01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2072 Kartik

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I *(SH401)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $y = (\sin^{-1} x)^2$ then show that:
   i) $(1-x^2)y_2 - xy_1 - 2 = 0$
   ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$
2. State Rolle's Theorem and verify the theorem for $f(x) = \frac{x(x+3)}{e^{x/2}}; x \in [-3,0]$
3. Evaluate: $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$
4. Find the asymptotes of the curve: $(a+x)^2(b^2+x^2) = x^2y^2$
5. Find the pedal equation of the curve $r^2 = a^2 \cos 2\theta$
6. Evaluate $\int_0^{\pi/4} \frac{(\sin x + \cos x)}{(9+16\sin 2x)}\,dx$
7. Use Beta Gamma function to evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$
8. Evaluate by using the rule of differentiation under the sign of integration.
   $\int_0^\infty \frac{e^{-x}\sin bx}{x}\,dx$
9. Find the area of one loop of the curve $r = a\sin 3\theta$

OR

Find the volume of the solid formed by the revolution of the cardioid $r = a(1+\cos\theta)$ about the initial line.

Find center and eccentricity of conic $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$

OR

Describe and sketch the graph of the equation $r = \frac{10}{3+2\cos\theta}$

10. Find the condition that the line $lx + my + n = 0$ may be a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

11. Show that the pair of tangents drawn from the center of a hyperbola are its asymptotes.

12. Solve the differential equation: $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$

13. Solve: $y-2px+ayp^2=0$ where $p=\frac{dy}{dx}$

14. Solve the differential equation: $x\frac{dy}{dx}+y\log y=xye^x$

15. Solve the differential equation: $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=x^2$

***

01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2071 Shawan

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	I/AI	Time	3 hrs.

Subject: - Engineering Mathematics I *(SH401)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. If $y = \log\left(x + \sqrt{a^2 + x^2}\right)$, then show that $\left(a^2 + x^2\right)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0$ [5]
2. State and prove Lagrange's Mean Value theorem. [5]
3. Evaluate: $\lim_{x \to \frac{\pi}{4}} (\sin x)^{\tan x}$ [5]
4. Find the asymption of the curve $a^2y^2 + x^2y^2 - a^2x^2 + 2ax^3 - x^4 = 0$ [5]
5. Find the radius of curvature at the origin for the curve $x^3 + y^3 = 3axy$
6. Evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}}\, dx$ [5]
7. Apply differentiation under integral sign to evaluate $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x}\, dx$ [5]
8. Using Gamma function show that $\int_0^{\frac{\pi}{4}} \sin^6 x \cos^2 x\, dx = \frac{3\pi - 4}{192}$ [5]
9. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

OR

Find the volume of the solid generated by the revolution of the cardioid $r = a(1 - \cos\theta)$ about the initial line.

10. Solve: $\sin x \frac{dy}{dx} + y\cos x = x\sin x$ [5]
11. Solve: $xp^2 - 2yp + ax = 0$ where $p = \frac{dy}{dx}$ [5]
12. Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^{3x}$ [5]
13. Solve: $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ [5]
14. Transform the equation $x^2 - 2xy + y^2 + x - 3y = 0$ to axes through the point (-1,0) parallel to the lines bisecting the angles between the original axes. [5]
15. Find the center, length of axes and the eccentricity of the ellipse $2x^2 + 3y^2 - 4x - 12y + 13 = 0$ [5]
16. Find the length of axes and ecentricity of the conic [5]

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$$

OR

Describe and sketch the conic $r = \frac{12}{2 - 6\cos\theta}$

01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2071 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I *(SH401)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. State Leibnity's theorem on Leigher derivatives:

   If $y = \sin(m \sin^{-1} x)$ then show that

   $(1-x^2)\, y_{n+2} - (2n+1)\, x y_{n+1} + (m^2-n^2) y_n = 0$

2. Assuming the validity of expansion, find the expansion of the function $\frac{e^x}{1+e^x}$ by Maclaurin's theorem.

3. Evaluate $\lim_{x\to 0} \frac{xe^x - (1+x)\log(1+x)}{x^2}$

4. Find the asymptotes of the curve $y^3 + 2xy^2 + x^2 y - y + 1 = 0$

5. Find the radius of curvature of the curve $y = x^2(x-3)$ at the points where the tangent is parallel to x-axis

   OR

   Find the pedal equation of the curve $r^2 = a^2 \cos 2\theta$

6. Show that $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}} = \frac{\Pi}{4}$

7. Apply differentiation under integral sign to evaluate $\int_0^{\Pi/2} \frac{dx}{(a^2\sin^2 x + b^2\cos^2 x)^2}$

8. Use gamma function to prove that $\int_0^1 \frac{dx}{(1-x^6)^{1/4}} = \Pi/3$

9. Find the volume or surface area of solid generated by revolving the cycloid $x = a(\theta+\sin\theta)$, $y = a(1+\cos\theta)$ about its base.

