Subject: - Discrete Structure (CT551)

✓ Candidates are required to give their answers in their own words as far as practicable.
✓ Attempt **All** questions.
✓ The figures in the margin indicate **Full Marks**.
✓ Assume suitable data if necessary.

1. Using resolution principle, prove that the hypotheses "If today is Tuesday then I will have a test in Discrete Structure or Microprocessor". If my Microprocessor teacher is sick then I will not have a test in Microprocessor." and "Today is Tuesday and my Microprocessor teacher is sick." lead to the conclusion that "I will have a test in Discrete Structure". [8]

2. Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction. Draw the tableau for the formula $(T \vee S) \rightarrow \neg Q$ where $\neg$ denotes the negation of variable, $\vee$ denotes the disjunction of variables and $\rightarrow$ is the symbol for implication. [5+3]

3. State the contrapositive and inverse of the conditional statement, "If it snows tonight then I will stay at home". Using mathematical induction technique, prove that the following statement is true: $3+3*5+3*5^2+....3*5^n = 3(5^{n-1}-1)/4$ whenever n is nonnegative integer. [2+6]

4. Differentiate between a Finite State Machine and a Finite State Automation. Design a Finite State Automata that accepts precisely those string over {a,b} that contains an even no. of a's. Your design should include the proper definition of the Finite State Automata, transition table and the transition diagram. [2+6]

5. Consider the regular grammar $G = (N, T, P, \sigma)$ where N = set of non-terminal symbols = {σ, C}, T = set of terminal symbols = {a,b}, P is the set of production rules = {$\sigma \rightarrow b\sigma$, $\sigma \rightarrow aC$, $C \rightarrow bC$, $C \rightarrow b$} and σ being the starting symbol. Construct a non-deterministic finite state automaton equivalent to given regular grammar. Use this non-deterministic finite state automaton to generate equivalent deterministic finite state automaton. [4+4]

6. Find all the solutions of the recurrence relation: [8]
$a_n = 5a_{n-1} - 6a_{n-2} + 2^n$ with initial conditions $a_0 = 1$ and $a_1 = 4$

7. Explain the Euler path and Euler circuit with the help of a diagram. State the necessary and the sufficient conditions for Euler circuits and paths. [5+3]

8. Draw neat and clean graphs of: C_7 (a cycle with 7 vertices), K_5 (a complete graph with 5 vertices), Q_3 (a 3 dimensional hypercube) and $K_{3,4}$ (complete bipartite graph). Use graph coloring technique to color each of these graphs and state their respective chromatic numbers. [4+4]

9. Use Dijkstra's algorithm to find the length of shortest path in the following weighted graph. Also highlight the shortest path/paths in the graph: [8]
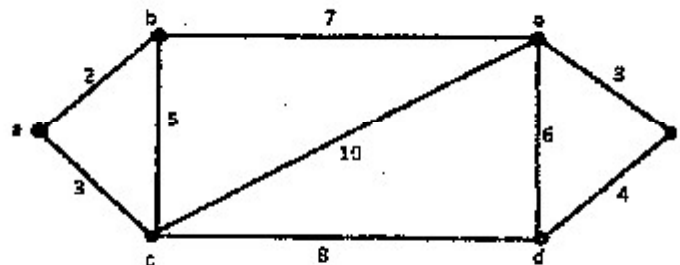
10. Write short notes on: [4+4]

   i) Maximum Flow Mincut Theorem
   ii) Handshaking Theorem

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Exam.		**Regular / Back**	
Level	BE	Full Marks	80
Programme	BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Discrete Structure (CT551)

1. Use resolution to show the hypothesis "It is note raining or Sita has her umbrella," "Sita does not have her umbrella or she does not get wet," and "It is raining or Sita does not get wet" imply that "Sita does not get wet."   [8]

2. Use mathematical induction to show that   [8]

$$1^3 + 2^3 + \ldots + n^3 = [n(n+1)/2]^2$$

whenever n is a positive integer.

3. State the converse, contrapositive and inverse for the conditional statement, "I go to the beach whenever it is a sunny summer day."   [3]

4. Why is a tableau method important in propositional logic? Draw the tableau for the formula   [2+3]

$$\Phi = (p \wedge \neg q) \rightarrow s$$

Where $\neg$ denotes the negation of a variable, $\wedge$ denotes the conjunction of variables and $\rightarrow$ denotes the implication.

