01 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2071 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Using the properties, evaluate the determinant: [5]

$$\begin{vmatrix} 1 & a & a^2 & a^3+bcd \\ 1 & b & b^2 & b^3+cda \\ 1 & c & c^2 & c^3+abd \\ 1 & d & d^2 & d^3+abc \end{vmatrix}$$

2. Prove that every square matrix can uniquely be expressed as the sum of a symmetric and a skew symmetric matrix. [5]

3. Test the consistency of the system: [5]

   $x-6y-z=10,\ 2x-2y+3z=10,\ 3x-8y+2z=20$

   And solve completely, if found consistent.

4. Find the eigen values and eigenvecters of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$. [5]

5. Using the line integral, compute the workdone by the force [5]

   $\vec{F}=(2x-y+2z)\vec{i}+(x+y-z)\vec{j}+(3x-2y-5z)\vec{k}$

   when it moves once around a circle $x^2+y^2=4; z=0$

6. State and prove Green's Theorem in plane. [5]

7. Verify Stoke's theorem for $\vec{F}=(x^2+y^2)\vec{i}-2xy\vec{j}$ taken around the rectangle bounded by the lines $x=\pm a$, $y=0$, $y=b$. [5]

8. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$ where $\vec{F}=(2xy+z)\vec{i}+y^2\vec{j}-(x+3y)\vec{K}$ by Gauss divergence theorem; where S is surface of the plane $2x+2y+z=6$ in the first octant bounding the volume V. [5]

9. Find the Laplace transform of the following: [2.5×2]

   a) $te^{-2t}\cos t$

   b) Sinhat.cost

10. Find the inverse Laplace transform of : [2.5×2]

a) $\dfrac{1}{S(S+1)}$

b) $\dfrac{S^2}{(S^2+b^2)^2}$

11. Solve the differential equation $y''+2y'+5y = e^{-t}\sin t, y(0)=0, y'(0)=1$, by using Laplace transform. [5]

12. Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \le x \le \pi$. [5]

13. Obtain half range sine series for the function $f(x) = x - x^2$ for $0 < x < 1$. [5]

14. Graphically maximize and minimize [5]

$z = 9x + 40y$ subjected to the constraints

$y - x \ge 1, y - x \le 3, 2 \le x \le 5$

15. Solve the following Linear Programming Problem by Simplex method: [10]

Maximize, $P = 20x_2 - 5x_1$

Subjected to, $10x_2 - 2x_1 \le 5$

$2x_1 + 5x_2 \le 10$ and $x_1, x_2 \ge 0$

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01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2070 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Using the properties of determinant prove [5]

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

2. Prove that $(AB)^T = B^T A^T$ where A is the matrix of size $m\times p$ and B is the matrix of size $p\times n$ [5]

3. Find the rank of the following matrix by reducing normal form. $\begin{bmatrix} 1 & 3 & -2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{bmatrix}$ [5]

4. Find the eigen values and eigen vectors of the following matrix. $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$ [5]

5. Prove that the line intergral $\int_A^B \vec{F}.d\vec{r}$ is independent of the path joining any two points A and B in a region if $\int_C \vec{F}.d\vec{r} = 0$ for any simple closed curve C in the region. [5]

6. Evaluate $\iint_S \vec{F}.\hat{n}\, ds$ where $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and S is the finite plane $x + y + z = 1$ between the coordinate planes. [5]

OR

Evaluate $\iint_S \vec{F}.\hat{n}\, ds$ for $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ where S is the surface of sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

7. Evaluate, $\iint_S \vec{F}.\hat{n}\, ds$ for $\vec{F} = x\vec{i} - y\vec{j} + (z^2 - 1)\vec{k}$ where S is the surface bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 1$ [5]

8. Verify the stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the upper part of the sphere $x^2 + y^2 + z^2 = a^2$ C is its boundary. [5]

9. Find the Laplace transform of (a) $t^2 \sin zt$ and (b) $\frac{1-e^t}{t}$ [2.5×2]

10. Find the inverse Laplace transform of (a) $\frac{2s+3}{s^2+5s-6}$ (b) $\frac{s^3}{s^4-a^4}$ [2.5×2]

11. Solve the following differential equation by using Laplace transform [5]

$y'' + y' - 2y = x$, $y(0) = 1$, $y'(0) = 0$

12. Obtain the Fourior series for $f(x) = x^2$ in the interval $-\pi < x < \pi$ and hence prove that

$$\sum \frac{1}{x^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \ldots\ldots = \frac{\pi^2}{6}$$ [5]

