Comphys Homework 2

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I. PROGRAMMING ASSIGNMENTS

$N = 10^5$ takes too much time, so I run for $N = 10^4$, and $m = 20/N$. I have tried barnes-hut tree, but njit cannot work when there is a customize class. Moreover, njit for two for loop is faster than direct numpy operation. So I still use two for loop.

A. Problem 1 and 2

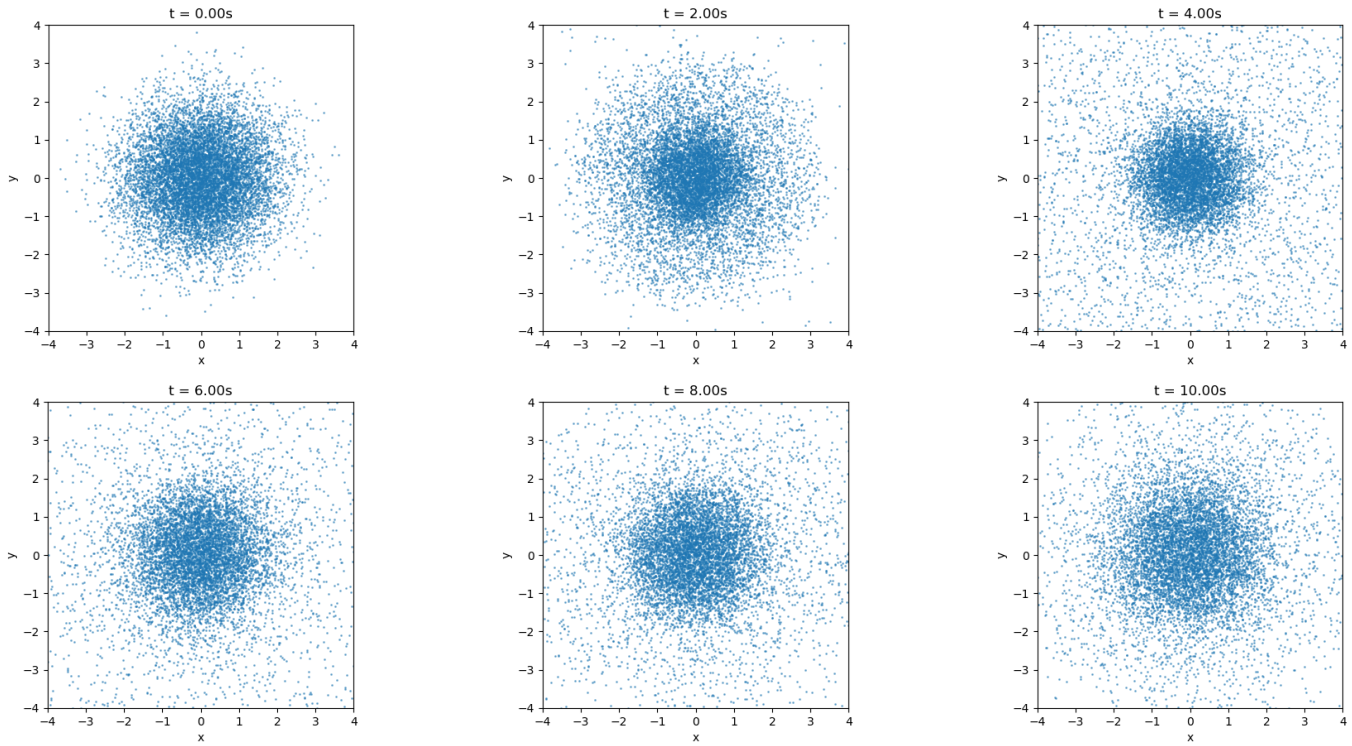


FIG. 1: Particles run in Euler method.

