Lecture16_QHardware_class11

quinta-feira, 22 de fevereiro de 2024 19:16

Lecture16...

Quantum Hardware – Optical Models

Class XI

Carlos Bessa *

February 23, 2024

* Quantum Formalism, Zaiku Group Ltd.

Based on the book, Quantum Computation and Quantum Information, by M. Nielsen and I. Chuang

Optical Photon Quantum Computer – Non-Linear Kerr Media

GLOSSARY

Ψ) Phase-Shifter

1)

$$P|0\rangle = |0\rangle$$

2)

$$P(\theta)|1\rangle = e^{i\theta}|1\rangle \Rightarrow P(\theta=\pi)|1 = -|1\rangle$$

,

Ψ) Beamsplitter (50/50)

1)

$$B|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

2)

$$B|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

GLOSSARY

Ψ) Kerr

1)

$$K|00\rangle = |00\rangle$$

2)

$$K|01\rangle = |01\rangle$$

3)

$$K|10\rangle = |10\rangle$$

4)

$$K|11\rangle = e^{i\xi}|11\rangle \Rightarrow K(\xi=\pi)|11\rangle = -|11\rangle$$

Optical Photon Quantum Computer – Non-Linear Kerr Media

GLOSSARY

Ψ) Fredkin (when $\xi = \pi$)

1) $$F|0ba\rangle = |0ba\rangle \rightarrow a, b \in \{0,1\}$$

2) $$F|100\rangle = |100\rangle$$

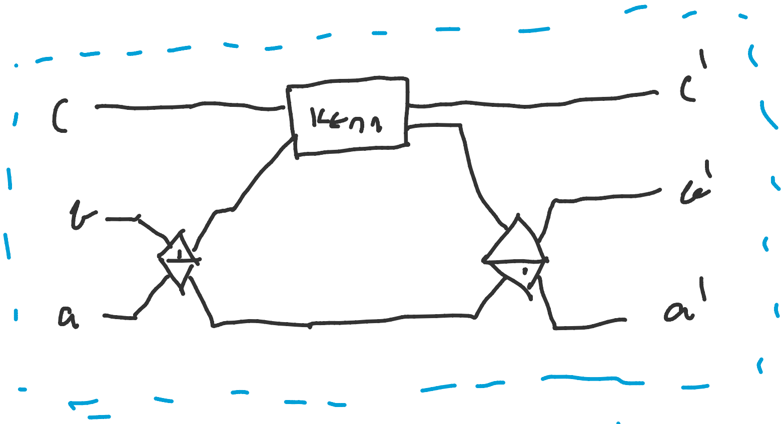
3) $$F|101\rangle = -|110\rangle$$

4) $$F|110\rangle = |101\rangle$$

5) $$F|111\rangle = |111\rangle$$

3

BOX 7.4: THE QUANTUM FREDKIN GATE (CONTINUATION)



$$U = B K B^\dagger$$

$$U = e^{i\xi c^\dagger c \frac{(b^\dagger - a^\dagger)(b-a)}{2}} \qquad (7.47)$$

$$U = e^{i\frac{\pi}{2} b^\dagger b}\, e^{\frac{\xi}{2} c^\dagger c (a^\dagger b - b^\dagger a)}\, e^{-i\frac{\pi}{2} b^\dagger b}\, e^{i\frac{\xi}{2} a^\dagger a c^\dagger c}\, e^{i\frac{\xi}{2} b^\dagger b c^\dagger c} \qquad (7.48)$$

$e^{i\frac{\pi}{2} b^\dagger b}$, $e^{-i\pi/2\, b^\dagger b}$ > PHASE-SHIFTERS IN b MODE

REMEMBER! $P|1\rangle = e^{i\delta}|1\rangle$, $P|0\rangle = |0\rangle$

- $$e^{\pm i\pi/2\, b^\dagger b}|1\rangle = \left(1 \pm i\frac{\pi}{2} b^\dagger b + \dots\right)|1\rangle = |1\rangle \pm i\frac{\pi}{2} b^\dagger b|1\rangle + \dots$$
$$= \left(1 \pm i\frac{\pi}{2} + \dots\right)|1\rangle = e^{\pm i\frac{\pi}{2}}|1\rangle = \pm i|1\rangle$$

