi c^\dagger c\, e^{\lambda G} b^\dagger b\, e^{-\lambda G} + \cdots$$

$$= 1 + i\xi c^\dagger c b^\dagger b + i\xi c^\dagger c (a^\dagger a - b^\dagger b)\sin^2\lambda - i\xi c^\dagger c (b^\dagger a + a^\dagger b) \sin\lambda\cos\lambda + \cdots$$

$$\lambda = \frac{\pi}{4}$$

$$U = 1 + i\xi c^\dagger c \left[\frac{(b^\dagger - a^\dagger)(b - a)}{2} \right] + \cdots$$

$$U = e^{i\xi c^\dagger c \frac{(b^\dagger - a^\dagger)(b - a)}{2}}$$