ECE257A Review Problems Lecture 5-8

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I. Lecture 5. DSSS; OFDM

1. How does DSSS spreading work? How is it implemented in a practical communication system (major signal processing blocks and their roles)?
2. Why can DSSS improve SNR? What's the quantitative improvement as a function of spreading factor?
3. How is DSSS implemented in ZigBee?
4. How is DSSS implemented in 802.11b? Why is sampling time synchronization needed and how is it achieved?
5. What are the advantages of OFDM compared with single-carrier (SC) FDM?
6. Mathematical model of OFDM implementation. How is orthogonality achieved among subcarriers?
7. Why is cyclic prefix needed? How is it implemented in OFDM?

Answer:

1. Basic idea: Using spread spectrum modulation to convert a narrow-band signal into a wide-band symbol. This explores frequency diversity, even if some parts of the channel suffer from deep fading, other parts can still carry sufficient signal energy.

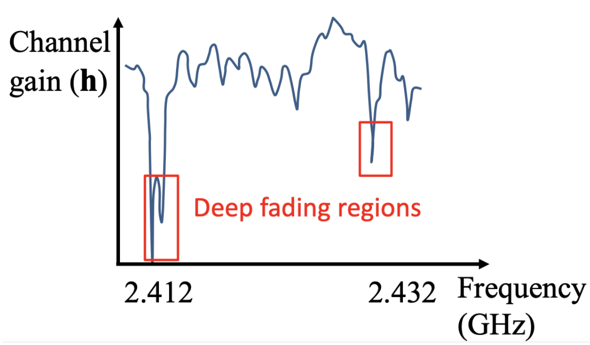

Fig. 1. Frequency selective fading

Basic approach: One symbol is spread to multiple "chips". A symbol is a basic signal unit, the number of chips per symbol is called the spread factor.

2. The amplitude of the noise becomes $\frac{1}{N}$ when de-spreading occurs and the amplitude of the signal recovers to its original amplitude after spreading and de-spreading, so the noise is degraded to $\frac{1}{N}$, where N is the length of the chip sequence. So the SNR is improved by N times. If without spreading, the amplitude of the noise will not recover to its original amplitude after de-spreading.

3. Zigbee: every 4 bits form a symbol, and each symbol is mapped to a 32-bit chip sequence. So $250Kbp \rightarrow 62.5kS/s \rightarrow$

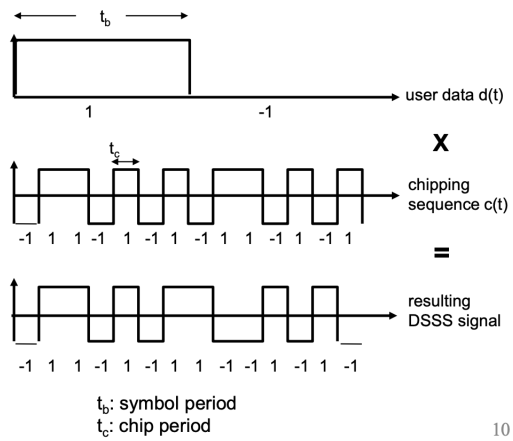

Fig. 2. DSSS modulation

Fig. 3. DSSS demodulation

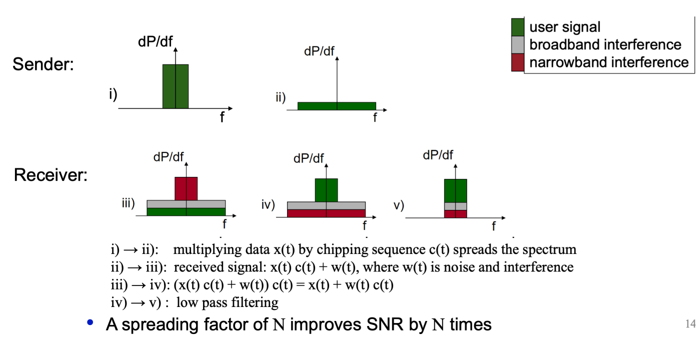

Fig. 4. Network of networks

$2MChips/s$. The signal is later modulated with O-QPSK (orthogonal QPSK) modulation and sent out I phase (real) and Q phase (imaginary) signals simultaneously. The sampling period is $0.5\mu S$, so bandwidth is $1/0.5MHz = 2MHz$. Decoding is done doing cross-correlation to the chip sequences.