- $$e^{\pm \pi/2\, b^\dagger b}|0\rangle = \left(1 + \pm \frac{\pi}{2} b^\dagger b + \dots\right)|0\rangle = |0\rangle$$

– THE LAST TWO TERMS ARE PHASE SHIFTERS THAT COME FROM THE CROSS PHASE MODULATION

- $e^{i\xi/2\, a^+a c^+c}|10\rangle_{ca} = |10\rangle_{ca}$
- $e^{i\frac{\xi}{2}a^+a c^+c}|01\rangle_{ca} = |01\rangle$
- $e^{i\frac{\xi}{2}a^+a c^+c}|00\rangle_{ca} = |00\rangle_{ca}$
- $e^{i\xi/2\, a^+a c^+c}|11\rangle_{ca} = e^{i\xi/2}|11\rangle_{ca}$

– THE SAME FOR MODE b $\quad e^{i\xi/2\, b^+b c^+c}|10\rangle_{cb} = |10\rangle$

$\vdots$

– ALL THESE EFFECTS ARE NOT FUNDAMENTAL AND CAN BE COMPENSATE

– THE MOST IMPORTANT TERM IS THE SECOND IN EQ. (7.48), WHICH DEFINES THE QUANTUM FREDKIN OPERATOR

$$F(\xi) = e^{\frac{\xi}{2}c^+c\,(a^+b - b^+a)} \qquad (7.49)$$

THE USUAL (CLASSICAL) FREDKIN GATE IS RECOVER WHEN $\xi = \pi$

– IN WHICH CASE WHEN NO PHOTON ARE INPUT AT c, THEN $a' = a$, $b' = b$

$$\rightarrow F|010\rangle_{cba} = e^{\theta c^+c\,(a^+b - b^+a)}|010\rangle_{cba}, \quad \theta = \frac{\xi}{2}$$

$$= [1 + \theta(a^+b - b^+a)c^+c + \dots]|010\rangle_{cba}$$

$$= |010\rangle + 0$$

$[c, a] = [c, b] = 0$

$\rightarrow F|0ba\rangle = |0ba\rangle$

– NOW CONSIDER THAT A SINGLE PHOTON INPUT AT c, THEN

$a' = b, \quad b' = a$

$$\rightarrow F|110\rangle_{cba} = e^{\theta c^\dagger c (a^\dagger b - b^\dagger a)} |110\rangle$$

$$= \Big[1 + \theta (a^\dagger b - b^\dagger a) c^\dagger c + \frac{\theta^2}{2!} (a^\dagger b - b^\dagger a)^2 (c^\dagger c)^2$$

$$+ \frac{\theta^3}{3!} (a^\dagger b - b^\dagger a)^3 (c^\dagger c)^3 + \frac{\theta^4}{4!} (a^\dagger b - b^\dagger a)^4 (c^\dagger c)^4 + \cdots \Big] |110\rangle$$

- $\theta (a^\dagger b - b^\dagger a) c^\dagger c |110\rangle_{cba} = \theta (a^\dagger b - b^\dagger a) |110\rangle_{cba} = \theta |101\rangle$

- $\frac{\theta^2}{2!} (a^\dagger b - b^\dagger a)(a^\dagger b - b^\dagger a) c^\dagger c c^\dagger c |110\rangle_{cba} = \frac{\theta^2}{2!} (a^\dagger b - b^\dagger a)(a^\dagger b - b^\dagger a) |110\rangle$

$$= \frac{\theta^2}{2!} (a^\dagger b - b^\dagger a) |101\rangle = -\frac{\theta^2}{2!} |110\rangle$$

- $\frac{\theta^3}{3!} (a^\dagger b - b^\dagger a)(a^\dagger b - b^\dagger a)^2 (c^\dagger c)^3 |110\rangle = -\frac{\theta^3}{3!} |101\rangle$

