Lecture22_QHardware_class17

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Lecture22...

Quantum Hardware – Optical Models

Class XVII

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Based on the book, Quantum Computation and Quantum Information, by M. Nielsen and I. Chuang

Optical Cavity Quantum Electrodynamics (QED) – Two-level atoms

Single-photon single-atom absorption and refraction

- Ψ) The most interesting regime in the cavity QED, for our purpose, is that in which single photons interact with single atoms
- Ψ) This is an unusual regime, in which traditional concepts such as the index of refraction and permittivity in classical theories of electromagnetism break down
- Ψ) In particular, we would like to utilize a single atom to obtain a non-linear interaction between photons
- Ψ) Without loss of generality we may neglect in Eq. (7.71) N, since it will only contribute with a fixed phase (HOMEWORK)
- Ψ) Recalling that time evolution is given by

$$U = e^{-iHt/\hbar} = e^{-iHt}, \quad \hbar = 1$$

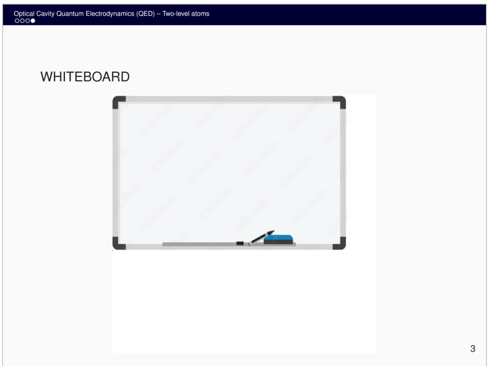
here and in the following, it will often be convenient to drop $\hbar$, and we show do so freely

- Ψ) Focusing on the case of at most a single excitation in the field mode, where we are interested in the regime where single photons interact with single atoms
- Ψ) In this case, our basis are: $|Field, Atom\rangle \rightarrow |0,0\rangle, |1,0\rangle and |0,1\rangle$

REMEMBER

- We are working with a two-level atom, thus it exists only two possible states (ground state $|0\rangle$) and excited state $|1\rangle$
- In the cavity we wish to have at maximum one photon (zero or one), all other possible states would give us a number of photons > 1

Let's see this on the whiteboard.



7.5.3 SINGLE-PHOTON SINGLE-ATOM ABSORPTION AND INTERACTION

OUR BASIS : $\begin{cases} |\text{FIELD}, \text{ATOM}\rangle = |\text{FIELD}\rangle \otimes |\text{ATOM}\rangle \\ \langle \text{FIELD}, \text{ATOM}| = \langle \text{FIELD}| \otimes \langle \text{ATOM}| \end{cases}$

$$(7.71) \Rightarrow H = \underbrace{\hbar \omega N}_{} + \hbar \delta z + g(\sigma^+ a + \sigma a^+)$$

↳ NEGLECT N SINCE IT IS CONTRIBUTES WITH ONLY A PHASE (HOMEWORK)

$\hbar = 1$

$$H = \delta z + g(\sigma^+ a + \sigma a^+), \quad \delta = \frac{\omega_0 - \omega}{2}, \quad \begin{cases} \omega_0 \to \text{ATOM} \\ \omega \to \text{FIELD} \end{cases}$$

BASIS: $|00\rangle, |10\rangle, |01\rangle, \cancel{|11\rangle}$, $\begin{cases} z|0\rangle = |0\rangle \\ z|1\rangle = -|1\rangle \end{cases}$

$z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

- $\delta z|00\rangle = \delta|00\rangle$, $\delta z|01\rangle = -\delta|01\rangle$
- $\delta z|10\rangle = \delta|10\rangle$

REMEMBER: $\begin{cases} \sigma|0\rangle_{\text{ATOM}} = 0, & \sigma^+|0\rangle_{\text{ATOM}} = |1\rangle_{\text{ATOM}} \\ \sigma|1\rangle_{\text{ATOM}} = |0\rangle_{\text{ATOM}}, & \sigma^+|1\rangle_{\text{ATOM}} = 0 \end{cases} \rangle$ LAST LECTURE

$\begin{cases} a|0\rangle_{\text{FIELD}} = 0, & a^+|0\rangle_{\text{FIELD}} = |1\rangle_{\text{FIELD}} \end{cases} \rangle$ SHO

$$\left\{\begin{array}{ll} a|0\rangle_{\text{FIELD}} = 0 \,, & a^{\dagger}|0\rangle_{\text{FIELD}} = |1\rangle_{\text{FIELD}} \\ a|1\rangle_{\text{FIELD}} = |0\rangle_{\text{FIELD}} \,, & a^{\dagger}|1\rangle_{\text{FIELD}} = \sqrt{2}\,|2\rangle_{\text{FIELD}} \end{array}\right. \quad > \text{SHO}$$

- $g(\sigma^{+}a + \sigma a^{\dagger})|00\rangle = 0 + 0 = 0$
- $g(\sigma^{+}a + \sigma a^{\dagger})|10\rangle = g(|01\rangle + 0) = g|01\rangle$
- $g(\sigma^{+}a + \sigma a^{\dagger})|01\rangle = g(0 + |10\rangle) = g|10\rangle$

