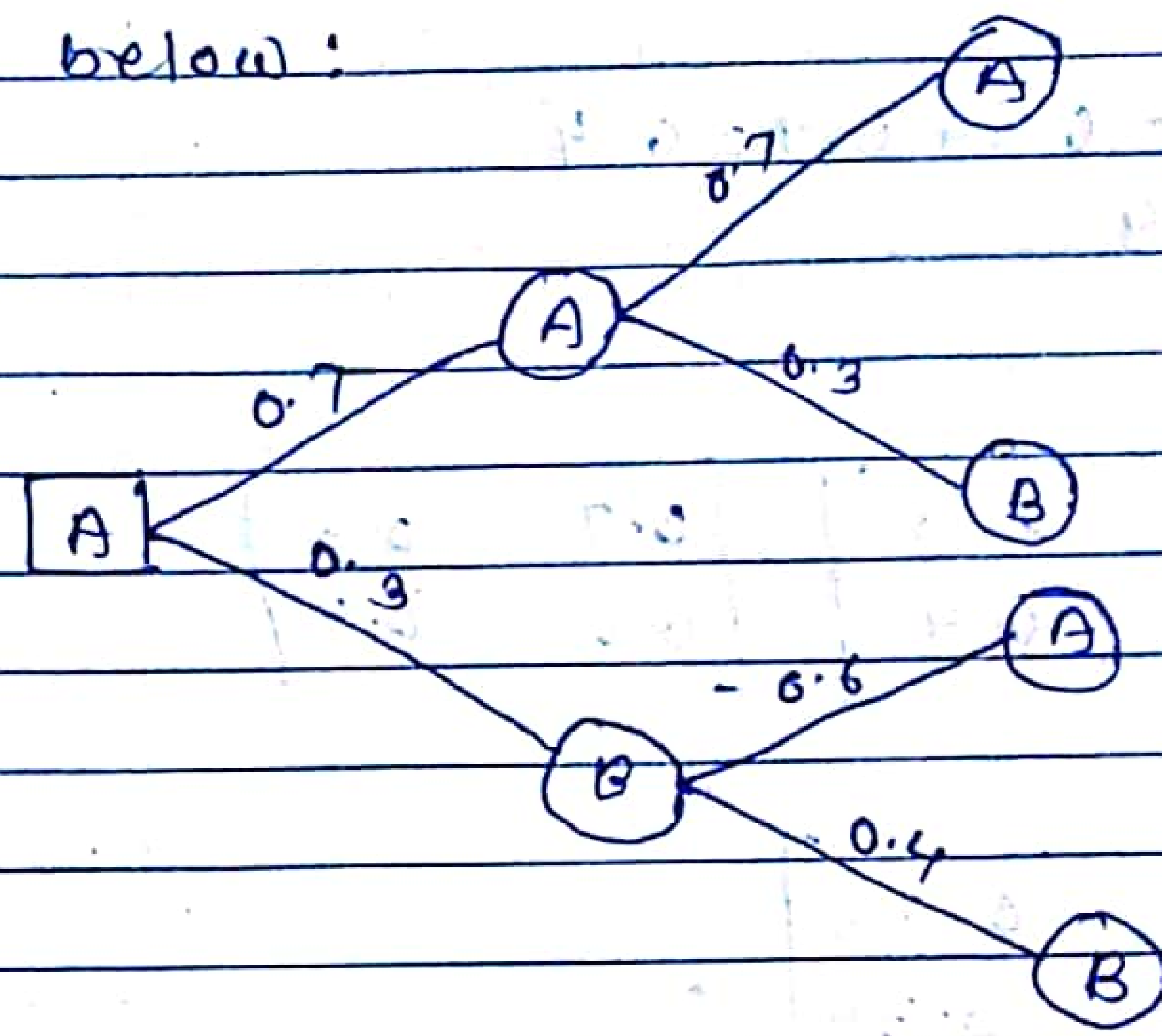
Assignment - 2

1) The transition of state can also be depicted in the form of probability tree which is shown below:



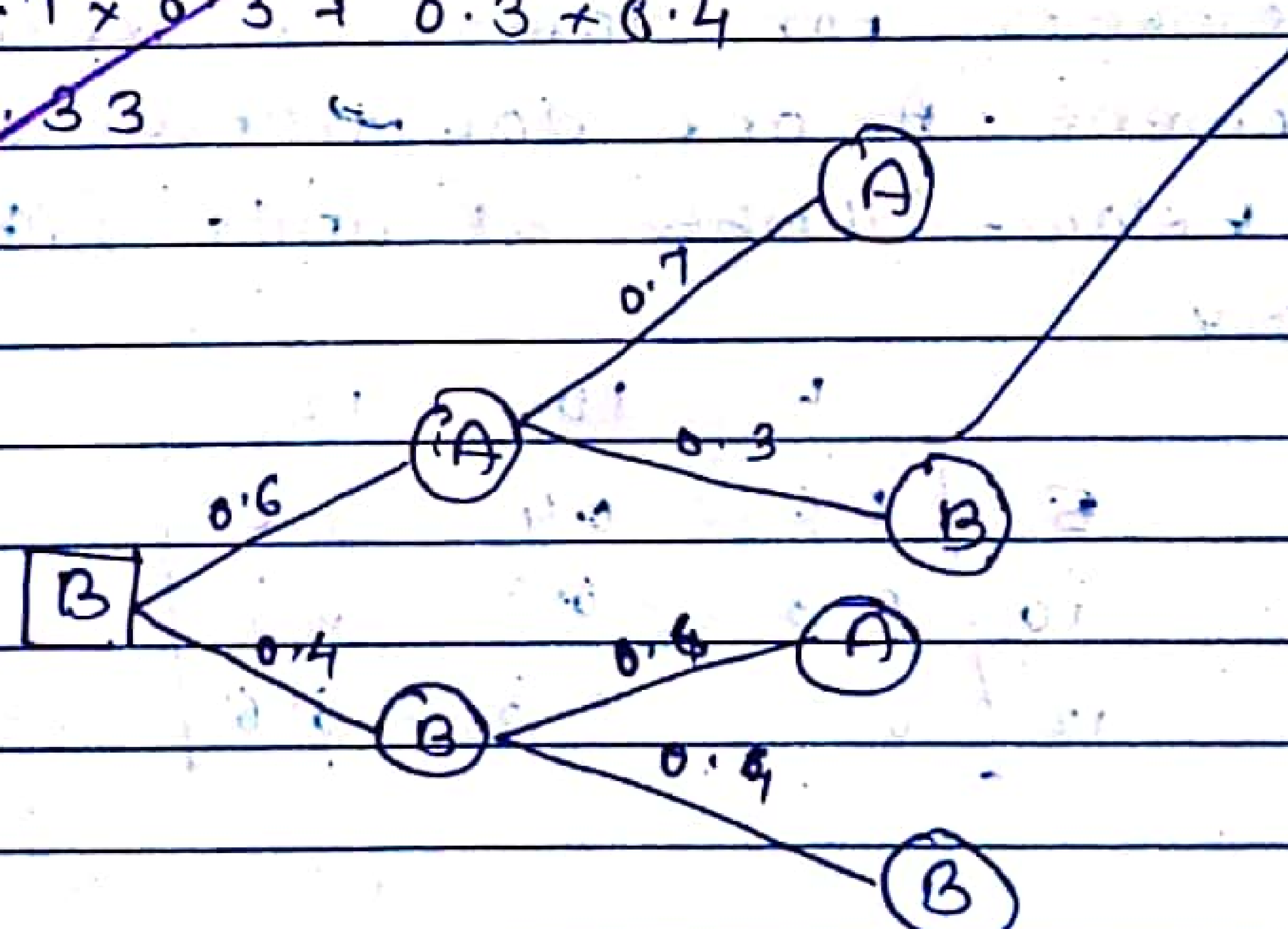
work out the state probabilities

$P_{11}^{(2)} = 0.7 \times 0.7 + 0.3 \times 0.6$

$= 0.67$

$\cancel{P_{12}^{(2)} = 0.7 \times 0.3 + 0.3 \times 0.4}$

$\cancel{= 0.33}$



$P_{21}^{(2)} = 0.6 \times 0.7 + 0.4 \times 0.6$
$= 0.66.$

$P_{22}^{(2)} = 0.6 \times 0.3 + 0.4 \times 0.4$
$= 0.34$

$$P = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$$

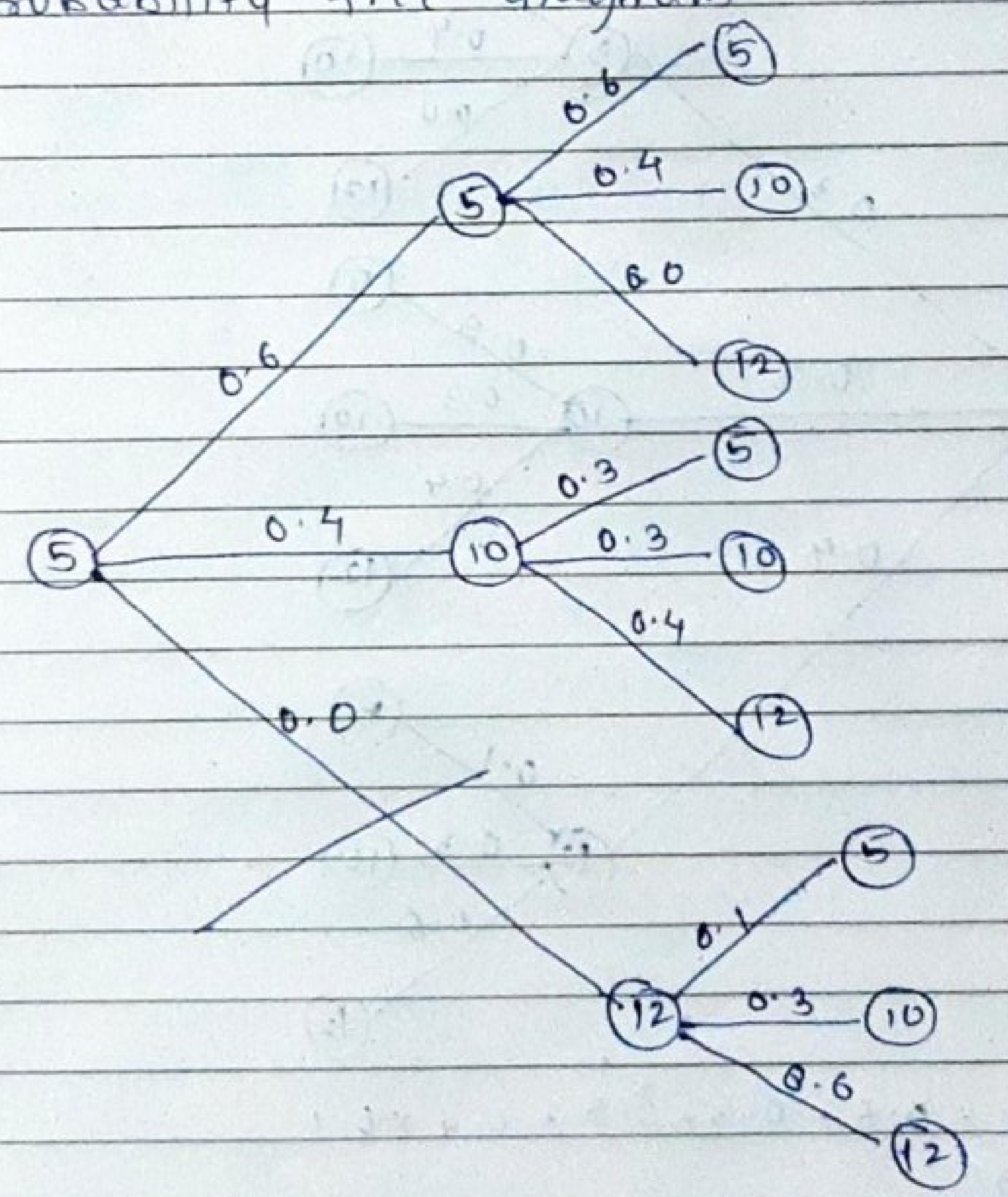
$$P = \begin{bmatrix} 0.67 & 0.33 \\ 0.66 & 0.34 \end{bmatrix}$$

2) The number of units of an item that are withdrawn from inventory on day-to-day basis follow a markov chain process in which requirements for tomorrow depend on today's requirement. A one day transaction matrix is given below. Number of units withdraw from the inventory

$$\begin{array}{c|ccc} & 5 & 10 & 12 \\ \hline 5 & 0.6 & 0.4 & 0.0 \\ 10 & 0.3 & 0.3 & 0.4 \\ 12 & 0.1 & 0.3 & 0.6 \end{array}$$

(a) Construct a tree diagram showing inventory requirements on two consecutive days
(b) Develop a two-day transition matrix
(c) Comments on how two-day matrix might be helpful to manager who is responsible for inventory management.

