02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2071 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem for a homogeneous function of two independent variables and verify it for the function $u = x^n \sin\left(\frac{y}{x}\right)$. [1+4]

2. Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 1$ and $xyz + 1 = 0$. [5]

3. Evaluate $\iint xy(x+y)\,dx\,dy$ over the area between $y = x^2$ and $y = x$. [5]

4. Evaluate the integral by changing to polar coordinates $\int_0^1\int_x^{\sqrt{2x-x^2}} (x^2+y^2)\,dy\,dx$. [5]

OR

Find by triple integration the volume of sphere $x^2 + y^2 + z^2 = a^2$. [5]

5. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $4x - 3y + 1 = 0 = 5x - 3z + 2$ are coplanar. Also find their point of intersection. [5]

6. Find the length and equation of the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $2x - 3y + 27 = 0, 2y - z + 20 = 0$. [5]

7. Find the centre and radius of the circle $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$, $x - 2y + 2z - 3 = 0$. [5]

8. Find the equation of right circular cone whose vertex at origin and axis the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ with the vertical angle 30°. [5]

OR

Find the equation of the right circular cylinder having for its base the circle $x^2 + y^2 + z^2 = 9, x - y + z = 3$. [5]

9. Solve by the power series method the differential equation $y'' - 4xy' + (4x^2 - 2)y = 0$. [5]

10. Test whether the solutions of $y''' - 2y'' - y' + 2y = 0$ are linearly independent or dependent. [5]

11. Show that: $J_{\left(\frac{5}{2}\right)}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{3}{x}\sin x + \frac{3-x^2}{x^2}\cos x\right)$ [5]

12. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are the reciprocal system of vectors, then prove that

$$\vec{a}'\times\vec{b}' + \vec{b}'\times\vec{c}' + \vec{c}'\times\vec{a}' = \frac{\vec{a}+\vec{b}+\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]},\quad [\vec{a}\,\vec{b}\,\vec{c}] \neq 0.$$ [5]

13. The necessary and sufficient condition for the function $\vec{a}$ of scalar variable t to have a constant direction is $\vec{a}\times\frac{d\vec{a}}{dt} = 0$. [5]

14. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of vector $2\vec{i} - \vec{j} - 2k$. [5]

OR

If $\vec{a}$ is a constant vector and $\vec{r}$ be the position vector, then, prove that $\nabla\times(\vec{a}\times\vec{r}) = 2\vec{a}$. [5]

15. Determine whether the series is convergent or divergent $\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1} - n\right)$ [5]

16. Find the interval and radius of convergence of the power series: $\sum_{n=1}^{\infty}\frac{2^n.(x-3)^n}{n+3}$. [5]

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2070 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II ***(SH451)***

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $u = \log \frac{x^2 + y^2}{x + y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$.
2. Find the extreme value of $x^2 + y^2 + z^2$ connected by the relation $ax + by + cz = p$.
3. Evaluate $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2\,dy\,dx}{\sqrt{y^4 - a^2x^2}}$ by changing order of integration.
4. Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z}\,dz\,dy\,dx$.
5. Find the length of the perpendicular from the point (3, −1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also obtain the equation of perpendicular.
6. Find the magnitude and the equation of S.D. between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $2x - 3y + 27 = 0,\ 2y - z + 20 = 0$.
7. Find the equation of the sphere through the circle $x^2 + y^2 = 4$, $z = 0$ and is intersected by the plane $x + 2y + 2z = 0$ is a circle of radius 3.

   OR

   Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 + 6x - 2z + 1 = 0$ which passes through the line $x + z - 16 = 0,\ 2y - 3z + 30 = 0$.
8. Find the equation of the right circular cone whose vertex at origin and axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ with vertical angle 30°.

   OR

   Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2}$.
9. Solve the differential equation $y'' - 4xy' + (4x^2 - 2)y = 0$ by power series method.
10. Express $f(x) = x^3 - 5x^2 + x + 2$ interms of Legendre polynomials.

11. Show that $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$.

12. Find a set of vectors reciprocal to the following vectors $2\vec{i}+3\vec{j}-\vec{k}$, $\vec{i}-\vec{j}-2\vec{k}$, $-\vec{i}+2\vec{j}+2\vec{k}$.

13. Prove that the necessary and sufficient condition for the vector function of a scalar variable t to have constant magnitude is $\vec{a}\cdot\frac{d\vec{a}}{dt}=0$.

14. A particle moves along the curve $x = 4\cos t$, $y = t^2$, $z = 2t$. Find velocity and acceleration at time $t = 0$ and $t = \frac{\pi}{2}$.