13. Obtain half range sine series for $f(x) = \pi x - x^2$ in $(0, \pi)$ [5]

14. Graphically minimize $z = 4x_1 + 3x_2 + x_3$ [5]

Subject to $x_1 + 2x_2 + 4x_3 \geq 12$

$3x_1 + 2x_2 + x_3 \geq 8$ and $x_1, x_2, x_3 \geq 0$

15. Minimize $z = 8x_1 + 9x_2$ [10]

Subject to $x_1 + 3x_2 \geq 4$

$2x_1 + x_2 \geq 5$ with $x_1, x_2 \geq 0$

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62 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2069 Ashad

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B. Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *The figures in the margin indicate* ***Full Marks.***
- ✓ *Assume suitable data if necessary.*

1. Find the value of the determinant: [5]

$$\begin{vmatrix} 1 & a & a^2 & a^3+bcd \\ 1 & b & b^2 & b^3+cda \\ 1 & c & c^2 & c^3+dab \\ 1 & d & d^2 & d^3+abc \end{vmatrix}$$

2. Prove that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrices. [5]

3. Find the rank of matrix: $\begin{bmatrix} 1 & 3 & -2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{bmatrix}$ reducing to echelon form. [5]

4. Verify Cayley-Hamiltan theorem for the matrix: $\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ [5]

5. Find the Laplace transforms of: (a) $te^{-t}\sin t$ (b) $\dfrac{e^{at}-\cos 6t}{t}$ [5]

6. If $L[f(t)]=F(s)$, then prove that $L[f^1(t)]=SF(s)-f(o)$. [5]

7. Use Laplace transform to solve: $x''+2x'+5x=e^{-t}\sin t$ given $x(0)=0$; $x'(0)=1$. [5]

8. Obtain the Fourier series for $f(x)=x^3$ in the interval $-\pi\le x\le\pi$. [5]

9. Obtain half-range sine series for e^x in (0, 1). [5]

10. Maximize $z=2x_1+3x_2$ subject to constraints $x_1-x_2\le 2$, $x_1+x_2\ge 4$ and $x_1, x_2\ge 0$ graphically. [5]

11. Solve the linear programming problems by simplex method constructing the duality [10]

Minimize $Z = 3x_1+2x_2$
Subject to $2x_1+4x_2\ge 10$
$4x_1+2x_2\ge 10$
$x_2\ge 4$ and $x_1, x_2\ge 0$

12. Prove that $\vec{F}=(2xz^3+6y)\vec{i}+(6x-2yz)\vec{j}+(3x^2z^2-y^2)\vec{k}$ is conservative vector field and find its scalar potential function. [5]

13. Evaluate $\iint_S \vec{F}.\hat{n}\,ds$ where $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ and S is the finite plane x+y+z=1 between the co-ordinate planes. [5]

14. Using Green's theorem, find the area of the hypocycloid $\frac{x^{2/3}}{a^{2/3}}+\frac{y^{2/3}}{b^{2/3}}=1$. [5]

15. Evaluate $\iint_S \vec{F}.\hat{n}\,ds$ where $\vec{F}=2x\vec{i}+3y\vec{j}+4z\vec{k}$ and S is the surface of sphere $x^2+y^2+z^2=1$ by Gauss divergence theorem. [5]

OR

Verify Stoke's theorem for $\vec{F}=2y\vec{i}+3x\vec{j}-z^2\vec{k}$ where S is the upper half of the sphere $x^2+y^2+z^2=9$ and 'C' is its boundary. [5]

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2069 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks.***
- ✓ *Assume suitable data if necessary.*

1. Find the value of the determinant $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$ [5]

2. Show that the matrix $B^\theta AB$ is Hermitian or skew-Hermitian according as A is Hermitian and skew- Hermitian. [5]

3. Find the rank of the matrix $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$ reducing this into the triangular form. [5]

4. Obtain the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and verify that it is satisfied by A. [5]

5. Evaluate $\int_C \vec{F}.d\vec{r}$, where $\vec{F} = (x-y)\vec{i} + (x+y)\vec{j}$ along the closed curve C bounded by $y^2 = x$ and $x^2 = y$ [5]