10. If the line $lx+my+n=0$ is normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then show that $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$

11. Solve the locus of a point which moves in such a way that the difference of its distance from two fixed points is constant is Hyperbola.

12. Solve the differential equation $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=6x$

13. Solve $(x^2D^2+xD+1)y=\sin(\log x^2)$

14. Solve $y=yp^2+2px$ where $p=\frac{dy}{dx}$

15. Solve: $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{2x}\sin x$

16. Describe and sketch the graph of the equation $r=\frac{10}{2-3\sin\theta}$

OR

Show that the conic section represented by the equation

$14x^2-4xy+11y^2-44x-58y+71=0$ is an ellipse. Also find its center, eccentricity, latus recturn and foci

***

01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2071 Shawan

Exam.	New Back (2066 & Later Batch)		
Level	BE	**Full Marks**	80
Programme	All (Except B.Arch)	**Pass Marks**	32
Year / Part	I / I	**Time**	3 hrs.

Subject: - Engineering Mathematics I *(SH401)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. If $y = \log\left(x + \sqrt{a^2 + x^2}\right)$, then show that $\left(a^2 + x^2\right)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0$ [5]
2. State and prove Lagrange's Mean Value theorem. [5]
3. Evaluate: $\lim_{x \to \frac{\pi}{4}} (\sin x)^{\tan x}$ [5]
4. Find the asymption of the curve $a^2y^2 + x^2y^2 - a^2x^2 + 2ax^3 - x^4 = 0$ [5]
5. Find the radius of curvature at the origin for the curve $x^3 + y^3 = 3axy$
6. Evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}}\, dx$ [5]
7. Apply differentiation under integral sign to evaluate $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x}\, dx$ [5]
8. Using Gamma function show that $\int_0^{\frac{\pi}{4}} \sin^4 x \cos^2 x\, dx = \frac{3\pi - 4}{192}$ [5]
9. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

   OR

   Find the volume of the solid generated by the revolution of the cardioid $r = a(1 - \cos\theta)$ about the initial line.
10. Solve: $\sin x \frac{dy}{dx} + y\cos x = x\sin x$ [5]
11. Solve: $xp^2 - 2yp + ax = 0$ where $p = \frac{dy}{dx}$ [5]
12. Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^{3x}$ [5]
13. Solve: $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ [5]
14. Transform the equation $x^2 - 2xy + y^2 + x - 3y = 0$ to axes through the point (-1,0) parallel to the lines bisecting the angles between the original axes. [5]
15. Find the center, length of axes and the eccentricity of the ellipse $2x^2 + 3y^2 - 4x - 12y + 13 = 0$ [5]
16. Find the length of axes and ecentricity of the conic [5]

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$$

OR

Describe and sketch the conic $r = \frac{12}{2 - 6\cos\theta}$

***

01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division

2070 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I *(SH401)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $Y = \sin(m \sin^{-1}x)$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$
2. Apply Maclaurin's series to find the expansion of $\frac{e^x}{1+e^x}$ as far as the term in x^3
3. Evaluate: $\lim_{x \to a} \left(2 - \frac{x}{a}\right)^{\tan\frac{\pi x}{2a}}$
4. Find the asymptotes of the curve $x(x-y)^2 - 3(x^2 - y^2) + 8y = 0$
5. Find the pedal equation of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$
6. Apply the method of differentiation under integral sign to evaluate $\int_0^\infty \frac{\log(1+a^2x^2)}{1+b^2x^2}dx$
7. Show that $\int_0^\infty \frac{\log(1+x^2)}{1+x^2}dx = \pi \log 2$
8. Use Gamma function to prove that $\int_0^1 \frac{dx}{(1-x^6)^{\frac{1}{6}}} = \frac{\pi}{3}$
9. Find the area of two loops of the curve $a^2y^2 = a^2x^2 - x^4$

OR

Find the volume of the solid formed by the revolution of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ about the tangent at the vertex.

