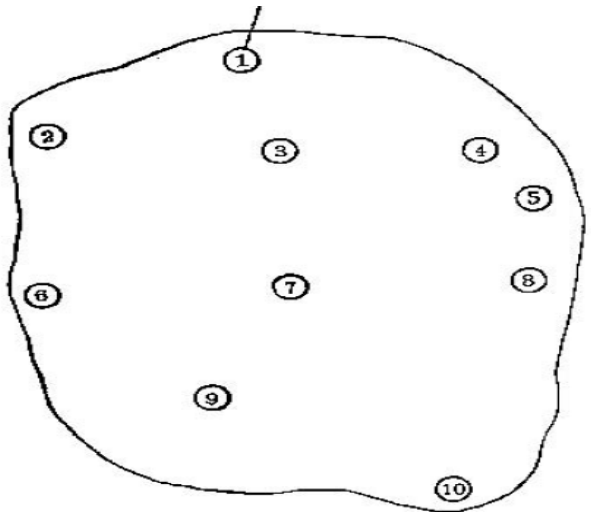
Hamilton 圈问题

Hamilton 圈：包含图$G(V,E)$中所有顶点的圈。

例

下图是包含 10 个顶点的图，各个顶点之间的距离见下表。现要找一条最短的 Hamilton 圈。



	2	3	4	5	6	7	8	9	10
1	8	5	9	12	14	12	16	17	22
2		9	15	17	8	11	18	14	22
3			7	9	11	7	12	12	17
4				3	17	10	7	15	18
5					8	10	6	15	15
6						9	14	8	16
7							8	6	11
8								11	11
9									10

最短 Hamilton 圈的数学表达式

设d_{ij}为顶点i到顶点j的距离，$x_{ij}=1$表示边(i,j)在所求路上，$x_{ij}=0$表示边(i,j)不在所求路上。

$$\min \sum_{(i,j)\in E} d_{ij}x_{ij}$$

$$s.t. \quad \sum_{j\in V} x_{ij} = 1, i = 1,\cdots 10;$$

$$\sum_{j\in V} x_{ji} = 1, i = 1,\cdots 10;$$

$$u_i - u_j \le (n-1) - n * x_{ij}$$

$$u_i \ge 0$$

$$u_j \ge 0$$

$$(i, j = 2,3,\cdots,n;)$$

LINGO 程序求解

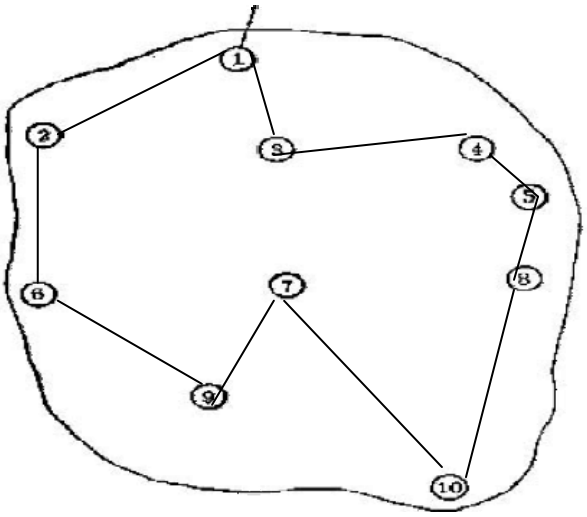
```
model:
sets:
cities/1..10/:u;
link(cities, cities): d,x;
endsets
data:
d = 0  8  5  9 12 14 12 16 17 22
    8  0  9 15 16  8 11 18 14 22
    5  9  0  7  9 11  7 12 12 17
    9 15  7  0  3 17 10  7 15 15
   12 16  9  3  0  8 10  6 15 15
   14  8 11 17  8  0  9 14  8 16
```

```
    12 11  7 10 10  9  0  8  6 11
    16 18 12  7  6 14  8  0 11 11
    17 14 12 15 15  8  6 11  0 10
    22 22 17 15 15 16 11 11 10  0;
enddata
n=@size(cities);
min=@sum(link(i,j): d(i,j)*x(i,j));
@for(cities(i):
   @sum(cities(j)|j#ne#i: x(j,i))=1;
   @sum(cities(j)|j#ne#i: x(i,j))=1;
);
@for(link(i,j)|i#ne#1 #and#
j#ne#1:u(i)-u(j)<=n-1-n*x(i,j));
@for(link : @bin(x));
end
```

结果

```
Global optimal solution found.
Objective value:                              73.00000
Objective bound:                              73.00000
Infeasibilities:                              0.000000
Extended solver steps:                               1
Total solver iterations:                           827

                     Variable           Value        Reduced Cost
                     X( 1, 2)       1.000000            8.000000
                     X( 2, 6)       1.000000            8.000000
                     X( 3, 1)       1.000000            5.000000
                     X( 4, 3)       1.000000            7.000000
                     X( 5, 4)       1.000000            3.000000
                     X( 6, 9)       1.000000            8.000000
                    X( 7, 10)       1.000000            11.00000
                     X( 8, 5)       1.000000            6.000000
                     X( 9, 7)       1.000000            6.000000
                    X( 10, 8)       1.000000            11.00000
```



结果不唯一