Optimization: Transportation Network Analysis

University of Illinois at Urbana Champaign
IE 533 Project

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I. Project Overview

The primary objective of our project is to determine the shortest routes between the top three populated elementary schools and the two hospitals located in the East District of Hsinchu City, Taiwan. Hsinchu City is a province-administrative city that is often referred to as the Silicon Valley of Taiwan. As elementary school students are highly active and prone to injury and illness, it is crucial for parents and teachers to be aware of the most efficient routes to the nearest hospitals. There are two well-equipped hospitals in the East District of Hsinchu City.

To address the routing optimization problem, we utilized a graph of the highway transportation network in the area and implemented Kruskal's Algorithm and Network Simplex Algorithm to determine the shortest path. Our project aims to provide a valuable resource to parents and teachers, enabling them to quickly and effectively reach medical facilities in case of an emergency.

II. Background and Assumptions

(1) Three elementary schools with the most students in Hsinchu City also come from different areas:

- Rural Area: Guandong Elementary School, Guandong Village, 1822 students
- Suburbs: Dongyuan Elementary School, Dongxi Village, 2166 students
- Downtown Area: Sanmin Elementary School, Fuxin Village, 1925 students

(2) The two largest hospitals in the East district, Hsinchu City:

- MacKay Memorial Hospital, located in Guangfu Village with 1200 beds
- Cathay General Hospital, located in Fude Village with 839 beds

(3) Assumptions

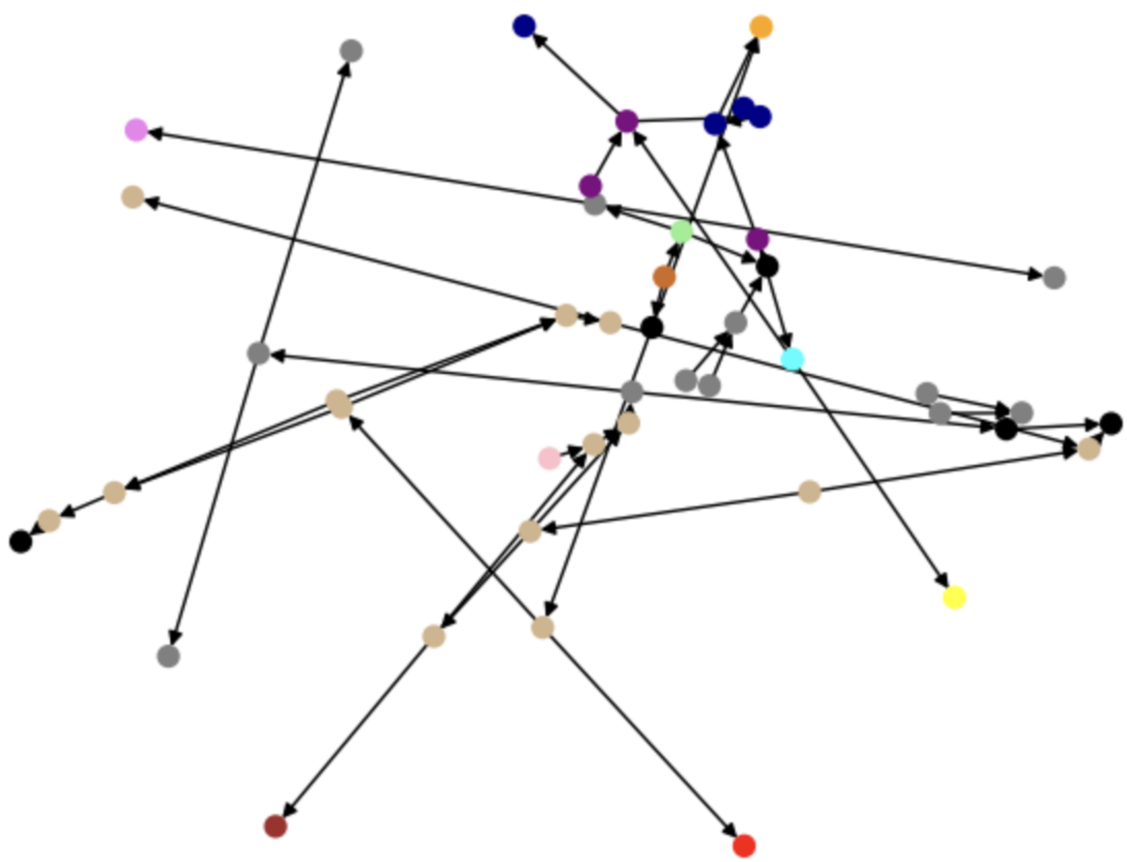
- We assumed that Dongyuan and Guandong Elementary schools would be in closer proximity to MacKay Hospital, while Sanmin elementary school would be nearer to Cathay Hospital.
- We chose the highway system as the basis for our analysis due to the absence of traffic lights on the highway, which we believed would result in more efficient routing.
- Our target selection is based on a combination of the number of students and the location of each school to ensure that our analysis was reflective of the needs of