10. Solve the differential equation $(1+y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$
11. Solve: $y - 3px + ayp^2 = 0$
12. Solve: $(D^2 - 2D + 5)y = e^{2x}.\sin x$
13. A resistance of 100 Ohms, an inductance of 0.5 Henry are connected in series with a battery 20 volts. Find the current in the circuit as a function of time.
14. What does the equation $3x^2 + 3y^2 + 2xy = 2$ becomes when the axes are turned through an angle 45° to the original axes.
15. Show that the locus of a point which moves in such a way that the differences of its distance from two fixed points is constant is a hyperbola.
16. Find the center, length of the axes and eccentricity of the conic $2x^2 + 3y^2 - 4x - 12y + 13 = 0$

OR

Describe and sketch the graph of the polar equation of conic $r = \frac{10\,\text{cosec}\theta}{2\,\text{cosec}\theta + 3}$

***

01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2069 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I *(SH401)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $y = \log\ (x + \sqrt{(a^2 + x^2)}$ show that $(a^2+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$
2. State and prove Lagrange's Mean Value theorem.
3. If $\lim_{x \to 0} \frac{a \sin x - \sin 2x}{\tan^3 x}$ is finite, find the value of a and the limit.
4. Find asymptotes of $(x^2-y^2)^2 - 2(x^2+y^2) + x - 1 = 0$
5. Find the radius of curvature at any point (x,y) for the curve $x^{2/3}+y^{2/3}=a^{2/3}$
6. Prove that $\int_0^\infty \frac{\sin bx}{x} dx = \frac{\pi}{2} (b > 0)$
7. Use Beta and Gamma function to evaluate $\int_0^{2a} x^5 \sqrt{2ax - x^2}\ dx$
8. Evaluate $\int_0^\infty \frac{e^{-x} \sin bx}{x} dx$ by using the rule of differentiation under the sign of integration.
9. Find the volume of the solid formed by the revolution of the cardiod $r = a(1-\cos\theta)$ about initial line.

OR

Find the area bounded by the curve $x^2y = a^2(a-y)$ and the x-axies

10. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$
11. Solve the differential equation $x\frac{dy}{dx} + y \log y = xye^x$
12. Solve the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = e^x + e^{-x}$
13. Solve $y = px - \sqrt{m^2 + p^2}$ where $p = \frac{dy}{dx}$

OR

A resistance of 100 ohms, an inductance of 0.5 henry are connected in series with a battery of 20 volts. Find the current in the circuit as a function of time.

14. Solve that locus of a point which moves in such a way that the differences of it distance from two fixed point is constant is Hyperbola.
15. Find the equation of ellipse of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where a>b
16. Describe and sketch the graph of the equation $r = \frac{4\sec\theta}{2\sec\theta - 1}$

***

01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2068 Shrawan

Exam.	New Back (2066 Batch & Later)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt <u>All</u> questions.*
- ✓ *<u>All</u> questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $y = \log(x + \sqrt{a^2 + x^2})$, show that $(a^2 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0$.
2. State and prove Lagrange's mean value theorem.
3. Evaluate: $\lim_{x \to 0}\left(\frac{\tan x}{x}\right)^{1/x}$.
4. Find the asymptotes of the curve $(x^2 - y^2)(x + 2y + 1) + x + y + 1 = 0$.
5. Show that for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the radius of curvature at the extremity of the major axis is equal to half of the latus rectum.
6. Evaluate: $\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}$.
7. Use Gamma function to prove that $\int_0^1 \frac{dx}{(1 - x^5)^{1/5}} = \frac{\pi}{5}$.
8. Using method of differentiation under integral sign, evaluate: $\int_0^\infty \frac{e^{-x}\sin bx}{x}\,dx$.
9. Find the area bounded by the cardioid, $r = a(1 + \cos\theta)$.

OR

Find the volume of the solid formed by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ about its base.

10. Find the angle through which the axes must be turned so that the equation $ax^2 + 2hxy + by^2 = 0$ may become an equation having no term involving xy.
11. Obtain the equation of an ellipse in the standard form.
12. Find the centre of the conic $2x^2 + 8xy - 3y^2 - 40x - 20y + 50 = 0$.
13. Solve the differential equation $(x + y + 1)\frac{dy}{dx} = 1$.
14. Find the general solution of the differential equation: $p^2 - 4xyp + 8y^2 = 0$.
15. Find the general solution of the differential equation: $(D^2 + 2D + 1)y = e^x \cos x$.
16. Newton's law of cooling states that "The temperature of an object changes at a rate proportional to the difference of temperatures between the object and its surroundings". Supposing water at a temperature 100°C cools to 80°C in 10 minutes, in a room maintained at 30°C, find when the temperature of water will become 40°C.

