Lecture12_QHardware_class7

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Lecture12...

Quantum Hardware – Optical Models

Class VII

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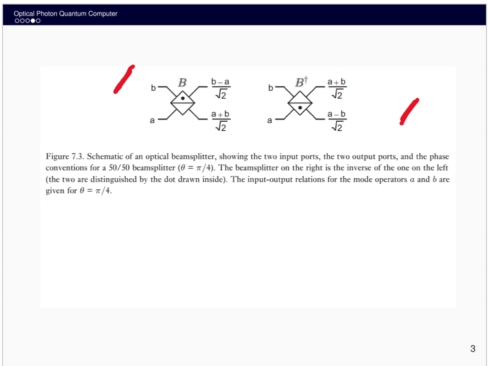
Based on the book, Quantum Computation and Quantum Information, by M. Nielsen and I. Chuang

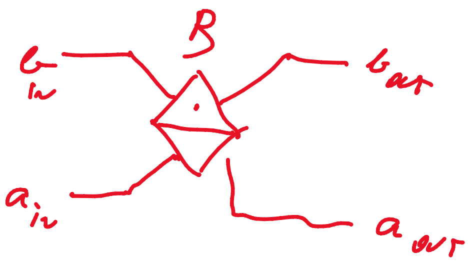
Optical Photon Quantum Computer

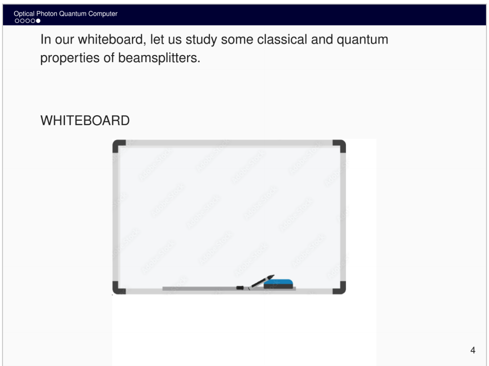
In the last lecture, we discussed that the most experimentally accessible devices for manipulating photon states using optical instruments are: Mirrors, Phase shifters, Beamsplitters, etc. Today, we will focus on investigating the quantum properties of beamspliters.

- Ψ) Compared with other fundamental experiments in physics, optical tests of QM are often distinguished by their simplicity. Most of quantum optical experiments do not require a whole industry
- Ψ) A simple optical beamsplitter is a device that demonstrates the quantum nature of light
- Ψ) Quite several puzzling quantum effects have been seen by splitting or recombining photons at a small cube of glass
- Ψ) Additionally, the beamsplitter serves as a theoretical paradigm for other linear optical devices
- Ψ) Interferometers, semi-transparent mirrors, dielectric interfaces, waveguide couplers, and polarizers are all described sufficiently well by a simple beamsplitter model

- Ψ) An ideal beamsplitter is a reversible, lossless device in which two incident beams may interfere to produce two outgoing beams that reflect a fraction R of incident light and transmit $T = 1 - R$
- Ψ) They are partially silvered pieces of glass that split a light beam into two
  - Ψ This situation may be reversed by sending the two beams back to the glass, where they interfere constructively to restore the original beam
- Ψ) However, if we change the phase of the two beams, their mutual interference generates two outgoing beams in general
- Ψ) So four beams might involved, two incident and two outgoing light modes that can be represented schematically in Fig. 7.3







BEAM SPLITERS

– WHAT HAPPEN IF TWO COHERENT LIGHT BEAMS WITH COMPLEX AMPLITUDES a_{in} AND b_{in} INTERFERE?

– IN CLASSICAL OPTICS THE AMPLITUDES ARE SIMPLY SUPERIMPOSED ACCORDING TO THE LINEAR TRANSFORMATION

$$\begin{pmatrix} a_{OUT} \\ b_{OUT} \end{pmatrix} = B \begin{pmatrix} a_{in} \\ b_{in} \end{pmatrix} \quad , \quad E \sim a\, e^{i\vec{k}\cdot\vec{r} - i\omega t}$$

B IS DESCRIBED BY THE MATRIX: $B = \begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix}$

$$B_{clas} = \begin{pmatrix} T & R \\ R & T \end{pmatrix} \sim \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

THIS IS PHENOMENOLOGICALLY CONFIRMED!