the local community. These assumptions were integral to our project and were made based on our research and understanding of the local context.

III. Data Pre-processing

(1) Get the transportation network graph

First, we extracted every node and edge from the main transportation network, which is the undirected road map of the highway for East District, Hsinchu. Additionally, we extracted a smaller transportation network to specify the nodes that represent each village in East District, Hsinchu. Second, we generated a graph of the highway system in East District, Hsinchu. For each highway, we produced a graph in which the locations of starting points and destinations were colored. Similarly, the source and sink in each highway were considered to be the set of nodes, as the highway itself can be regarded as the set of edges. This approach enabled us to clearly visualize the transportation network and identify the shortest routes between the elementary schools and hospitals.



(2) Add edges to the graph with their respective weights to get the adjacency matrix

```
[[0.     0.5697 0.585  ... 0.     0.     0.    ]
 [0.5697 0.     0.     ... 0.     0.     0.    ]
 [0.585  0.     0.     ... 0.     0.     0.    ]
 ...
 [0.     0.     0.     ... 0.     0.     0.    ]
 [0.     0.     0.     ... 0.     0.     0.    ]
 [0.     0.     0.     ... 0.     0.     0.    ]]
```

(3) Compute the distance matrix by using Floyd–Warshall Algorithm base on the adjacency matrix

The Floyd-Warshall algorithm is a well-known algorithm used to solve the all-pairs shortest path problem in a weighted directed graph. The algorithm generates a distance matrix from an adjacency matrix in the following steps:

