Assignment No 4

1) Apply rule for Dominance to the following matrix

	B1	B2	B3
A1	9	8	-7
A2	3	-6	4
A3	6	7	7.

→ Step ① -: Find saddle point

	B1	B2	B3	min	Max
A1	9	8	-7	-7	
A2	3	-6	4	-6	6
A3	6	7	7	6	
max	9	8	7		
min		7			

here minimax ≠ maxmin

∴ Saddle point does not exit

step ② : - Apply Dominance Rule

from player A's point of view

A1		A2	A1		A3	A2		A3
9	>	3	9	>	6	-6	<	7
8	>	-6	8	>	7	4	<	7
-7	<	4	-7	<	7	A2	<	A3

Can't Compare   Can't Compare   Delete A2.

Reduce Maintain 3

	B1	B2	B2
A1	9	8	-7
A2	6	7	7.

Step 3: From B1 main av view

B_1	B_2	B_1	B_2	B_2	B_3
$9 > 8$		$9 > -7$		$8 > -7$	
$6 < 7$		$6 < 7$		$7 = 7$	
				$B_2 = B_3$	
Can't compare		Can't compare		delete B_2	

	B_1	B_3
A_1	9	-7
A_2	6	7

Step 4: By Algebric Rule

Player A player with strategy A_1 with prob P_1, & A with prob $(1-P)$ then B plays with B_1

$$= 9 \times p - (1-P) \times 6$$
$$= 9p - 6 - 6p$$
$$= 3p - 6 \quad \text{--- (I)}$$

~~If player A playes with A_1 & B_3 then Player B plays with B_3~~

$$= 7p = (1-P) \times 7$$
$$= 7p + 7 - 7p$$
$$= -14p + 7 \quad \text{--- (II)}$$

Step 5: Compare as (2) & II

$$9p + 6 = -14p + 7$$
$$3p + 14p = 7 - 6$$
$$17p = 1$$
$$\boxed{p = \frac{1}{17}}$$

$\therefore (1-p) = 1 - \frac{1}{17} = \frac{16}{17}$

Step(6) :- player B player'1 strategy B_1 with prob q and B_2 with prob $(1-q)$ then A player with A_2

$$= 9 \times q + (1-q) \times (-7)$$
$$= 9q + (-7 + 7q)$$
$$= 9q + 7q - 7$$
$$= 16q - 7 \quad \text{— III}$$

It playes B plays with B_1 & B_3 then plays with A_3.

$$= 6 \times q + (1-q) \times 7$$
$$= 6q + 7 - 7q$$
$$= 6q - 7q + 7$$
$$= -q + 7 \quad \text{— IV}$$

Step (7) consider eqn III & IV

$$16q - 7 = -q + 7$$
$$16q + q = 7 + 7$$
$$17 = 14$$
$$q = \frac{14}{17}$$

$\therefore (1-q) = 1 - \frac{14}{17} = \frac{3}{17}$

$\therefore$ player A plays strategy A_1 & A_3 with prob $\frac{1}{17}$ & $\frac{16}{17}$

Player B plays strategy B_1 & B_3 with prob $\frac{14}{17}$ & $\frac{3}{17}$

Step (8)

$$v = 9 \times \frac{1}{17} + (6) \times \frac{16}{17}$$
$$= \frac{9}{17} + \frac{96}{17}$$

$$\boxed{V = \frac{105}{17}}$$

Q 2) let us take example with reduce matrix given below

	B_1	B_3
A_1	9	-7
A_3	6	7

→ Step ①: Find the saddle point

	B_1	B_3	min	max
A_1	9	-7	-7	6
A_3	6	7	6	
max	9	7		
min		7		

here minmax ≠ maxmin

∴ saddle point does not exist

Step ②:- player A1 plays with strateg A1 with prob p & A3 with prob $(1-p)$ then player B plays

$$= 9 \times p + (1-p) \times 6$$

$$= 9p + 6 - 6p$$

$$= 3p + 6 \quad \text{—— I}$$

II player A plays with A1 & A3 then plays B plays with B3

$$= \cancel{-7 \times p + (1-p) \times 7}$$

$$= -7p + 7 - 7p$$

$$= -14p + 7 \quad \text{—— II}$$

step ③ :- Consider eqⁿ I & II

$$3p + 6 = -14p + 7$$

$$3p + 14p = 7 - 6 \qquad p = \frac{1}{17}$$

$$17p = 1$$

$\therefore (1-p) = 1 - \frac{1}{17} = \frac{16}{17}$

Step ④ :- If players B plays strategy B_1 with q Prob and Strategy B_3 with $(1-q)$ Prob then A plays with A_1