6. Find the value of the normal surface integral $\iint_S \vec{F}.\vec{n}\,ds$ for $\vec{F} = x\vec{i} - y\vec{j} + (z^2-1)\vec{k}$, where S is the surface bounded by the cylinder $x^2+y^2 = 4$ between the planes Z = 0 and Z = 1. [5]

7. Using Green's theorem, find the area of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ [5]

8. Verify stoke's theorem for $\vec{F} = 2y\vec{i} + 3x\vec{j} - z^2\vec{k}$ where S is the upper half of the sphere $x^2+y^2+z^2 = 9$ and C is its boundary. [5]

OR

Evaluate the volume intergral $\iiint_V \vec{F}\,dv$, where V is the region bounded by the surface $x = 0,\ y = 0,\ y = 6,\ z = x^2,\ z = 4$ and $\vec{F} = 2xz\vec{i} - x\vec{j} + y^2\vec{k}$

9. Find the Laplace transforms of the following functions [2.5×2]
   a) $t\,e^{-4t}\sin 3t$
   b) $\dfrac{\cos at - \cos bt}{t}$

10. State and prove the second shifting theorem of the Laplace transform. [5]

11. Solve the following differential equation using Laplace transform. [5]

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=x \text{ given } y(0)=1,\ y'(0)=0$$

12. Obtain the Fourier series for $f(x) = x^2$ in the interval $-\pi < x < \pi$ and hence show that

$$\sum\frac{1}{n^2}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots\dots=\frac{\pi^2}{6}$$ [5]

13. Express $f(x) = x$ as a half-range sine series in $0 < x < 2$ [5]

14. Maximize $Z = 4x_1+5x_2$ subject to constraints [5]

$2x_1 + 5x_2 \le 25$

$6x_1 + 5x_2 \le 45$

$x_1 \ge 0$ and $x_2 \ge 0$

$\frac{4\times 45}{6}$ $(0, 9)$

graphically

15. Solve the following linear programming problem using the simplex method. [10]

Maximize $P = 50x_1+80x_2$

Subject to $x_1+ 2x_2 \le 32$

$3x_1 + 4x_2 \le 84$

$x_1, x_2 \ge 0$

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2068 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BCE, BEL, BEX, BCT, BME, BIE, B. AGRI.	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Prove that: $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$. [5]

2. Define Hermition and Skew Hermition matrix. Show that every square matrix can be uniquely expressed as the sum of a Hermition and a skew Hermition. [5]

3. For what value of λ the equation $x + y + z = 1$, $x + 4y + 10z = \lambda^2$ and $x + 2y + 4z = \lambda$ have a solution? Solve them completely in each case. [5]

4. Find the eigen values and eigen vectors of $A = \begin{vmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{vmatrix}$. [5]

5. Evaluate $\int_C \vec{F}.\vec{dr}$, Where C: $x^2 = y$ and $y^2 = x$ and $\vec{F} = (x-y)\vec{i} + (x+y)\vec{j}$. [5]

6. State and prove Green theorem in a plane. [5]

7. Verify Guess divergence theorem for $\vec{F} = x^2\vec{i} + 3\vec{j} + yz\vec{k}$. Taken over the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. [5]

8. Find the Laplace transform of the given function (i) $t^2 \sin t$ (ii) $\cos at \sinh at$ [5]

9. Evaluate $\iint_s \vec{F}.\hat{n}ds$ where $\vec{F} = 3\vec{i} + x\vec{j} - yz\vec{k}$ and s is the surface of the cylinder $x^2 + y^2 = 9$ included in the first octant between the plane $z = 0$, $z = 4$. [5]

10. Find the inverse Laplace transform: (a) $\frac{1}{(S-2)(S+4)}$ (b) $\log\left(\frac{s^2+a^2}{s^2}\right)$ [5]

11. Solve the equation using Laplace transform $y'' + 4y' + 3y = t$, $t>0$ $y(0) = 0$, $y'(0) = 1$. [5]

13

12. Obtain a Fourier series to represent the function f(x) = /x/ for $-\pi \le x \le \pi$ and hence deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$ [5]

13. Obtain the half Range Sine Series f(x) = ex in 0<x<1. [5]

OR

Obtain the Fourier series for $f(x) = x - x^2$ where $-1 < x < 1$ as a Fourier series of period 2.

