Solving Nurse Rostering Problem by Penalty-Based Single Objective Genetic Algorithm

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***Abstract*—Hospitals often rely on experienced nurses to arrange the shifts of nurses, but manual scheduling often takes a lot of time. Therefore, this paper mainly uses genetic algorithms to assist computer operations to speed up the process of nurse scheduling, so as to output objective scheduling results that comply with regulations.**

This paper mainly divides the shift types into four types: Early shift, Afternoon shift, Night shift, and Day-off. In the definition of hard constraint and soft constraint, the definition method is referred to in the related literature.

In this experiment, the Binary Representation method is used to express the genotype of the scheduling results, the penalty-based method is used to design the objective function, and the results of the different combinations of Selection, Crossover, and Mutation methods are analyzed.

The final experiment found that using Fitness best selection, FlipBit mutation with Uniform Crossover or Ordered Crossover method can get the best results that meet all hard constraints requirements.

Keywords—Genetic Algorithms, Nurse Rostering Problem

I. Introduction

Nurse rostering problem is a common problem in every hospital. A good nurse shift schedule has a great impact on the quality of health care. Common shift types are shown in Table 1. According to the literature [1], in medical institutions with many nursing staff or when the roster changes, nurse officers need to spend a lot of time scheduling the nurse's shifts.

TABLE I. Common shift timetable

Shift types	Early	Afternoon	Night	Day-off
Time (24)	7:00~15:00	15:00~23:00	23:00~7:00	

Since 1970, the nurse rostering problem has been regarded as an NP-hard problem. With the gradual increase of legal and hospital hard constraints and the soft constraint requirements that additionally consider nurses' preferences, the nurse rostering problem became more complicated.

Overseas literature has enough complete definitions of hard and soft constraints on nurse rostering problems [2][3][4], so in our paper, we will refer to the relevant hard limits that are close to domestic scheduling in the implementation process, and combined with the nurse preference soft constraints [5] for nurse's shift scheduling.

And using the genetic algorithm with penalty method to let the scheduling results more closely conform to our defined constraints.

The organization of this paper is as follows: Section 2 provides Hard constraints, soft constraints, and objective function definition. Section 3 describes the nurse rostering initial data, rostering algorithms, and its parameters. Section 4 presents the experimental results of our proposed approach. Section 5 gives the conclusion and future work.

II. Constraints and Algorithm

In this section, the content of the definition of hard constraints and soft constraints in this paper will be explained. And define the objective functions that let the Genetic Algorithm converge according to the literature [6][7].

The formula expressions of Hard constraints and Soft constraints are referenced from the literature [8] and the constraint formula expressions in "Computation results on new staff scheduling benchmark instance" by Tim Curtois and Rong Qu.

The variable names that will be used in the constraints formula are shown in Table 2.

TABLE II. Variable table

ID	Description
$\boldsymbol{I}$	Set of Nurses.
$\boldsymbol{T}$	Set of the work shift types.
$\boldsymbol{T}'$	Day-off.
D	Set of days n the rostering horizon={1…h}
$\boldsymbol{X_{idt}}$	1 if nurse i is assigned shift type t on day d, 0 otherwise.
$\boldsymbol{n_{it}^{min}}$	A minimum number of nurses can be assigned to the shift type t
$\boldsymbol{n_{it}^{max}}$	A maximum number of nurses can be assigned to the shift type t

A. Hard Constraints

This paper chooses the legal and hospital-related regulations as hard constraints that cannot be violated in nurse scheduling. Table 3 presents the full list of current hard constraints.

H_4, H_5, and H_6 are designed for the experiment of this paper, and the scope of these three constraints can be adjusted according to the actual needs of the hospital.

TABLE III. HARD CONSTRAINTS TABLE

ID	Description
$\boldsymbol{H_1}$	Each nurse can only work one shift a day.
$\boldsymbol{H_2}$	Each nurse can't schedule two consecutive shifts.
$\boldsymbol{H_3}$	Each nurse must take two day-off during the week.
$\boldsymbol{H_4}$	During Early shifts, there must be at least two to four nurses.
$\boldsymbol{H_5}$	During Afternoon shifts, there must be at least two to four nurses.
$\boldsymbol{H_6}$	During Night shifts, there must be at least one to two nurses.