TESTING THE STATE $|11\rangle$

- $g(\sigma^{+}a + \sigma a^{\dagger})|1\cancel{1}\rangle = g(0 + \sqrt{2}\,\overset{\text{TWO-PHOTONS}}{|20\rangle})$ X

LET'S BUILD H IN TERMS OF A MATRIX

$$H = \delta z + g(\sigma^{+}a + \sigma a^{\dagger})$$

$$H = \begin{array}{c} \\ \langle 00| \\ \langle 01| \\ \langle 10| \end{array} \begin{array}{c} \begin{array}{ccc} |00\rangle & |01\rangle & |10\rangle \end{array} \\ \begin{pmatrix} \delta & 0 & 0 \\ 0 & \delta & g \\ 0 & g & -\delta \end{pmatrix} \end{array}$$

$$\left\{\begin{array}{l} \langle i|i\rangle = 1 \\ \langle i|j\rangle = 0 \end{array}\right.$$

- $\langle 00|H|00\rangle = \langle 00|\left(\delta|00\rangle + 0\right) = \delta\langle 00|00\rangle = \delta$
- $\langle 00|H|01\rangle = \langle 00|\left(-\delta|01\rangle + g|10\rangle\right) = -\delta\overset{=0}{\langle 00|01\rangle} + g\overset{=0}{\langle 00|10\rangle} =$

$\vdots$ (HOMEWORK)!

IN THE BOOK: $H = \overset{-}{+}\begin{pmatrix} \delta & 0 & 0 \\ 0 & \delta & g \\ 0 & g & -\delta \end{pmatrix}$ (7.76)

IN THE BOOK: $H = t\begin{pmatrix} 0 & \delta & g \\ 0 & g & -\delta \end{pmatrix}$ (7.76)

— WE CAN WRITE THIS MATRIX IN THE FOLLOWING FORM

$$U = e^{-i\delta t}|00\rangle\langle 00| + \left[\cos(\Omega t) - i\frac{\delta}{\Omega}\sin(\Omega t)\right]|01\rangle\langle 01| +$$

$$+ \left[\cos(\Omega t) + i\frac{\delta}{\Omega}\sin(\Omega t)\right]|10\rangle\langle 10| - \frac{gi}{\Omega}\sin(\Omega t)\left[|10\rangle\langle 01| + |01\rangle\langle 10|\right] \quad (7.77)$$

$U = e^{-iHt}$, WHERE H IS GIVEN BY (7.76)

EXERCISE <u>7.18</u>, TO FIND Eq. (7.77), USE FORMULA

(7.78) (ERRATUM), $e^{i\vec{n}\cdot\vec{\sigma}} = \cos|n| + i\hat{n}\cdot\vec{\sigma}\sin|n|$

$\vec{n} = -t(g, 0, \delta)$, $\vec{\sigma} = (X, Y, Z)$

$|n| = \sqrt{\vec{n}\cdot\vec{n}} = \Omega t$

$\hat{n} = \frac{\vec{n}}{|n|} = \frac{-t(g, 0, \delta)}{\Omega t} = -\frac{1}{\Omega}(g, 0, \delta)$

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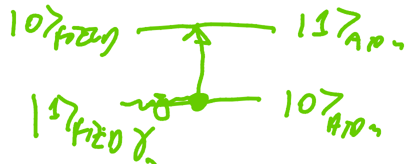
$$(7.77) \Rightarrow \quad U = e^{-i\delta t}|00\rangle\langle 00| + \left[\cos(\Omega t) - i\frac{\delta}{\Omega}\sin(\Omega t)\right]|01\rangle\langle 01|$$

$$+ \left[\cos(\Omega t) + i\frac{\delta}{\Omega}\sin(\Omega t)\right]|10\rangle\langle 10| - i\frac{g}{\Omega}\sin(\Omega t)\left[|01\rangle\langle 10| + |10\rangle\langle 01|\right]$$

— FIND THE PROBABILITY (χ_r) THAT AN INITIAL PHOTON $|1\rangle_{FIELD}$ IS <u>ABSORBED</u> BY ATOM (WHICH WE ASSUME STARTS IN THE GROUND STATE $|0\rangle_{ATOM}$

$$\chi_r = \sum_k \left|\langle 0k|U|10\rangle\right|^2 \qquad (7.79)$$

USING EQ. (7.77) FOR U WE WILL FIND THAT ONLY THE LAST TERM SURVIVES WHEN $k=1$



$$\chi_r = \left|\langle 01|U|10\rangle\right|^2 = \left|-\frac{ig}{\Omega}\sin(\Omega t)\right|^2$$

$$= \frac{g^2}{\Omega^2}\sin^2(\Omega t) = \frac{g^2}{g^2+\delta^2}\sin^2(\Omega t) \qquad (7.79)$$

$\Omega = \sqrt{g^2+\delta^2}$

EXERCISE 7.19: PLOT EQ. (7.79) FOR $t=1$, $g=1.2$

EXERCISE 7.20: DERIVE EQUATION (7.80)

$\}$ <u>HOMEWORK</u>