Exercise 1.1

Problem

Consider the sum-of-squares error function given by (1.2) in which the function $y(x, w)$ is given by the polynomial (1.1). Show that the coefficients $w = w_i$ that minimize this error function are given by the solution to the following set of linear equations: $\sum_{j=0}^{M} A_{ij} w_j = T_i$, where $A_{ij} = \sum_{n=1}^{N} (x_n)^{i+j}$, and $T_i = \sum_{n=1}^{n} (x_n)^i t_n$

Formulae

(1.1): $y(x, \mathbf{w}) = w_0 + w_1 x + w_2 x^2 + ... + w_M x^M = \sum_{j=0}^{M} w_j x^j$
(1.2): $E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} y(x_n, \mathbf{w}) - t_n{}^2$

Solution

Substitute (1.1) into (1.2), and then let the derivative of the error term with respect to $\mathbf{w}$ equal 0:

$$\frac{\delta E}{\delta w} = \sum_{n=1}^{N} (\sum_{j=0}^{M} w_j x_n^j - t_n) x_n^i = 0$$

$$\sum_{n=1}^{N} \sum_{j=0}^{M} w_j x_n^{i+j} = \sum_{n=1}^{N} x_n^i t_n$$

This can be simplified to resemble equation (1.222)

Exercise 1.2