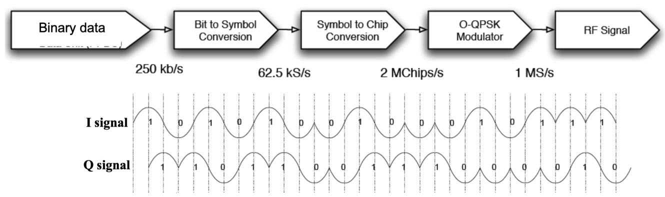

Fig. 5. ZigBee modulation workflow

4. 802.11b: On transmitter each chip is converted into BPSK symbol using DBPSK. Further spread out the BPSK symbols using barker code, later shaped/ filtered into waveform in order to pass the FCC certificate. On the waveform 2 samples (I and Q) represent 1 chip, with chip rate 11 MHz. This yields sampling rate 22Mbps and bandwidth 22MHz, with bit-rate 1Mbps. On the receiver side the despreading process is similar. Symbol level synchronization in this phase is extremely important in this phase because if they are not aligning perfectly, the correlation result may be really weak which is not good for recovering the signal. It can be done by oversampling to make sure it can successfully recover the waveform.

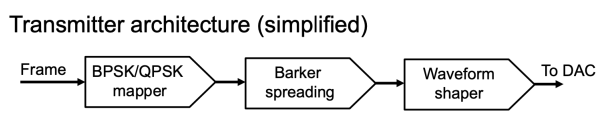

Fig. 6. 802.11b transmitter architecture

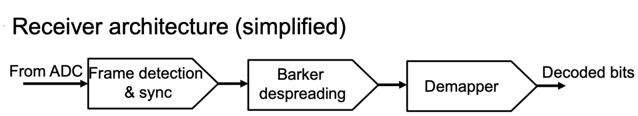

Fig. 7. 802.11b receiver architecture

5. Orthogonal frequency division multiplexing is a modulation scheme to spread information over a wide-band spectrum. Compared to Single Carrier modulation it occupies more frequency band and more efficient.

6.

N point IFFT :

$$x(t) = \sum_{k=-\frac{N}{2}} k = \frac{N}{2} - 1X[k]e^{j2\pi k\Delta Ft}, 0 \leq t \leq T_{symbol}$$

Orthogonality:

$$\frac{1}{T_b}\int_{t_0}^{t_0+T_b} e^{j2_k t}e^{j2_l t}\,dt =$$

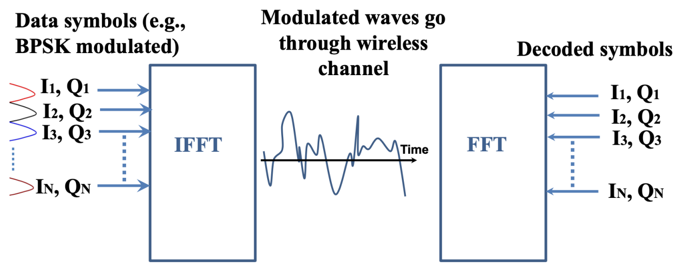

Fig. 8. OFDM implementation is basically doing IFFT at transmitter and FFT at receiver

$$\frac{1}{T_b}\int_{t_0}^{t_0+T_b} e^{j2\pi(k-l)\Delta Ft}dt = \begin{cases} 1 & \text{if } k = l \\ 0 & \text{if } k \neq l \end{cases}$$

7. Cyclic prefix is a guard interval between symbols to contain inter-symbol interference, it is long enough to ensure the multipath from one symbol does not affect the next. The implementation is just copying the G tailing samples to the front and form a $(G+N)$ sample OFDM symbol.