15. Test the convergence of the series $1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\dots$

16. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-1)^n(x-3)^n}{n+1}$.

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2070 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Find $\frac{du}{dt}$ if $u = \sin\left(\frac{x}{y}\right)$, $x = e^t$ & $y = t^2$

2. Find the extreme value of $x^2 + y^2 + z^2$ connected by the relation $x+z = 1$ and $2y+z = 2$

3. Evaluate: $\iint_R xy\,dx.dy$ where R is the region over the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ in the first quadrant.

4. Evaluate the integral by changing to polar coordinates $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 .\sqrt{x^2+y^2}\,.dy.dx$

   OR

   Evaluate: $\iiint x^{l-1}.y^{m-1}.z^{n-1}.dx.dy.dz$, where x,y,z are all positive but $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \le 1$

5. Find the equation of the plane through the line $2x+3y-5z = 4$ and $3x-4y+5z = 6$ and parallel to the coordinates axes.

6. Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z-3}{5}$ & $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ are coplanar. Find their point of intersection and equation of plane in which they lie.

7. Find the centre and radius of the circles $x^2+y^2+z^2-8x+4y+8z-45 = 0$, $x-2y+2z-3=0$

8. Find the equation of a right circular cone with vertex (1,1,1) and axis is the line $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ and semi vertical angle 30°.

9. Solve by power series method the differential equation $y'' + xy' + y = 0$

10. Find the general solution of the Legendre's differential equation.

11. Prove Bessel's Function $\frac{d\left[x^{-n}J_n(x)\right]}{dx} = -x^{-n}J_{n+1}$

12. Prove that: $\left[\vec{b}\times\vec{c} \;\; \vec{c}\times\vec{a} \;\; \vec{a}\times\vec{b}\right] = \left[\vec{a} \;\; \vec{b} \;\; \vec{c}\right]^2$

10/2

13. Find n so that $r^n \vec{r}$ is solonoidal.

14. Prove that the necessary and sufficient condition for a function $\vec{a}$ of scalar variable to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$

15. Test the series for convergence or divergence

$$x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \frac{15}{17}x^4 + \ldots\ldots + \frac{n^2-1}{n^2+1}x^n + \ldots\ldots(x > 0)$$

16. Find the radius of convergence and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n.2^n}$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2069 Bhadra

Exam.	Regular (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II ***(SH451)***

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $\sin u = \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$, show that $x\dfrac{\delta u}{\delta x}+y\dfrac{\delta x}{\delta y}=0$.
2. Obtain the maximum value of xyz such that $x + y + z = 24$.
3. Evaluate: $\iint xy(x+y)dxdy$ over the area between $y = x^2$ and $y - x$.
4. Evaluate $\iiint_V x^2\, dx\, dy\, dz$ over the region V bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$.
5. Find the image of the point (2, −1, 3) in the plane $3x-2y-z-9=0$.
6. Find the S.D. between the line $\dfrac{x-6}{3}=\dfrac{7-y}{1}=\dfrac{z-4}{1}$ and $\dfrac{x}{-3}=\dfrac{y+9}{2}=\dfrac{2-z}{-4}$. Find also equation of S.D.
7. Obtain the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle.
8. Find the equation of cone with vertex (3, 1, 2) and base $2x^2 + 3y^2 = 1$, $z = 1$.

   OR

   Find the equation of right circular cylinder whose axis is the line $\dfrac{x-\alpha}{\ell}=\dfrac{y-\beta}{m}=\dfrac{z-r}{n}$ and whose radius 'r'
9. Solve the initial value problem $y'' + 2y' + 5y = 0$, given $y(0) = 1$, $y'(0) = 5$.
10. Define power series. Solve by power series method of differential equation, $y' + 2xy = 0$.
11. Prove the Bessell's function $\dfrac{d}{dx}\left[x^n J_n(x)\right]= x^n J_{n-1}(x)$.
12. Prove if $\vec{\ell}, \vec{m}, \vec{n}$ be three non-coplanar vectors then

$$\left[\vec{\ell}\,\vec{m}\,\vec{n}\right]\left(\vec{a}\times\vec{b}\right)=\begin{vmatrix} \vec{\ell}.\vec{a} & \vec{\ell}.\vec{b} & \vec{\ell} \\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m} \\ \vec{n}.\vec{a} & \vec{n}.\vec{b} & \vec{n} \end{vmatrix}.$$

3

13. Prove that the necessary and sufficient condition for the vector function of a scalar variable t have a constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$.

