NEH Algorithm

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- **NEH – Nawaz, Enscore, Ham**

  - NEH heuristic
  - NEH insertion technique

Flow-Shop Scheduling Problem - FSP

Determining the best sequence of *n* jobs to be processed on *m* machines in the same order.

Possible No. of sequences = $(n!)^m$

Permutation Flow-Shop Scheduling Problem - PFSP

Same job sequence on all machines

Possible No. of sequences = *(n!)*

For makespan minimisation, PFSP is *NP-complete,* if *m* is greater than *2*

NEH Algorithm

- *Stage 1:*

  The generation of an initial order of jobs with respect to an indicator value.

- *Stage 2:*

  The iterative insertion of jobs into a partial sequence according to the initial order of stage 1.

NEH Algorithm for Makespan Minimisation

- *Step 1:* Order the jobs by non-increasing sums of processing times on the machines;
- *Step 2:* Take the first two jobs and schedule them in order to minimise the partial makespan as if there were only these two jobs
- *Step 3:* For $k = 3$ to n do Step 4
- *Step 4:* Insert the kth job at the place, which minimises the partial makespan among the k possible ones.

Total No. of sequences to be enumerated $= n(n+1)/2 - 1$

Example

4 job x 5 machine flow-shop problem

Process times

		Machines (*m*)				
		M1	M2	M3	M4	5
Jobs (*n*)	J1	5	9	8	10	1
	J2	9	3	10	1	8
	J3	9	4	5	8	6
	J4	4	8	8	7	2

- Step 1:

$T_1 = 5 + 9 + 8 + 10 + 1 = 33$

$T_2 = 9 + 3 + 10 + 1 + 8 = 31$

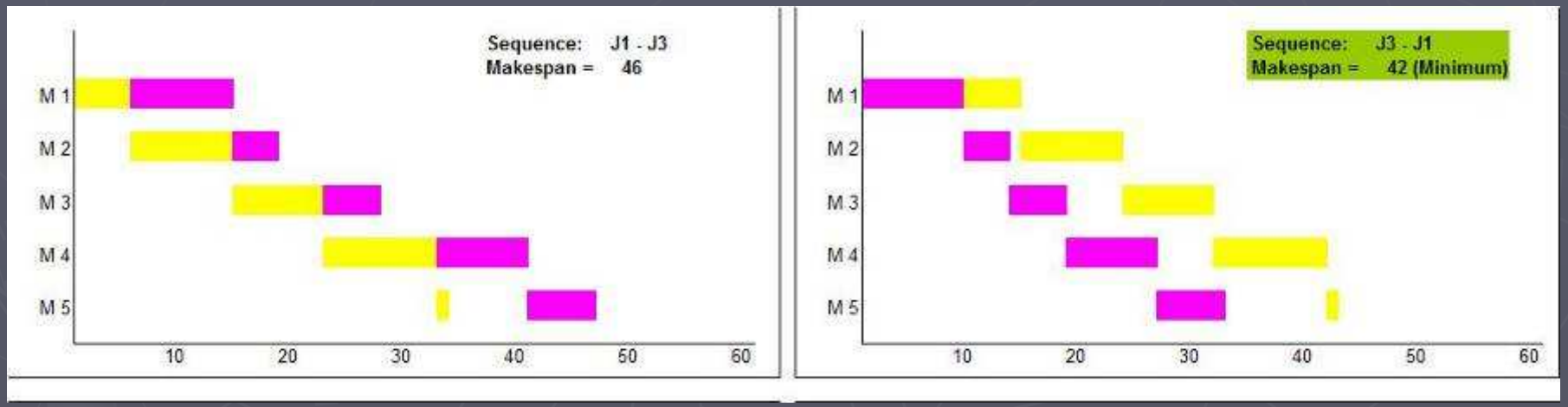
$T_3 = 9 + 4 + 5 + 8 + 6 = 32$

$T_4 = 5 + 9 + 8 + 10 + 1 = 29$

- Step 2: J1, J3, J2, J4

Step 3: Select J1 and J3, and find the best partial sequence for only these two jobs.