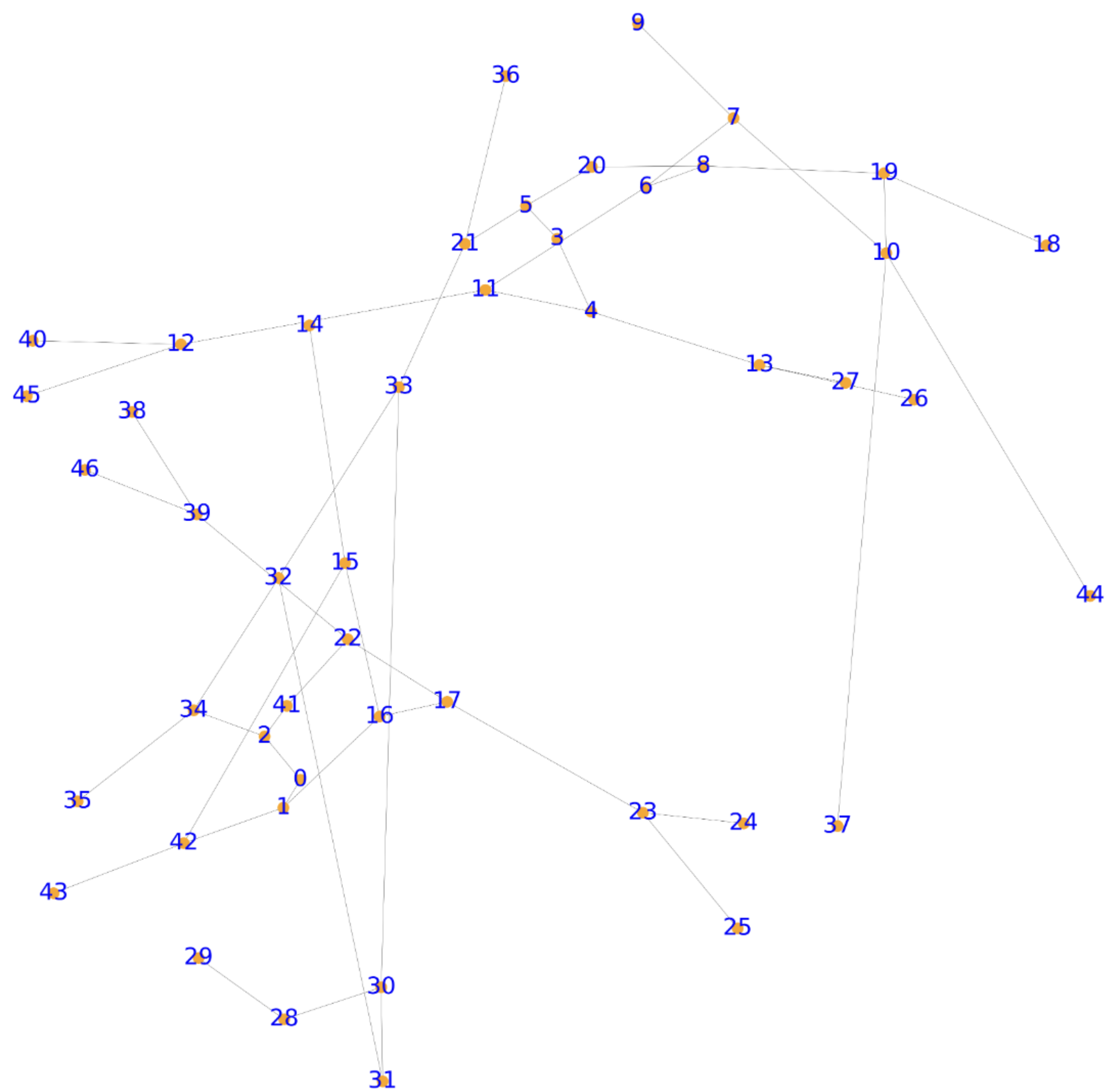
1. Initialize the distance matrix to be the same as the adjacency matrix.
2. For each vertex in the graph, set the distance to itself to be 0.
3. For each pair of vertices (i,j) in the graph, check if there is a shorter path that goes through a vertex k. If such a path exists, update the distance between i and j to be the sum of the distance between i and k and the distance between k and j.
4. Repeat step 3 for all possible vertices k.
5. After all pairs have been checked, the resulting distance matrix will contain the shortest distances between all pairs of vertices.

```
[[0.     0.5697 0.585  ... 0.9081 3.6028 1.8283]
 [0.5697 0.     1.1547 ... 0.3384 4.1725 1.2586]
 [0.585  1.1547 0.     ... 1.4931 3.0178 1.6065]
 ...
 [0.9081 0.3384 1.4931 ... 0.     4.5109 1.2609]
 [3.6028 4.1725 3.0178 ... 4.5109 0.     4.6243]
 [1.8283 1.2586 1.6065 ... 1.2609 4.6243 0.    ]]
```