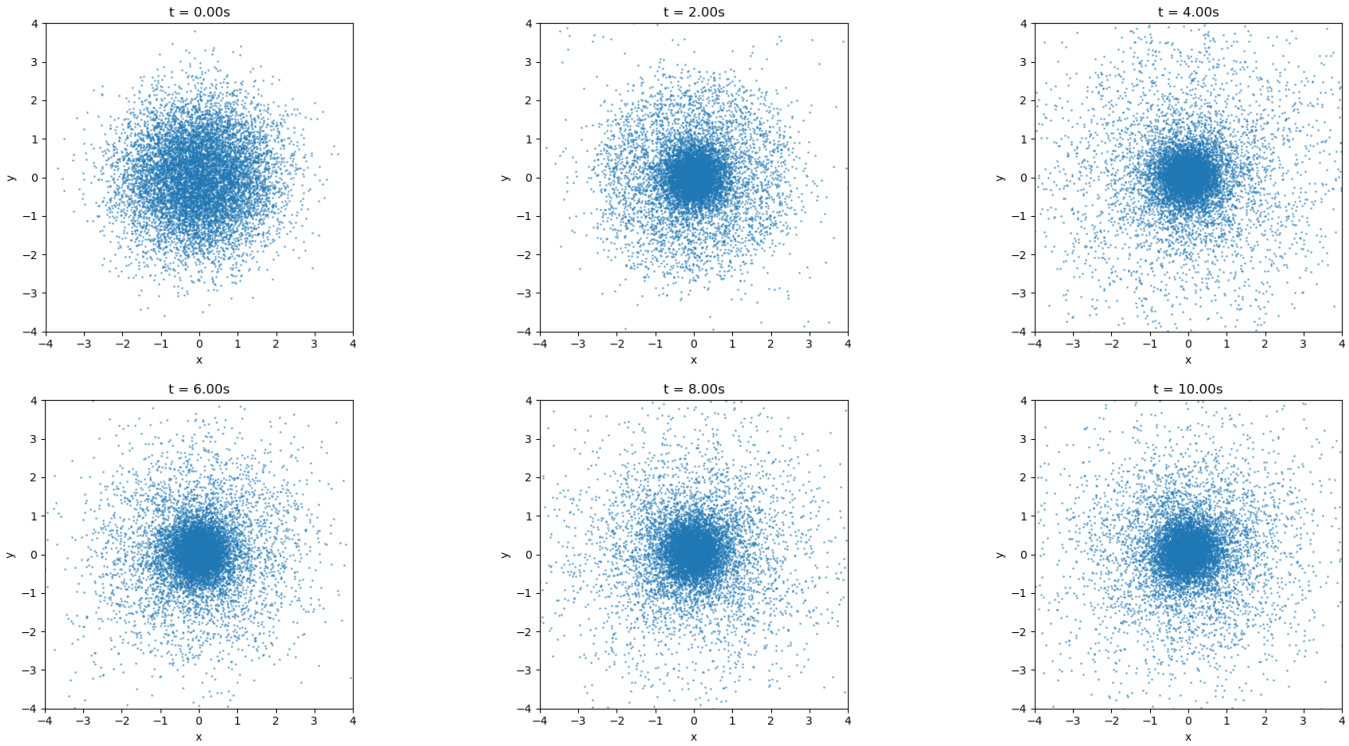


FIG. 2: Particles run in RK2 method.