- $\frac{\theta^4}{4!} = \frac{\theta^4}{4!} |110\rangle$

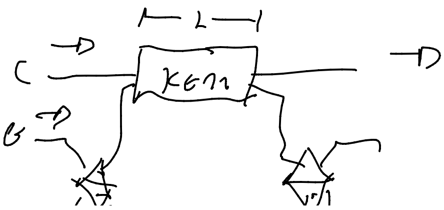
$$F|110\rangle = \Big[|110\rangle + \theta|101\rangle - \frac{\theta^2}{2!}|110\rangle - \frac{\theta^3}{3!}|101\rangle + \frac{\theta^4}{4!}|110\rangle + \cdots \Big]$$

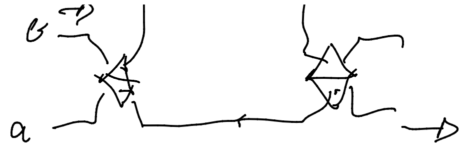
$$= |110\rangle \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots\right) + |101\rangle \left(\theta - \frac{\theta^3}{3!} + \cdots\right)$$

$F|110\rangle = \cos\theta \, |110\rangle + \sin\theta \, |101\rangle \quad , \quad \theta = \xi/2$

$F|110\rangle = \cos\frac{\xi}{2} |110\rangle + \sin\frac{\xi}{2} |101\rangle$, WHEN $\xi = \pi$

$$\boxed{F(\pi)|110\rangle = |101\rangle}$$



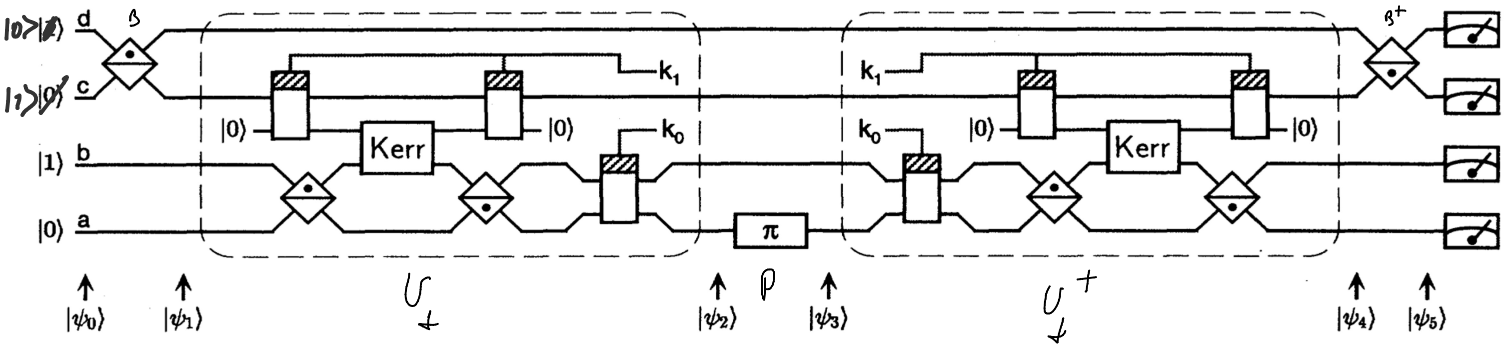


— you can check that

$$F\,|101\rangle_{cba} = \cos\frac{\xi}{2}\,|101\rangle_{cba} - \sin\frac{\xi}{2}\,|110\rangle_{cba}\ ,\quad \text{when } \xi = \pi$$

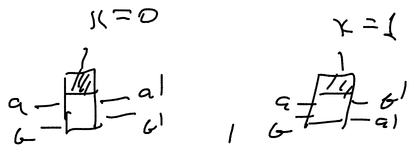
$$F\,|101\rangle_{cba} = -\,|110\rangle_{cba} \qquad\qquad \xi = \chi L$$

