No.
Date : :

例 6.4

$E(\hat{\theta}) = \theta$

$E(\hat{\theta}) = E\left(\frac{\Sigma(x_i-\bar{x})^2}{n}\right)$

化簡 ⇒ $E(\Sigma(x_i-\bar{x})^2)$

$= E(\Sigma(x_i^2 - 2x_i\bar{x} + \bar{x}^2))$

$= E(\Sigma(x_i^2 - 2(n\bar{x})(\bar{x}) + \bar{x}^2))$

$E(\hat{\theta}) = \frac{1}{n}E(\Sigma x_i^2 - n\bar{x}^2)$

① $E(\Sigma x_i^2) = n\sigma^2 + nu^2$

② $E(n\bar{x}^2) = nE(\bar{x}^2)$

$E(x_i^2) = \sigma^2 + u^2$

$E(\bar{x}) = u$, $V(\bar{x}) = \frac{\sigma^2}{n} = E(\bar{x}^2) - u^2$

⇔ $E(\bar{x}^2) = \frac{\sigma^2}{n} + nu^2$

$E(\hat{\theta}_1) = E\left(\frac{\Sigma(x_i-\bar{x}^2)}{n}\right)$

$= \frac{1}{n}(n\sigma^2 + nu^2 - \sigma^2 - nu^2)$

$= \frac{n-1}{n}\sigma^2 \neq \sigma^2$

非不偏估計量 #

$E(\hat{\theta}_2) = E\left(\frac{\Sigma(x_i-\bar{x})^2}{n-1}\right)$

$= \frac{1}{n-1}E(\Sigma x_i^2 - n\bar{x}^2)$

$= \frac{1}{n-1}(n\sigma^2 + nu^2 - \sigma^2 - nu^2)$

$= \sigma^2$

不偏估計量 #