CE 311K: Newton Raphson and Differentiation

Krishna Kumar

University of Texas at Austin

krishnak@utexas.edu

October 10, 2020

1 Newton Raphson

Newton Raphson

Assuming r is a root of f and that f is continuously differentiable in the vicinity of r with $f'(r) \neq 0$, then a sequence (x_n) that converges to r for $n \to \infty$ can be found using the Taylor expansion of f:

$$f(r) = f(x_n + \varepsilon_n) = f(x_n) + f'(x_n)\varepsilon_n + \frac{f''(x_n)}{2!}\varepsilon_n^2 \ldots$$

$$\varepsilon_n \approx -\frac{f(x_n)}{f'(x_n)}$$

$$r = x_n + \varepsilon_n \approx x_n - \frac{f(x_n)}{f'(x_n)}$$

in other words $x_n - \frac{f(x_n)}{f'(x_n)}$ is the next iteration of r, and hence we write:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Newton Raphson

Assuming r is a root of f and that f is continuously differentiable in the vicinity of r with $f'(r) \neq 0$, then a sequence (x_n) that converges to r for $n \to \infty$ can be found using the Taylor expansion of f:

$$
\begin{aligned}
f(r) = f(x_n + \varepsilon_n) &= f(x_n) + f'(x_n)\varepsilon_n + \frac{f''(x_n)}{2!}\varepsilon_n^2 \dots \\
\varepsilon_n &\approx -\frac{f(x_n)}{f'(x_n)} \\
r = x_n + \varepsilon_n &\approx x_n - \frac{f(x_n)}{f'(x_n)}
\end{aligned}
$$

in other words $x_n - \frac{f(x_n)}{f'(x_n)}$ is the next iteration of r, and hence we write:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

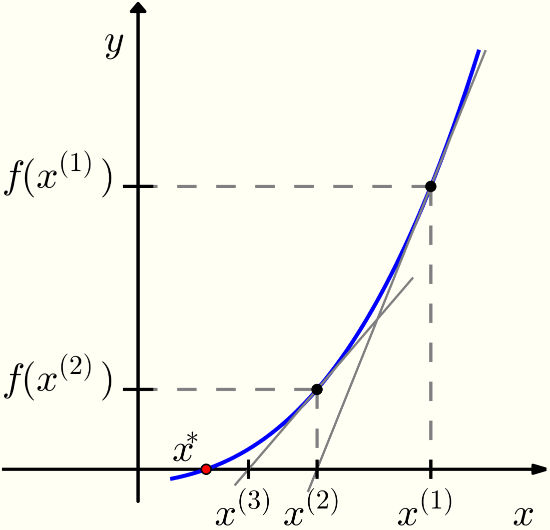
We wish to find roots of $f(x)$ using a converging sequence (x_n). But we want to do it faster.

Newton's original method (1685) was purely algebraic, which he applied only to polynomials and used a sequence of polynomials instead of successive approximations x_n.

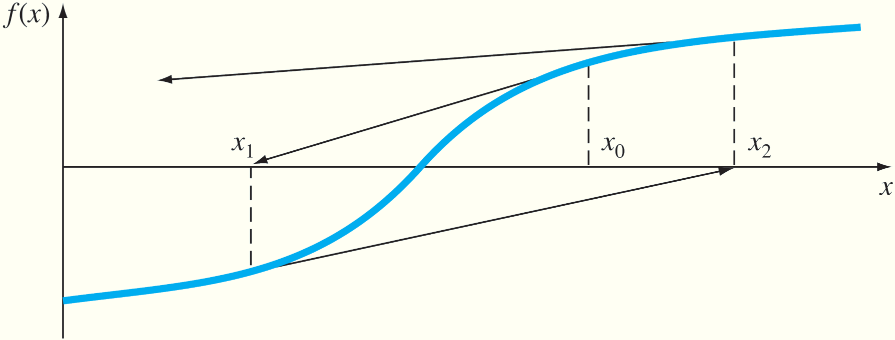
Raphson's simplified version (1690) was also only algebraic and he applied it only to polynomials but used x_n approximations.

Simpson gave the form used today 50 years later (1740), along with other important results in the same paper.

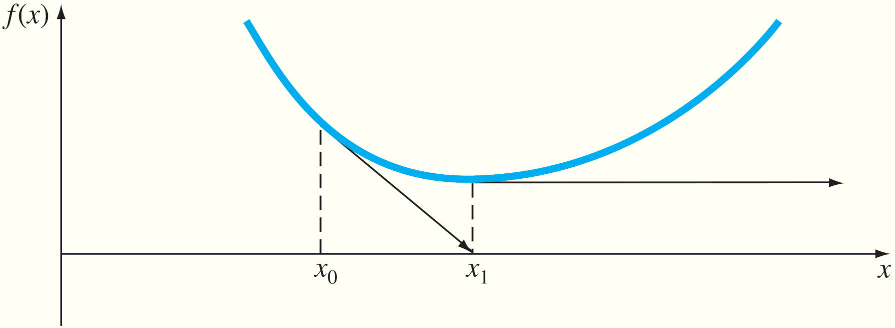
Newton-Raphson graphical expression



Newton-Raphson failure



Newton-Raphson failure



Newton-Raphson failure