COMP2804 – Winter 2020 – Assignment 1 – Andy Chia – 101111058

A *dec-string* is a sequence of characters from the 10-character alphabet $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
For example, these are dec-strings:

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0
36562342320
49548362729
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Let $n \geq 0$ be an integer.

2.	1.	What is the number of dec-strings of length n? $$10^n$$
	2.	What is the number of dec-strings $d1,\ldots,dn$ of length n such that $d1d2 \neq 00$. In other words, what is the number of dec-strings of length n that don't begin with 00? - d_1 -> 9 ways to fill - d_2 -> 9 ways to fill - $d_3, \ldots, d_n$ -> 10 ways to fill d_1 and d_2 from the dec-string is: $$10^{n-2}$$ Removing d_1 and d_2 from all the possibilities of length n: $$10^n - 10^{n-2}$$

	3.	What is the number of dec-strings $d1, \ldots, dn$ of length n such that $d1d2 \neq 00$ *and* $d2d3 \neq 11$?
		- A = $d_1d_2 \neq 00$, B = $d_2d_3 \neq 11$
		- to figure out so that $\|A \cap B\|$ is true
		- you must add up $\|A \cap \bar{B}\|$ and $\|\bar{A} \cap B\|$ so that you can find how many to remove from dec-strings (Product rule)
		  ○ Both A and B can't be false at the same time since they occupy two different characters and have d_2 in common
		  ○ Find the numbers that can't be used and remove it from the results $(\neg(A \cap B) = (A \cap \bar{B}) \cup (\bar{A} \cap B))$
		Both A and B can be explained with:
		$$10^{n-2}$$
		To have A and B true from dec-strings, possible results to get A or B false will be removed:
		$$10^n - 2(10^{n-2})$$

	4.	What is the number of dec-strings $d1, \ldots, dn$ of length n such that $d1d2 \neq 00$ *and* $d2d3 \neq 01$? - A = $d_1d_2 \neq 00$, B = $d_2d_3 \neq 01$ - A and B both have d_2 in common unlike question 2.3 To find $\|A \cap \bar{B}\|$ using compliment rule: $\bar{B} = d2d3 = 01$ 1. fill d_2 with 0 2. fill d_3 with 1 3. fill d_1 with anything except 0 so that A is not false (9 possibilities) 4. fill the rest {0, 1, …, 9} (10 possibilities) product rule: $$9 * 10^{n-3}$$ $$10^n - 9 * 10^{n-3}$$

	5.	What is the number of dec-strings d1, …, dn of length n such that d1d2=00 *or* d1d2d3=111?
		- A = d_1d_2 = 00, B = $d_1d_2d_3$ = 111
		- Find: $\lvert A \cup B \rvert$
		  ○ $\lvert A \cup B \rvert = \lvert A \rvert + \lvert B \rvert - \lvert A \cap B \rvert$
		  ○ $\lvert A \rvert = 10^{n-2}$
		  ○ $\lvert B \rvert = 10^{n-3}$
		  ○ $\lvert A \cap B \rvert = 0$ (Impossible always since A share the characters that is in B)
		$$10^{n-2} + 10^{n-3}$$

	6.	What is the number of dec-strings d1, …, dn of length n≥4 such that d1d2≠00 *or* d3d4≠11.
		- $n \geq 4$
		- A = $d_1d_2 \neq 00$, B = $d_3d_4 \neq 11$
		- X = set of all dec-strings
		- Find: $\lvert A \cup B \rvert$
		  ○ $\lvert A \cup B \rvert = \lvert A \rvert + \lvert B \rvert - \lvert A \cap B \rvert$
		  ○ $\lvert A \rvert = 10^{n-2}$
		  ○ $\lvert B \rvert = 10^{n-2}$
		  ○ $\lvert A \cap B \rvert = 10^{n-4}$
		  ○ $\lvert A \cup B \rvert = 2(10^{n-2}) - 10^{n-4}$
		  ○ Good Ones for n = 4 (first 4 characters)
		$$10^n - (2(10^{n-2}) - 10^{n-4})$$

	7.	A dec-string $d_1, \ldots, d_n$ is *bad* if $d_i = d_{i+1}$ or $d_i + d_{i+1} = 9$ for at least one $I \in \{1, \ldots, n-1\}$ and it is *good* otherwise. What is the number of good dec-strings of length n? - d_n will have full range since 1. d_1 has 10 possibilities since it is unrestricted by anything previous 2. $d_2, \ldots, d_n$ has 8 possibilities for being restricted by the previous number. $$10 * 8^{n-1}$$
	8.	A dec-string d1, …, dn is *2-bad* if, di=dj or di+dj=9 for some i<j≤i+2 and it is *2-good* otherwise. What is the number of 2-good dec-strings? -
	9.	