OR

Solve: $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x$

***

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2068 Baishakh

Exam.		Regular / Back	
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt <u>All</u> questions.*
- ✓ *<u>All</u> questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $y = a\cos(\log x) + b\sin(\log x)$. Prove that $x^2 y_{n+2} + (2n+1)x y_{n+1} + (n^2+1)y_n = 0$.
2. State and prove Rolle's theorem.
3. Determine the values of a, b, c, so that $\lim_{x \to 0} \frac{(a + b\cos x)x - c\sin x}{x^5} = 1$.
4. Find the asymptotes of the curve $(x + y)^2 (x + 2y + 2) = x + 9y - 2$.
5. If e_1 and e_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2 = 4ax$, prove that $e_1^{-2/3} + e_2^{-2/3} = (2a)^{-2/3}$.
6. Prove that $\int_0^{\pi} \frac{x\tan x}{\sec x + \cos x} dx = \frac{\pi^2}{4}$.
7. Apply the method of differentiation under integral sign to prove:

$$\int_0^{\pi/2} \frac{dx}{(a^2\sin^2 x + b^2\cos^2 x)^2} = \frac{\pi(a^2+b^2)}{4a^3b^3}.$$

8. Use Gamma function to prove that $\int_0^1 \frac{dx}{(1-x^6)^{1/6}} = \frac{\pi}{3}$.
9. Find the area bounded by the curve $x^2y = a^2(a-y)$ and the x axis.

OR

Find the volume of the solid formed by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ about its base.

10. Solve the differential equation: $(1 + y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$.
11. Solve: $xy^2(p^2 + 2) = 2py^3 + x^3$
12. solve : $(D^2 - 2D + 5)y = e^{2x}\sin x$
13. Solve the differential equation: $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$
14. What does the equation $3x^2 + 3y^2 + 2xy = 2$ becomes when the axes are turned through an angle 45° to the original axis.

OR

Describe and Sketch the graph of the conic $r = \frac{10\cos ec\theta}{2\cos ec\theta + 3}$.

15. Derive the equation of Ellipse in the standard form.
16. Find the equation of tangents to the hyperbola $3x^2 - 4y^2 = 12$ which are perpendicular to the line $x - y + 2 = 0$. Also find the point of contact.

***

01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2067 Ashadh

Exam.	Regular/Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. If $y = e^{a\tan^{-1}x}$, prove that $(1+x^2)y_{n+2} + (2nx + 2x - a)y_{n+1} + n(n+1)y_n = 0$. 5
2. State and prove Lagrange's mean value theorem.
3. Evaluate $\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$
4. Find the asymptotes of the curve $(x+y)^2(x+2y+z) = x + 9y - 2$.
5. Find the radius of curvature of the curve $r = a(1 - \cos\theta)$.
6. Apply the method of differentiation under integral sign to evaluate $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)}dx$.
7. Prove that $\int_0^{\pi/2} \frac{\sin^2 x dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$.
8. Use Gamma function to prove $\int_0^{\pi/6} \cos^4 3\theta . \sin^2 6\theta = \frac{5\pi}{192}$. 5
9. Find, by method of integration, the area of the loop of the curve $ay^2 = x^2(a - x)$.
10. Solve the differential equation $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$. 5
11. Solve $y = yp^2 + 2px$, where $p = dy/dx$ 5
12. Solve $(D^2 - 3D + 2)y = x^2 + x$ 5
13. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference of temperature between the object and its surroundings. Supposing water at 100°C cools to 80°C in 10 minutes, in a room temperature of 30°C, find when the temperature of water will become 40°C?

OR

Solve the differential equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$.

14. Find the condition that the line $lx + my + n = 0$ may be the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 5
15. Derive the equation of a hyperbola in standard form. 5
16. Find the centre, length of axes and eccentricity of the conic $2x^2 + 3y^2 - 4x - 12y + 13 = 0$.

OR 5

Identify and sketch the conic $r = \frac{10}{3 + 2\cos\theta}$.