(a) probability tree diagram



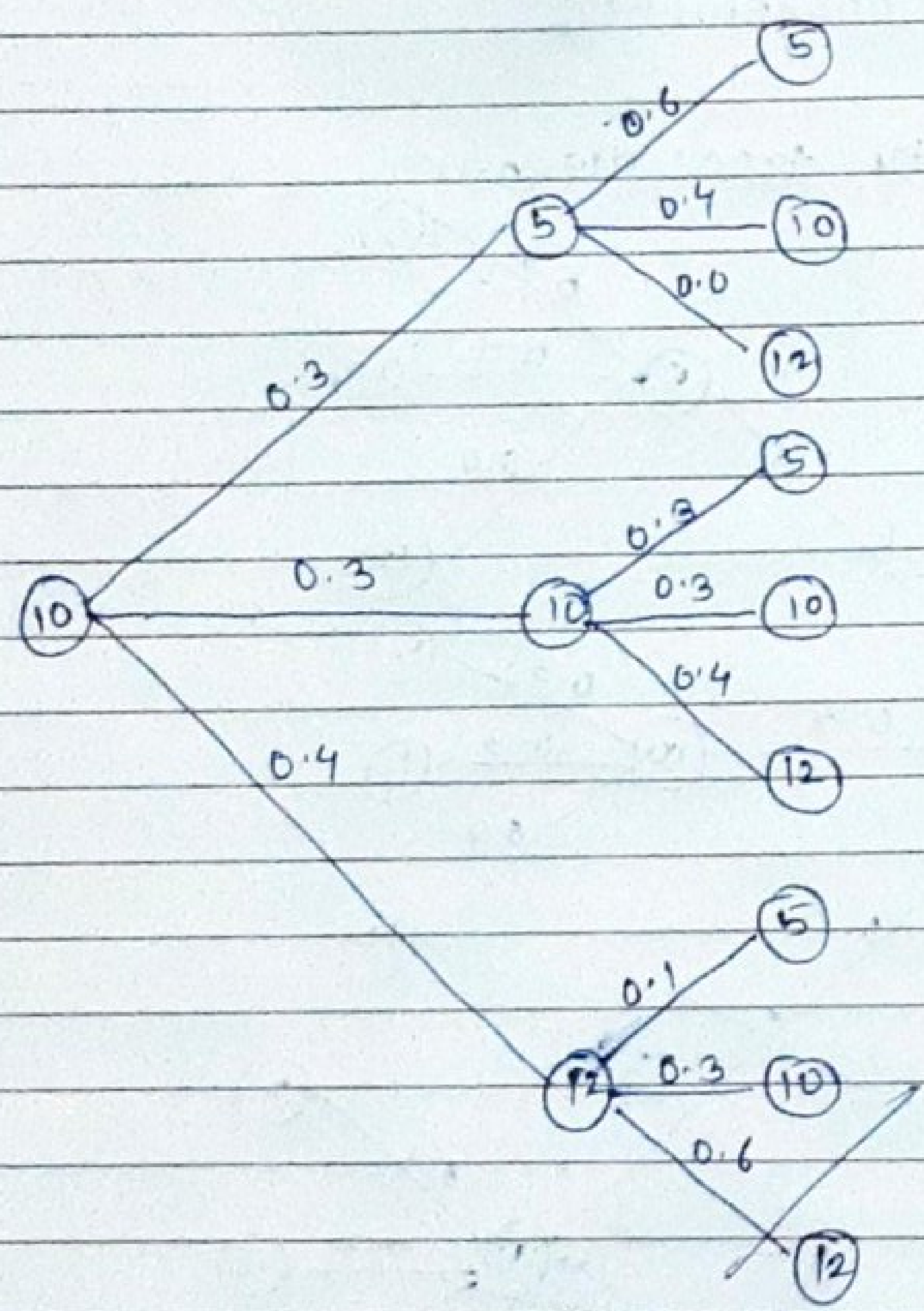
$P_{11}^{(2)} = 0.6 \times 0.6 + 0.4 \times 0.3 + 0.0 \times 0.1$
$= 0.48$