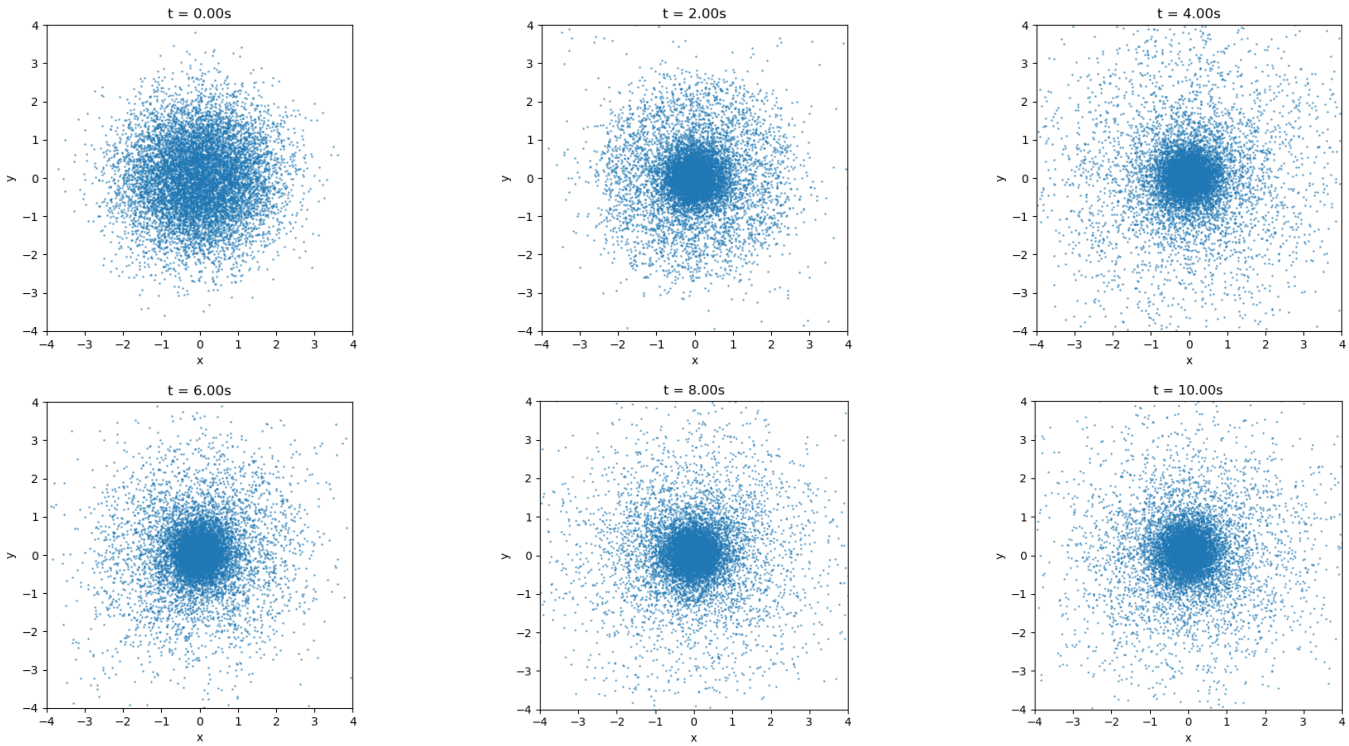


FIG. 3: Particles run in RK4 method.