***

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2066 Shrawan

Exam.		Regular / Back	
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Mathematics I

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt <u>All</u> questions.*
- ✓ *The figures in the margin indicate <u>Full Marks</u>.*
- ✓ *Assume suitable data if necessary.*

1. Find the angle of intersection of the pair of curves $r^n = a^n \cos n\theta$ and $r^n = a^n \sin n\theta$. [5]

OR

If $y = a\cos(\log x) + b\sin(\log x)$. Prove that $x^2 y_{n+2} + (2n+1)x\, y_{n+1} + (x^2+1)y_n = 0$

2. State Rolle's theorem and verify it for the function $f(x) = x.(x+3).e^{-(x/2)}$, $x \in [-3, 0]$ [5]

3. Evaluate: $\lim_{x \to 0} \frac{(1+x)^{-1/x} - e}{x}$ [1+4]

4. A cone is circumscribed to a sphere of radius r. Show that when the volume of the cone is least its altitude is 4r and its semivertical angle is $\sin^{-1}(1/3)$. [5]

5. Find the asymptotes of the curve $(x-y)^2.(x-2y+2) = x+9y-2$. [5]

OR

Find the radius of curvature at any point (x, y) for the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

6. Integrate any three [10]

a) $\int \frac{x e^x}{(1+x)^2} dx$ b) $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$

c) $\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}} dx$ d) $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{1+\sqrt{\cot x}} dx$

7. Evaluate $\int_1^4 x^3 dx$ by the method of summation. [5]

8. Obtain reduction formula for $\int \cot^n x\, dx$ and hence integrate $\int \cot^7 x\, dx$. [5]

OR

Using Gamma function show that $\int_0^\infty e^{-x^2} . x^2 dx \times \int_0^\infty e^{-x^4} . dx = \frac{\pi}{8\sqrt{2}}$

9. Find the area bounded by the cardioid $r = a(1 + \cos\theta)$ [5]

OR

Find the volume of the solid formed by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base.

10. Solve any three of the following differential equations. [15]

a) $x\,dy + y\,dx = \sqrt{x^2 - y^2}\,.dx$ b) $x\frac{dy}{dx} + y\log y = xy.e^x$

c) $y - 2px + ap^2.y = 0$ d) $(D^2 - 3D + 2)y = e^x$

11. If the axes be turned through an angle $\tan\theta = 2$. What does the equation $4xy - 3x^2 = a^2$ becomes? [5]

12. Find the equation of an ellipse in the standard form. [5]

13. If e_1 and e_2 are the eccentricities of the hyperbola, and it conjugate respectively. Then prove that $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$. [5]

03 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2065 Shrawan

Exam		Regular/Back	
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / I	Time	3 hrs

Subject: - Mathematics I

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Find the angle between the curves $r = a \sin 2\theta$, $r = a \cos 2\theta$. [5]

OR

If $y = (x^2 - 1)^n$, prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

2. State and prove Lagrange's mean value theorem. [5]

3. Evaluate: $\lim_{x \to 0} (\cot x)^{\frac{1}{\log x}}$ [5]

4. Find the surface of the right circular cylinder of greatest surface which can be inscribed in a sphere of radius r. [5]

5. Find the asymptotes of the curve $(x^2 - y^2)(x + 2y + 1) + x + y + 1 = 0$. [5]

OR

Show that the radius of curvature for the curve $r^m = a^m \cos m\theta$ is $\frac{\theta^m}{(m+1)r^{m-1}}$.

6. Integrate any three: [10]

a) $\int \frac{\cos x dx}{(1+\sin x)(2+\sin x)}$ b) $\int_0^{\pi/4} \frac{\sin 2\theta d\theta}{\sin^4\theta + \cos^4\theta}$

c) $\int_0^{\pi/2} \frac{\sqrt{\cot x}dx}{1+\sqrt{\cot x}}$ d) $\int_{-1}^{2} \frac{dx}{x^3}$

7. Evaluate $\int_0^1 \sqrt{x}dx$ by the method of summation. [5]

8. Obtain a reduction formula for $\int \sec^n x\, dx$ and hence find $\int \sec^6 x\, dx$. [5]

OR

Evaluate $\int_0^1 \frac{dx}{(1-x^6)^{1/6}}$

9. Find the area of a loop of the curve $a^2y^2 = a^2x^2 - x^4$. [5]

OR

Find the volume of the solid generated by revolving the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the axis of x.

10. Solve any three of the following differential equations. [15]

a) $(3y - 7x + 7)dx + (7y - 3x + 3)dy = 0$
b) $\cos x\, dy = y(\sin x - y)dx$
c) $p^2 - py + x = 0$; where $p = \frac{dy}{dx}$
d) $(D^2 - 3D + 2)y = x^2 + x$

11. Find the changed form of the equation $3x^2 + 3y^2 + 2xy = 2$ when the axes are turned through 45° the origin remaining fixed. [5]

12. The line $x + y = 0$ is a directrix of an ellipse, the point (2,2) is the corresponding focus. If the eccentricity be 1/3, find the equation of the other directrix. [5]

13. Find the equation of the hyperbola in the standard form [5]