$$H_1: \sum_{t\in T} \boldsymbol{X_{idt}} \leq 1, \forall i \in I, d \in D \tag{1}$$

$$H_2: \boldsymbol{X_{idt}} + \boldsymbol{X_{i(d+1)t}} < 2, \forall t \in T, i \in I, d \in D \tag{2}$$

$$\boldsymbol{H_3}: \sum_{d\in T'} \boldsymbol{X_{idt}} = 2, \forall i \in I, d = 7 \tag{3}$$

$$H_4: \boldsymbol{n_{it}^{min}} = 2 \textit{ and } \boldsymbol{n_{it}^{max}} = 4, \forall \boldsymbol{t} = \text{Early shift} \tag{4}$$

$$H_5: \boldsymbol{n_{it}^{min}} = 2 \textit{ and } \boldsymbol{n_{it}^{max}} = 4, \forall\ \boldsymbol{t} = \text{Afternoon shift} \tag{5}$$

$$H_6: \boldsymbol{n_{it}^{min}} = 1 \textit{ and } \boldsymbol{n_{it}^{max}} = 2, \forall\ \boldsymbol{t} = \text{Night shift} \tag{6}$$

B. *Soft Constraints*

In this paper, the nurse's preference is used as the soft constraint of scheduling. In the experiment, each nurse's scheduling preference is stored in its array. There are three elements in the array, which represent the morning shift, evening shift, and night shift from left to right, and use 0 or 1 to express the nurse's preference or not.

TABLE IV. SOFT CONSTRAINTS TABLE

ID	Description
$\boldsymbol{S_1}$	Each nurse shifts preference.

As shown in Figure 1, there are ten nurses in the picture. The first nurse prefers the morning shift, the second nurse prefers not to be assigned to the night shift, the third nurse prefers the night shift, etc.

Fig. 1. Example of nurse's preferences

C. *Objective function*

This paper hopes that the results of nurses' shift scheduling can better meet the defined hard constraints. Referring to related literature [9][10][11], in this paper, our hard constraints are given a larger penalty value than soft constraints in the objective function.

$\{H_1^+, H_2^+, H_3^+, H_4^+, H_5^+, H_6^+\}$ and $\{H_1^-, H_2^-, H_3^-, H_4^-, H_5^-, H_6^-\}$ Indicates the number of times the hard constraints has been met and the number of times the hard constraints has been violated according to the Eq.(1)(2)(3)(4)(5)(6).

S_1^+, S_1^- Indicates the number of times the soft constraint has been met and the number of times the soft constraint has been violated.

In this paper, the variables P_h and P_s are used to represent the penalty values of hard constraints and soft constraints respectively.

Objective function:

$$\text{Min. Fitness} = P_h \times \Big(\sum_{i\in I}\sum_{d=0}^{D} \boldsymbol{H}_{1\,id}^{-} + \sum_{i\in I}\sum_{d=0}^{D} \boldsymbol{H}_{2\,id}^{-} + \sum_{i\in I}\sum_{d=0}^{D} \boldsymbol{H}_{3\,id}^{-} + \sum_{i\in I}\sum_{d=0}^{D} \boldsymbol{H}_{4\,id}^{-} + \sum_{i\in I}\sum_{d=0}^{D} \boldsymbol{H}_{5\,id}^{-} + \sum_{i\in I}\sum_{d=0}^{D} \boldsymbol{H}_{6\,id}^{-}\Big) + P_s \times \sum_{i\in I}\sum_{d=0}^{D} \boldsymbol{S}_{1\,id}^{-} \tag{7}$$

III. DATA AND ALGORITHM

This section will describe the data representation, parameter setting, selection, crossover, and mutation methods used in the genetic algorithm experiments in this paper.

A. *Data Representation*

This paper adopts the 0-1 programming method to present the scheduling result of each nurse in a 0-1 manner, as shown in Figure 2. The figure shows one of the nurse's 31-day initial shift results.

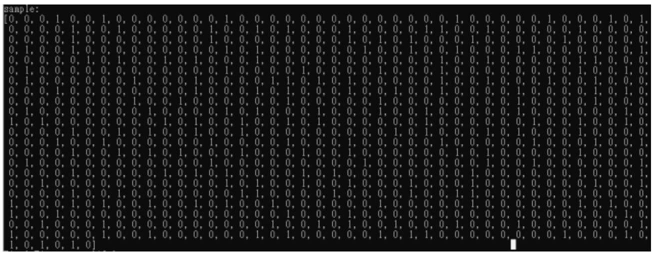

Fig. 2. Example of nurse's scheduling result

The elements in the array are in groups of three, which sequentially represent the scheduling results of the day, as shown in Table 5.

