

CE 311K: Control flow - Branching and Iterations

Krishna Kumar

University of Texas at Austin

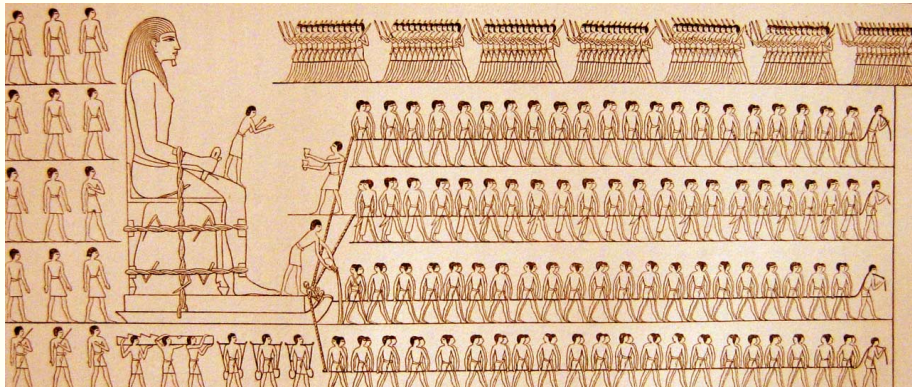
krishnak@utexas.edu

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1 Numerical solution of a sliding block

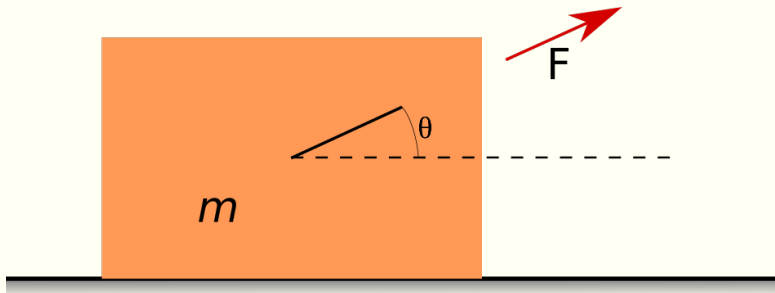
2 Bisection method

What is the optimal angle to pull the statue?



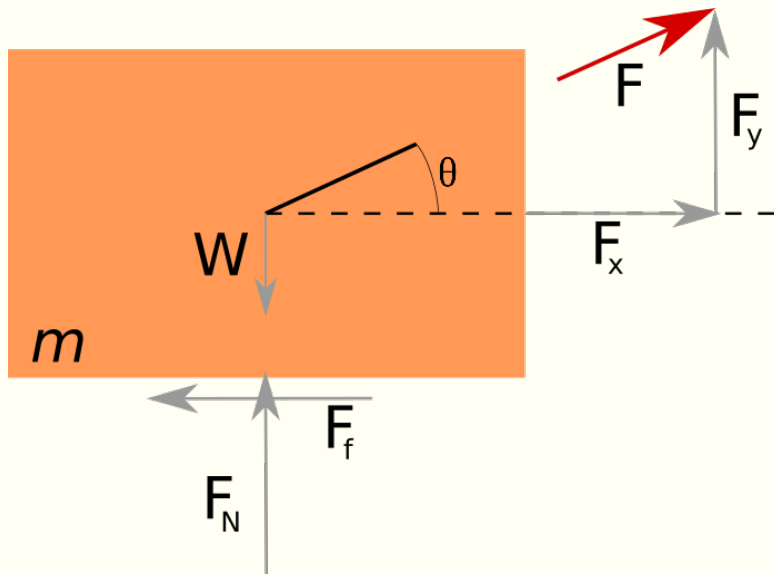
A wall painting from the tomb of Djehutihotep (credit: martinhumanities.com)

Numerical solution of a sliding block: Approximation



What is the optimal angle to pull the block applying the least amount of force?

