Kết quả nghiên cứu tuần 3

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Reference documents

The document used for study in this lab report is: Antenna Theory (chapter 2 + 3 + 4).

Magnetic vector potential

In the previous lab slide, we only derive some very basic antenna parameters are P, U, D, G based on the given $W(\phi,\theta)$ function. So, how to determine $W(\phi,\theta)$ function?
From the Maxwell's equation in differential form:

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{H} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -j\omega\vec{B} = -\frac{\partial}{\partial t}\mu_0\vec{H}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \vec{J} + j\omega\varepsilon_0\vec{E} = \vec{J} + j\omega\vec{D}$$

We introduce the new parameter: $\vec{D} = \varepsilon_0\vec{E}$ (electric displacement).
So how to design current density $\vec{J}$ to satisfies our required antenna characteristic? Or equivalently, given $\vec{J}$, determine $\vec{E}$ and $\vec{H}$; and finally use the complex power density of Poynting vector (irradiance) equation to yield $W(\phi,\theta)$.
Set an auxiliary vector $\vec{A}$ (also can be named as magnetic vector potential) defined as:

$$\vec{B} = \nabla \times A$$

From the third equation:

$$\nabla \times \vec{E} = -j\omega(\nabla \times \vec{A}) \Rightarrow \nabla \times (\vec{E} + j\omega\vec{A}) = 0$$

The right term is equal to the gradient descent of ϕ:

$$\vec{E} + j\omega\vec{A} = -\nabla\phi$$

Magnetic vector potential

From the fourth equation:

$$\begin{aligned}
\nabla \times \vec{H} &= \vec{J} + j\omega\varepsilon_0\vec{E} = \vec{J} + j\omega\varepsilon_0(-j\omega\vec{A} - \nabla\phi) \\
\Rightarrow \frac{1}{\mu_0}(\nabla \times \nabla \times \vec{A}) &= \vec{J} + j\omega\varepsilon_0(-j\omega\vec{A} - \nabla\phi) \\
\Rightarrow \frac{1}{\mu_0}[\nabla(\nabla \cdot \vec{A}) - \nabla^2\vec{A}] &= \vec{J} + \omega^2\varepsilon_0\vec{A} - j\omega\varepsilon_0\nabla\phi \\
\Rightarrow \nabla(\nabla \cdot \vec{A}) - \nabla^2\vec{A} &= \mu_0(\vec{J} + \omega^2\varepsilon_0\vec{A} - j\omega\varepsilon_0\nabla\phi)
\end{aligned}$$

After some reduction steps I don't understand, we yield:

$$\nabla^2\vec{A} + k_0^2\vec{A} = -\mu_0\vec{J} \quad (k_0 \text{ is a constant})$$

This equation has one solution:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r'})\frac{e^{-jk|\vec{r}-\vec{r'}|}}{|\vec{r}-\vec{r'}|}dv'$$

Somehow we can neglect gradient of $\nabla\phi$, so $\vec{E}$ and $\vec{H}$ can easily be determined in term of $\vec{A}$:

$$\begin{cases}
\vec{E} = -j\omega\vec{A} \\
\vec{H} = \frac{1}{\mu_0}\nabla \times \vec{A} = \frac{\hat{r}\times\vec{E}}{\eta} = \frac{\hat{r}\times(\vec{E_\theta}\hat{\theta}+\vec{E_\phi}\hat{\phi})}{\eta}
\end{cases}$$

Magnetic vector potential

In short, we summarize 4 steps to find $\vec{E}$ and $\vec{H}$ in far-field zone with given current density $\vec{J}$:

1. Find magnetic vector potential $\vec{A}(\vec{r'})$ from $\vec{J}$ and distance $\vec{r'}$.
2. Seperate $\vec{A_{theta}}$ and $\vec{A_\phi}$ components.
3. Substitute results into each $\vec{E}$ components:

$$\vec{E_r} = 0$$
$$\vec{E_\theta} = -j\omega\vec{A_\theta}$$
$$\vec{E_\phi} = -j\omega\vec{A_\phi}$$

4. Find the magnetic intensity function:

$$\vec{H} = \frac{\vec{r} \times \vec{E}}{\eta}$$

From $\vec{E}$ and $\vec{H}$ above, we can easily deduce:

$$\vec{W} = \frac{1}{2}\vec{E} \times \vec{H^*}$$

Now we apply our steps to analyze the dipole, which is the simplest antenna.