(4) Get the coordinates (latitude, longitude) for each node, use the coordinates to generate the distance matrix for villages in East District, Hsinchu City

```
{59841243: (121.003667, 24.7768238),
 1978206250: (121.0044473, 24.7866504),
 1978206256: (121.0043583, 24.7869458),
 311080350: (121.009676, 24.8047102),
 313164209: (121.0088576, 24.8010811),
 382954328: (121.0066285, 24.7929114),
 311080356: (121.0067684, 24.7935742),
 311080358: (121.0055276, 24.7914947),
 338641979: (121.0045261, 24.7884144),
 311080367: (121.0045872, 24.7910325),
 1983162853: (121.0053015, 24.7904898),
 313164190: (121.0074571, 24.7954034),
 313164197: (121.0077582, 24.7966884),
 1168830664: (121.0082732, 24.7988829),
 314947099: (121.0063313, 24.7896862),
 314947113: (121.0062476, 24.7889482),
 314947102: (121.005472, 24.7895001),
 413339632: (121.0078575, 24.7953315),
 338641974: (121.0046879, 24.7908094),
 338641976: (121.0048262, 24.7899693),
 1714279107: (121.0036301, 24.7845537),
 1714279093: (121.0031839, 24.7822091),
 1747621095: (121.0094441, 24.801035),
 413339633: (121.0084083, 24.7965816),
 1747620923: (121.0100934, 24.7967764),
...
 1978206287: (121.0062905, 24.7887926),
 1978206324: (121.0071676, 24.7891221),
 1726059983: (121.0052975, 24.7903819),
 3340976219: (121.0086998, 24.7976513),
 4461256993: (121.0080639, 24.7978206)}
```

(5) Generate the undirected network for villages in East District, Hsinchu City



(6) Get the node numbers of the villages where the target elementary schools and hospitals are located.

- Dongshi Village (Dongyuan Elementary School): Node 9
- Fuxin Village (Sanmin Elementary School): Node 7
- Guandong Village (Guandong Elementary School): Node 24
- Guanfu Village (MacKay Memorial Hospital): Node 27
- Fude Village (Cathay General Hospital): Node 36

IV. Implementations of Algorithms

(1) Dijkstra's Algorithm

- Dijkstra's algorithm generates the shortest path from a starting vertex to all other vertices in a weighted graph. It does this by maintaining a set of visited vertices and a set of unvisited vertices, initially containing only the starting vertex.
- At each iteration, the algorithm chooses the unvisited vertex with the smallest distance from the starting vertex and marks it as visited. It then examines all of the unvisited neighbors of the current vertex and updates their distances if a shorter path to them is found through the current vertex.
- This process continues until all vertices have been visited or until the shortest path to the target vertex has been found. Once the shortest path has been found, the algorithm can trace back through the table of distances to identify the intermediate vertices used in the path.
- Dijkstra's algorithm is a greedy algorithm, which means that it always chooses the path that looks the most promising at the current stage, without worrying about the global optimal solution. However, it guarantees finding the shortest path in a weighted graph with non-negative edge weights.