– IN QUANTUM OPTICS THE COMPLEX AMPLITUDES a_{in}, b_{in}

- IN QUANTUM OPTICS THE COMPLEX AMPLITUDES a_{in}, b_{in} CORRESPOND TO THE ANNIHILATION OPERATORS $\hat{a}_{in}, \hat{b}_{in}$ OF THE INCIDENT FIELDS $(\hat{E}_{in} \sim \hat{a}_{in})$

- THE OUTGOING BEAMS ARE CHARACTERIZED BY THE OPERATORS $\hat{a}_{out}, \hat{b}_{out}$

- WE ASSUME THAT THE LINEAR RELATION GIVEN ABOVE IS ALSO VALID TO THE QUANTUM FIELDS

$$\begin{pmatrix} \hat{a}_{out} \\ \hat{b}_{out} \end{pmatrix} = B \begin{pmatrix} \hat{a}_{in} \\ \hat{b}_{in} \end{pmatrix}, \quad B = \begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix}$$

- IN THIS MODEL WE CAN DESCRIBE MOST OF PASSIVE AND LOSSLESS DEVICES IN WHICH TWO BEAMS INTERFERE TO PRODUCE OUTGOING FIELDS

- LET'S DETERMINE SOME PROPERTIES OF THE BEAMSPLITTER

- CONSIDER THAT INCOMING LIGHT BEAMS AND OUTGOING BEAMS ARE BOTH <u>INDEPENDENT</u> BOSONIC MODES

- WE ALREADY KNOW THE FOLLOWING RELATIONS:

$$[a_{out}, a^\dagger_{out}] = [a_{in}, a^\dagger_{in}] = [b_{out}, b^\dagger_{out}] = [b_{in}, b^\dagger_{in}] = 1$$

$$[a_{out}, b_{out}] = [a_{in}, b_{in}] = 0$$

- $[a_{out}, a^\dagger_{out}] = [a_{in} B_{00} + b_{in} B_{01}, a^\dagger_{in} B^*_{00} + b^\dagger_{in} B^*_{01}] = 1$

$$= [a_{in} B_{00}, a^\dagger_{in} B^*_{00}] + [a_{in} B_{00}, b^\dagger_{in} B^*_{01}] + [b_{in} B_{01}, a^\dagger_{in} B^*_{00}] +$$

$$+ [b_{in} B_{01}, b^\dagger_{in} B^*_{01}] = 1$$

$$= |B_{00}|^2 \underbrace{[a_{in}, a^\dagger_{in}]}_{=1} + B_{00} B^*_{01} \underbrace{[a_{in}, b^\dagger_{in}]}_{=0} + B_{01} B^*_{00} \underbrace{[b_{in}, a^\dagger_{in}]}_{=0}$$

$$= |B_{00}|^2 \overset{=1}{[a_{in}, a^\dagger_{in}]} + B_{00}B_{01}^* \overset{=0}{[a_{in}, b^\dagger_{in}]} + B_{01}B_{00}^* \overset{=0}{[b_{in}, a^\dagger_{in}]}$$

$$+ |B_{01}|^2 \overset{=1}{[b_{in}, b^\dagger_{in}]} = 1$$

$$\Rightarrow |B_{00}|^2 + |B_{01}|^2 = 1$$

THE SAME FOR

- $[b_{OUT}, b^\dagger_{OUT}] = |B_{10}|^2 + |B_{11}|^2 = 1$
- $[a_{OUT}, b^\dagger_{OUT}] = 0 \Rightarrow B_{00}B_{10}^* + B_{01}B_{11}^* = 0$

IN OTHER WORDS, B IS UNITARY

$$B^{-1} = B^\dagger \qquad \text{(HOMEWORK)}$$

– APART FROM THE UNITARITY, THE BEAM SPLITTER COEFFICIENTS ARE FREE PARAMETERS THAT DEPEND ON THE PARTICULAR EXPERIMENTAL SITUATION

– FOR OUR PURPOSE AND APPLICATIONS, WE ARE INTERESTED ON REAL ROTATIONS MATRIZES

– IN THIS CASE, WE COULD REPRESENT B IN TERMS OF THE TRANSMISSIVITY T AND THE REFLECTIVITY R AS

$$B = \begin{pmatrix} T & R \\ -R & T \end{pmatrix}, \quad B^\dagger = B^{-1} = \begin{pmatrix} T & -R \\ R & T \end{pmatrix}$$

AND THE RELATION $T^2 + R^2 = 1$ (HOMEWORK)

