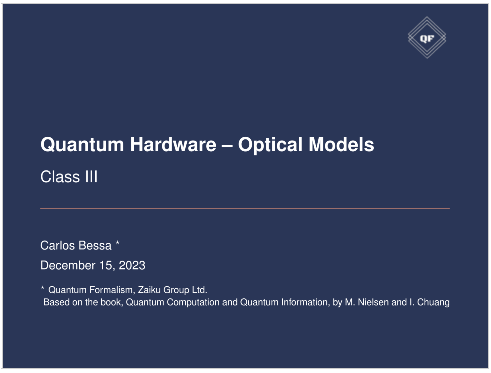
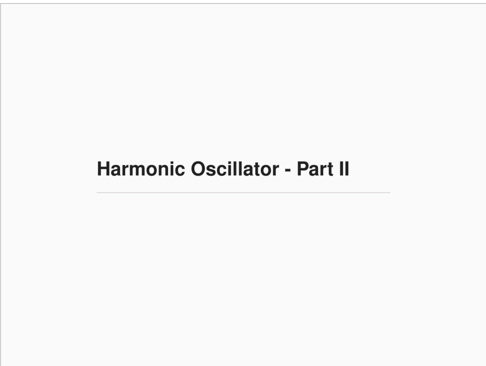
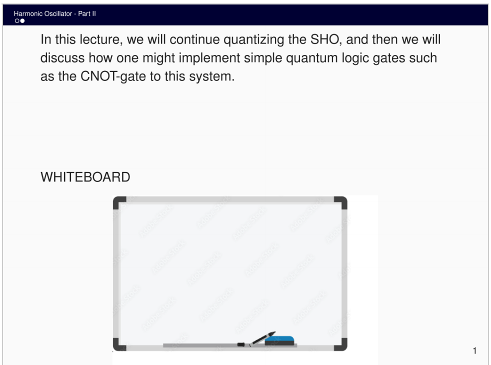
Lecture8_QHardware_class3

sexta-feira, 15 de dezembro de 2023 14:40

Lecture8_...







– THE SINGLE HARMONIC OSCILLATOR (CONTINUATION ...)

$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2$$

$\rightarrow E_n = \hbar\omega(n + 1/2)$, $n = 0, 1, 2, 3, 4, \ldots$

$\boxed{E_0 = \frac{\hbar\omega}{2}}$ (GROUND STATE)

• $a|n\rangle = \sqrt{n}\,|n-1\rangle$ (ANNIHILATION OPERATOR)

$a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$ (CREATION OPERATOR)

LAST LECTURE

- $a|n\rangle = \sqrt{n}\,|n-1\rangle$ (ANNIHILATION OPERATOR)
- $a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$ (CREATION OPERATOR)

– WHAT'S HAPPEN WHEN APPLYING a^+ SUCCESSIVELY TO THE GROUND STATE $|0\rangle$

$a^+|0\rangle = |1\rangle$

$(a^+)^2|0\rangle = a^+|1\rangle = \sqrt{2}\,|2\rangle \Rightarrow |2\rangle = \dfrac{(a^+)^2|0\rangle}{\sqrt{2}}$

$(a^+)^3|0\rangle = \sqrt{2}\,a^+|2\rangle = \sqrt{6}\,|3\rangle \Rightarrow |3\rangle = \dfrac{(a^+)^3|0\rangle}{\sqrt{6}}$

$\vdots \qquad\qquad \vdots$

$$|n\rangle = \frac{(a^+)^n}{\sqrt{n!}}|0\rangle$$

– IN THIS WAY WE HAVE SUCCEEDED IN CONSTRUCTING SIMULTANEOUS EIGENKETS OF $N = a^+a$, AND H WITH EIGENVALUES

$$E_n = (n + 1/2)\hbar\omega, \quad n = 0, 1, 2, 3, \dots$$

FROM $\begin{cases} a|n\rangle = \sqrt{n}\,|n-1\rangle \\ a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle \end{cases}$, AND THE ORTHONORMALITY CONDITION FOR $\{|n\rangle\}$ WE OBTAIN THE FOLLOWING MATRIX ELEMENTS

- $\langle n'|a|n\rangle = \sqrt{n}\,\langle n'|n-1\rangle = \sqrt{n}\,\delta_{n',n-1}$
- $\langle n'|a^+|n\rangle = \sqrt{n+1}\,\langle n'|n+1\rangle = \sqrt{n+1}\,\delta_{n',n+1}$

Obs.: $\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$

~USING THIS TOGETHER WITH THE DEFINITION ...