14. Solve the following by using the simplex method: [7.5]

Maximize $P = 15x_1 + 10x_2$,
Subject to
$2x_1 + x_2 \le 10$,
$x_1 + 3x_2 \le 10$,
$x_1, x_2 \ge 0$.

15. Solve by using the dual method: [7.5]

Minimize $C = 21x_1 + 50x_2$,
Subject to $2x_1 + 5x_2 \le 12$,
$3x_1 + 7x_2 \le 17$,
$x_1, x_2 \ge 0$.

OR

Solve the following LPP by using the big M-method:

Maximize $P = 2x_1 + x_2$,
Subject to
$x_1 + x_2 \le 10$,
$-x_1 + x_2 \ge 2$,
$x_1, x_2 \ge 0$.

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2068 Baishakh

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Using the properties of determinant prove that: [5]

$$\begin{vmatrix} x & 1 & y & 1 \\ 1 & y & 1 & x \\ 1 & x & 1 & y \\ y & 1 & x & 1 \end{vmatrix} = (x+y+2)(x-y)^2(x+y-2)$$

2. If A and B are two non singular matrices of the same order, prove that $(AB)^{-1} = B^{-1}.A^{-1}$. [5]

3. Find the rank of the following matrix reducing to normal form $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$. [5]

4. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$. [5]

5. Find the Laplace transform of the following functions: [5]

   a) $te^{-3t}\cos 2t$ b) $\dfrac{e^{3t} - \cos 6t}{t}$

6. Find the inverse Laplace transform of the following functions: [5]

   a) $\dfrac{1}{(s-2)(s+2)^2}$ b) $\dfrac{1}{s^2(s+2)}$

7. Solve using Laplace transform $(D^2 + 4D + 3)x = e^{-t}$, where $x(0) = x'(0) = 1$. [5]

8. Obtain a Fourier series for $f(x) = x^3$ in the interval $-\pi \le x \le \pi$. [5]

9. Find the half range sine series for the function $f(x) = x - x^2$ in the interval $0 < x < 1$. [5]

10. Maximize $Z = x_1 + 1.5\,x_2$ subject to constraints [5]

    $2x_1 + 2x_2 \le 160$
    $x_1 + 2x_2 \le 120$
    $4x_1 + 2x_2 \le 280$
    $x_1 \ge 0$ and $x_2 \ge 0$ graphically.

11. Solve the following linear programming problems by simplex method [10]

Maximize $Z = 15x_1 + 10x_2$

Subject to $2x_1 + 2x_2 \leq 10$

$x_1 + 3x_2 \leq 10$ and $x_1, x_2 \geq 0$

12. Show that the vector field $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational. Find the sector function $\varphi(x, y, z)$ such that $\vec{F} = \nabla\varphi$. [5]

13. If S be the part of the surface $Z = 9 - x^2 - y^2$ with $Z \geq 0$ and $\vec{F} = 3x\,\bar{i} + 3y\bar{j} + Z\bar{k}$, find the flux of F through S. [5]

14. State and prove that Green's theorem in the plane. [5]

15. Evaluate by Stoke's theorem: [5]

$$\int_c (e^x dx + 2y dy - dz)$$

Where c is the curve: $x^2 + y^2 = 4$, $z = 2$.

OR

Verify Gauss divergence theorem for the vector function $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$, taken over the unit cube bounded by the planes: $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

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05 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING

Examination Control Division

2067 Ashadh

Exam.	Regular/Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Mathematics III

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Using the properties of determinant prove:

$$\begin{vmatrix} a^2+1 & ba & ca & da \\ ab & b^2+1 & cb & db \\ ac & bc & c^2+1 & dc \\ ad & bd & cd & d^2+1 \end{vmatrix} = a^2+b^2+c^2+d^2+1$$

2. Show that every square matrix can be uniquely expressed as the sum of hermitian and a skew-hermitian matrix.

3. Reduce to normal form and find the rank of the matrix:

$$\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

4. Find the eigen values and eigne vectors of the matrix

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

5. Find the Laplace transform of:

a) cosh at sin at   b) $\dfrac{\cos 2t - \cos 3t}{t}$

6. Find the inverse Laplace transform of:

a) $\dfrac{1}{s^2(s^2+a^2)}$ b) $\log\dfrac{s+1}{s-1}$

7. State and prove the integral theorem of the Laplace transform.

8. Solve the following differential equation using the Laplace transform.

$y''' + 2y'' - y' - 2y = 0$ where $y(0) = y'(0) = 0$ and $y''(0) = 6$

9. Find a Fourier series to represent $x - x^2$ [illegible]. Hence show that

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$$

10. Express $f(x) = x$ as a cosine half range series in $0 < x < 2$.

11. The acceleration of a moving particle at any time t is given by $\frac{d^2\vec{r}}{dt^2} = 12\cos 2t\hat{i} - 8\sin 2t\hat{j} + 16t\hat{k}$. Find the velocity $\vec{v}$ and displacement $\vec{r}$ at anytime t if

$t = 0$, $\vec{v} = 0$ and $\vec{r} = 0$.