$P_{11}^{(2)} = 0.6 \times 0.4 + 0.4 \times 0.3 + 0.0 \times 0.3$

$= 0.36$

$P_{13}^{(2)} = 0.6 \times 0.0 + 0.4 \times 0.4 + 0.0 \times 0.6$

$= 0.16$



$P_{21}^{(2)} = 0.3 \times 0.6 + 0.3 \times 0.3 + 0.4 \times 0.1$

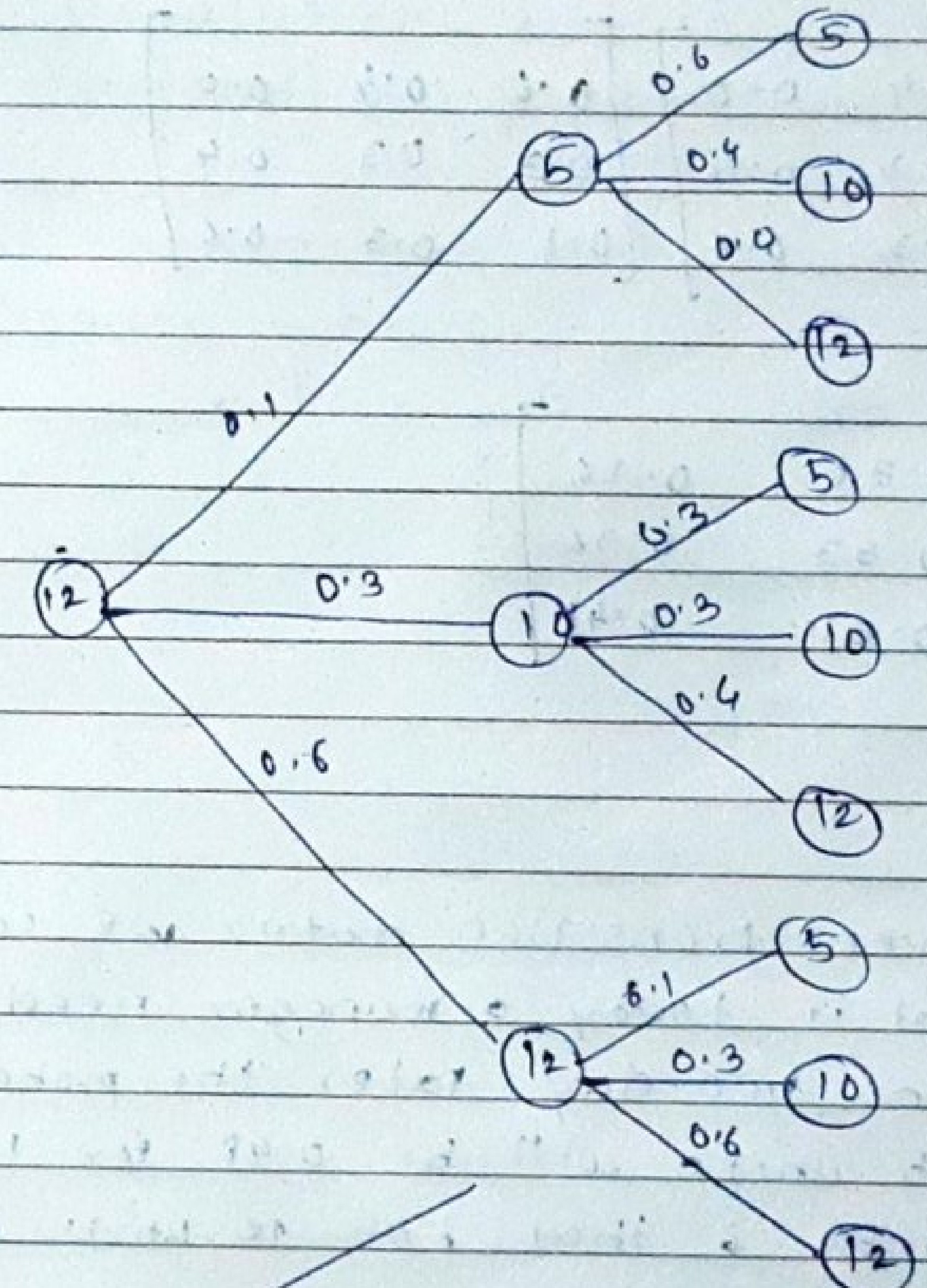
$= 0.31$

$P_{22}^{(2)} = 0.3 \times 0.4 + 0.3 \times 0.3 \times 0.4 \times 0.3$

$= 0.33$

$P_{23}^{(2)} = 0.3 \times 0.0 + 0.3 \times 0.4 + 0.4 \times 0.6$

$= 0.36$



$P_{31}^{(2)} = 0.1 \times 0.6 + 0.3 \times 0.3 + 0.6 \times 0.1$

$= 0.21$

$P_{32}^{(2)} = 0.1 \times 0.4 + 0.3 \times 0.3 + 0.6 \times 0.3$

$= 0.31$

$P_{33}^{(2)} = 0.1 \times 0.0 + 0.3 \times 0.4 + 0.6 \times 0.6$

$= 0.48$

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(b) Develope a two day transition matrix