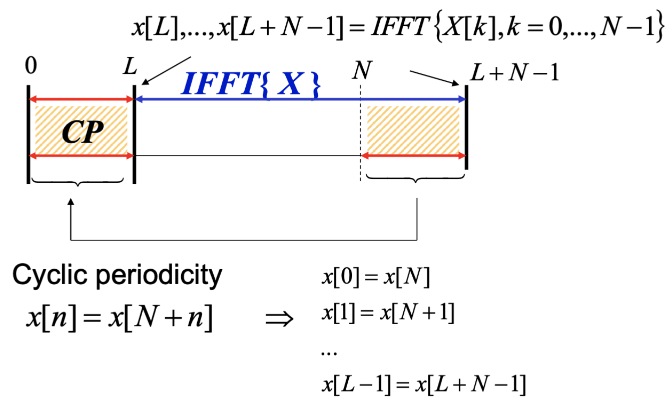

Fig. 9. Implementation of cyclic prefix

II. Lecture 6. WiFi packet processing

1. In 802.11 OFDM, how are bits mapped to subcarriers? Why is it done in this way? How is an OFDM symbol constructed?

2. How is the STF preamble constructed to facilitate packet detection? How to detect the STF preamble? How is time synchronization done based on STF?

3. How is the LTF preamble constructed? How to use the LTF to estimate frequency offset?

4. How is channel equalization done for 802.11 OFDM packets?

Answer:

1. 96 bits are mapped to 48 symbols using QPSK modulation, 48 symbols is then mapped to 64 samples. Subcarrier allocation: 48 data + 4 pilots + 12 nulls = 64. c_1 to c_{26} , and $c_{-26}$to c_{-1} are null subcarriers which serve as guardband to avoid leakage interference to adjacent channels. c_{-21}, c_{-7}, c_7 and c_{26} are pilots which are used for fine-grained channel

estimation error correction. The rest are data subcarriers. And then do 64 point IFFT to time domain signal.

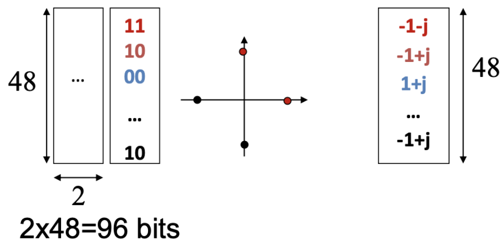

Fig. 10. 802.11 QPSK modulation

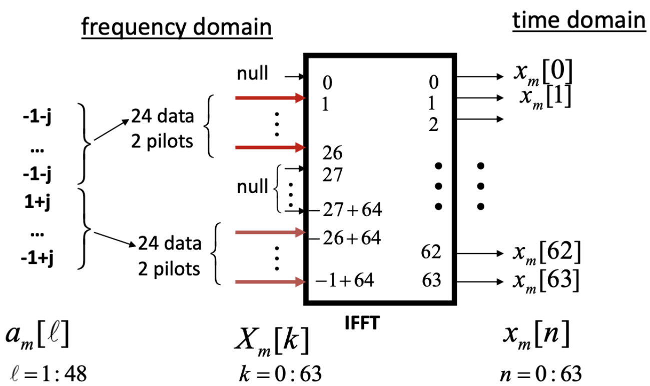

Fig. 11. 802.11 subcarrier allocation

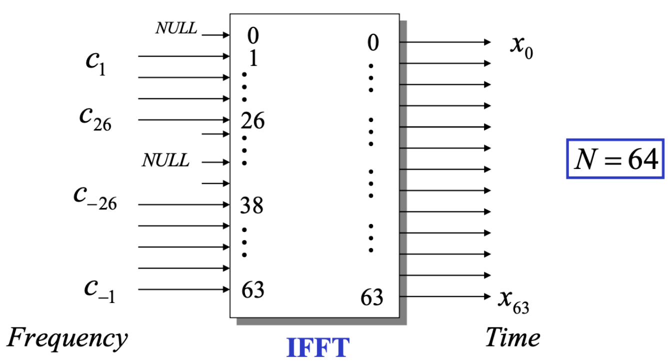

Fig. 12. OFDM subcarrier allocation

An OFDM packet is constructed of STF, LTF and OFDM data symbols. OFDM data symbols contains 64 FFT samples pretended by a cyclic prefix of 16 samples.