5. Differentiate between Finite State Machines and Finite State Automata. Design a Finite State Automata that accepts precisely those strings over {a, b} that contain an odd number of b's. Your design should include the proper definition of the finite-state automation, transition table and the transition diagram.   [2+6]

6. Consider the regular grammar $G = (N, T, P, \sigma)$ where N = Set of Non-Terminals = {σ, A, B}, T = Set of Terminals = {a, b} with productions.   [4+4]

σ→aA, σ→bB, A→a, B→a and starting symbol σ.

Construct a Non-Deterministic Finite State Automata equivalent to the above given regular grammar and convert this into equivalent Deterministic Finite State Automata.

7. Find all solutions of the recurrence relation   [8]

$$a_n = 3a_{n-1} + 2^n$$

with initial condition $a_0 = 5$.

8. Use Dijkstra's algorithm to find the length of the shortest path between the vertices a and z in the weighted graph displayed below. [8]
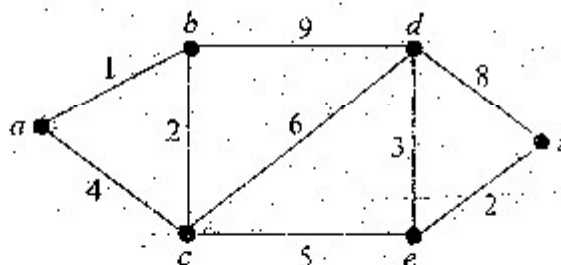
9. Draw the figure for the complete graph with 6 vertices (This is usually denoted by K_6). Define the term graph coloring and the chromatic number of a graph coloring. What is the chromatic number of the complete graph K_6? [2+2+2+2]

10. Explain the Hamiltonian path and Hamiltonian circuit with the help of a diagram. State the necessary and sufficient conditions for Euler circuits and paths. How is Euler circuit different from the Hamiltonian circuit? [3+2+2]

11. Write short notes on: [3+3+3]

   a) Spanning tree
   b) Cutsets and Cutvertices
   c) Application of trees

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Exam.		Regular	
Level	BE	Full Marks	80
Programme	BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Discrete Structure *(CT551)*

✓ Candidates are required to give their answers in their own words as far as practicable.
✓ Attempt **All** questions.
✓ The figures in the margin indicate **Full Marks**.
✓ Assume suitable data if necessary.

1. If $P = F$, $Q = T$, $S = T$, $R = F$, then find truth value of: [4+4]
   a) $(S \to (P \wedge \overline{R}) \wedge ((P \to (R \vee Q)) \wedge S)$
   b) $((P \wedge \overline{Q}) \leftrightarrow (Q \wedge R)) \to (S \vee \overline{Q})$

2. Using rules of inferences, show that the hypothesis "It is not rainy today and its hotter than yesterday", "We will go for movie only if it is rainy", "If we do not go for movie, then we will go for shopping", and "If we go for shopping, then we will be home by sunset" lead to the conclusion "We will be home by sunset". You are required to show each steps and give reasons for those steps before you come to desired conclusion from the hypothesis.   [8]

3. Prove by Mathematical Induction: [8]
   $1.2.3 + 2.3.4 + 3.4.5 + \ldots + n(n + 1)(n + 2) - n(n + 1)(n + 2)(n + 3) / 4$

4. Design a Finite State Machines (FSM) that performs binary serial addition. Define DFA and NDFA. Construct DFA that recognize the language "The set of bit brings that do not contain three consecutive 0's. Show only necessary figures and state diagrams.   [3+2+3]

5. Define and differentiate between context-sensitive, context free and regular grammars with suitable examples. Explain in short the role of regular expressions.   [6+2]

6. What do you understand by recurrence relation? Explain in brief. Derive and solve the recurrence relation for Tower of Hanoi puzzle.   [2+6]

7. Is $K_{3,3}$ graph a planar graph? Explain it with suitable reasons.   [4+4]

8. Define Regular and Bipartite graphs with suitable examples.   [3+3]

9. Define level and height of tree? What is full m-ary tree and balanced tree?   [2+2]

10. State the handshaking theorem for the undirected graph and use it to prove the theorem that an undirected graph has an even number of vertices of odd degree.   [2+4]

11. Write down the short notes on the following: [4+4]
   a) Maximum Flow Mincut Theorem
    b) Graph Coloring

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Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Discrete Structure (CT551)

✓ Candidates are required to give their answers in their own words as far as practicable.
✓ Attempt **All** questions.
✓ The figures in the margin indicate **Full Marks**.
✓ Assume suitable data if necessary.