14. Find the angle between the normal to the surfaces $x \log z = y^2 - 1$ and $x^2 y + z = 2$ at the point (1, 1, 1).

15. Test the convergence of the series $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \ldots\ldots\ldots\ldots$

16. Find the interval of cgt, radius of cgt and centre of cgt of power series $\sum \frac{2^n x^n}{n!}$

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02 TRIBHUVAN UNIVERSITY	Exam.	New Back (2066 & Later Batch)		
INSTITUTE OF ENGINEERING	Level	BE	Full Marks	80
Examination Control Division	Programme	All except B.Arch.	Pass Marks	32
2069 Poush	Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *__All__ questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem on homogeneous functions of two independent variables. And if $\text{Sin } u = \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ then prove $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=0$

2. Find the minimum value of the function $F(x,y,z)=x^2+y^2+z^2$ when $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=1$

3. Evaluate: $\iint r^3 dr\, d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$

4. Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dz\, dx\, dy$

   OR

   Find the volume of sphere $x^2+y^2+z^2=a^2$ using Diritchlet's integral.

5. Prove that the lines

   $\dfrac{x+1}{-3}=\dfrac{y-3}{2}=\dfrac{z+2}{1}$ and $x=\dfrac{y-7}{-3}=\dfrac{z+7}{2}$ are coplanar and find the equation of plane in which they lie.

6. Show that the shortest distance between two skew lines

   $\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ and $\dfrac{x-2}{3}=\dfrac{y-4}{4}=\dfrac{z-5}{5}$ is $1/\sqrt{6}$

7. A variable plane is parallel to the plane $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ and meets the axes in A, B, C.

   Prove that the circle ABC lies on the cone $\left(\dfrac{b}{c}+\dfrac{c}{b}\right)yz+\left(\dfrac{c}{a}+\dfrac{a}{c}\right)zx+\left(\dfrac{a}{b}+\dfrac{b}{a}\right)xy=0$

8. Find the equation of the right circular cylinder of radius 4 and axis the line $x = 2y = -z$

9. Show that the solutions of $x^2y''' - 3xy'' + 3y' = 0, (x > 0)$ are linearly independent.

OR

Solve the equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 4)y = 0$ in series form.

10. Prove that $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$ where the symbols have their usual meanings.

11. Apply the power series method to the following differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

OR

Find the general solution of Legendre's differential equation.

12. Show that $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}) = \left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{c}$ and deduce $\left[\vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\ \ \vec{a}\times\vec{b}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

13. Prove that the necessary and sufficient condition for the function $\vec{a}$ of scalar variable to have a constant direction is $\vec{a}\times\frac{d\vec{a}}{dt} = 0$

14. Find the angle between the surface $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$ at the point (2,-1,2)

15. Test the convergence of the series $\sum\frac{(n+1)^n x^n}{n^{n+1}}$

16. Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-1)^n (x-1)^2}{\sqrt{n}}$

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2069 Poush

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All except B.Arch.	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem on homogeneous functions of two independent variables. And if $\sin u = \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ then prove $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$

2. Find the minimum value of the function $F(x,y,z)=x^2-y^2+z^2$ when $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

3. Evaluate: $\iint r^3 dr\, d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$

4. Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\, dz\, dx\, dy$

   OR

   Find the volume of sphere $x^2+y^2+z^2 = a^2$ using Dirichlet's integral.

5. Prove that the lines

   $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and $x=\frac{y-7}{-3}=\frac{z+7}{2}$ are coplanar and find the equation of plane in which they lie.

6. Show that the shortest distance between two skew lines

   $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is $1/\sqrt{6}$

7. A variable plane is parallel to the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and meets the axes in A, B, C. Prove that the circle ABC lies on the cone $\left(\frac{b}{c}+\frac{c}{b}\right)yz+\left(\frac{c}{a}+\frac{a}{c}\right)zx+\left(\frac{a}{b}+\frac{b}{a}\right)xy=0$

8. Find the equation of the right circular cylinder of radius 4 and axis the line $x = 2y = -z$

9. Show that the solutions of $x^2y'''-3xy''+3y'=0, (x>0)$ are linearly independent.

OR

Solve the equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-4)y=0$ in series form.

10. Prove that $4J''_n(x)=J_{n-2}(x)-2J_n(x)+J_{n+2}(x)$ where the symbols have their usual meanings.

11. Apply the power series method to the following differential equation $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

OR

Find the general solution of Legendre's differential equation.

