AC Machines Problems and Solutions

Group 14

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Problem 16.4

A 60 *Hz* induction motor is needed to drive a load at approximately 850 *rpm*. How many poles should the motor have? What is the slip of this motor for a speed of 850 *rpm*?

Solution:

$$P = \frac{120f}{n_s} = \frac{120.60}{850} \approx 8,47$$

$\Rightarrow$ The motor should have 10 poles, the slip:

$$s = \frac{\omega_s - \omega_m}{\omega_s} = \frac{\frac{120.60}{10} - 850}{\frac{120.60}{10}} = \frac{5}{90}$$

Problem 16.6

Prepare a table that shows synchronous speeds for three-phase induction motors operating at 50 Hz. Consider motors having eight or fewer poles. Repeat for 400 Hz motors.

Solution:

With $f = 50Hz$, we have:

P	n_s
2	3000
4	1500
6	1000
8	750

Problem 16.6

With $f = 400Hz$, we have:

P	n_s
2	24000
4	12000
6	8000
8	6000

Problem 16.10

A 10 *hp* six-pole 60 *Hz* three-phase induction motor runs at 1160 rpm under full-load conditions. Determine the slip and the frequency of the rotor currents at full load. Also estimate the speed if the load torque drops in half.

Solution:

Synchronous speed:

$$n_s = \frac{120f}{p} = \frac{120.60}{6} = 1200$$

Slip:

$$s = \frac{n_s - n_m}{n_s}.100\% = 3,33\%$$

Problem 16.10

Frequency of rotor current:

$$f_{rot} = 3f = \frac{3,33\%}{100\%}.60 = 2(Hz)$$

We have:

$$T_1 = \frac{T_{load}}{2} \Rightarrow s_1 = \frac{s}{2} = 1,66\%$$

New speed:

$$s_1 = \frac{n_s - n_m}{n_s}.100\% \Rightarrow n_{m1} = n_s - \frac{s_1 n_s}{100\%} = 1200 - \frac{1,665\%.1200}{100\%} = 1180(rpm)$$

Problem 16.14

A two-pole 60 *Hz* induction motor produces an output power of 3 *hp* at a speed of 1700 *rpm*. With no load, the speed is 1798 *rpm*. Assume that the rotational torque loss is independent of speed. Find the rotational power loss at 1700 *rpm*.

Problem 16.14

Solution: From the problem discription we have:

$P = 2, f = 60Hz, P_{out} = 5Hp, n_{m1} = 3500rpm, n_{n0-load} = 3598rpm, T_{loss} = constant.$

The output power in Watt is:

$$P_{out} = 5.746 = 3730$$

The output power:

$$P_{out} = T_{out}.\omega.m_1 = T_{out}.n.m_1.\frac{2\pi}{60}$$

The output torque is:

$$T_{out} = \frac{P_{out}}{nm_1}.\frac{60}{2\pi} = \frac{3730}{3500}.\frac{60}{2\pi} = 10,177$$

Problem 16.14

Furthermore, we know that T_{out} is equal to:

$$T_{out} = T_{dev} - T_{rot} \Rightarrow T_{dev} = T_{out} + T_{rot}$$

From the previous we know that the developed torque is proportional to slip for the small values of slip. In our case the slip is:

$$s = \frac{n_s - n_{m1}}{n_s}.100\% \Rightarrow n_s = \frac{120f}{p} = 3600rpm$$

$$s = \frac{3600 - 3500}{3600}.100\% = 2,78\% = 0,0278$$

Problem 16.14

We can conclude that the previous claim is correct. Then we can write:

$$T_{dev} = Ks$$

$$T_{out} + T_{rot} = 0,0278K$$

$$10,177 + T_{rot} = 0,0278K$$

Under no load condition, we have:

$$n_s = 3600 rpm, n_{n0-load} = 3598 rpm$$

$$T_{out} = T_{dev} - T_{rot} = 0 \Rightarrow T_{dev} = T_{rot}$$

Problem 16.14

The developed torque is proportional to slip, so we can write again:

$$T_{dev} = T_{rot} = Ks = K\frac{n_s - n_{n0} - load}{n_s} = K.\frac{3600 - 3598}{3600} = \frac{2K}{3600}$$

Substituting T_{dev} value, we have:

$$10.177 + \frac{2K}{3600} = 0,0278K \Rightarrow K = 373,55$$

Problem 16.14

Then, the torque loss is:

$$T_{rot} = \frac{2K}{3600} = 0,2075$$

Finally, the rotational power is:

$$P_{rot} = T_{rot}.\omega m_1 = T_{rot}.nm_1\frac{2\pi}{60} = 0,2075.3500.\frac{2\pi}{60} = 76,05W$$

Problem 16.15

A certain four-pole $230 - V - rms$ 60 Hz delta connected three-phase induction motor has $R_s = 1\Omega, X_s = 1,5\Omega, X_m = 40\Omega, R_r' = 0,5\Omega, X_r' = 0,8\Omega$. Under load, the machine operates at 1740 rpm and has rotational losses of 300 W. Neglecting the rotational losses, find the no-load speed, line current, and power factor for the motor.