TABLE V. SHIFTS REPRESENTATION TABLE

Representation	[1,0,0]	[0,1,0]	[0,0,1]	[0,0,0]
Shift Types	Early	Afternoon	Night	Day-off

B. *Rostering Algorithms*

The flow of the genetic algorithm in this paper is shown in Figure 3. In the beginning, the shift schedule of each nurse will be presented in a 0-1 manner, and the size of the population and Generation will be initialized. Then, the fitness values will be calculated according to the defined objective function and the operation of the genetic operator will be performed. After that, we will calculate the fitness value to evaluate the scheduling result. Such steps will be repeated until the maximum generation and the final scheduling result will be output.

In this paper, the genetic operator chooses different selection, crossover, and mutation methods for implementation, and the methods used in this paper are shown in Table 6.

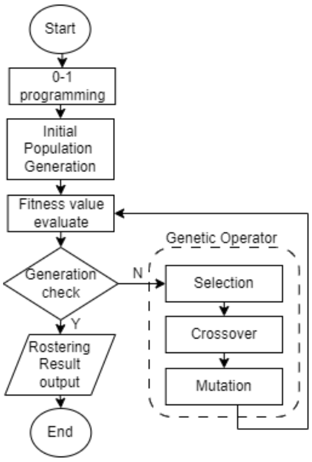

Fig. 3. Genetic Algorithm flow chart

TABLE VI. GENETIC OPERATOR METHOD TABLE

Operator	Method
Selection	Random, Roulette Wheel, Fitness Best, Fitness Worst
Crossover	Two Point, Uniform, Partially Matched, Ordered
Mutation	FlipBit

Because the objective function used in this paper hopes to get the minimum value, this means that the smaller the fitness value is, the better the scheduling result will be.

Next, we are going to explain the details of the genetic operator's method. First, the selection method used in the paper is explained, as shown in Figure 4.

Assuming that each selection will select k chromosomes, **Random Selection** will randomly select k chromosomes for subsequent calculations.

The **Roulette Wheel Selection** means that the smaller the fitness value calculated based on each chromosome, the greater the probability that the chromosome will be selected for subsequent calculations.

In addition, **Fitness Best Selection** will select k chromosomes with the best Fitness value calculation results for subsequent calculations, while **Fitness Worst Selection** will select k chromosomes with the worst Fitness Value calculation results for subsequent calculations.

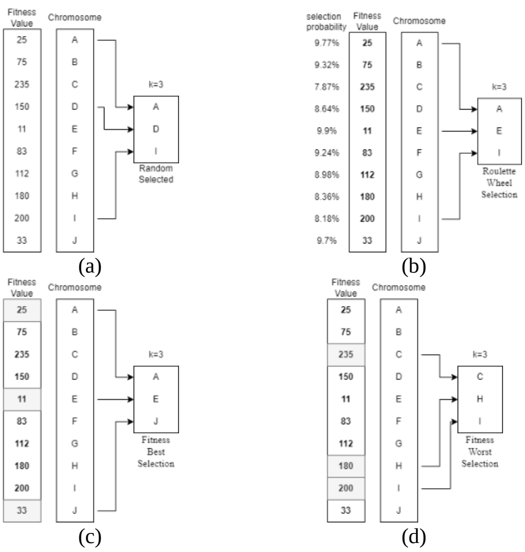

Fig. 4. Selection Functions: (a)Random selection,(b)Roulette Wheel selection, (c)Fitness Best selection, (d)Fitness Worst selection

Second, the Crossover method used in the paper is explained, as shown in Figure 5. **Two Point Crossover** is to exchange the gene which between two points from the two parent chromosomes and to generate the children chromosomes.

Uniform Crossover will generate a set of masks. Whether the gene at a certain position of the parent chromosome needs to be exchanged will be determined by whether the value of the position in the mask is 1, and the exchanged chromosome is the children chromosome.

The **Partially Matched Crossover** method remembers the corresponding relationship between the genes to be exchanged and generates new offspring chromosomes.

The **Ordered Crossover** method remembers the order in which each gene was exchanged in the original parent chromosome, and after the child chromosome is generated, the exchanged genes are rearranged in the order of the parent chromosome.

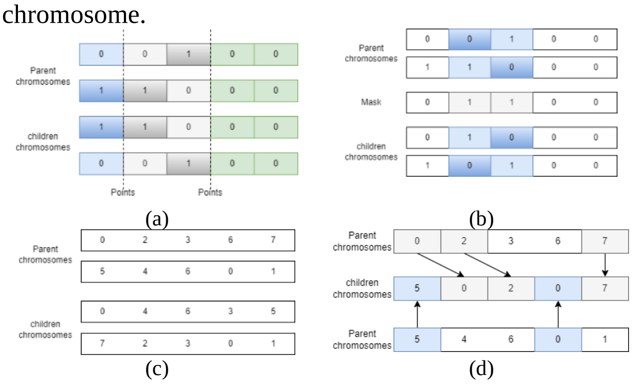

Fig. 5. Crossover Functions: (a) Two Point Crossover,(b) Uniform Crossover, (c) Partially Matched Crossover, (d) Ordered Crossover

Finally, the Mutation method used in the paper is explained. **FlipBit Mutation** mainly randomly converts genes in chromosomes from 1 to 0, and from 0 to 1, as shown in Figure 6.