Collective Arts Brewing currently makes 30 types of IPA and 6 types of Lager.

3.	1.	The manager at Mike's Place needs to choose 4 types of IPA and 4 types of Lager. How many options does the manager have? 1. chose 4 IPA -> $\binom{30}{4}$ ways 2. chose 4 Lager -> $\binom{6}{4}$ ways By product rule: $\binom{30}{4} * \binom{6}{4}$
	2.	The 8 beers (4 IPA and 4 Lager) selected in the previous question must be placed in a line on a display shelf so that no two IPA are adjacent, and no two Lager are adjacent. How many ways are there to do this? - no 2 IPA adjacent, no 2 Lager adjacent 1. First slot there are 8 possibilities. 2. second slot there are 4 possibilities a. this is because if we have an IPA or a Lager in the first slot we cant have one of the same type following it

		3. 3 possibilities of the same type as the first, then 3 of the next type 4. 2 and 2 5. 1 and 1 $$8 * 4 * 3 * 3 * 2 * 2 * 1 * 1 = 1152$$
	3.	Continuing from the previous question, suppose that two of the beers selected were All Together Now (an IPA) and Hot Pink (a Lager). Since both cans are pink, the manager doesn't want to place them adjacent to each other. How many ways are there to do this (while still alternating between IPA and Lager)? - No 2 IPA adjacent, no 2 Lager adjacent - All Together Now and Hot Pink - the two cans can't be adjacent 1. Set ATN and HP as pairs -> 2 ways 2. Place the pairs on the shelves anywhere -> 7 ways 3. place the rest: 3,3,2,2,1,1 $$2 * 7 * 3 * 3 * 2 * 2 * 1 * 1 = 504$$ $$1152 - 504 = 648$$
	4.	How many of the arrangements from the previous question have the All Together Now among from the 4 leftmost bottles and the bottle of Hot Pink among the 4 rightmost bottles? -

Consider all permutations of the integers $1, \ldots, 1000$.

4.	1.	In how many of these permutations do 1,2,3,4 appear consecutively and in this order? 1. (1, 2, 3, 4) must be added into the premutation in this order -> 997 ways 2. add up the possibilities for the following permutations -> 996! $$997 * 996! = 997!$$
	2.	In how many of these permutations do 1,2,3,4 appear consecutively, but not necessarily in order? (For example, they may appear as 1,2,3,4, or 4,2,3,1, or 3,1,2,4, or so on.) - Like the previous one but with an added way to have {1, 2, 3, 4} -> 4! $$997! * 4!$$
	3.	In how many of these permutations does 1 appear before 2, 2 appear before 3, and 3 appear before 4? (In other words, 1,2,3,4 appear in order, but not necessarily consecutively.) - 1, 2, 3, 4 must appear in this order - but any number can appear between them Chose 4 slots out of 1000 for the 4 numbers $$\binom{1000}{4}$$ Add the rest of the other numbers with any order $$996!$$ Result $$\binom{1000}{4} * 996!$$
	4.	In how many of these permutations do 1,2,3,4 appear in order but no two are adjacent? - 1, 2, 3, 4 In order - can't be following each other consecutively 1. inserting a number between 1 and 2 -> 996 options 2. inserting a number between 2 and 3 -> 995 options

		3. inserting a number between 3 and 4 -> 993 options 4. insert the remaining numbers -> $\frac{1000!}{7!}$ • out of the 1000, 7 spots are already taken up by 1, 2, 3, 4 and the separating numbers from 1. 2. 3. $$996 * 995 * 994 * (\frac{1000!}{7!})$$

A certain friend of mine has spent the better part of a lifetime testing recreational drugs. After thorough testing, this friend has identified 20 recreational drugs $D_1, \ldots, D_{20}$ and determined (experimentally) that any 3 of these drugs can be taken simultaneously with no adverse effects.

5.	1.	Assuming my friend determined this entirely by testing, how many experiments did my friend have to perform? - 20 drugs - 3 at a time $$\binom{20}{3} = 1140$$
	2.	A new designer drug called D21 has just hit the streets and my friend wants to know if D21 can be added to their list. That is, can any triple of D1, ..., D21 be safely taken together? How many *additional* experiments does my friend need to determine this? $$\binom{21}{3} - \binom{21}{3} = 1330 - 1140 = 190$$
	3.	Suppose my friend survives the experience and D21 makes it onto the list. My friend takes scrupulous notes about all experiments and notices something peculiar about the answers to the preceding two questions. What combinatorial identity did my friend just discover? -
	4.	