1. Construct an argument using rules of inference to show that the hypotheses "Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job." [8]

2. Use mathematical induction to show that [8]
$1^2 + 2^2 + \cdots \cdots + n^2 = n(n+1)(2n+1)/6$
whenever n is a positive integer.

3. State the converse, contrapositive and inverse for the conditional statement, "A positive integer is a prime only if it has no divisors other than 1 and itself." [3]

4. Define satisfiable and unsatisfiable formulas. Draw the tableau for the formula [2+3]
$\Phi = \neg((p \wedge q) \vee r)$
where $\neg$ denotes the negation of a variable, $\vee$ denotes the disjunction of variables and $\wedge$ denotes the conjunction of variables.

5. Define Finite State Machines. Design a Finite State Automata that accepts precisely those strings over {a, b} that contain two consecutive a's. Your design should include the proper definition of the finite-state automaton, transition table and the transition diagram. [2+6]

6. Consider the regular grammar $G = (N, T, P, \sigma)$ where N= Set of Non-Terminals = {σ, A, B}, T= Set of Terminals = {a, b} with productions [4+4]
$\sigma \rightarrow a$, $\sigma \rightarrow bB$, $A \rightarrow bA$, $A \rightarrow aB$, $A \rightarrow b$, $A \rightarrow a$, $B \rightarrow b$ and starting symbol σ.
Construct a Non-Deterministic Finite State Automata equivalent to the above given regular grammar and convert this into equivalent Deterministic Finite State Automata.

7. Find all solutions of the recurrence relation [8]
$a_n = 2a_{n-1} + 2^n$
with initial condition $a_0 = 2$.

8. Use Dijkstra's algorithm to find the length of the shortest path   [8]
   between the vertices a and z in the weighted graph displayed
   below.
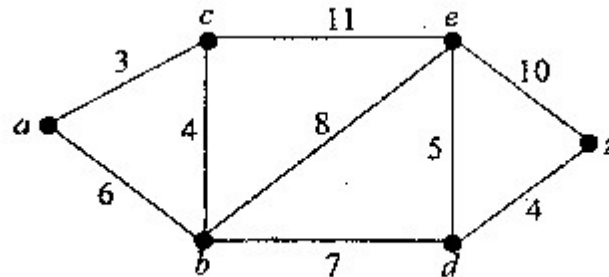
9. Draw the figure for the complete graph with 5 vertices (This is   [2+2+2+2]
   usually denoted by K_5). Define the term graph coloring and the
   chromatic number of a graph in graph coloring. What is the
   chromatic number of the complete graph K_5.

10. Construct an influence graph for the board members of a company   [4]
    if the President can influence the Director of Research and
    Development, the Director of Marketing, and the Director of
    Operations; the Director of Research and Development can
    influence the Director of Operations; the Director of Marketing
    can influence the Director of Operations; and no one can
    influence, or be influenced by, the Chief Financial Officer.

11. How is Euler circuit different from the Hamiltonian circuit?   [3]
    Explain

12. Write short notes on   [3+3+3]
    a) Spanning tree and its applications
    b) Network Flows
   c) Regular graphs

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Subject: - Discrete Structure

✓ Candidates are required to give their answers in their own words as far as practicable.
✓ Attempt *All* questions.
✓ The figures in the margin indicate *Full Marks*.
✓ Assume suitable data if necessary.

1) Using rules of inferences, show that the hypotheses "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed." You are required to show each steps and give reasons for those steps before you come to the desired conclusion from the hypotheses. **(8)**

2) Use mathematical induction to prove that **(8)**
$3 + 3 \cdot 5 + 3 \cdot 5^2 + \ldots + 3 \cdot 5^n = 3(5^{n+1} - 1)/4$
whenever n is a nonnegative number.

3) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction. Draw the tableau for the formula $(T \vee S) \rightarrow \neg Q$ where $\neg$ denotes the negation of a variable, $\vee$ denotes the disjunction of variables and $\rightarrow$ is the symbol for implication. **(5+3)**

4) Design a finite-state automaton that accepts only those set of strings over $\{a, b\}$ which starts with *baa*. Precisely, only those strings which begin with *baa* should be accepted and other strings over $\{a, b\}$ should be rejected. Your design should include the proper definition of the finite-state automaton, transition table and the transition diagram. **(3+2+3)**

5) Discuss regular expressions and regular languages in detail with suitable examples. Explain the different properties of regular languages. **(4+4)**

6) Find all solutions of the recurrence relation **(8)**
$a_n = 2a_{n-1} + 3^n$
with initial condition $a_1 = 5$.