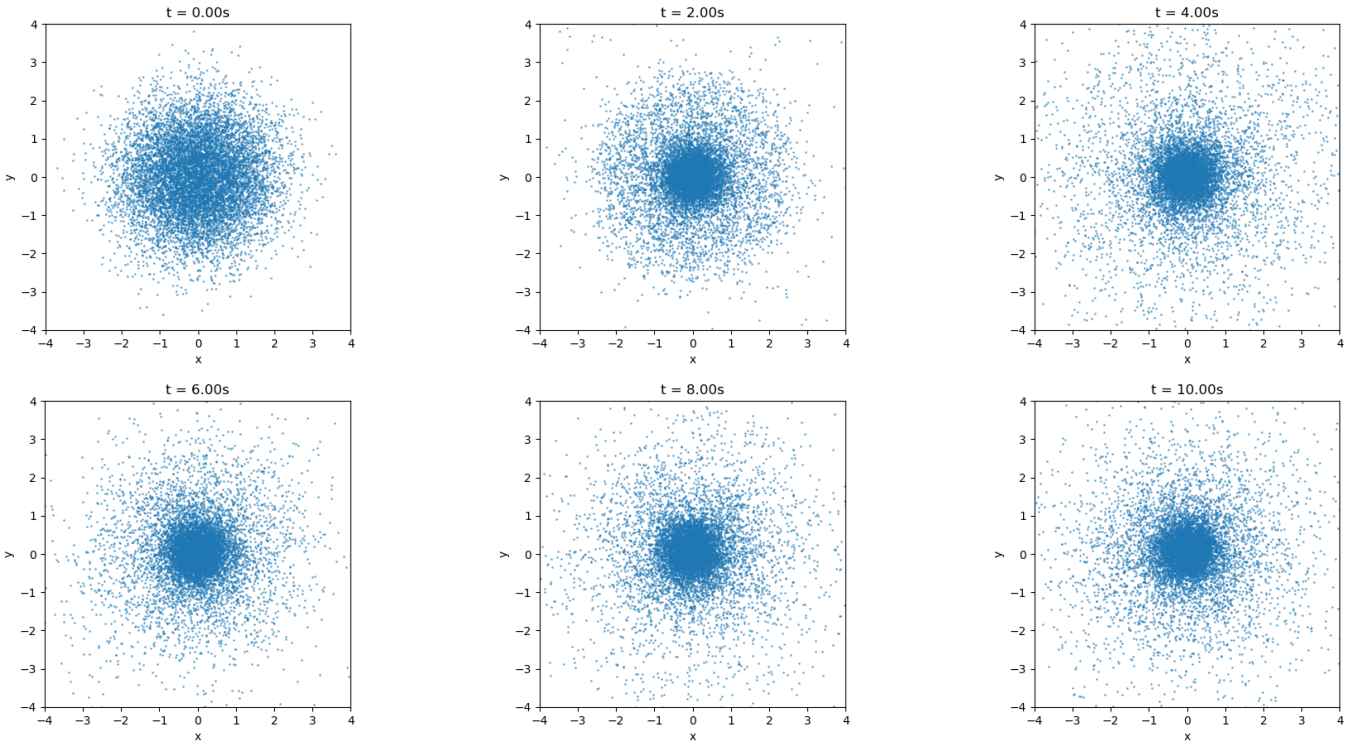
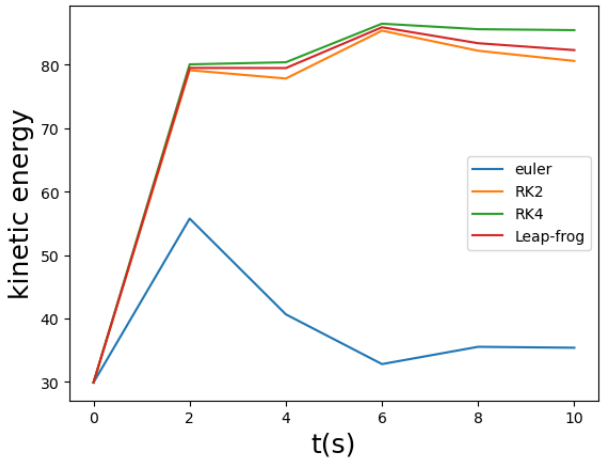


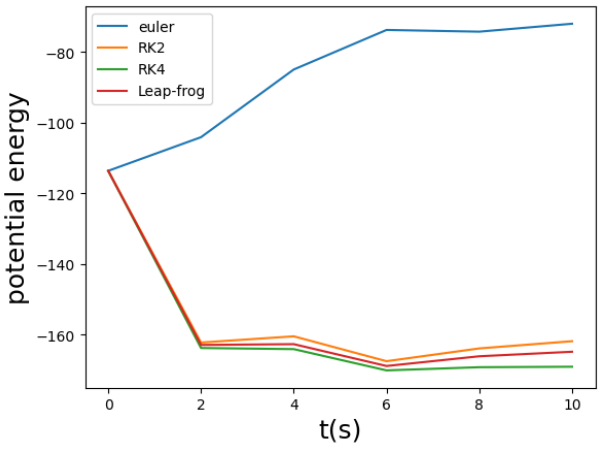
FIG. 4: Particles run in Leap-frog method.