Problem 16.15

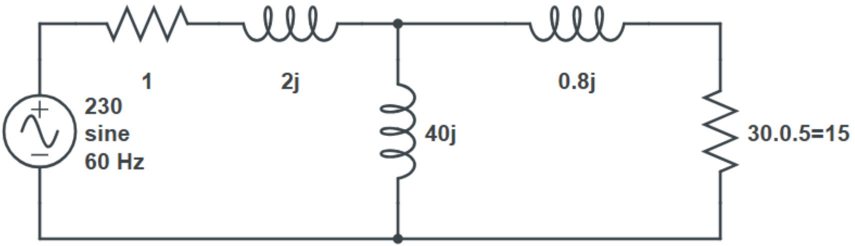

Figure: Circuit for problem 16.15

Problem 16.15

Solution: Synchronous speed (no-load speed):

$$n_s = \frac{120.60}{4} = 1800$$

The slip:

$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1740}{1800} = \frac{1}{30}$$

$$Z_s = 1 + 2j + \frac{(0,8j + 15)40j}{0,8j + 15 + 40j} = 13,7 + 7,45j = 15,6\angle 28,55^\circ$$

Power factor $= \cos 28,55^\circ = 87,84\%$ lagging

$$I_s = \frac{V_s}{Z_s} = \frac{230\angle 0^\circ}{15,6\angle 28,55^\circ} = 14,74\angle - 28,55^\circ$$

Problem 16.21

A 3 *hp* six-pole 60 *Hz* delta-connected three-phase induction motor is rated for 1140 *rpm*, 220 *V rms*, and $8,58$ *A rms* (line current) at an 80 percent lagging power factor. Find the full-load efficiency.

Solution:

$$P_{out} = 2.746 = 1492$$

We have:

$$V_s = V_{line}$$

$$I_s = \frac{I_{line}}{\sqrt{3}}$$

Problem 16.21

$$P_{in} = 3I_s V_s \cos\theta = \frac{3I_{line}}{\sqrt{3}} V_{line} \cos\theta = \sqrt{3} I_{line} V_{line} \cos\theta$$
$$= \sqrt{3}.5,72.220.0,8 = 1743,69(W)$$

The efficiency is:

$$\eta = \frac{1492}{1743,69}.100\% = 85,57\%$$

Problem 16.31

A certain four-pole 440 V rms 60 Hz three phase delta-connected induction motor has $R_s = 0,12\Omega, X_s = 0,30\Omega, X_m = 7.5\Omega, R_r' = 0,10\Omega, X_r' = 0,20\Omega$. Under load, the machine operates with a slip of 4 percent and has rotational losses of 2 kW.

Determine the power factor, output power, copper losses, output torque, and efficiency.

Solution:

$$P_{out} = 2.746 = 1492W$$

Synchronous speed:

$$n_s = \frac{120f}{p} = \frac{120.60}{8} = 900(rpm)$$

Problem 16.31

Slip:

$$s = \frac{n_s - n_m}{n_s} = \frac{900 - 850}{900} = 0,0556$$

Frequency of motor currents:

$$f_r = s.f_s = 0,0556.60 = 3,33(Hz)$$

Developed power:

$$P_{dev} = 3\frac{1-s}{s}.R_r^{'}I_r^2 = \frac{1-s}{s}P_r$$

Also: $P_{dev} = P_{out} + P_{rot} = 1492 + 100 = 1592(W)$

$$\Rightarrow P_r = \frac{s}{1-s}P_{dev} = \frac{0,0556}{1-0,0556}.1592 = 93,726(W)$$

Problem 16.42

A six-pole 60 Hz synchronous motor is operating with a developed power of 5 hp and a torque angle of 5°. Find the speed and developed torque. Suppose that the load increases such that the developed torque doubles. Find the new torque angle. Find the pull-out torque and maximum developed power for this machine.

Problem 16.42

Solution:

From the problem description we have:

$$P = 6W, f = 60Hz, P_{dev1} = 5hp, \delta_1 = 5^\circ$$

The developed power in watts is:

$$P_{dev1} = 5.746 = 3730$$

The synchronous speed is:

$$n_s = \frac{120f}{p} = \frac{120.60}{6} = 1200rpm$$

Problem 16.42

The developed torque is:

$$T_{dev1} = \frac{P_{dev1}}{\omega s} = \frac{P_{dev1}}{ns} . \frac{60}{2\pi} = \frac{3730.1200}{1200} . \frac{60}{2\pi} = 29,682 Nm$$

The developed torque is doubled, so we can write:

$$T_{dev2} = 2T_{dev1} = 2.29,682 = 59,635 Nm$$

Recall that developed torque is given by following equation:

$$T_{dev} = K.B_r.B_{total}. \sin \delta$$

Problem 16.42

We can use the previous equation to determine new torque angle δ_2. But first, we have to calculate the value of constants $K.B_r.B_{total}$. For the T_{dev1} and δ_1, we have:

$$T_{dev1} = K.B_r.B_{total}.\sin\delta_1 \Rightarrow K.B_r.B_{total} = \frac{T_{dev1}}{\sin\delta_1} = \frac{29,682}{\sin 5^\circ} = 340,563$$

For the T_{dev2} and δ_2, we can write:

$$T_{dev2} = K.B_r.B_{total}.\sin\delta_2 \Rightarrow \sin\delta_2 = \frac{59,635}{340,563} = 0,175$$

Finally, the torque angle is:

$$\delta_2 = \arcsin 0,175 = 10,08^\circ$$

Problem 16.42

The pull out torque occurs for torque angle $\delta = 90^\circ$

$$T_{pull-out} = K.B_r.B_{total}.\sin 90^\circ = 340,563$$

The maximum developed power is defined as follows:

$$P_{max} = T_{pull-out}\omega.s = T_{pull-out}.ns\frac{2\pi}{60} = 42976,41W$$