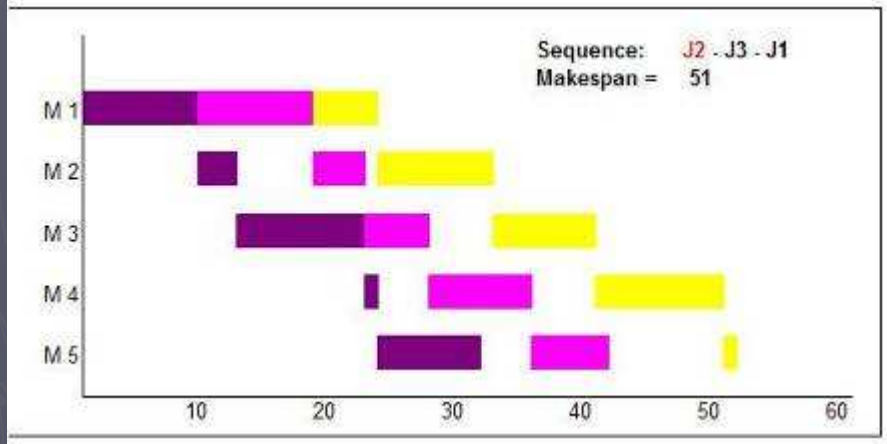
The best sequence is: J1 – J3 with makespan = 42

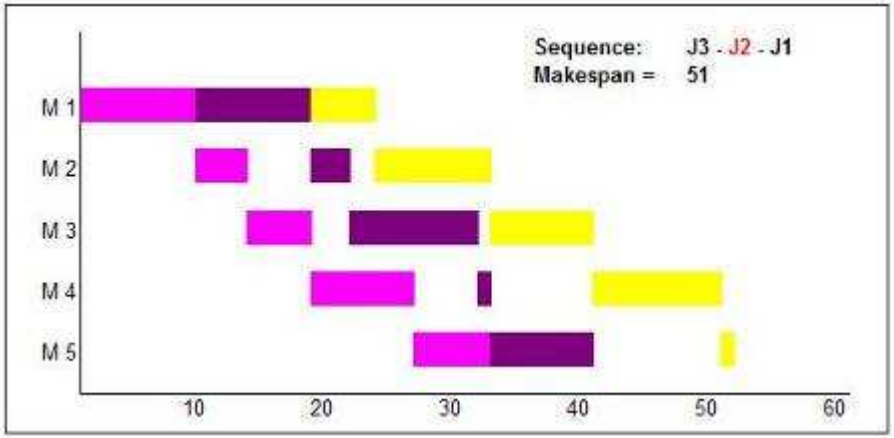


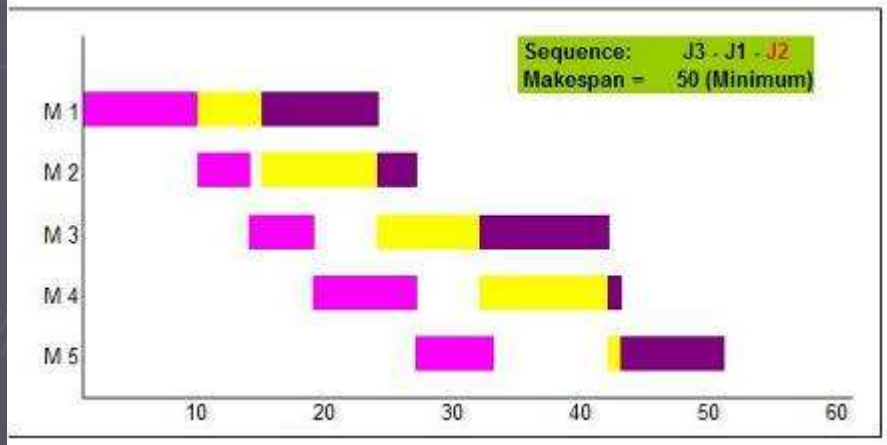
(In the next step, the relative position of J1 and J3 should always be J3 – J1, i.e. J3 before J1)

Step 4: Select J2 (job in the 3rd position of the list of Step 2) and find the best sequence by inserting J2 at all three possible positions in the partial sequence J3 - J1 obtained in the previous step.