$= 9 \times q + (1-q) \times (-7)$

$= 9q + (-7 + 7q)$

$= 9q + 7q - 7$

$= 16q - 7$ —— III

If player B plays with B_1 & B_3 then A plays with A_3

$= 6 \times q + (1-q) \times 7$

$= 6q + 7 - 7q$

$= 6q - 7q + 7$

$= -q + 7$ —— IV

Step ⑤ :- Consider eqn III & IV

$16q - 7 = -q + 7$

$16q + q \quad -7 + 7$

$17q = 14$

$\boxed{q = \frac{14}{17}}$

$\therefore (1-q) = 1 - \frac{14}{17} = \frac{3}{17}$

$\therefore$ Player A plays Strategy A_1 & A_3 with prob. $\frac{1}{17}$ & $\frac{16}{17}$

$\therefore$ player B plays Strategy B_1 & B_3 with prob $\frac{14}{17}$ & $\frac{3}{17}$

Step ⑥ :-

$$v = 9 \times \frac{1}{17} + (6) \times \frac{16}{17}$$

$$= \frac{9}{17} + \frac{96}{17} \qquad v = \frac{105}{17}$$

Q 3) Solve the following game

B's strategy

A's strategy	b1	b2	b3
a1	12	-8	-2
a2	6	7	3
a3	-10	-6	2

Step ① :- Find saddle point

B's Strategy

A's strategy	b1	b2	b3	min	Max
a1	12	-8	-2	-8	
a2	6	7	3	3	3
a3	-10	-6	2	-10	
Max	12	7	3		
min		3			

here minimax = Maxmin

∴ seddle point exist.

value of game = 3

∴ A is using a_1 strategy &

B is using b2 strategy.

Q4) Solve the following 2-person zero-sum game.

Player B

Player A	8	-3	7
	6	-4	5
	-2	2	-3

step ①:- find saddle point

	b_1	b_2	b_3	min	max
a_1	8	-3	7	-3	
a_2	6	-4	5	-4	-4
a_3	-2	2	-3	-3	
max	8	2	7		
min		2			

here minimax ≠ maxmin

∴ Saddle point does not exist

Step ② :- Apply dominance Rule

∴ from player A's point of view

a_1	a_2	a_1	a_3
8 > 6		8 > -2	
-3 > -4		-3 < 2	
7 > 5		7 > -3	

$a_2 < a_1$ | cant compare

Delete a_2

Reduce matrix

	b_1	b_2	b_3
a_1	8	-3	7
a_3	-2	2	-3

step ③ :- from B's point of view

b_1 b_2	b_1 b_3	b_2 b_3
8 > 3	8 > 7	-3 < 7
-2 > 2	-2 > -3	2 > -3

Can't Compare $b_1 > b_3$ Can't Compare.
delete b_1

Reduce matrix

$$\begin{array}{c|cc} & b_2 & b_3 \\ \hline a_1 & -3 & 7 \\ a_3 & 2 & -3 \end{array}$$

Step ④ Apply add meny. rule

∴ Column difference

For $b_2 = -3 - 2 = 5$

For $b_3 = 7 - (-3) - 10$

Row difference

For $a_1 = -3 - 7 = 10$

For $a_3 = 2 - (-3) = 5$

	b_2	b_3	
a_1	-3	7	5 1/3
	2	-3	10 2/3

Step ⑤: Probabilities for row & column

for $a_1 = \frac{5}{5+10} = \frac{5}{15} = \frac{1}{3}$

for $a_3 = \frac{10}{5+10} = \frac{10}{15} = \frac{2}{3}$.

for $b_2 = \frac{10}{10+5} = \frac{10}{15} = \frac{2}{3}$

for $b_3 = \frac{5}{10+5} = \frac{5}{15} = \frac{1}{3}$

∴ player A play with a_1 & a_3 strategy with prob $\frac{1}{3}$ & $\frac{2}{3}$

∴ Player B plays with b_2 & b_3 strategy with prob $\frac{2}{3}$ & $\frac{1}{3}$

Step ⑥ :- Find value of game.

$$v = -3 \times \frac{1}{3} + 2 \times \frac{2}{3}$$

$$= \frac{-3}{3} + \frac{4}{3}$$

$$\boxed{v = \frac{1}{3}}$$