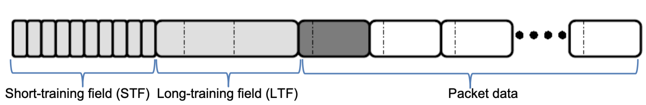

Fig. 13. Packet structure (802.11a/g/n/ac OFDM packets)

2. STF is constructed of 10 repeated training symbols, each containing 16 samples. It is used for packet detection and synchronization.

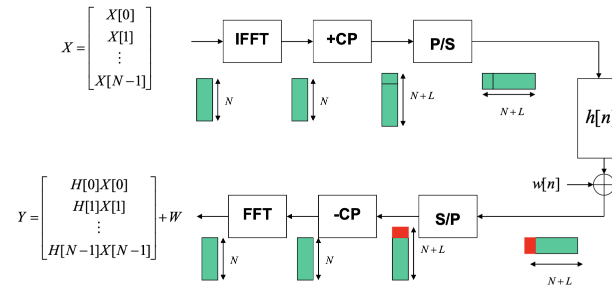

Fig. 14. Overall architecture of an OFDM radio

Packet detection

Self-correlation of STF symbols

$$R(m) = \sum_{i=m}^{m+16-1} y(i)y^*(i-16) \approx \sum_{i=m}^{m+16-1} |y(i)|^2$$

Energy of 16 samples

$$E(m) = \sum_{i=m}^{m+16-1} |y(i)|^2$$

Self-correlation of non-STF signals

$$R(m) = \sum_{i=m}^{m+16-1} y(i)y^*(i-16) \approx 0$$

Energy of non-STF samples

$$E(m) = \sum_{i=m}^{m+16-1} |y(i)|^2$$

Packet is detected if $R(m) \gg 0andR(m) \approx E(m)$ and this remains true across 160-16 samples.

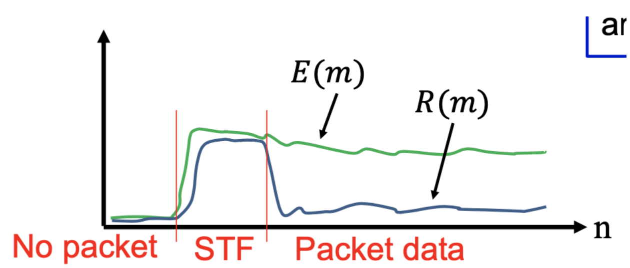

Fig. 15. Packet detection

Time synchronization

Cross-correlation, spanning across 16 samples (one STF symbol)

$$R(m) = \sum_{i=m}^{m+16-1} y(i)S^*(i-m) \approx 16h(m)$$

Note: channel is coherent within short period, so $h(m) \approx h(m+1) \approx h(m+15)...$

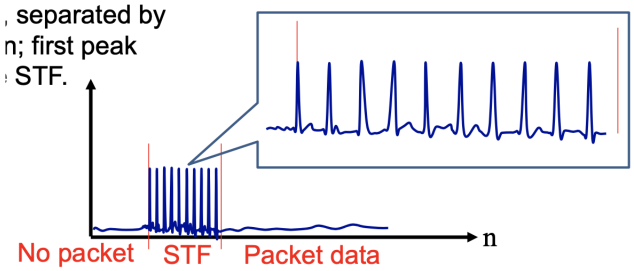

Fig. 16. Time synchronization

Cross-correlation of non-STF samples

$$R(m) = \sum_{i=m}^{m+16-1} y(i)S^*(i-m) \approx 0$$

STF is detected if 10 consecutive peaks separated by 16 samples in between, first peak point is the start of the STF.

3. LTF are 2 repeated training symbols each containing 64 samples, prepended by a 32-sample cyclic prefix. It is used for frequency offset and channel estimation.

Frequency offset estimation

Transmitter's carrier frequency is not exactly equal to the receiver's frequency, this will destroy orthogonality and cause inter-subcarrier interference.