– USING THIS TOGHETER WITH THE DEFINITION OF a AND $a^\dagger$

$$a = \frac{1}{\sqrt{2m\hbar\omega}}(m\omega x + ip)$$

$$a^\dagger = \frac{1}{\sqrt{2m\hbar\omega}}(m\omega x - ip)$$

$$\bullet\ a + a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\, 2x$$

$$\bullet\ a^\dagger - a = \frac{-2ip}{\sqrt{2m\hbar\omega}}$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^\dagger)\ ; \quad p = i\sqrt{\frac{m\hbar\omega}{2}}\,(a^\dagger - a)$$

– WE DEFINE MATRIX ELEMENTS OF x AND p OPERATORS

$$\bullet\ \langle n'|x|n\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n}\,\delta_{n',n-1} + \sqrt{n+1}\,\delta_{n',n+1}\right)$$

$$\bullet\ \langle n'|p|n\rangle = i\sqrt{\frac{m\hbar\omega}{2}}\left(\sqrt{n+1}\,\delta_{n',n+1} - \sqrt{n}\,\delta_{n',n-1}\right)$$

– NOTICE THAT NEITHER x AND p IS DIAGONAL IN THE N–REPRESENTATION WE ARE USING, BECAUSE x AND p DO NOT COMMUTE WITH N

– WE CAN ALSO USE THIS TO COMPUTE THE ENERGY EIGENFUNCTIONS IN THE POSITION SPACE

– TO DO THIS, LET'S START WITH THE GROUND STATE

$$a|0\rangle = 0$$

IN TERMS OF x– REPRESENTATION

$$\langle x'|a|0\rangle = \langle x'|\frac{1}{\sqrt{2m\hbar\omega}}(m\omega x + ip)|0\rangle = 0$$

$$p = \frac{\hbar}{i}\frac{d}{dx}$$

$\sqrt{2m\hbar\omega}$ $\qquad p = \frac{\hbar}{i}\frac{d}{dx}$

$$\langle x'|a|0\rangle = \frac{1}{\cancel{\sqrt{2m\hbar\omega}}}\left(m\omega\langle x'|x|0\rangle + \hbar\langle x'|\frac{d}{dx}|0\rangle\right) = 0$$

$$= \left(m\omega x' + \hbar\frac{d}{dx'}\right)\underbrace{\langle x'|0\rangle}_{\psi_0(x')} = 0$$

$$\left(x' + x_0^2\frac{d}{dx'}\right)\psi_0(x') = 0 \quad , \quad x_0 = \sqrt{\frac{\hbar}{m\omega}}$$

$$x_0^2\frac{d}{dx'}\psi_0(x') = -x'\psi_0(x') \Rightarrow \int\frac{d\psi_0(x')}{\psi_0(x')} = -x_0^{-2}\int x'dx'$$

$$\psi_0(x') = c\, e^{-x'^2/2x_0^2}$$

$$\int|\psi_0|^2 dx' = 1 \Rightarrow c = \frac{\sqrt{2}}{\pi^{1/4}x_0^{1/2}}$$

$$\psi_0(x') = \frac{1}{\pi^{1/4}\sqrt{x_0}}\, e^{-\frac{x'^2}{2x_0^2}} \quad , \text{ GROUND STATE}$$

– WE CAN OBTAIN THE ENERGY EIGENFUNCTIONS FOR THE EXCITED STATES

• $\langle x'|1\rangle = \psi_1(x') = \langle x'|a^+|0\rangle = \frac{1}{\sqrt{2m\hbar\omega}}\langle x'|\left(m\omega x - ip\right)|0\rangle$

$$\psi_1(x') = \frac{1}{\sqrt{2}\, x_0}\left(x' - x_0^2\frac{d}{dx'}\right)\psi_0(x') \quad , \quad \psi_0(x') = \langle x'|0\rangle$$

• $\langle x'|2\rangle = \psi_2(x') = \langle x'|(a^+)^2|0\rangle = \frac{1}{\ldots}\langle x'|(m\omega x - ip)^2|0\rangle$

$\bullet \langle x'|2\rangle = \psi_2(x') = \langle x'|(a^+)^2|0\rangle = \frac{1}{2\sqrt{2}\, m\hbar\omega} \langle x'|(m\omega x - ip)^2|0\rangle$

$\vdots$

$$\psi_2(x') = \frac{1}{\sqrt{2!}} \left(\frac{1}{\sqrt{2}\, x_0}\right)^2 \left(x' - x_0^2 \frac{d}{dx'}\right)^2 \psi_0(x')$$

IN GENERAL,

$$\psi_n(x') = \frac{1}{\pi^{1/4}\sqrt{2^n n!}} \left(\frac{1}{x_0^{n+1/2}}\right) \left(x' - x_0^2 \frac{d}{dx'}\right)^n e^{-1/2\left(\frac{x'}{x_0}\right)^2}$$