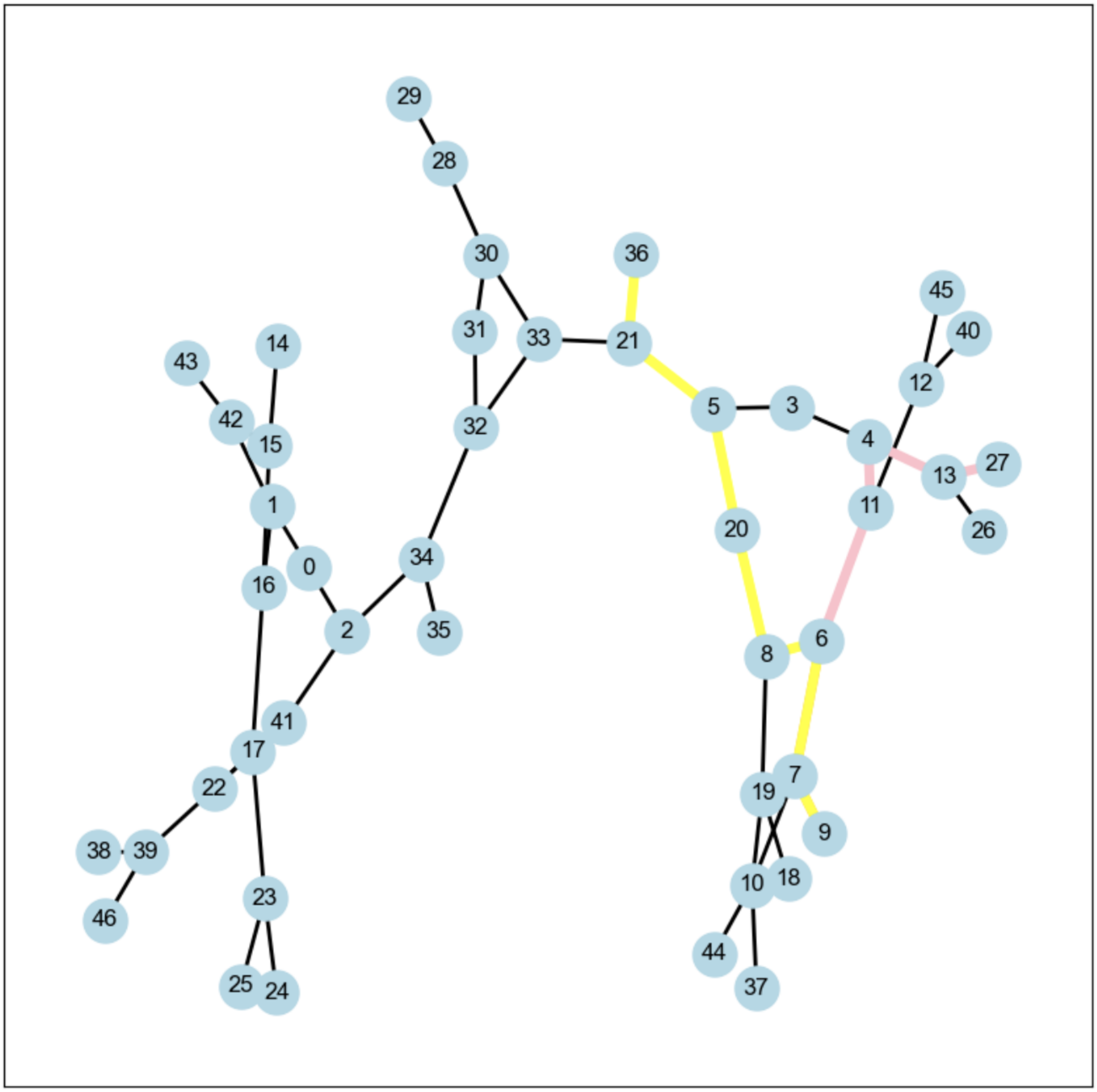
(i) Dongyuan Elementary School (Node 9)

- The distance to MacKay Memorial Hospital (Node 27) is 4.62929.

The shortest path from Node 9 to Node 27 is: 9 -> 7 -> 6 -> 8 -> 20 -> 5 -> 21 ->36

- The distance to Cathay General Hospital (Node 36) is 2.3446.