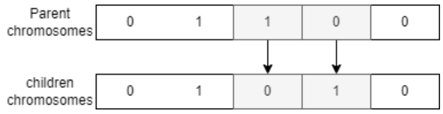

Fig. 6. FlipBit Mutation

The parameter settings related to the genetic algorithm are shown in Table 7.

TABLE VII. GENETIC ALGORITHM PARAMETERS SETTING TABLE

Parameters	Values
Crossover ratio	0.9
Mutation ratio	0.1
Population size	300
Generation size	500
Hard Constraints Penalty value	100

IV. EXPERIMENT

This section will describe the analysis and comparison of the experimental results of different genetic operator methods in this paper, and will finally give an optimal method combination and output the best nurse shift scheduling results.

In this paper, a total of nine sets of experiments were implemented. First, the Selection method was adjusted. After the best selection method was found, then the Crossover was adjusted, after that, we can finally have the best genetic operator combination in the experimental data and the final scheduling result simultaneously. The data results are shown in Table 8, and the convergence curve is shown in Figure 7.

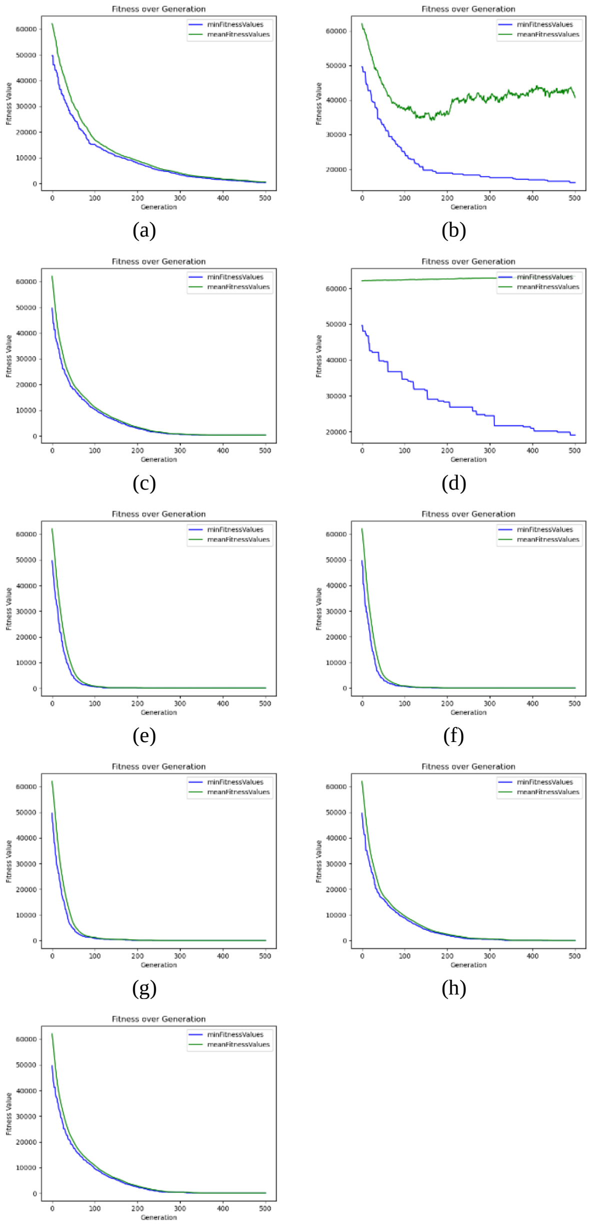

Fig. 7. Convergence Process Curve: (a)-(i) represents Table 8 from top to bottom genetic operator combination

From Figure 7, it can be found that if the Roulette wheel Selection method or the Fitness worst selection method is chosen for the Selection method, poor convergence results will be obtained. Other combinations can get smoother and normally converged curves.