— YOU CAN CHECK THE FIRST FIVE SOLUTIONS FOR ψ_n IN PAGE 285 (NIELSEN-CHUANG)

— THESE WAVE-FUNCTIONS DESCRIBE THE PROBABILITY AMPLITUDE THAT A PARTICLE IN THE SHO WILL BE FOUND AT DIFFERENT POSITIONS WITHIN THE POTENTIAL

— IN GENERAL WE WILL BE MORE INTERESTED IN THE FOLLOWING PROPERTIES OF THE STATES

$$[H, a^+] = \left[\hbar\omega\left(a^+a + 1/2\right), a^+\right] = \hbar\omega\left[a^+a, a^+\right] + \frac{\hbar\omega}{2}\underbrace{\left[1, a^+\right]}_{=0}$$

$$= \hbar\omega\left(a^+\underbrace{[a, a^+]}_{=1} + \underbrace{[a^+, a^+]}_{=0} a\right) = \hbar\omega a^+$$

APLYING THIS TO $|\psi\rangle$

$$H a^+|\psi\rangle = \left([H, a^+] + a^+H\right)|\psi\rangle = \left(\hbar\omega a^+ + a^+H\right)|\psi\rangle$$

REMEMBER: $H|\psi\rangle = E|\psi\rangle$

$$\underbrace{H a^\dagger |\psi\rangle} = (\hbar\omega + E)\underbrace{a^\dagger|\psi\rangle}$$

$a^\dagger|\psi\rangle \rightarrow$ is an eigenstate of H, with energy

$E + \hbar\omega$

- Similarly, $a|\psi\rangle$ is an eigestate with energy
$E - \hbar\omega$ (homework)

- It follows that $(a^\dagger)^n |\psi\rangle$ are eigenstates for any n with energies: $E + n\hbar\omega$ (homework)

where, $E = \frac{1}{2}\hbar\omega$, is the ground state energy

$$\boxed{E_n = E + n\hbar\omega}$$

QUANTUM COMPUTATION

- Suppose we want to perform computation with the SHO described before

What can be done?

Qubits: $|0\rangle, |1\rangle, |2\rangle, \ldots$

- A natural choice for qubits representation are the energy eigenstates $|n\rangle$

- Let's perform, for instance, a CNOT-gate in the following way:

Recall that a CNOT-gate performs the mapping

— RECALL THAT A CNOT-GATE PERFORMS THE MAPPING

control → target

- $CNOT\,|00\rangle_L \rightarrow |00\rangle_L$
- $CNOT\,|01\rangle_L \rightarrow |01\rangle_L$
- $CNOT\,|10\rangle_L \rightarrow |11\rangle_L$
- $CNOT\,|11\rangle_L \rightarrow |10\rangle_L$

, $|\ \rangle_L \rightarrow$ LOGICAL STATES

$|\ \rangle \rightarrow$ SHO STATES

— SO, FOR THE SHO QUANTUM COMPUTER MODEL WE NEED SIMILAR LOGIC TO REPRESENT A CNOT-GATE

— SUPPOSE THAT AT $t=0$, THE SYSTEM IS STARTED IN STATE SPANNED BY THE SHO BASIS STATES ($|n\rangle$)

— AND THEN WE EVOLVE THE SYSTEM FORWARD IN TIME $t = \pi/\omega$

— FROM THE SCHRÖDINGER EQUATION TIME EVOLUTION OF THE EIGENSTATES IS GIVEN BY

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle, \qquad |\psi(t)\rangle = e^{-iHt/\hbar}$$

$$|\psi(t,x)\rangle = e^{-iHt/\hbar}|\psi(0,x)\rangle$$

$$|\psi(0)\rangle = \sum_n c_n(0)\,|n\rangle, \quad \text{BUT}$$

$$|\psi(t)\rangle = \underbrace{e^{-iHt/\hbar}}_{T}|\psi(0)\rangle$$

$T \rightarrow$ TIME EVOLUTION OPERATOR (UNITARY)

$$|\psi(t)\rangle = \sum_n C_n\, e^{-iHt/\hbar}|n\rangle, \quad \text{AND} \quad H = \hbar\omega(aa^\dagger + \tfrac{1}{2})$$

$$|\psi(t)\rangle = \sum_n C_n\, e^{-i\cancel{\hbar}\omega(aa^\dagger+1/2)t/\cancel{\hbar}}|n\rangle$$

$$= \sum_n C_n\, \underbrace{e^{-i\omega t/2}}_{}\, e^{-ia^\dagger a\omega t}|n\rangle$$

$$\boxed{|\psi(t)\rangle = \sum_n C_n(t)\, \underbrace{e^{-ia^\dagger a\omega t}}_{}\, |n\rangle}$$