The shortest path from Node 9 to Node 36 is: 9 -> 7 -> 6 -> 11 -> 4 -> 13 -> 27



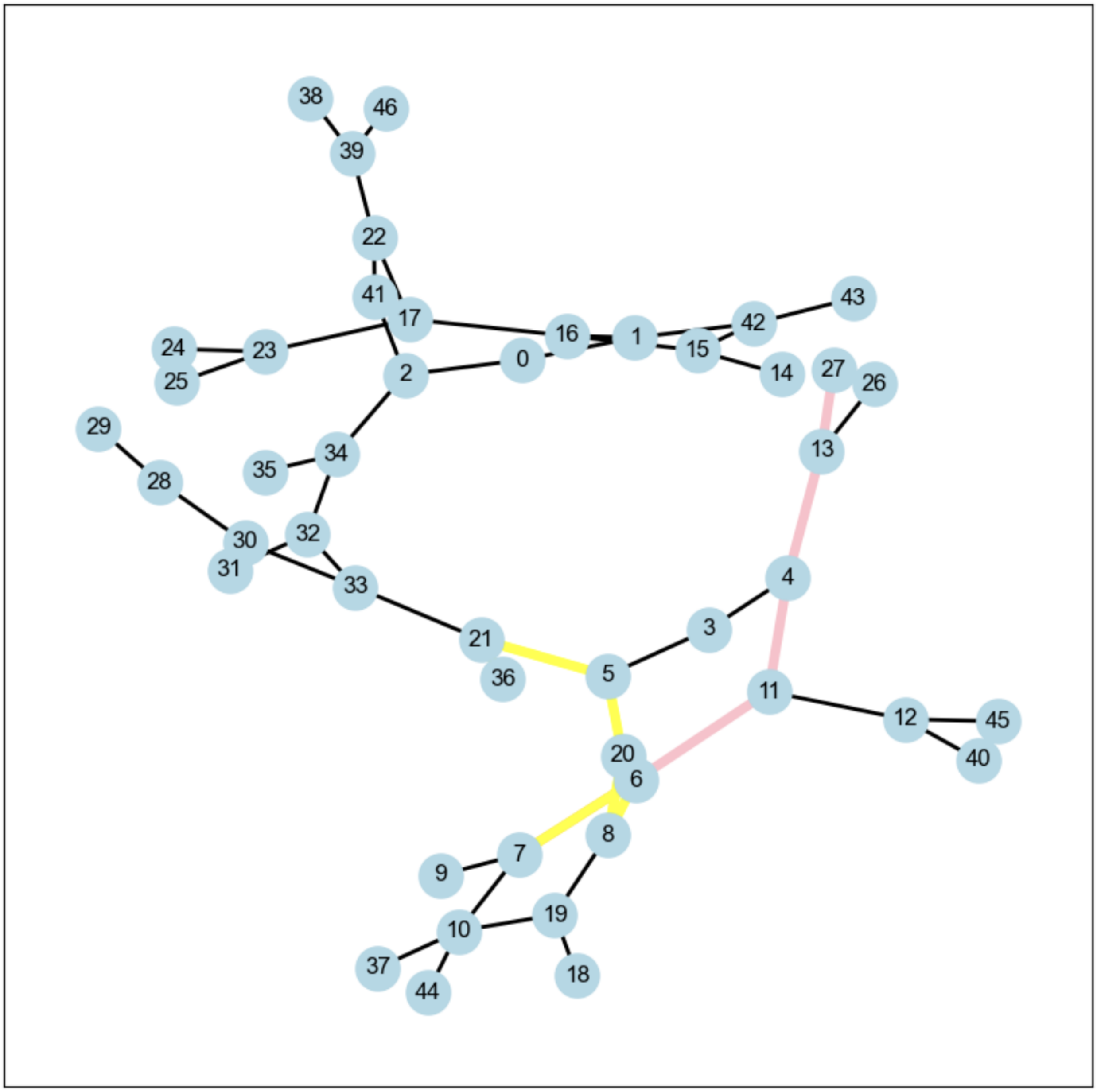
(ii) Sanmin Elementary School (Node 7)

- The distance to MacKay Memorial Hospital (Node 27) is 4.269799.

The shortest path from Node 7 to Node 27 is: 7 -> 6 -> 8 -> 20 -> 5 -> 21 ->36

- The distance to Cathay General Hospital (Node 36) is 1.985099.

The shortest path from Node 7 to Node 36 is: 7 -> 6 -> 11 -> 4 -> 13 -> 27



(iii) Guandong Elementary School (Node 24)

- The distance to MacKay Memorial Hospital (Node 27) is 0.6129.