TABLE VIII. EXPERIMENT RESULT TABLE

Selection	Crossover	Mutation	Min Fitness Value	Mean Fitness Value
Random	Two Point	FlipBit	319.0	542.48
Roulette wheel	Two Point	FlipBit	16128.0	40727.8
Fitness best	Two Point	FlipBit	313.0	322.6
Fitness worst	Two Point	FlipBit	19029.0	63359.1
Fitness best	Uniform (0.3)	FlipBit	114.0	128.0
Fitness best	Uniform (0.5)	FlipBit	11.0	23.01
Fitness best	Uniform (0.7)	FlipBit	8.0	17.67
Fitness best	Partially Matched	FlipBit	10.0	22.01
Fitness best	Ordered	FlipBit	111.0	124.01

From Table 8, selecting a parent chromosome with a better fitness value for each generation will be better than selecting a parent chromosome with a poorer fitness value. After subsequent crossover and mutation operator processing. Better fitness values can be obtained by using the Fitness Best selection method.

In addition, if the Roulette wheel with the same number of generations is used, a better fitness value cannot be obtained.

It can also be analyzed from the experimental results that choosing Uniform Crossover or Partially Matched Crossover among the crossover methods can obtain better fitness value than Two-point Crossover and Ordered Crossover.

Based on the experimental results, it can be learned that the Fitness Best Selection method and the FlipBit Mutation method are chosen, and the Uniform Crossover or Partially Matched Crossover method is used can obtain the best fitness value.

We already know that we set the Penalty value of Hard Constraints to 100, because in the end, whether it is Min Fitness Value or Mean Fitness Value, we can get a value less than 100. Therefore, the results of nurse scheduling shifts can fully comply with the laws and regulations and hospital regulations selected in this article.

Finally, we choose the best combination (Fitness Best Selection + Uniform Crossover (0.7) + FlipBit Mutation) in the experimental data to output the best 31-day nurse scheduling results, as shown in Figure 8. And in order to make it easier for nurses to view their shift schedules respectively, we also output the results of the nurses' shift schedule in Excel, as shown in Figure 9.

Fig. 8. Nurse's shifts rostering Result (Terminal)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	
1	Name	1	2	3	4	5	6	7	8	9	10	11	12	13	
2	Allson	Afternoon	Day-off	Day-off	Early	Day-off	Early	Early	Day-off	Day-off	Early	Day-off	Early	Day-off	Ni
3	Amy	Early	Early	Day-off	Early	Day-off	Afternoon	Night	Day-off	Early	Day-off	Afternoon	Day-off	Afternoon	Ni
4	Betty	Day-off	Night	Day-off	Day-off	Day-off	Day-off	Day-off	Day-off	Afternoon	Afternoon	Afternoon	Afternoon	Day-off	Af
5	Cindy	Day-off	Day-off	Early	Afternoon	Early	Early	Afternoon	Day-off	Day-off	Early	Night	Afternoon	Early	Da
6	Ellie	Night	Day-off	Early	Night	Day-off	Night	Day-off	Early	Day-off	Day-off	Day-off	Afternoon	Early	Af
7	Janey	Early	Early	Afternoon	Day-off	Night	Afternoon	Day-off	Early	Afternoon	Early	Day-off	Day-off	Afternoon	Da
8	Marilyn	Afternoon	Afternoon	Afternoon	Afternoon	Afternoon	Day-off	Day-off	Day-off	Day-off	Day-off	Early	Early	Night	Da
9	Robin	Early	Day-off	Night	Afternoon	Afternoon	Night	Day-off	Afternoon	Early	Night	Day-off	Day-off	Day-off	Ea
10	Ruby	Day-off	Afternoon	Day-off	Day-off	Afternoon	Day-off	Afternoon	Day-off	Day-off	Afternoon	Early	Day-off	Day-off	Af
11	Tina	Day-off	Night	Day-off	Day-off	Early	Day-off	Early	Afternoon	Day-off	Afternoon	Day-off	Early	Day-off	Ea

Fig. 9. Nurse's shifts rostering Result (Excel)

V. CONCLUSION AND FUTURE WORK

This paper uses 0-1 programming to construct nurses' schedules for implementation and refers to relevant literature to select and define relevant constraints that meet domestic nurses' schedules. It also considers nurses' schedule preferences, and finally designs in Objective Among the functions, the experiment shows that the combination of Fitness Best Selection + Uniform Crossover (0.7) + FlipBit Mutation of the Genetic Operator of the Genetic Algorithm can get the best scheduling results.

In this paper, only the scheduling of nurses is considered, so the single objective method is used to deal with it. In the future, in addition to adding more constraints to make the scheduling results closer to real life, the multi-objective method can also be used to implement more limited scheduling algorithms with the limited operating room resources of the hospital, to construct and improve the scheduling system of the hospital.

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