The best sequence is J3 – J1 – J2 with makespan of 50

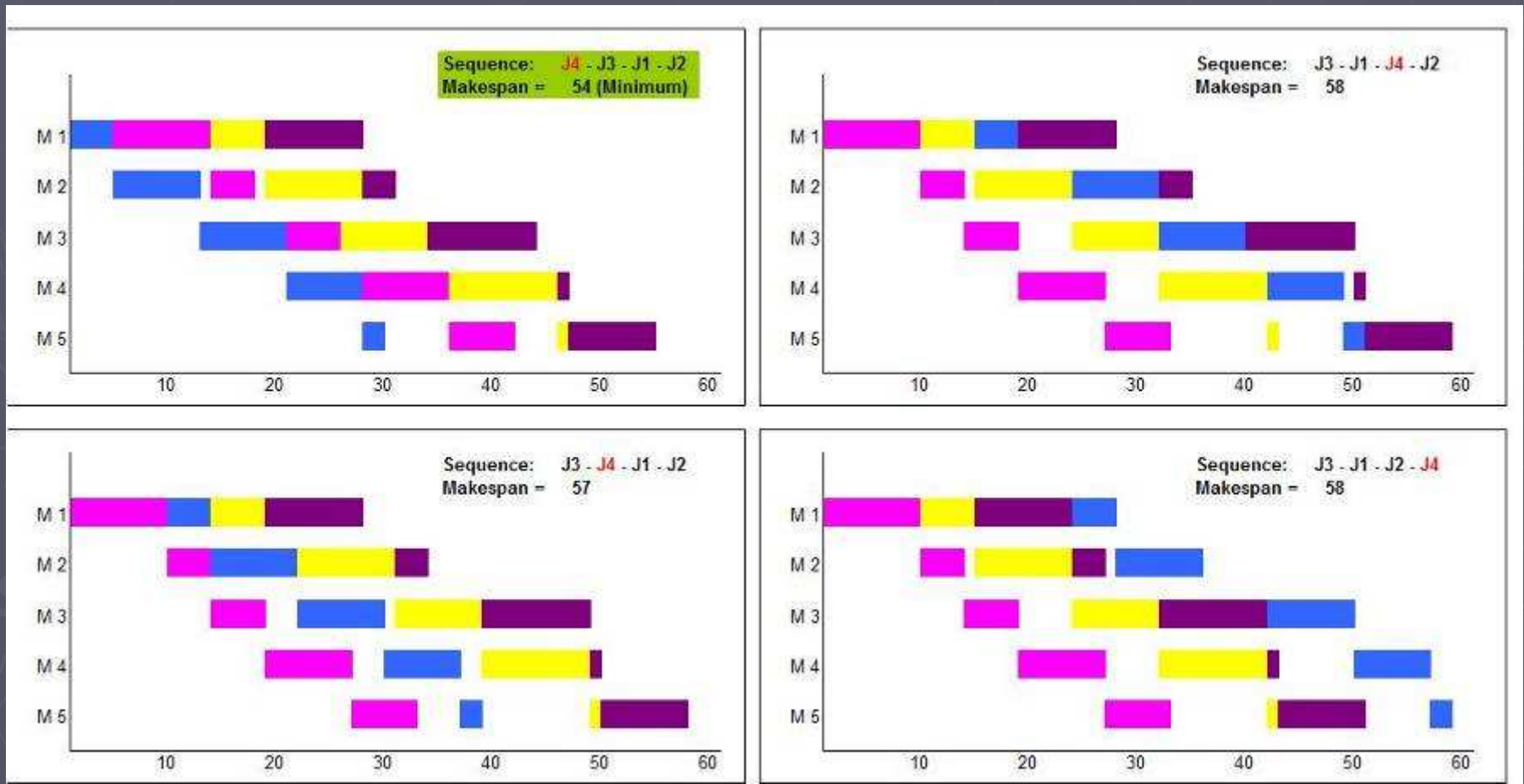






Step 4 a: Select J4 (job in the 4th position of the list of Step 2) and find the best sequence by inserting J4 at all four possible positions in the partial sequence J3 - J1 - J2 obtained in the previous step.

The best sequence is J4 - J3 – J1 – J2 with makespan of 54



- NEH algorithm is the most efficient algorithm to minimise makespan for Permutation Flow-Shop Problem (PFSP).

- Research has shown that, on the average, NEH algorithm returns makespan close to 3% of the optimal for
- $n = 5, \ldots 500$, and $m = 5, \ldots 25$.

- NEH algorithm can be used for
- - Flow time
- - Lateness/Tardiness, etc.
- minimisation problems.

- Almost all metaheuristics are using NEH results as a seed for the next stage improvement.

- Significance and applications of NEH algorithm can be highlighted by the number of citations reported in the scheduling research literature.

- SCOPUS database lists 375 citations; at least 70 in 2008 alone.