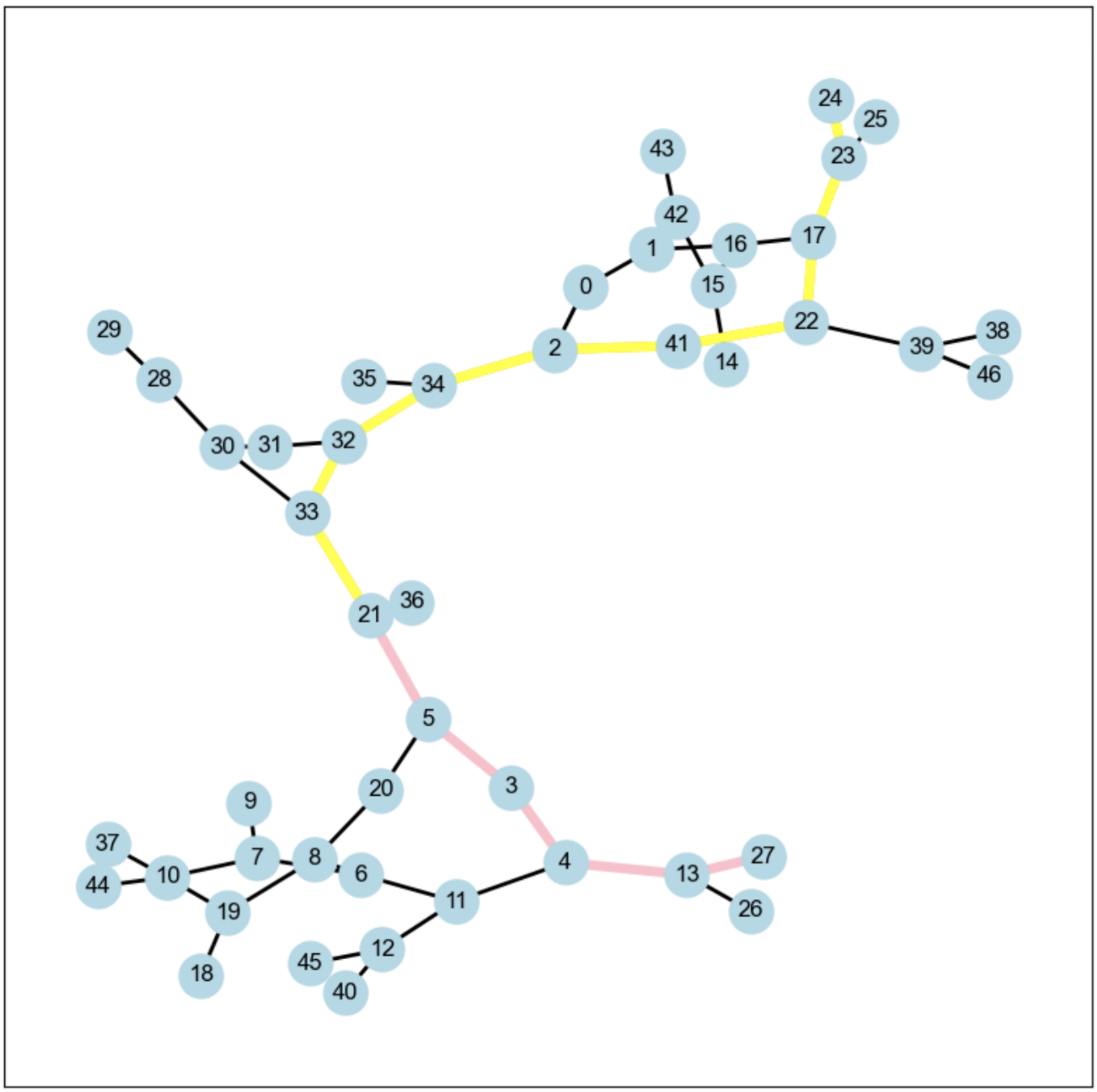
The shortest path from Node 24 to Node 27 is:

24 -> 23 -> 17 -> 22 -> 41 -> 2 ->34 -> 32 -> 33 -> 21 -> 36

- The distance to Cathay General Hospital (Node 36) is 1.8874.

The shortest path from Node 24 to Node 36 is:

24 -> 23 -> 17 -> 22 -> 41 -> 2 ->34 -> 32 -> 33 -> 21 -> 36 -> 5 -> 3 -> 4 -> 13 -> 27



(2) Floyd–Warshall Algorithm

- The Floyd-Marshall algorithm generates the shortest path between all pairs of vertices in a weighted graph. It works by gradually building up a table of distances between pairs of vertices, where the entry at position (i, j) in the table represents the shortest path between vertex i and vertex j.
- The algorithm begins by initializing the distance table to reflect the distances given by the graph's adjacency matrix. It then iteratively updates the distance table by considering all possible intermediate vertices on a path between each pair of vertices. Specifically, for each pair (i, j), the algorithm considers all vertices k and determines whether the path from i to j through k would be shorter than the current distance between i and j. If it is, then the distance table is updated accordingly.
- By the end of the algorithm, the distance table will contain the shortest path between all pairs of vertices. The actual shortest paths can be reconstructed by tracing back through the table to identify the intermediate vertices used in each path. The Floyd-Marshall algorithm is a dynamic programming algorithm, which means that it builds up solutions to subproblems and reuses those solutions to solve larger problems.