$$P^{(2)} = \begin{bmatrix} 0.6 & 0.4 & 0.0 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.3 & 0.6 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 & 0.0 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$$

$$P^{(2)} = \begin{bmatrix} 0.48 & 0.36 & 0.16 \\ 0.31 & 0.33 & 0.36 \\ 0.21 & 0.31 & 0.48 \end{bmatrix}$$

(c)
From the given transition matrix we can conclude that in today a manager needs 5 units then two day later the probability of needing 5 units will be 0.48 for 10 units it is 0.36 & then for 12 units it is 0.16

a) Two traders are in for tough comptition one trader has been lucky to retain his customer 70% of the time and also in attracting the customer earlier using the product of his compititor trader to switch over two his product 50% of the time

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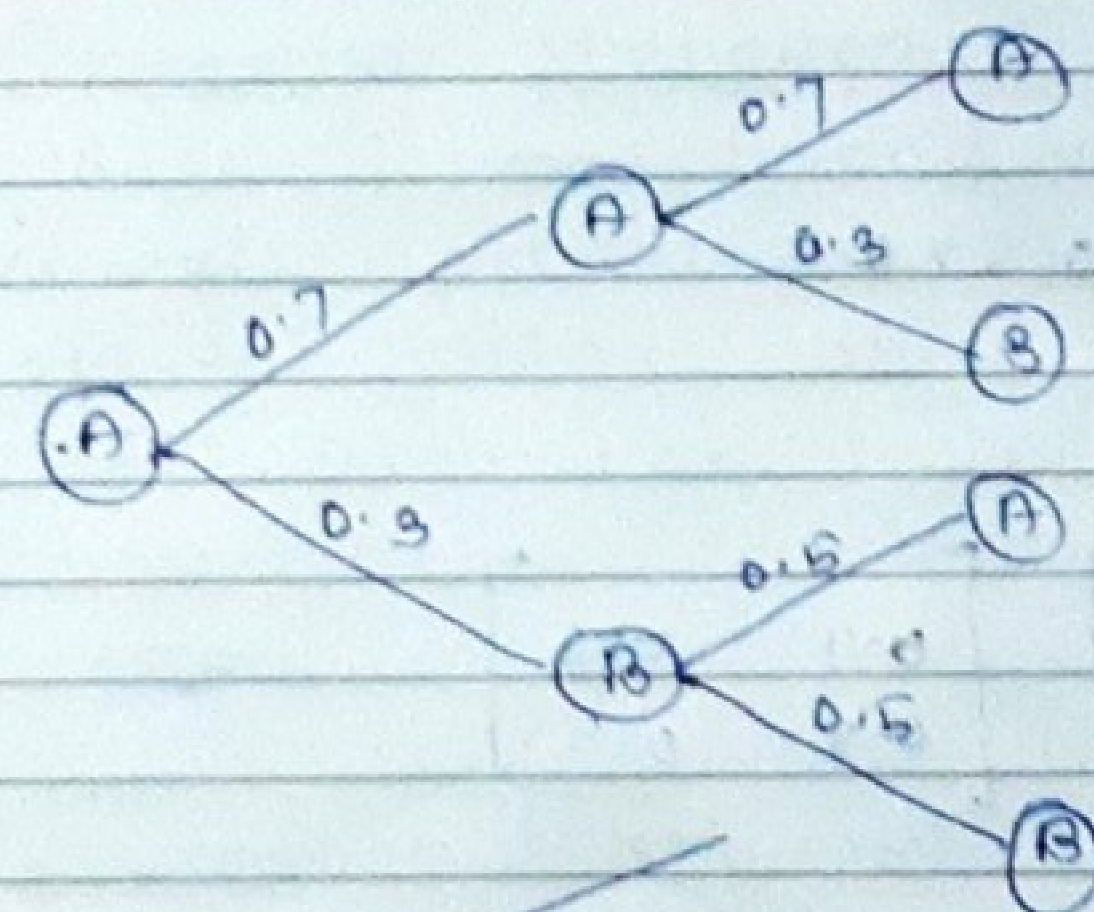
Construct and interpret a transition matrix in terms of
(a) Retaintion & loss
(b) Retaintion & gain

