Kết quả nghiên cứu tuần 3 Phòng thí nghiệm Thông tin Vô tuyến

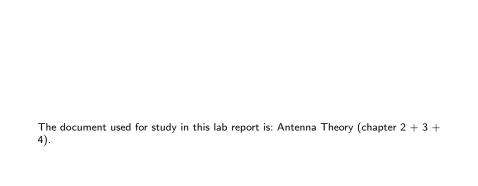
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Magnetic vector potential

In the previous lab slide, we only derive some very basic antenna parameters are $P,\ U,\ D,\ G$ based on the given $W(\phi,\theta)$ function. So, how to determine $W(\phi,\theta)$ function? From the Maxwell's equation in differential form:

$$\begin{split} \nabla \cdot \vec{D} &= \rho \\ \nabla \cdot \vec{H} &= 0 \\ \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} = -j\omega \vec{B} = -\frac{\partial}{\partial t} \mu_0 \vec{H} \\ \\ \nabla \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} = \vec{J} + j\omega \varepsilon_0 \vec{E} = \vec{J} + j\omega \vec{D} \end{split}$$

We introduce the new parameter: $\vec{D} = \varepsilon_0 \vec{E}$ (electric displacement).

So how to design current density \vec{J} to satisfies our required antenna characteristic? Or equivalently, given \vec{J} , determine \vec{E} and \vec{H} ; and finally use the complex power density of Poynting vector (irradiance) equation to yield $W(\phi,\theta)$.

Set an auxiliary vector \vec{A} (also can be named as magnetic vector potential) defined as:

$$\vec{B} = \nabla \times A$$

From the third equation:

$$\nabla \times \vec{E} = -j\omega(\nabla \times \vec{A}) \Rightarrow \nabla \times (\vec{E} + j\omega \vec{A}) = 0$$

The right term is equal to the gradient descent of ϕ :

$$\vec{E} + i\omega \vec{A} = -\nabla \phi$$

Magnetic vector potential

From the fourth equation:

$$\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon_0 \vec{E} = \vec{J} + j\omega\varepsilon_0 (-j\omega\vec{A} - \nabla\phi)$$

$$\Rightarrow \frac{1}{\mu_0} (\nabla \times \nabla \times \vec{A}) = \vec{J} + j\omega\varepsilon_0 (-j\omega\vec{A} - \nabla\phi)$$

$$\Rightarrow \frac{1}{\mu_0} [\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}] = \vec{J} + \omega^2\varepsilon_0 \vec{A} - j\omega\varepsilon_0 \nabla\phi$$

$$\Rightarrow \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 (\vec{J} + \omega^2\varepsilon_0 \vec{A} - j\omega\varepsilon_0 \nabla\phi)$$

After some reduction steps I don't understand, we yield:

$$abla^2 \vec{A} + k_0^2 \vec{A} = -\mu_0 \vec{J}$$
 (k_0 is a constant)

This equation has one solution:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \vec{J}(\vec{r'}) \frac{e^{-jk|\vec{r}-\vec{r'}|}}{|\vec{r}-\vec{r'}|} dv'$$

Somehow we can neglect gradient of $\nabla \phi$, so \vec{E} and \vec{H} can easily be determined in term of \vec{A} :

$$\begin{cases} \vec{E} = -j\omega\vec{A} \\ \vec{H} = \frac{1}{\mu_0}\nabla\times\vec{A} = \frac{\hat{r}\times\vec{E}}{\eta} = \frac{\hat{r}\times(\vec{E_\theta}\,\hat{\theta} + \vec{E_\phi}\,\hat{\phi})}{\eta} \end{cases}$$

Magnetic vector potential

In short, we summarize 4 steps to find \vec{E} and \vec{H} in far-field zone with given current density \vec{J} :

- 1 Find magnetic vector potential $\vec{A}(\vec{r'})$ from \vec{J} and distance $\vec{r'}$.
- 2 Seperate $\vec{A_{theta}}$ and $\vec{A_{\phi}}$ components.
- 3 Substitute results into each \vec{E} components:

$$\vec{E}_r = 0$$

 $\vec{E}_\theta = -j\omega \vec{A}_\theta$
 $\vec{E}_\phi = -j\omega \vec{A}_\phi$

4 Find the magnetic intensity function:

$$\vec{H} = \frac{\vec{r} \times \vec{E}}{\eta}$$

From \vec{E} and \vec{H} above, we can easily deduce:

$$ec{W} = rac{1}{2} ec{E} imes ec{H^*}$$

Now we apply our steps to analyze the dipole, which is the simplest antenna.