7) Use Dijkstra's algorithm to find the length of the shortest path between the vertices a and z in the weighted graph displayed below. **(8)**
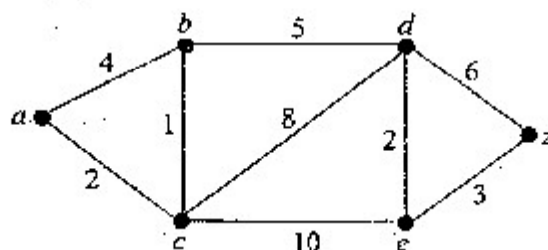
8) Draw the figure for the complete bipartite graph $K_{3,4}$ and the cycle graph with 5 vertices (This is usually denoted by C_5). What is the chromatic number of the drawn complete bipartite graph $K_{3,4}$ and the cycle graph C_5. **(2+2+2+2)**

9) State the handshaking theorem for the undirected graph and use it to prove the theorem that an undirected graph has an even number of vertices of odd degree. **(2+4)**

10) Write short notes on: - **(4+3+3)**
    a) Eulerian graph
    b) Hamiltonian graph
    c) Spanning tree

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Subject: - Discrete Structure *(CT551)*

✓ Candidates are required to give their answers in their own words as far as practicable.
✓ Attempt __All__ questions.
✓ The figures in the margin indicate __Full Marks__.
✓ Assume suitable data if necessary.

1. Construct an argument using rules of inference to show that the [8]
   hypotheses "If it does not rain or if it is not foggy, then the
   sailing race will be held and the lifesaving demonstration will go
   on," "If the sailing race is held, then the trophy will be awarded,"
   and "The trophy was not awarded" imply the conclusion "It
   rained." You are required to show each step and give reasons for
   those steps before you come to the desired conclusion from the
   hypotheses.

2. Use mathematical induction to prove the inequality $n < 2^n$ for [8]
   all positive integers n.

3. Why tableau method is important in the propositional logic? Draw the [2 + 6 = 8]
   tableau for the formula set
   $\Phi = \{(p \wedge \neg q) \rightarrow s, \neg q \vee \neg r, p \wedge t\}$
   where ¬ denotes the negation of a variable, ∨ denotes the
   disjunction of variables, ∧ denotes the conjunction of variables
   and → denotes the implication.

4. Differentiate between Deterministic Finite State Automata and [2+6 = 8]
   Non-Deterministic Finite State Automata. Design a Finite State
   Automata that accepts precisely those strings over {a, b} that
   contain an even number of a's. Your design should include the
   proper definition of the finite-state automaton, transition table and
   the transition diagram.

5. Consider the regular grammar defined by T={a, b}, N={σ, C} [4 + 4 = 8]
   with productions
   σ→bσ, σ→aC, C→bC, C→b and starting symbol σ.
   Construct a Non-Deterministic Finite State Automata equivalent to
   the above given regular grammar and convert this into equivalent
   Deterministic Finite State Automata.

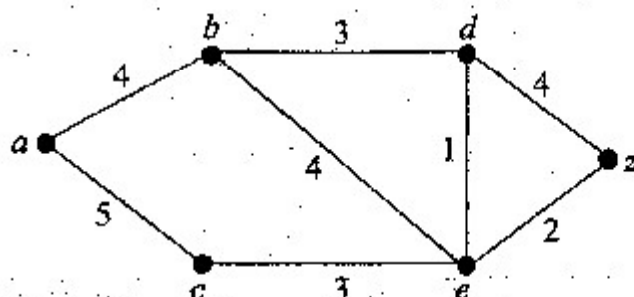6. Find all solutions of the recurrence relation [8]
$$a_n = 7a_{n-1} - 16a_{n-2} + 12a_{n-3} + n4^n$$
with initial condition $a_0 = -2$, $a_1 = 0$ and $a_2 = 5$.

7. Use Dijkstra's algorithm to find the length of the shortest path between the vertices a and z in the weighted graph displayed below. [8]

8. Draw the figure for the complete bipartite graph $K_{4,5}$ and the cycle graph with 6 vertices (This is usually denoted by C_6). What is the chromatic number of the drawn complete bipartite graph $K_{4,5}$ and the cycle graph C_6. [2+2+2+2]

9. Define a tree and discuss its various properties as well as applications of trees. [1+2+4=7]

10. Write short notes on: – [3+3+3=9]
    a) Eulerian graph
    b) Max flow, min cut theorem
    c) Planar and regular graphs

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