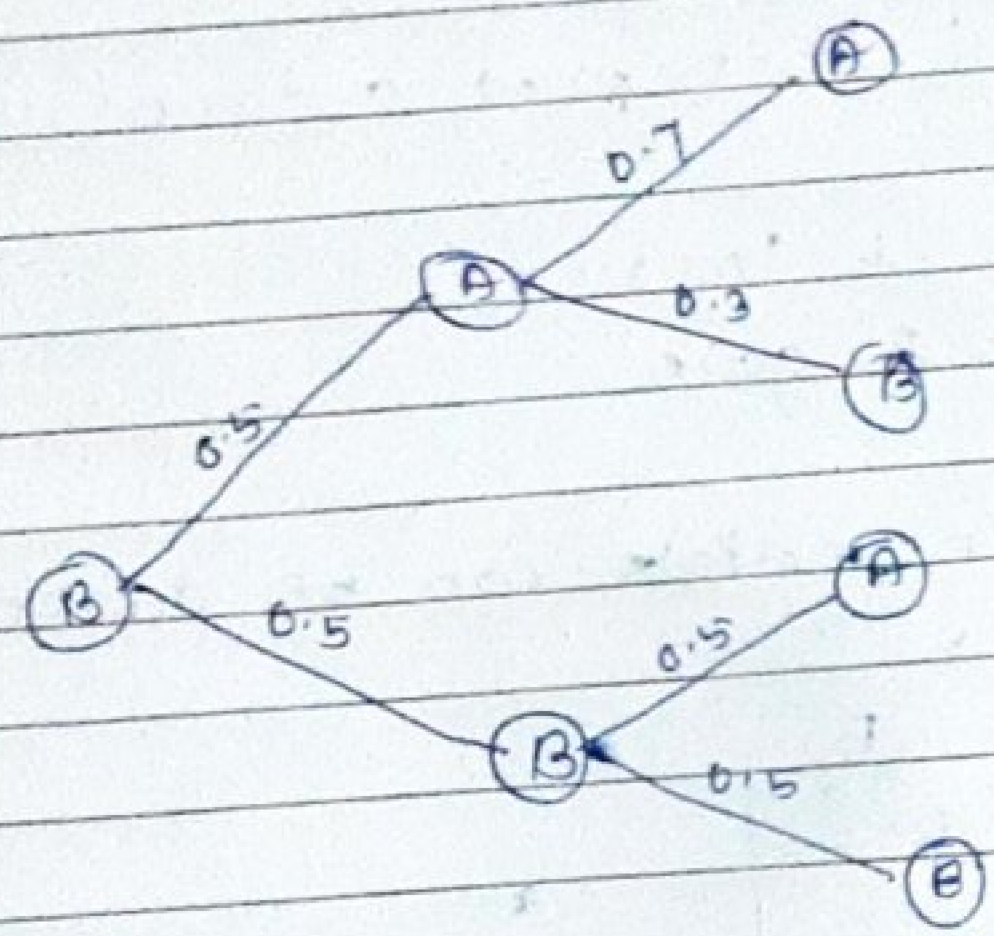
The transition matrix can be arrange in following manner

$$\begin{array}{c|cc} & A & B \\ \hline A & 0.7 & 0.3 \\ B & 0.5 & 0.5 \end{array}$$



