

Total No. of Questions : 8]

SEAT No. :

P-3389

[Total No. of Pages : 4

[6027]-305

S.Y. M.C.A. (Management Faculty)

MT-31 : PROBABILITY AND COMBINATORICS

(2019 Pattern) (Semester - III)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates :

- 1) Question No. 1 and question No. 8 are compulsory.
- 2) Solve any four questions from question no. 2 to question no. 7.
- 3) Figures to the right indicate full marks.
- 4) Use of non-programmable calculator and statistical table is allowed.

Q1) Solve any three sub-questions out of five :

[3 × 5 = 15]

- a) A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways this can be done? How many committees would consist of 1 man and 2 women?
- b) In how many ways can you put 7 letters into their respective envelopes such that exactly 3 go into the right envelope?
- c) A juggler has seven red, five green and four blue balls. During his stunt, he accidentally drops a ball & then picks it up. As he continues another ball falls. What is the probability that the first ball dropped is blue and the second ball is green.
- d) Let X be a continuous random variable with probability density function

$$f(x) = x^2 \left(2x + \frac{3}{2} \right) \quad 0 < x \leq 1$$
$$= 0 \quad \text{otherwise}$$

If $y = 2X + 3$ find $V(y)$.

- e) The number of Industrial Injuries per working week in a particular factory is known to follow a Poisson distribution with mean 0.5. Find the probability that in a particular week there will be
 - i) no accident
 - ii) less than 2 accidents
 - iii) more than 2 accidents

P.T.O.

- Q2) a)** Find the number of integer solutions to the following
 $X_1 + X_2 + X_3 = 28$, so that $3 \leq X_1 \leq 19$, $4 \leq X_2 \leq 20$, $5 \leq X_3 \leq 21$. [5]
- b)** Prove by combinational arguments [5]

i)
$$\binom{2n}{n} = 2 \binom{n}{2} + n^2$$

ii)
$$\sum_{k=0}^r \binom{m}{k} \binom{n}{r-k} = \binom{m+n}{r}$$

- Q3) a)** Three persons A, B and C have applied for a job in a private company. The chance of their selection is in the ratio 1:2:4. The probability that A, B and C can introduce changes to improve the profit of the company are 0.8, 0.5 and 0.3 respectively. If change take place, find the probability it is due to B. [5]
- b)** Explain the following terms with examples [5]
- Random variable
 - Sample space
 - Events

- Q4) a)** A discrete random variable X has the following probability distribution. [5]

X :	-2	-1	0	1	2	3
P(X) :	0.14	0.16	0.2	0.24	0.18	0.08

Find (i) $P(X > -1)$ (ii) $P(-2 < X \leq 2)$ (iii) $P(|X| > 1)$

- b)** We will assume that smiling times of eight week old baby follows uniform distribution between zero to 23 seconds. Inclusive let X denote smiling time in seconds. Find (i) $P(2 < X < 18)$ (ii) $P(X > 12/X > 8)$ [5]

- Q5) a)** Show that Poisson distribution is a limiting condition of Binomial distribution? State the assumptions related to it. [5]
- b)** A continuous random variable X has the following probability distribution function [5]

$$f(x) = k(1+x) \quad 2 \leq x \leq 5$$

$$= 0 \quad \text{otherwise}$$

Find (i) K (ii) $P(X < 4)$ (iii) $P(X > 3)$

- Q6) a)** I roll a die twice and obtain two numbers X_1 - result on first die
 X_2 = result on second die [5]

Find (i) $P(X_2 = 4)$ (ii) $P(X_1 + X_2 = 7)$

(iii) $P(X_1 = 2 \text{ or } 3 \text{ and } X_2 \geq 4)$.

- b) All boys at a military school must run a fixed course as fast as they can as a part of physical examination with mean 29 minutes and standard deviation 2.5 minutes. Find the probability that boy selected at random run a fixed course in (i) less than 25 minutes (ii) between 26 and 30 minutes (iii) greater than 27 minutes. [5]

- Q7) a)** The joint probability mass function of (X, Y) is given by [5]

$$f(x, y) = \begin{cases} x^2 + \frac{xy}{3} & 0 \leq x \leq 1 \quad 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find : i) $P\left(X > \frac{1}{3}\right)$ ii) $P\left(X < \frac{1}{4}, Y < \frac{3}{2}\right)$

- b) Scores on a standardized College Entrance Examination (CEE) are normally distributed with mean 510 and standard deviation 60. A selective university considers for admission only applicants with CEE score above 650. Find the percentage of all individuals who took the CEE who meet the university's CEE requirement. [5]

- Q8)** Solve any 3 sub-questions out of 5 : [3 × 5 = 15]

- a) Determine the number of 5 card combinations out of a deck of 52 cards, if there is exactly one are in 5 cards.
- b) A pin consists of 4 digits. What is the probability that any 4 digit pin selected will contain no repeated digits.
- c) You can take an examination which contains 10 multiple choice questions. Each question has 4 possible options. Your score is X in the examination which is total number of correct answers. Find the probability of
- i) All correct answers
- ii) All wrong answers
- iii) Exactly 4 correct answers

- d) A large software development company employs 100 computer programmers of them 45 are proficient in Java, 30 in C#, 20 in Python, 6 in C# and Java, one in Java and Python, five is C# and python and just one programmer is proficient in all three languages above. Determine the number of computer programmers that are not proficient in any of these three languages.
- e) The following table represents the joint probability distribution of discrete variables X & Y.

X \ Y	Y		
	1	2	3
1	k	2k	3k
2	4k	5k	6k
3	8k	9k	11k

Find :

- k
- Conditional probability distribution X given $Y = 1$.
- Conditional probability distribution of Y given $X = 2$.