Suppose received signal in time-domain

$$y(n) = x(n)e^{j2\pi n\Delta f}$$

$$Y(k) = FFTy(n)... = \sum_{n=1}^{N-1} y(n)e^{-j2\pi n\frac{k}{N}}$$

$$= \sum_{n=1}^{N-1} x(n)e^{-j2\pi n\frac{k}{N}}e^{j2\pi n\Delta f}$$

Utilize the LTF, which has 2 repeating blocks, each N=64 samples. We can compute a Δf for each sample in an LTF symbol and then take the average.

$$y(n+N) = y(n)e^{2\pi jn\Delta f}$$

So

$$e^{2\pi jn\Delta f} = \frac{y(n+N)}{y(n)}, \Delta f = img\{\frac{y(n+N)}{y(n)}\}/(2\pi n)$$

Frequency offset compensation

i-th sample needs to multiply $e^{-2\pi ji\Delta f}$

4.

Channel estimation

Assume flat fading (each subcarrier k is distorted by a single multiplier $H(k)$ which is the channel gain). Use LTF to estimate $H(k)$, for LTF received signal on subcarrier k is

$$Y(k) = FFT(LTFsymbol1) or FFT(LTFsymbol2)$$

The signal sent on subcarrier k is known, denoted as $Sl(k)$, then the channel gain on subcarrier k is $H(k) = Y(k)/Sl(k)$.

Channel equalization

To decode the data symbol carried on subcarrier k, simply do $X(k) = Y(k)/H(k)$.

III. Lecture 7. MIMO

1. Understand the fundamental differences between diversity gain (from SIMO or MISO) and multiplexing gain (from MIMO).

2. How does SIMO improve wireless link SNR?

3. How does open-loop transmit diversity improve SNR?

4. How does close-loop transmit diversity improve SNR and link capacity?

Answer:

1. Diversity gain from receiver diversity (SIMO) or trasmitter diversity (MISO) improves link SNR. Multiplexing gain from spatial multiplexing (MIMO) improves link concurrency.

Antenna diversity or spatial diversity employs multiple antennas either at the transmitter or receiver, the antennas are physically separated to overcome the multipath fading. MIMO employs multiple antennas at the transmitter and receiver to take advantage of antenna diversity, spatial multiplexing and beamforming. It improves signal quality and data rates (link concurrency).

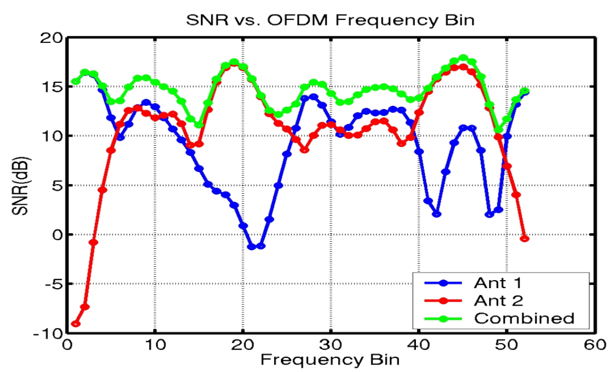

Fig. 17. Receiver diversity mitigates frequency-selective fading

2. Receiver diversity increases SNR proportionally to N_r (number of receiver antennas). Capacity gain can be analyzed based on Shannon's equation

$$C = Blog2(1 + SNR)$$

When SNR is low, $log2(1+SNR) \approx SNR$, so gain is almost linear wrt N_r. When SNR is high, gain is approximately log wrt N_r.

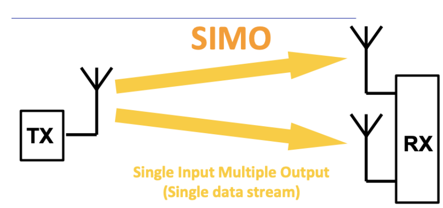

Fig. 18. Receiver diversity

Suppose receiver i receives signal $y_i = h_i x + n_i = hx + n_i$, then corresponding SNR on antenna i is $r_i = \frac{|h|^2}{\sigma^2}$. Adding up all the antenna paths

$$y = \sum_{i=1}^{N_r} y_i = N_r hx + \sum_{i=1}^{N_r} n_i$$

Assuming noise on each antenna is independent Gaussian variables, combined SNR is

$$r_{\sum} = \frac{|N_r h|^2}{N_r \sigma^2} = \frac{N_r |h|^2}{\sigma^2}$$

3. In transmitter diversity transmitter sends multiple versions of the same signal through multiple antennas. There are two modes of transmitter diversity, open-loop transmitter diversity and closed-loop transmitter diversity.