Exercise 7.13



$$|dcba\rangle = |0\,1\,1\,0\rangle$$

$|0\rangle_L$ $|1\rangle_L$



i) $k_1 = k_0 = 0$

$$H|0\rangle_L = \frac{1}{\sqrt{2}}\left(|0\rangle_L + |1\rangle_L\right)$$

$$H|1\rangle_L = \frac{1}{\sqrt{2}}\left(|0\rangle_L - |1\rangle_L\right)$$

- $|\psi_0\rangle = |0110\rangle_{dcba}$
- $|\psi_1\rangle = B_{dc}\,|\psi_0\rangle = \dfrac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)_{dc} \otimes |10\rangle_{ba}$
- $|\psi_2\rangle = U_{f_{00}}\,|\psi_1\rangle$, since $k_1 = k_0 = 0$, switch 0 and switch 1 are off, so $U_{f_{00}} = I$

$|\psi_2\rangle = |\psi_1\rangle$

- $|\psi_3\rangle = P_a |\psi_2\rangle$, BUT $P|0\rangle = |0\rangle$
  $P|1\rangle = -|1\rangle$

$|\psi_3\rangle = |\psi_2\rangle$

- $|\psi_4\rangle = U_f^{\dagger} |\psi_3\rangle = |\psi_3\rangle$
- $|\psi_5\rangle = B_{dc}^{\dagger} |\psi_4\rangle = B_{dc}^{\dagger} |\psi_3\rangle = B_{dc}^{\dagger} \frac{1}{\sqrt{2}} \left(|01\rangle + |10\rangle\right)_{dc} \otimes |10\rangle_{ba}$

$|\psi_5\rangle = |01\rangle_{dc} \otimes |10\rangle_{ba} = |01\,10\rangle_{dc}$

$\begin{array}{cc} \vee & \vee \\ 0_L & 1_L \end{array}$

f_{00} = CONSTANTE

- ii) $k_1 = 0$, $k_0 = 1$

  - $|\psi_0\rangle = |01\,10\rangle_{dcba}$
  - $|\psi_1\rangle = B_{dc} |\psi_0\rangle = \frac{1}{\sqrt{2}} \left(|01\rangle + |10\rangle\right)_{dc} \otimes |10\rangle_{ba}$
  - $|\psi_2\rangle = U_{f_{01}} |\psi_1\rangle$, SWITCH 1 is OFF
    SWITCH 0 is ON $\Rightarrow$ $a \xrightarrow{\sigma} b$

$|\psi_2\rangle = \frac{1}{\sqrt{2}} \left(|01\rangle + |10\rangle\right)_{dc} \otimes |01\rangle_{ba}$

- $|\psi_3\rangle = P_a |\psi_2\rangle = -\frac{1}{\sqrt{2}} \left(|01\rangle + |10\rangle\right)_{dc} \otimes |01\rangle_{ba}$
- $|\psi_4\rangle = U_{f_{01}}^{\dagger} |\psi_3\rangle$, SWITCH 1 is OFF
  SWITCH 0 is ON $a \rightarrow b$

• $|\Psi_4\rangle = U_{t_{01}} |\Psi_3\rangle$, SWITCH 1 is OFF
SWITCH 0 is ON $a \to b$

$$|\Psi_4\rangle = -\frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)_{dc} \otimes |10\rangle_{ba}$$

• $|\Psi_5\rangle = B^{\dagger}_{dc} |\Psi_4\rangle = -|\underset{0_L}{01}\underset{1_L}{10}\rangle$, t_{01} is CONSPANTE

iii) $K_1 = 1$, $K_0 = 0$

• $|\Psi_0\rangle = |0110\rangle$
• $|\Psi_1\rangle = B_{dc} |\Psi_0\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)_{dc} \otimes |10\rangle_{ba}$

• $|\Psi_2\rangle = U_{t_{10}} |\Psi_1\rangle$, SWITCH 1 is ON
SWITCH 0 is OFF

$\zeta = \pi$

$$\begin{cases} F\,|110\rangle_{cba} = |101\rangle_{cba} \\ F\,|101\rangle_{cba} = -|110\rangle_{cba} \end{cases}$$