12. Show that $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})=\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{c}$ and deduce $\left[\vec{b}\times\vec{c}\;\;\vec{c}\times\vec{a}\;\;\vec{a}\times\vec{b}\right]=\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$

13. Prove that the necessary and sufficient condition for the function $\vec{a}$ of scalar variable to have a constant direction is $\vec{a}\times\frac{d\vec{a}}{dt}=0$

14. Find the angle between the surface $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$

15. Test the convergence of the series $\sum\frac{(n+1)^n x^n}{n^{n+1}}$

16. Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-1)^n(x-1)^2}{\sqrt{n}}$

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2068 Bhadra

Exam.		**Regular**	
Level	BE	**Full Marks**	80
Programme	All (Except B.Arch.)	**Pass Marks**	32
Year / Part	I / II	**Time**	3 hrs.

Subject: - Engineering Mathematics II

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem for homogeneous function of two variables. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\operatorname{Cot} u$. [1+4]

2. Find the minimum value of $x^2 + xy + y^2 + 3z^2$ under the condition $x + 2y + 4z = 60$. [5]

3. Change the order of integration and hence evaluate the same.

   $$\int_0^a \int_0^x \frac{\cos y \, dy dx}{\sqrt{(a-x)(a-y)}}$$ [5]

4. Find by double integration, the volume bounded by the plane $z = 0$, surface $z = x^2 + y^2 + 2$ and the cylinder $x^2 + y^2 = 4$. [5]

5. Prove that the plane through the point (α, β, γ) and the line $x = py + q = rz + s$ is given by:

   $$\begin{vmatrix} x & py+q & rz+s \\ \alpha & p\beta+q & r\gamma+s \\ 1 & 1 & 1 \end{vmatrix} = 0.$$ [5]

6. Find the magnitude and equation of the shortest distance between the lines: [5]

   $$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

7. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 - 3x + 4y - 2z - 5 = 0$, $5x - 2y + 4z + 7 = 0$ as a great circle. [5]

   OR

   Find the equation which touches the sphere $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ at $(1, 2, -2)$ and passes through the point $(1, -1, 0)$. [5]

8. Find the equation of the cone with vertex (α, β, γ) and base $y^2 = 4ax$, $z = 0$ [5]

9. Solve the initial value problem

   $y'' - 4y' + 3y = 10e^{-2x}$, $y(0) = 1$, $y'(0) = 3$. [5]

10. Solve by power series method the differential equation $y'' - 4xy' + (4x^2 - 2)y = 0$. [5]

11. Express $f(x) = x^3 - 5x^2 + 6x + 1$ in terms of Legendre's polynomials. [5]

OR

Prove that $\frac{d}{dx}\left[x^{-n} J_n(x)\right] = -x^{-n} J_{n+1}(x)$. [5]

12. Find a set of vectors reciprocal to the following vectors: [5]

$-\vec{i}+\vec{j}+\vec{k},\ \vec{i}-\vec{j}+\vec{k},\ \vec{i}+\vec{j}-\vec{k}$

13. Prove that $\vec{b}\times\vec{c},\ \vec{c}\times\vec{a}$ and $\vec{a}\times\vec{b}$ are coplanar or non-coplanar according as $\vec{a},\ \vec{b},\ \vec{c}$ are coplanar or non-coplanar.

14. Prove that curl $(\vec{a}\times\vec{b}) = \vec{a}\operatorname{div}\vec{b} - (\vec{a}.\nabla)\vec{b}$ [5]

OR

If $u = x + y + z$, $v = x^2 + y^2 + z^2$ and $w = xy + yz + zx$, show that [gradu gradv gradew] = 0

15. Test the convergence of the series: [5]

$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \ldots\ldots\ldots + \frac{(n+1)}{n^3}x^n + \ldots\ldots\ldots$

16. Find the radius of convergence and the interval of convergence of the power series: [5]

$\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{n+1}$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2067 Mangsir

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's Theorem for a homogeneous function of two independent variables and verify it for the function: [1+4]

$$u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$$

2. Find the extreme value of $\phi = x^2+y^2+z^2$ connected by the relation $ax + by + cz = p$ [5]

3. Evaluate: $\iint_R xy\,dx\,dy$ where R is the region over the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant. [5]

4. Transform to polar coordinates and complete the integral $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2)\,dy\,dx$. [5]

OR

Evaluate: $\iiint x^{l-1}.y^{m-1}.z^{n-1}\,dx\,dy\,dz$

where x, y, z are all positive but $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1$.