ACCOUNTS FOR ENERGY CONSERVATION OF LOSSLESS PORTS

– THE SIMPLEST CHOICE TO SATISFY THESE CONDITIONS IS

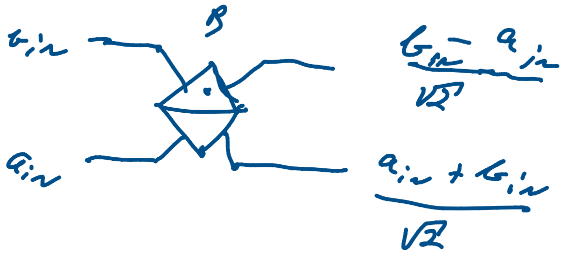
$T = \cos\theta$, $R = \sin\theta$

THEN,

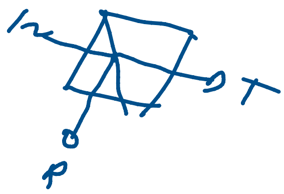
$$B = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad , \quad B^{\dagger} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

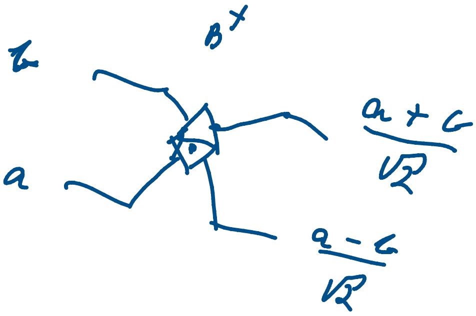
$$\begin{pmatrix} a_{out} \\ b_{out} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in} \end{pmatrix}$$

$$\begin{cases} a_{out} = a_{in}\cos\theta + b_{in}\sin\theta & (7.17) \\ \\ b_{out} = -a_{in}\sin\theta + b_{in}\cos\theta & (7.18) \end{cases}$$



, $\theta = \pi/4$





– WE COULD FIND THE SAME RESULT USING THE FOLLOWING HAMILTONIAN

$$H_{BS} = \theta\left(a b^{\dagger} - a^{\dagger} b\right)$$

AND THE BEAM SPITTER PERFORMS THE UNITARY OPERATION

$$B = e^{H} = e^{\theta(a b^{\dagger} - a^{\dagger} b)}$$

– THE TRANSFORMATIONS EFFECTED BY B ON a AND b WHICH WILL BE LATTER USEFUL ME

$$\begin{cases} B a B^\dagger = a\cos\theta + b\sin\theta \\ B b B^\dagger = -a\sin\theta + b\cos\theta \end{cases} \qquad (7.26)$$

TO VERIFY THESE RELATIONS WE USE THE BAKER–CAMPBELL–HAUSDORF FORMULA

EXERCISE 4.49 : (HOMEWORK)

$$e^{\lambda G} A e^{-\lambda G} = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} C_n \qquad (7.27)$$

$\lambda \to$ COMPLEX NUMBER

$A, G, C_n \to$ OPERATORS

C_n IS DEFINED RECURSIVELY AS

- $C_0 = A$, $C_1 = [G, C_0]$, $C_2 = [G, C_1]$, $C_3 = [G, C_2]$ ...

$C_n = [G, C_{n-1}]$ (I)

→ SINCE WE ALREADY PROVE THAT $[a, a^\dagger] = 1 = [b, b^\dagger]$

ERRATA (NIELSEN–CHUANG) $\boxed{G = a b^\dagger - a^\dagger b}$

$$[G, a] = [ab^\dagger - a^\dagger b, a] = [ab^\dagger, a] - [a^\dagger b, a] = a\underbrace{[b^\dagger, a]}_{=0} + \underbrace{[a, a]}_{=0} b^\dagger - a^\dagger [b, a] - \underbrace{[a^\dagger, a]}_{=-1} b$$

- $[G, a] = b$ (II)
- $[G, b] = -a$ (III)

$$C_{n,\text{EVEN}} = i^n a \quad , \quad C_{n,\text{ODD}} = -i^{n+1} b$$

TO SEE THIS USE EQUATIONS (I), (II), (III)

SO $\quad \theta\overbrace{(a b^\dagger - a^\dagger b)}^{G} \quad -\theta\overbrace{(a b^\dagger - a^\dagger b)}^{G} \quad \theta G \quad -\theta G$

So, $\mathcal{B} a \mathcal{B}^{\dagger} = e^{\theta \overbrace{(a b^{\dagger} - a^{\dagger} b)}^{G}} a\, e^{-\theta \overbrace{(a b^{\dagger} - a^{\dagger} b)}^{G}} = e^{\theta G} a\, e^{-\theta G}$

Using BCH-formula in the form of Eq. (7.27)

$$\mathcal{B} a \mathcal{B}^{\dagger} = e^{\theta G} a\, e^{-\theta G} = \sum_{n=0}^{\infty} \frac{\theta^n}{n!} c_n$$

$$\mathcal{B} a \mathcal{B}^{\dagger} = \sum_{n\,\text{even}} \frac{(i\theta)^n}{n!} a - i \sum_{n,\,\text{odd}} \frac{(i\theta)^n}{n!} b = a\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots\right) - ib\left(i\theta - \frac{i\theta^3}{3!} + \frac{i\theta^5}{5!} + \cdots\right)$$