12. Find the angle between the normals to the surface $xy = z^2$ at the points $(1,4,2)$ and $(-3,-3,3)$

13. Find the work done in moving a particle once round the circle $x^2 + y^2 = 9$, $z = 0$ under the force field $\vec{F}$ given by $\vec{F} = (2x - y + z)\vec{i} + (x + y - z^2)\vec{j} + (3x - 2y + 4z)\vec{k}$.

14. Evaluate $\iint_s \vec{F}.\vec{n}\, ds$ where s is the upper side of triangle with vertices $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ where $\vec{F} = \cancel{(x-2z)}\vec{i} + (x + 3y + z)\vec{j} + (5x + y)\vec{k}$.

15. State Green's theorem in a plane. Using Green's theorem find the area of $x^{2/3} + y^{2/3} = a^{2/3}$.

16. Verify Stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where s is the upper part of the sphere $x^2 + y^2 + z^2 = a^2$ and c is its boundary.

OR

Verify Gauss theorem for $\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$ over the region bounded by $x^2 + y^2 = a^2$, $z = 0$ and $z = h$.

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04 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2067 Magh

Exam.		Back	
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Mathematics III

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Show that $\begin{vmatrix} a & b & b & b \\ a & b & a & a \\ a & a & b & a \\ b & b & b & a \end{vmatrix} = -(b-a)^4$.

2. If P and Q are two orthogonal matrices of the same order, prove that their product is also orthogonal.

3. Reducing to normal form, find the rank of matrix $\begin{vmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{vmatrix}$

4. Find the eigen values and eigen vectors of the matrix $\begin{vmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{vmatrix}$.

5. Find a Fourier series for $f(x) = x^3$, $-\pi < x < \pi$.

6. Find the half range sine series for the function $f(x) = e^x$ for $0 < x < \pi$.

7. Find the Laplace transform of

   a) $t^2 \cos at$

   b) $t^3 e^{3t}$

8. Find the Inverse Laplace transform of

   a) $\frac{s}{(s-3)(s^2+4)}$

   b) $\log \frac{s(s+1)}{(s^2+4)}$

9. If $L\{f(t)\} = F(s)$, then prove $L\{e^{at} f(t)\} = F(s-a)$.

10. Use the Laplace transform to solve $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+3y=e^{-t}$, $y(0)=y^1(0)=1$.

11. The position vector of a moving particle at any time t is given by $\vec{r}=(t^2-1)\vec{i}+(4t-3)\vec{j}+(2t^2-6)\vec{k}$. Find the velocity and acceleration at t = 1. Also find their magnitudes.

12. Define divergence and curl of $\vec{V}$. Prove that div(Curl $\vec{V}$) = 0.

13. Evaluate $\int_c \vec{F}.\vec{dr}$ where $\vec{F}=Z\vec{i}+x\vec{j}+y\vec{k}$ and C is the arc of curve, $x=t^2+1$, $y=2t^2$, $z=t^2$ from t = 1 to t = 2.

14. Evaluate $\iint_s \vec{F}.\vec{n}\,ds$ where $\vec{F}=x\vec{i}-y\vec{j}+z\vec{k}$ and S is the outside of the lateral surface of circular cylinder, $x^2+y^2=a^2$ between planes z = 0 and z = 4.

15. Use Green's theorem to find the area of ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

16. Verify Stoke's theorem for $\vec{F}=x\vec{i}-z^2\vec{j}+y^2\vec{k}$ over the plane surface x + y + z = 1 lying in first octant.

OR

Verify Gauss's theorem for $\vec{F}=4x\vec{i}-2y^2\vec{j}+z^2\vec{k}$ taken over the region bounded by $x^2+y^2=4$, z =0 and z =3.

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