In an electric oven, transfer of energy from a heating element to food is governed by the heat equation. However, an approximate model is given by the following causal linear time-invariant discrete-time filter with input $x[n]$ and output $y[n]$

$$y[n] = \sum_{m=0}^{n-1} \frac{x[m]}{\alpha^{n-m}} \ for\ n \geq 0$$

where $x[n]$ represents the power delivered to the heating element, α is the thermal diffusivity, and $y[n]$ represents the temperature of the food where a temperature of 0 means room temperature. We expand the summation as follows

$$y[n] = \frac{1}{\alpha}x[n-1] + \frac{1}{\alpha^2}x[n-2] + \cdots \ldots . for\ n \geq 0$$

and convert $y[n]$ to a recursive difference equation

$$y[n] = \frac{1}{\alpha}x[n-1] + \frac{1}{\alpha}y[n-1]\ for\ n \geq 0$$

For this problem, assume that α = 2. We observe the system starting at n = 1

i. Assume that the food is initially at room temperature. What are the initial conditions and their values? Why?

ii. Give a formula for the impulse response of the filter $h[n]$. Simplify any summations.

iii. Is this a finite impulse response (FIR) or infinite impulse response (IIR) filter?

iv. Give the frequency selectivity of filter (lowpass, highpass, bandpass, bandstop, allpass, notch) and explain your reasoning.

Plot the response for (ii) in discrete domain (Use MATLAB).