– SO, WHAT IS IMPORTANT FOR US IS THAT THE SHO ENERGY EIGENSTATES UNDERGO TRANSFORMATIONS OF THE TYPE

$$|n\rangle \longrightarrow e^{-i\omega t a^\dagger a}|n\rangle$$

– SO, THE SYSTEM EVOLVE FORWARD IN TIME BY $t = \pi/\omega$

$$|n\rangle \longrightarrow e^{-ia^\dagger a\pi}|n\rangle$$

– USING THE DEFINITION OF THE EXPONENTIAL OPERATOR $\left(e^{\hat{A}} = \sum_{n=0}^{\infty} \frac{\hat{A}^n}{n!} = \hat{1} + \hat{A} + \frac{\hat{A}^2}{2!} + \cdots \frac{\hat{A}^n}{n!}\right.$

$$|n\rangle \longrightarrow (1 - i\pi\underbrace{a^\dagger a}_{N} + \cdots)|n\rangle = (1 - i\pi n + \cdots)|n\rangle$$

$$|n\rangle \longrightarrow e^{-i\pi n}|n\rangle = (\underbrace{\cos n\pi}_{\pm 1} - i\underbrace{\sin n\pi}_{=0})|n\rangle$$

$$|n\rangle \rightarrow (-1)^n |n\rangle$$

THE SHO AS A QUANTUM COMPUTER

$|0\rangle \rightarrow |0\rangle$; $|2\rangle \rightarrow |2\rangle$; $|4\rangle \rightarrow |4\rangle$

$|1\rangle \rightarrow -|1\rangle$; $|3\rangle \rightarrow -|3\rangle$ ⋮

So, THE CNOT-GATE FOR THE SHO COULD BE REPRESENTED BY

i) CNOT $|00\rangle_L \rightarrow |0\rangle$, SINCE $|0\rangle$ NEVER CHANGES

ii) CNOT $|01\rangle_L \rightarrow |2\rangle$, SINCE $|2\rangle$ " "

iii) CNOT $|10\rangle_L \rightarrow (|4\rangle + |1\rangle)/\sqrt{2}$

iv) CNOT $|11\rangle_L \rightarrow (|4\rangle - |1\rangle)/\sqrt{2}$

BECAUSE $|4\rangle$ NEVER CHANGES THE EIGENVALUES AND $|1\rangle \rightarrow -|1\rangle$ AFTER $t = \pi/\omega$

— THE RESULT OF THESE STATES (i) – iv)) IS THE DESIRED CNOT-GATE TRANSFORMATION

— IN GENERAL, A NECESSARY CONDITION FOR A PHYSICAL SYSTEM TO BE ABLE TO PERFORM UNITARY TRANSFORM IS SIMPLY THAT TIME EVOLUTION OPERATOR FOR THE SYSTEM : $T = e^{-iHt}$

— IN THE CASE ABOVE, THE CNOT GATES WAS SIMPLE TO IMPLEMENT BECAUSE IT ONLY HAS EIGENVALUES

SUMMARY FOR THE SHO QUANTUM COMPUTER

- QUBIT REPRESENTATION
  → ENERGY LEVELS $|0\rangle, |1\rangle, \dots$
- UNITARY EVOLUTION
  → ARBITRARY TRANSFORMS OF T REALIZED BY MATCHING THEIR EIGENVALUES SPECTRUMS TO THAT ~~[illegible]~~ GIVEN BY THE HAMILTONIAN: $H = a^\dagger a = N$
- INITIAL STATE PREPARATION
  → NOT CONSIDERED
- READOUT
  → NOT CONSIDERED

---- " ----

$$\left.\begin{aligned} \mathrm{CNOT}\,|10\rangle_L &= |11\rangle_L \\ \mathrm{CNOT}\,|11\rangle_L &= |10\rangle_L \end{aligned}\right\} \text{COMP. BASES}$$

$|0\rangle$

$$\left.\begin{aligned} \underset{\sim}{T}\,(|4\rangle + |1\rangle)/\sqrt{2} &\xrightarrow{\pi/4} (|4\rangle - |1\rangle)/\sqrt{2} \\ \underset{\sim}{T}\,(|4\rangle - |1\rangle)/\sqrt{2} &\longrightarrow (|4\rangle + |1\rangle)/\sqrt{2} \end{aligned}\right\} \text{SHO}$$

$$\begin{cases} |00\rangle_L \longrightarrow |0\rangle \\ |01\rangle_L \longrightarrow |2\rangle \end{cases}$$