5. Find the length of perpendicular from the point (3, −1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also obtain the equation of the perpendicular. [5]

6. Find the length and equation of the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$; $2x-3y+27 = 0 = 2y-z+20$. [5]

7. Find the centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$ is cut by the plane $x - 2y + 2z = 3$. [5]

8. Plane through OX and OY include an angle α. Show that their line of intersection lies on the cone $z^2(x^2 + y^2 + z^2) = x^2y^2 \tan^2 \alpha$. [5]

OR

Find the equation of the right circular cylinder whose guiding curve is the circle $x^2 + y^2 + z^2 - x - y - z = 0, x + y + z = 1$.

9. Solve in series: [5]

$$(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$$

10. Show that: [5]

$$J_{\frac{5}{2}}(x)=\sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x-\frac{3}{x}\cos x\right)$$

11. Show that: [5]

$$P_n(x)=\frac{1}{2^n n}\frac{d^n}{dx^n}\left(x^2-1\right)^n$$

12. Prove that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})+(\vec{a}\times\vec{c})\times(\vec{d}\times\vec{b})+(\vec{a}\times\vec{d})\times(\vec{b}\times\vec{c})=-2\times[\vec{b}\ \vec{c}\ \vec{d}\,]\,\vec{a}$ [5]

13. Prove that the necessary and sufficient condition for the vector function $\vec{a}$ of scalar variable λ to have a constant magnitude is $\left(\vec{a}\,\frac{d\vec{a}}{dt}\right)=0$. [5]

14. Apply the power series method to solve following differential equation [5]

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$$

15. Test the convergence of the series $\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+\ldots\ldots$ [5]

16. Show that $J_4(x)=\left(\frac{48}{x^3}-\frac{3}{x}\right)J_1(x)+\left(1-\frac{24}{x^2}\right)J_0(x)$. [5]

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03 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2067 Chaitra

Exam.	New Back (2066 Batch Only)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks.***
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem of homogeneous equation of two variables. If $u = \sin^{-1}\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$. Show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0$. [1+4]

2. Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 1$. [5]

3. Evaluate $\iint_R xy\,dx\,dy$ where R is the region over the area of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ in the first quadrant. [5]

4. Evaluate the integral by changing to polar co-ordinates. $\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2 + y^2)\,dy\,dx$.

OR

Find by triple integral, the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. [5]

5. Prove that $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}) = [\vec{a}\ \vec{b}\ \vec{c}]\,\vec{c}$ and deduce that $[\vec{b}\times\vec{c},\ \vec{c}\times\vec{a},\ \vec{a}\times\vec{b}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$. [5]

6. Prove that the necessary and sufficient condition for the vector function of a scalar variable t have constant magnitude is $\vec{a}\cdot\dfrac{d\vec{a}}{dt} = 0$. [5]

7. The position vector of a moving particle at any point is given by $\vec{r} = (t^2+1)\,\vec{i} + (4t-3)\,\vec{j} + (2t^2-6)\,\vec{k}$. Find the velocity and acceleration at $t = 1$. Also obtain the magnitudes. [5]

8. Prove that the lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular if $aa' + cc' + 1 = 0$. [5]

9. Prove that the lines $\dfrac{x-1}{2} = \dfrac{y+1}{-3} = \dfrac{z+10}{8}$ and $\dfrac{x-4}{1} = \dfrac{y+3}{-4} = \dfrac{z+1}{7}$ intersect. Find also their point of intersection and plane through them. [5]

10. Find the centre and radius of the circle $x^2 + y^2 + z^2 + x + y + z = 4$, $x + y + z = 0$. [5]

11. Show that the equation of a cone whose vertex is (α, β, γ) and base the parabola $z^2 = 4ax, y = 0$ is $(\beta z - \gamma y)^2 = 4a(\beta - y)(\beta x - \alpha y)$. [5]

OR

Find the equation of the right circular cylinder of radius 4 and axes of the line $x = 2y = -z$.

12. Test the convergence of the series $\frac{2}{1^p} + \frac{3}{2^p} + \frac{4}{3^p} + \frac{5}{4^p} + \frac{6}{5^p} + \ldots\ldots$ [5]

13. Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{(n+1)}$. [5]

14. Solve $(x+a)^2 \frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$. [5]

15. Solve the initial value problem
$y'' + y' - 2y = -6\sin 2x - 18\cos 2x = 0,\ y(0) = 0,\ y'(0) = 0.$ [5]

16. Show that $J_{-n}(x) = (-1)^n J_n(x)$.

OR

Find the general solution of Legendre's differential equation. [5]

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