B. Problem 3

Most of particles move towards center because of interaction, making potential energy reduce and kinetic energy increase. When we focus on total energy (Fig. 6), we can find the disadvantages of euler method. Also, Leap-frog scheme behaves better than RK2.



(a) kinetic energy



(b) potential energy

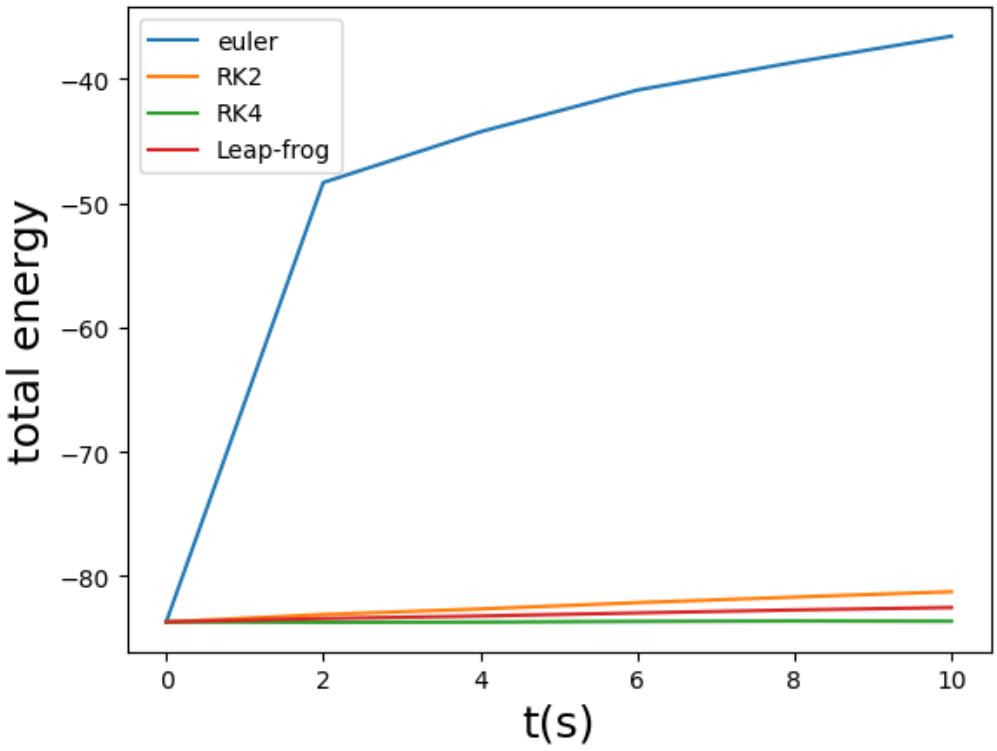


FIG. 6: total energy

For order of accuracy, assume real solution at $t + \Delta t$ is

$$r(t + \Delta t) = r(t) + v(t)\Delta t + \frac{a(t)}{2}\Delta t^2 + O(\Delta t^3) \tag{1}$$

For kick-drift-kick scheme, we predict the solution at $t + \Delta t$ is

$$\begin{aligned} r_k(t + \Delta t) &= r(t) + v_k(t + \frac{\Delta t}{2})\Delta t \\ &= r(t) + v(t)\Delta t + \frac{a(t)}{2})\Delta t^2 \end{aligned}$$

So truncation error is $r(t)$ is $r(t + \Delta t) - r_k(t + \Delta t) \sim O(\Delta t^3)$. For velocity:

$$\begin{aligned} v(t + \Delta t) &= (v(t) + \frac{a(t)}{2}\Delta t) + \frac{a(t + \Delta t)}{2} \\ &= v(t) + a(t)\Delta t + \frac{\dot{a}(t)}{2}\Delta t^2 + O(\Delta t^3) \end{aligned}$$

We get the same result.