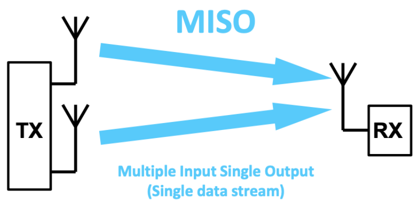

Fig. 19. Transmitter diversity

Open-loop transmitter diversity sends redundant versions of the same signal over multiple time slots and multiple antennas. The signals are encoded differently for different time slots and T_x antennas, this method is called Space-Time Block Code(STBC).

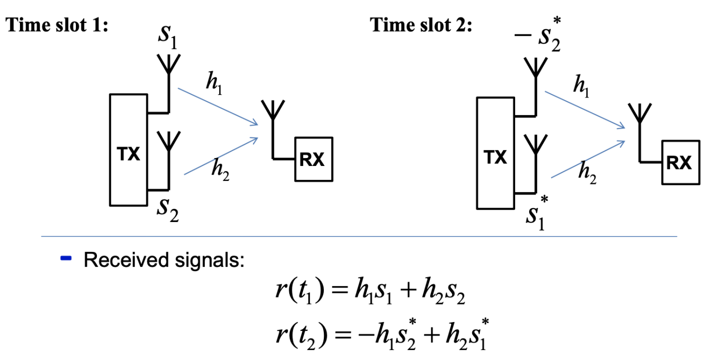

Fig. 20. Open-loop transmitter diversity STBC

Received signals at 2 time slots

$$r(t_1) = h_1 s_1 + h_2 s_2, r(t_2) = -h1 s_2^* + h_2 s_1^*$$

Combining them

$$y_1 = h_1^* r(t_1) + h_2 r^*(t_2) = (|h_1|^2 + |h_2|^2) s_1$$

$$y_2 = h_2^* r(t_1) - h_1 r^*(t_2) = (|h_1|^2 + |h_2|^2) s_2$$

With N_t transmitter antennas, combined SNR at receiver is (Note: total TX power, ϵ_x, is split among N_t transmit antennas)

$$r_{\sum} = \frac{\epsilon_x}{\sigma^2} \frac{|h_1|^2 + |h_2|^2 + ... + |h_{N_t}|^2}{N_t} = \frac{\epsilon_x}{\sigma^2} E[|h_1|^2]$$

Open-loop transmitter diversity causes the received SNR to harden to the average SNR. In other words, it eliminates the effects of small-scale fading but does not increase the average received SNR.

4. Closed-loop transmitter diversity sends redundant versions of the same signal over the same time slot. Each signal is encoded differently for different T_x antennas following a precoding algorithm, which requires feedback of channel state information(CSI).

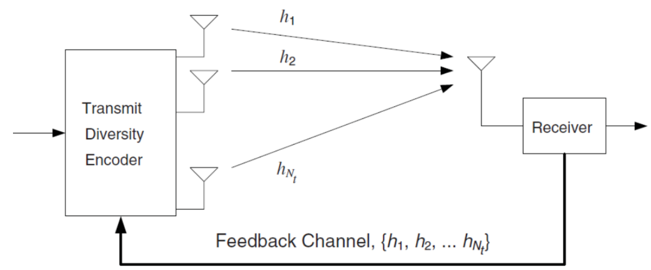

Fig. 21. Closed-loop transmitter diversity with CSI feedback

Signals from different antennas need to synchronize their phases, but different channels between T_x antennas and R_x antenna distort signals differently causing phase offset. T_x must know the phase offset from CSI feedback to compensate them and align the phases of the signals going through different channels.

Suppose we have 2 transmit antennas, then instead of receiving symbol x we receive $x + x = 2x$, received power becomes $4|x|^2$. But under practical constraint that total transmit power remains constant, we receive $\frac{x}{\sqrt{2]}} + \frac{x}{\sqrt{2]}} = \sqrt{2}x$. Received power becomes $2|x|^2$, SNR actually doubles in practice. More generally, with N_t T_x antennas, SNR increases to N_t times, the capacity gain is $log(N_t)$.