$P_{11}^{(2)} = 0.7 \times 0.7 + 0.3 \times 0.5$
$= 0.64$

$P_{12}^{(2)} = 0.7 \times 0.3 + 0.3 \times 0.5$
$= 0.36$



$P_{21}^{(2)} = 0.5 \times 0.7 + 0.5 \times 0.5$
$= 0.6$

$P_{22}^{(2)} = 0.5 \times 0.3 + 0.5 \times 0.5$
$= 0.4$

$$P = \begin{bmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.64 & 0.36 \\ 0.6 & 0.4 \end{bmatrix}$$ retaintion & loss

retaintion & gain

4) What do you understand by Markov chain? In what areas of management can it applied successfully

Ans) Markov chains, named after the Russian mathematician Andrew Markov, are stochastic models used to describe a sequence of events where the probability of each event depends only on the state attained in the previous event. In other words, it's a process where future states depend only on the current state & not on the events that preceded it.

→ Markov chains can be applied successfuly in various areas of management, including:-

(1) Operation Management :- Markov chains can be used to model and analyze the flow of materials, products, or information through different stages of a production or service process. This can help identify bottlenecks, optimize resource allocation, and improve process efficiency.

(2) Inventory Management :- Markov chains can be used to model inventory system, where the state represents the inventory level. By analyzing transition probabilities between states, businesses can better predict future inventory levels, optimize reorder points, and reduce holding costs.

(3) Project Management :- Markov chains can be used to model project workflows, where the

states represent different stages of project completion. This can help project manager forecast project timelines, identify critical paths, and allocate resources effectively.

(4) **Risk management**:- Markov chains can be used to model and analyze various types of risks, such as financial market fluctuation, credit default probabilities, or insurance claim pattern. By understanding transition probabilities between different risks states, businesses can develop robust risk management strategies and minimize potential losses.

5) Let us take a car service station where customers arrive randomly. The arrival and service pattern have been tabulated. Work out the simulated pattern for service to help design the service facility.

Service Pattern

Service time (min)	Frequency
0.8	11
1.0	31
1.5	36
2.0	19
2.5	7
3.0	5

Arrival Pattern for customers

Time betn arrival (min)	Frequency
0.5	2
1.0	6
1.5	10
2.0	25
2.5	20
3.0	14
3.5	10
4.0	7
4.5	4
5.0	2

Step 1:- Table for arrival pattern

Arrival Time (min)	Frequency	Cumulative frequency	Cumulative Probability	Random Number
0.5	2	2	0.02	00-01
1.0	6	8	0.08	02-07
1.5	10	18	0.18	08-17
2.0	25	43	0.43	18-42
2.5	20	63	0.63	43-62
3.0	14	77	0.77	63-76
3.5	10	87	0.87	77-86
4.0	7	94	0.94	87-93
4.5	4	98	0.98	94-97
5.0	2	100	1.00	98-99

Step 2-: Table for simulation system service pattern

Arrival Number	Random Number	Arrival Time	Time for Arrival betn
1	78	8.5	8.5
2	78	3.5	7.0
3	06	1.0	8.0
4	04	1.0	9.0
5	97	4.5	13.5
6	71	3.0	16.5
7	78	3.5	20.0
8	59	2.5	22.5
9	05	1.0	23.5
10	95	4.5	28.0

Service time	Frequency	Cumulative frequency	Cumulative Probability	Range
0.5	12	12	0.12	00-11
1.0	21	33	0.33	12-32
1.5	36	69	0.69	33-68
2.0	19	88	0.88	69-87
2.5	7	95	0.95	88-94
3.0	5	100	1.00	95-99

Service Number	Random Number	Service time
1	54	1.5
2	26	1.0
3	51	1.5
4	45	1.5
5	46	1.5
6	84	2.0
7	58	1.5
8	58	1.5
9	60	1.5
10	24	1.0
		14.5

= 14.5 min

Service time	Service start	Service finish
1.5	3.5	5.0
1.0	7.0	8.0
1.5	8.0	9.5
1.5	9.5	11.0
1.5	13.5	15.0
2.0	16.5	18.0
1.5	20.0	21.0
1.5	22.5	24.0
1.5	24.0	25.0
1.0	28.0	29.0

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Total elapsed time = 29 min
Waiting arrival time = 1.0 min
Percentage of customer time = $\frac{100}{29}$ = 3.44

$\cong$ 3%