$F|0ba\rangle = |0ba\rangle$

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}|0101\rangle_{dcba} + \frac{1}{\sqrt{2}}|1010\rangle_{dcba}$$

• $|\Psi_3\rangle = P_a |\Psi_2\rangle$ X

$$= +\frac{1}{\sqrt{2}}|0101\rangle_{dcba} + \frac{1}{\sqrt{2}}|1010\rangle_{dcba}$$

• $|\Psi_4\rangle = U^{\dagger}_{t_{10}} |\Psi_3\rangle = +\frac{1}{\sqrt{2}}|0\rangle_d \otimes \underbrace{F_{cba}|101\rangle}_{-|110\rangle} + \frac{1}{\sqrt{2}}|1\rangle_d \otimes \underbrace{F_{cba}|010\rangle_{cba}}_{|010\rangle}$

$$= -\frac{1}{\sqrt{2}}|0110\rangle + \frac{1}{\sqrt{2}}|1010\rangle = \frac{1}{\sqrt{2}}\Big(-\underbrace{|01\rangle}_{0_L} + \underbrace{|10\rangle}_{1_L}\Big)_{dL} \otimes |10\rangle_{Ga}$$

- $|\psi_5\rangle = B^{\dagger}_{dL}|\psi_4\rangle = -|\underbrace{10}_{1_L}\underbrace{10}_{1_L}\rangle$, f_{10} = BALANCED

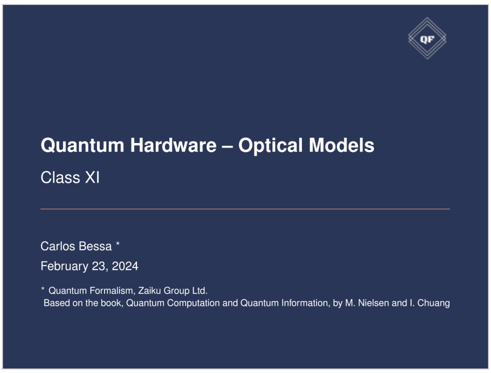
iv) $K_1 = 1 = K_2 = 1$ (WITHOUT THE PHASE-SHIFTER)

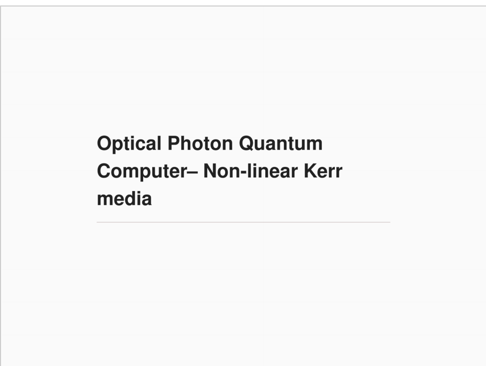
$$|\psi_5\rangle = -|\underbrace{10}_{1_L}10\rangle_{dLGa} \quad , \quad f_{11} \rightarrow \text{BALANCED}$$

obs.: TO HAVE A COMPLETE EXPLANATION WHY THE P OPERATOR SHOULD NOT BE APPLIED SEE THE VIDEO REFERENT TO THE CURRENT LECTURE

Beamer (29)

→ BELOW ARE THE DRAWBACKS





Drawbacks

- The single photon representation of a qubit is attractive. Single photons are relatively simple to generate and measure, and in the dual-rail representation, arbitrary single qubit operations are possible
- However, interacting photons is difficult
- The best non-linear Kerr media available are very weak, and may not provide a good cross phase modulation of π between single photons states
- Nevertheless, from studying this optical quantum computer, we have gained some valuable insight into the nature of architecture and system design of a QC
- We now can see what an actual QC, constructed nearly completely from optical interferometers, might look like in the laboratory
- However, it would be challenge to stabilize this type of devices when dealing with large quantum algorithms

- Despite these drawbacks, the theoretical formalism which describes these optical quantum computers is fundamental in all the other realizations we will study in the future
- In fact, the optical cavity QED computers is just another kind of optical quantum computer, but with a different (and better) kind of nonlinear medium