IV. Lecture 8. MU-MIMO and Network MIMO

1. How does MIMO spatial multiplex gain increase with the number of transmit or receive antennas?

2. How does channel rank and channel condition number affect MIMO spatial multiplexing gain? Understand the high-level mathematical reasons behind.

3. How does 802.11 MIMO estimate the channel matrix?

4. How does MU-MIMO enable concurrent transmission from a single AP to multiple users?

5. What's the asymptotic capacity gain of MU-MIMO, as a function of number of antennas/users?

6. Where does the MU-MIMO overhead come from? How does it affect the effective throughput?

7. Understand the differences between the 3 forms of network MIMO implementation in LTE.

Answer:

1. Spatial multiplexing form multiple independent links between T_x and R_x, and send data in parallel through them. Unfortunately, there is cross-talk between antennas which must be removed by digital signal processing algorithms.

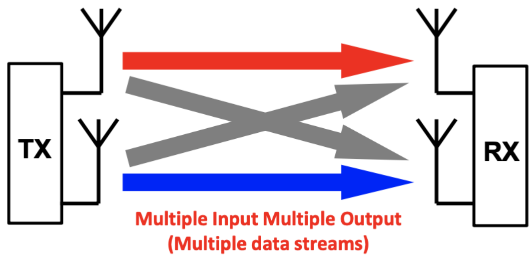

Fig. 22. Spatial multiplexing (MIMO)

Data received on to R_x antennas

$$y_1 = h_{11}x_1 + h_{12}x_2 y_2 = h_{21}x_1 + h_{22}x_2$$

Channel distortions h_{**} can be estimated by the receiver, the two unknowns x_1, x_2 can be easily obtained by solving the equations.

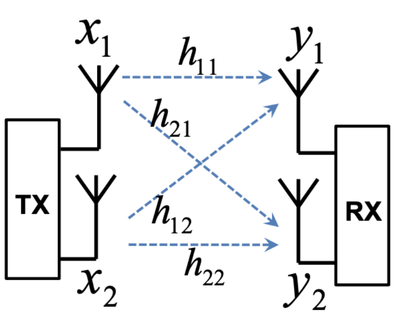

Fig. 23. Example 2x2 MIMO spatial multiplexing

In general, the capacity gain from spatial multiplexing scales linearly with $min(N_t, N_r)$. In practice, spatial multiplexing gain also depends on channel condition. If the channels between different antennas are correlated then you cannot solve the equations. Practical wireless devices' multiple antennas are separated sufficiently far (further than half-wavelength) in order to minimize the correlation.

2. The MIMO spatial multiplexing gain is just the channel rank. It is the rank of the matrix of the system of linear equations, which determines the dimension of the solution space of the system linear equations.

3. For 2×2 MIMO case, Antenna 1 sends packet with LTF1 to obtain estimation for h_{11}, h_{21} and antenna 2 sends packet with LTF2 to obtain estimation for h_{12}, h_{22}.

4. MU-MIMO uses Zero-Forcing Beamforming (ZFBF) to enable multiple streams of data to be sent to different users in parallel to remove cross-talk interference.

T_x antenna 1 sends

$$w_{11}x_1 + w_{12}x_2$$

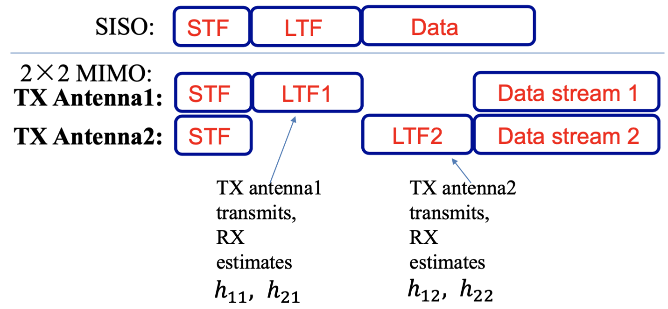

Fig. 24. 802.11 2×2 MIMO packet construction

T_x antenna 2 sends

$$w_{21}x_1 + w_{22}x_2$$

Data received by R_x1

$$\begin{aligned} y_1 &= h_{11}(w_{11}x_1 + w_{12}x_2) + h_{12}(w_{21}x_1 + w_{22}x_2) \\ &= (h_{11}w_{11} + h_{12}w_{21})x_1 + (h_{11}w_{12} + h_{12}w_{22})x_2 \end{aligned}$$

R_x1 only wants x_1, so ideally we force $h_{11}w_{12} + h_{12}w_{22} = 0$. Data received by R_x2

$$\begin{aligned} y_2 &= h_{21}(w_{11}x_1 + w_{12}x_2) + h_{22}(w_{21}x_1 + w_{22}x_2) \\ &= (h_{21}w_{11} + h_{22}w_{21})x_1 + (h_{21}w_{12} + h_{22}w_{22})x_2 \end{aligned}$$

R_x1 only wants x_1, so ideally we force $h_{21}w_{11} + h_{22}w_{21} = 0$. This is called Zero-Forcing Beamforming (ZFBF).

Zero-Forcing Beamforming

Stream i is pre-coded using weight

$$w_i = [w_{1i}w_{2i}...w_{Ni}]^T$$

The j-th T_x antenna transmits the signal

$$w_{j1}x_1 + w_{j2}x_2 + ...w_{jN}x_N$$

Receiver i receives

$$y_i = h_i w_i x_i + \sum_{j \neq i} h_i w_j x_j + n_i$$

Nullify the interference

$$h_i w_j = 0, \forall j \neq i$$

Matrix form

$$y = HWx + n$$

Let W be the pseudo inverse of H

$$W = H^\dagger = H^*(HH^*)^{-1}$$

T_x obtains channel state information(CSI) using CSI feedback scheduling

5. If the transmitter has N_t antennas then it can send N_t streams of data simultaneously to N_t users. MU-MIMO is essentially a form of spatial multiplexing so the channel must be well-conditioned (multipath rich and and full-rank channel matrix). It requires a closed-loop feedback, non-trivial overhead, making the net throughput gain lower than N_t times.

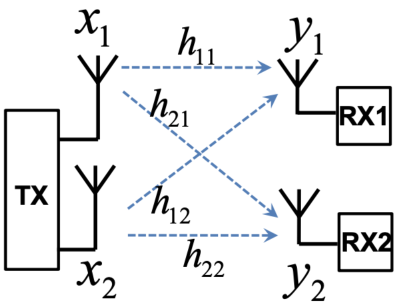

Fig. 25. MU-MIMO

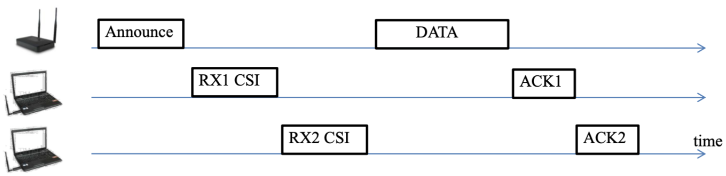

Fig. 26. 802.11 CSI feedback scheduling

6. Overhead comes from multiple dimensions. Time domain : CSI need to be updated over time. Frequency-domain : CSI needs to be updated for different subcarriers. Actual CSI values need to be represented in digital bits and carried through feedback packets. We can tame the feedback overhead through compression, in time domain update CSI intermittently, in frequency domain feedback CSI across a few subcarriers and then interpolate the rest, and quantize the channel gain values using fewer bits.

7.

3 forms of Coordinated Multi-Point Transmission (CoMP)	
Form	Function
CoMP-CS	**Coordinated Scheduling** 1. Allocate different time and frequency resources to cell edge users served by different cells
CoMP-CB	**Coordinated Beamforming** 1. Allocagte different spatial resources to users at cell edge, but time and frequency resources are reused. 2. Beamforming weights can be calculated in a way that cell edge users served by neighboring cells are nulled (allocate zero power)
CoMP-JT	**Joint Transmission** 1. Central precoding needed for multiple access nodes (or remote radio heads) 2. Data transmitted simultaneously from multiple access nodes or RRH 3. High demand onto the backhaul since data has to be in several places 4. Theoretically, JT can get rid of all inter-cell interference and scale network capacity with network density