Numerical solution of a sliding block: Forces



Numerical solution of a sliding block: Forces

$$F_x = F \cos \theta \quad \& \quad F_y = F \sin \theta$$

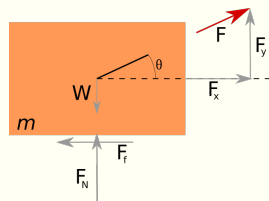
$$F_f = \mu \cdot F_N = \mu \cdot W - \mu F_y = \mu mg - \mu F \sin \theta$$

$$\text{Vertical forces } \sum F_{\text{vert}} \uparrow: F_y + F_N - W = 0$$

$$F_N = mg - F \sin \theta$$

$$\text{Horizontal forces } \sum F_{\text{hor}} \rightarrow: F_x - F_f = 0$$

$$F \cos \theta - \mu mg + \mu F \sin \theta = 0$$



$$F = \frac{\mu \cdot mg}{(\cos \theta + \mu \sin \theta)}$$

Numerical solution of a sliding block: Compute force

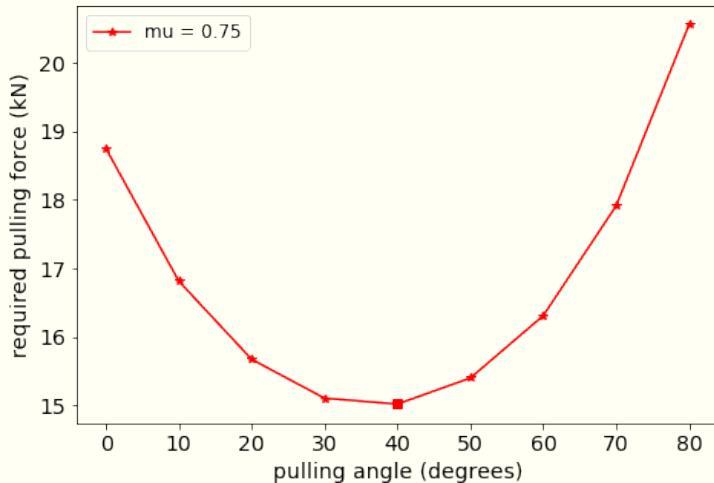
- Given $W = 25\text{kN}(2500\text{ kg})$, $\theta = 45^\circ$ and $\mu = 0.75$ (35°):

$$F = \frac{0.75 \times 25}{\cos(45) + 0.75 \sin(45)} = 15.15\text{ kN}.$$

- Given $W = 25\text{kN}(2500\text{ kg})$ and $\mu = 0.75$, what's the optimum θ ?

Numerical solution of a sliding block: Optimal theta?

Given $W = 25 \text{ kN}$ (2500 kg) and $\mu = 0.75$, what's the optimum θ ?



Lists

- A list is a sequence of data. (mutable)
- An 'array' in most other languages is a similar concept, but Python lists are more general than most arrays as they can hold a mixture of types.
- A list is constructed using square brackets:

```
>>> a = [0, 10, 20, 30, 40, 50, 60, 70, 80]
>>> print(a)
[0, 10, 20, 30, 40, 50, 60, 70, 80]
>>> type(a)
<class 'list'>
>>> len(a)
10
>>> a.append(90)
>>> print(a)
[0, 10, 20, 30, 40, 50, 60, 70, 80, 90]
```

Iterating through a list: for loops

Looping over each item in a list (or more generally a sequence) is called 'iterating'. We iterate over the members of the lab group using the syntax:

```
for each item in list do
    print(item)
```

```
for item in list:
    print(item)
```

 **Indentation matters in python!**

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- └ Numerical solution of a sliding block
 - └ Iterating through a list: for loops

Looping over each item in a list (or more generally a sequence) is called 'iterating'. We iterate over the members of the lab group using the syntax:

```
for each item in list do  
    print(item)
```

```
for item in list:  
    print(item)
```

▲ Indentation matters in python!

Variables defined inside a for loop is not accessible outside.

np.arange()

We need a new library called 'numpy', we are going to import the new library. `import numpy as np` The `np.arange()` returns a sequence of numbers:

`range(stop)`

stop: Number of integers (whole numbers) to generate, starting from zero.
eg. `np.arange(3)` yields a sequence of `[0, 1, 2]`.

`np.arange([start], stop[, step])`

- *start*: Starting number of the sequence.
- *stop*: Generate numbers up to, but not including this number.
- *step*: Difference between each number in the sequence.

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└ Numerical solution of a sliding block

└ np.arange()

np.arange()

We need a new library called 'numpy', we are going to import the new library. import numpy as np The np.arange() returns a sequence of numbers.

```
range(5)
```

stop: Number of integers (whole numbers) to generate, starting from zero. eg. np.arange(3) yields a sequence of [0, 1, 2].

```
np.arange([start], stop[, step])
```

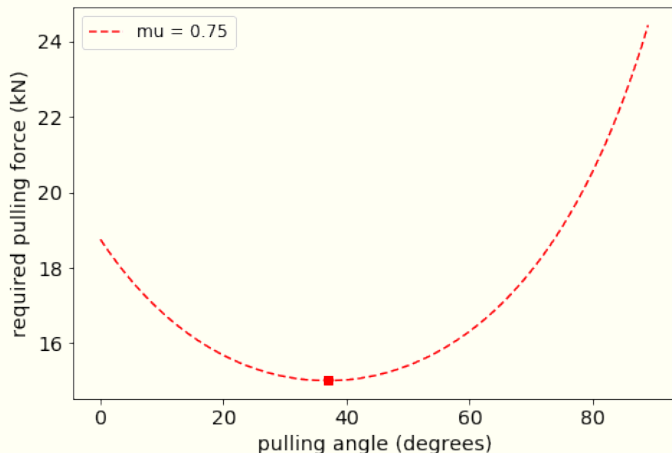
- start: Starting number of the sequence.
- stop: Generate numbers up to, but not including this number.
- step: Difference between each number in the sequence.

Note that:

- All parameters can be positive or negative.
- range() (and Python in general) is 0-index based, meaning list indexes start at 0, not 1

Numerical solution of a sliding block: Optimal theta?

Given $W = 25 \text{ kN}(2500 \text{ kg})$ and $\mu = 0.75$, what's the optimum θ ?



② Identifying optimum requires conditional statements

Comparison on int, float and strings

`i` and `j` are variable names and comparisons below evaluate to a Boolean

- `i > j`
- `i >= j`
- `i < j`
- `i <= j`
- `i == j`: equality test, True if `i` is the same as `j`
- `i != j`: in equality test, True if `i` is not the same as `j`

Logic operators on bools

a and b are variable names with Boolean values

- not a: True if a is False
False if a is True
- a and b: True if both are True.
- a or b: True if either or both are True.

A	B	A and B	A or B
True	True	True	True
True	False	False	True
False	True	False	True
False	False	False	False

Designing a smart window: if condition



- An electric window opener, attached to a rain sensor and a temperature gauge, might be controlled by the following program:
- If raining: close window
- If too hot (80F): open window
- If too cold (66F): close window
- Otherwise: do nothing and leave window as it is

Designing a smart window: if condition

```
# If raining, close the window
if raining:
    close_window()

# If the temperature is over 80 F, open window
elif temperature > 80: # else if
    open_window()

# If the temperature is below 66 F, close window
elif temperature < 66:
    close_window()

# Otherwise, do nothing and leave window as it is
else:
    continue
```

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└ Numerical solution of a sliding block

└ Designing a smart window: if condition

Designing a smart window: if condition

```
# If raining, close the window
if raining:
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# If the temperature is over 80 F, open window
elif temperature > 80: # else if
    open_window()

# If the temperature is below 66 F, close window
elif temperature < 66:
    close_window()

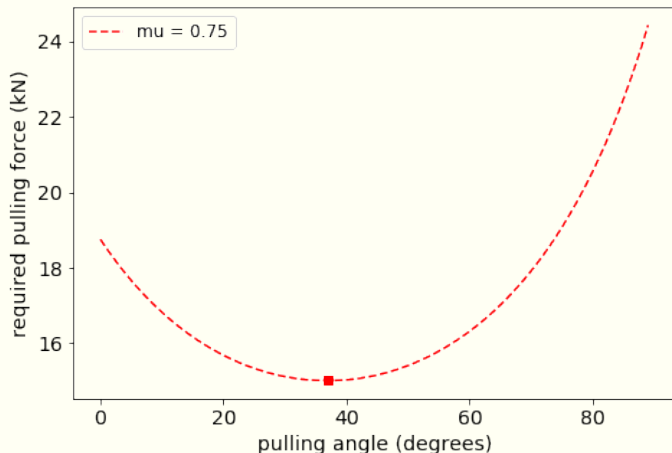
# Otherwise, do nothing and leave window as it is
else:
    continue
```

<condition> has a value True or False

evaluate expressions in that block if <condition> is True

Numerical solution of a sliding block: Optimal theta?

Given $W = 25 \text{ kN}(2500 \text{ kg})$ and $\mu = 0.75$, what's the optimum θ ?



Identify optimum with an if conditional statement

1 Numerical solution of a sliding block

2 Bisection method

Calculate the optimum angle to pull for a given force

- Given $F = 17.5 \text{ kN}$ (1750 kg), $W = 25 \text{ kN}$ and $\mu = 0.75$, what's θ ?

$$\text{Try } \theta = 30^\circ : F = \frac{0.75 \times 25}{\cos(30) + 0.75 \sin(30)} = 15.108 \text{ kN.}$$

$$\text{Try } \theta = 50^\circ : F = \frac{0.75 \times 25}{\cos(50) + 0.75 \sin(50)} = 15.403 \text{ kN.}$$

$$\text{Try } \theta = 40^\circ : F = \frac{0.75 \times 25}{\cos(40) + 0.75 \sin(40)} = 15.022 \text{ kN.}$$

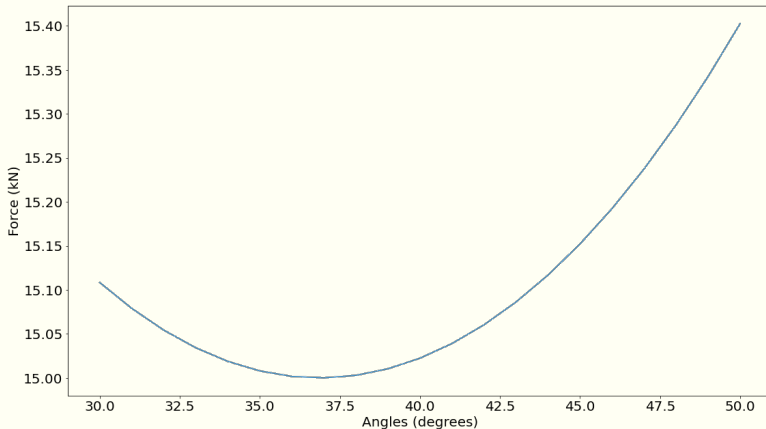
$$\text{Try } \theta = 35^\circ : F = \frac{0.75 \times 25}{\cos(35) + 0.75 \sin(35)} = 15.008 \text{ kN.}$$

$$\text{Try } \theta = 37.5^\circ : F = \frac{0.75 \times 25}{\cos(37.5) + 0.75 \sin(37.5)} = 15.001 \text{ kN.}$$

This is **bisection method**!

Calculate the optimum angle to pull for a given force

- Given $F = 17.5 \text{ kN}$ (1750 kg), $W = 25 \text{ kN}$ and $\mu = 0.75$, what's θ ?



What are the characteristics of a numerical solution?

- A numerical recipe is a *sequence of simple steps*
- *Flow of control* as each step is executed.
- Yields an *approximate* numerical answer (a finite number) for the problem
- These solutions can be very accurate
- Most answers are determined in an iterative approach (numerical method: mathematical / computer-aided technique) until a desired minimum/acceptable accuracy is obtained
- Typically, a finite set of iterations (steps) are used in the numerical method to obtain a solution. A means of determining *when to stop*.

Numerical solution of a sliding block: Friction angles