USE: $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\boxed{\mathcal{B} a \mathcal{B}^{\dagger} = a \cos\theta + b \sin\theta}$$

- $\mathcal{B} b \mathcal{B}^{\dagger} = b \cos\theta - a \sin\theta$ (HOMEWORK)

In terms of quantum logic gates, $\mathcal{B}$ performs a useful operation

$$\mathcal{B}|00\rangle = e^{\theta(a^{\dagger} b - a b^{\dagger})} |00\rangle = e^{\theta G} |00\rangle$$

$$= \left[1 + \theta G + \frac{(\theta G)^2}{2!} + \frac{(\theta G)^3}{3!} + \cdots\right] |00\rangle$$

- $G|00\rangle = (a^{\dagger} b - a b^{\dagger})|00\rangle = (a^{\dagger} b - a b^{\dagger})|0\rangle_b \otimes |0\rangle_a$

$$= |1\rangle_a \otimes 0 - 0 \otimes |1\rangle_b = 0$$

$$\mathcal{B}|00\rangle = (1 + 0 + 0 \cdots)|00\rangle = |00\rangle$$

– This means that when no photons in either input mode exist, no photons will exist in either output

– THIS MEANS THAT WHEN NO PHOTONS IN EITHER ...
MODE EXIST, NO PHOTONS WILL EXIST IN EITHER OUTPUT MODE

– LET'S SEE WHAT HAPPENS IF ONE SINGLE PHOTON EXISTS, FOR INSTANCE, IN MODE a

$$|1\rangle = a^{\dagger}|0\rangle$$

$$B|01\rangle = B|0\rangle_b \otimes |1\rangle_a = B|0\rangle_b \otimes a^{\dagger}|0\rangle_a$$

$$B|01\rangle = B a^{\dagger}|00\rangle \quad , \text{BUT}, \quad B^{\dagger}B = I$$

$$B|01\rangle = B a^{\dagger} B^{\dagger} \underbrace{B|00\rangle}_{|00\rangle} = B a^{\dagger} B^{\dagger}|00\rangle$$

AND FROM EQ. (7.33) $\Rightarrow$ $B a B^{\dagger} = a\cos\theta + b\sin\theta$

AND WE COULD PROVE THAT $B a^{\dagger} B^{\dagger} = a^{\dagger}\cos\theta + b^{\dagger}\sin\theta$ (HOMEWORK)

$$B|01\rangle = B a^{\dagger} B^{\dagger}|00\rangle = (a^{\dagger}\cos\theta + b^{\dagger}\sin\theta)|00\rangle$$

$$= a^{\dagger}|00\rangle\cos\theta + b^{\dagger}|00\rangle\sin\theta$$

$$\boxed{B|01\rangle = |01\rangle\cos\theta + |10\rangle\sin\theta}$$

$$\boxed{B|10\rangle = |10\rangle\cos\theta - |01\rangle\sin\theta}$$ (HOMEWORK)

– THUS ON THE $|0_L\rangle$ AND $|1_L\rangle$ MANIFOLD STATES

$$|10\rangle = |1_L\rangle \quad , \quad |01\rangle = |0_L\rangle$$

$$\to B|01\rangle = B|0_L\rangle = \cos\theta|0_L\rangle + \sin\theta|1_L\rangle$$

$$\to B|10\rangle = B|1_L\rangle = \cos\theta|1_L\rangle - \sin\theta|0_L\rangle$$

- MATRIX REPRESENTATION FOR B

$$B = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad (7.35)$$

- NOTE THAT, WE COULD WRITE (7.35) WITH

$$B = e^{-i\theta Y} \quad , \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

USING FORMULA (2.58) $\Rightarrow$ $e^{i\theta(\vec{v}\cdot\vec{\sigma})} = \cos\theta\, I + i\sin\theta\, \vec{v}\cdot\vec{\sigma}$

$$e^{-i\theta Y} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \quad , \quad \cancel{e^{i\theta Y}}$$

- PHASE SHIFTERS AND BEAMSPLITTERS TOGETHER ALLOW ARBITRARY SINGLE QUBIT OPERATIONS TO BE PERFORMED TO OPTICAL QUBIT

$P \to R_z(\alpha) = e^{-i\alpha Z/2}$ $\to$ PERFORMS A R_z - ROTATION

$B \to R_y(\alpha) = e^{-i\alpha Y/2}$ $\to$ " " R_y - ROTATION