(i) MacKay Memorial Hospital (Node 27)

Shortest Path from three elementary schools:

```
Shortest path from 9 (Dongyuan Elementary School) to 27: 9 -> 15 -> 2 -> 27
Shortest path from 7 (Sanmin Elementary School) to 27: 7 -> 2 -> 27
Shortest path from 24 (Guandong Elementary School) to 27: 24 -> 27
```

Unknown node numbers on the path:

```
position = number.index(15)
nodes[position]
```

```
'314947113'
```

```
position = number.index(2)
nodes[position]
```

```
'1978206256'
```

(ii) Cathay General Hospital (Node 36)

Shortest Path from three elementary schools:

```
Shortest path from 9 (Dongyuan Elementary School) to 36: 9 -> 36
Shortest path from 7 (Sanmin Elementary School) to 36: 7 -> 11 -> 36
Shortest path from 24 (Guandong Elementary School) to 36: 24 -> 36
```

Unknown node numbers on the path:

```
position = number.index(11)
nodes[position]

'313164190'
```

(3) Network Simplex Algorithm

- The network simplex algorithm is used to find the minimum cost flow in a directed network, which can be used to determine the shortest path from a source to a sink in a weighted graph. The algorithm works by maintaining a feasible flow on the network and iteratively adjusting the flow until the minimum cost flow is achieved.
- The first step is to initialize the flow with some feasible flow, which can be determined using other algorithms such as the Ford-Fulkerson method. Then, the algorithm performs a sequence of iterations, where each iteration involves finding a cycle in the residual graph that can be improved by increasing the flow along some arcs and decreasing it along others.
- During each iteration, the algorithm uses a pricing rule to determine the relative cost of each arc in the residual graph. This pricing rule is used to determine which arc should be used to increase the flow and which should be used to decrease it, in order to reduce the total cost of the flow.
- The algorithm terminates when no further improvement can be made to the flow, at which point it returns the minimum cost flow. The path from the source to the sink can then be determined by examining the flow along the arcs in the network.
- The network simplex algorithm is often used in transportation and logistics applications, where it is necessary to find the shortest path or minimum cost flow through a network of interconnected nodes and edges.

(i) MacKay Memorial Hospital (Node 27)

Shortest Path from three elementary schools:

```
[9, 7, 6, 11, 4, 13, 27]
[7, 6, 11, 4, 13, 27]
[24, 23, 17, 22, 41, 2, 34, 32, 33, 21, 5, 3, 4, 13, 27]
```

(ii) Cathay General Hospital (Node 36)

Shortest Path from three elementary schools:

```
[9, 7, 6, 8, 20, 5, 21, 36]
[7, 6, 8, 20, 5, 21, 36]
[24, 23, 17, 22, 41, 2, 34, 32, 33, 21, 36]
```

V. Conclusion

We found that the results from the Network Simplex Algorithm and Dijkstra's Algorithm were consistent. Our analysis indicated that Cathay Hospital would be the optimal choice for schools located in the downtown and suburban areas, while MacKay Hospital would be the preferable option for schools near rural areas.

Our findings deviated from our initial assumptions. We attribute this to the fact that we chose the highway system as the transportation network, which can be complex in some areas and result in redundant traveling time and costs. The importance of running the shortest path problem to determine optimal routes to medical facilities cannot be understated.

Looking ahead, we plan to address an outstanding issue in our study, which is identifying the villages represented by the node numbers on the shortest paths. This will enhance the practicality and applicability of our study. Overall, our study provides a valuable resource for parents and teachers, allowing them to quickly and effectively reach the nearest hospitals in case of an emergency.

VI. Appendix

The geographical map of Hsinchu City and the villages: