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Subject code: MAST20006

Subject name: Probability for Statistics

Assignment 1

Q1:

(a) Let P_1, P_2, P_3 be the probability of winning for each player

- The probability that the 1st player gets green is:

$$P_1 = \frac{2}{10} = 0.2 \Rightarrow P_1' = 1 - \frac{2}{10} = \frac{8}{10}$$

- 2nd player only wins if 1st player doesn't get green (P_1') AND 2nd player gets green, so:

$$P_2 = \frac{8}{10} * \frac{2}{9} = \frac{8}{45} = 0.1(7)$$

- 3rd player only wins if NEITHER 1st player nor 2nd player can get green AND 3rd player gets green,

so: $$P_3 = \frac{8}{10} * \frac{7}{9} * \frac{2}{8} = \frac{7}{45} = 0.1(5)$$

Since $P_1 (= 0.2) > P_2 (= 0.1(7)) > P_3 (= 0.1(5))$, the best choice is 1st position (1st player)

Q1 (b)

Let the number green cards be g

Then, $P_1 = \frac{g}{10}$

$$P_2 = \frac{10-g}{10} * \frac{g}{9}$$

Solving for g ($g \geq 1$), set $P_1 = P_2$:

$$\frac{g}{10} = \frac{10-g}{10} * \frac{g}{9} \iff \frac{g}{10} = \frac{10g - g^2}{90}$$

$$\iff 90g = 100g - 10g^2$$

$$\iff 10g^2 - 10g = 0$$

$$\iff 10g(g-1) = 0$$

$$\iff g = 1 \quad \text{OR} \quad g = 0$$

$$\iff g = 1 \ (g \geq 1)$$

Thus, if there's 1 green card in the deck, the chance to win is the same to draw first or second.

Q2:

(a) $P(A') = 1 - P(A) = 1 - 2/3 = 1/3$

(b) $P(A' \cap B') = P((A \cup B)')$ (De Morgan's law)

$= 1 - P(A \cup B)$

$= 1 - (P(A) + P(B) - P(A \cap B))$

(Inclusion-exclusion principle)

$= 1 - (\frac{2}{3} + \frac{2}{3} - \frac{2}{5}) = \frac{1}{15}$

(c) $P(B \cap A') = P(B) - P(A \cap B)$

($(B \cap A')$ is the part of B that doesn't overlap with A)

$= \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$

(d) $P(A \cup B') = P(A) + P(B') - P(A \cap B')$

(Inclusion-Exclusion principle)

$= \frac{2}{3} + \frac{1}{3} - P(B \cap A') = 1 - \frac{4}{15} = \frac{11}{15}$

(e) $P(A' \mid B) = \frac{P(A' \cap B)}{P(B)}$ (Definition of conditional probability)

$= \frac{P(B \cap A')}{2/3} = \frac{4/15}{2/3} = \frac{2}{5}$.

Q3: F: fail, D: defective, D': not defective

Given $P(F|D) = 0.1$

$P(F|D') = 0.02$

$P(D) = 0.005 \Rightarrow P(D') = 0.995$

(a) Now D and D' are mutually exclusive and exhaustive events, and thus form a partition for the sample space so we use the law of total probability:

$$P(F) = P(F|D)\,P(D) + P(F|D')\,P(D')$$

$$= 0.1 * 0.005 + 0.02 * 0.995$$

$$= 0.0204$$

(b) Use Bayes's theorem:

$$P(D|F) = \frac{P(D)\,P(F|D)}{P(F)} \quad \text{(From (a))}$$

$$= \frac{0.005 * 0.1}{0.0204} = \frac{5}{204} = 0.0245 \text{ (4d.p)}$$