AC machines Problems and Solutions

Group 14

May 2, 2024

Problem 16.4:

A 60 Hz induction motor is needed to drive a load at approximately 850 rpm. How many poles should the motor have? What is the slip of this motor for a speed of 850 rpm?

Solution:

$$P = \frac{120f}{n_s} = \frac{120.60}{850} \approx 8,47$$

$\Rightarrow$ The motor should have 10 poles, the slip:

$$s = \frac{\omega_s - \omega_m}{\omega_s} = \frac{\frac{120.60}{10} - 850}{\frac{120.60}{10}} = \frac{5}{90}$$

Problem 16.6:

Prepare a table that shows synchronous speeds for three-phase induction motors operating at 50 Hz. Consider motors having eight or fewer poles. Repeat for 400 Hz motors.

Solution:

With $f = 50Hz$, we have:

P	n_s
2	3000
4	1500
6	1000
8	750

With $f = 400Hz$, we have:

P	n_s
2	24000
4	12000
6	8000
8	6000

Problem 16.10:

A 10 hp six-pole 60 Hz three-phase induction motor runs at 1160 rpm under full-load conditions. Determine the slip and the frequency of the rotor currents at full load. Also estimate the speed if the load torque drops in half.

Solution:

Synchronous speed:

$$n_s = \frac{120f}{p} = \frac{120.60}{6} = 1200$$

Slip:

$$s = \frac{n_s - n_m}{n_s}.100\% = 3,33\%$$

Frequency of rotor current:

$$f_{rot} = 3f = \frac{3,33\%}{100\%}.60 = 2(Hz)$$

We have:

$$T_1 = \frac{T_{load}}{2} \Rightarrow s_1 = \frac{s}{2} = 1,66\%$$

New speed:

$$s_1 = \frac{n_s - n_m}{n_s}.100\% \Rightarrow n_{m1} = n_s - \frac{s_1 n_s}{100\%} = 1200 - \frac{1,665\%.1200}{100\%} = 1180(rpm)$$

Problem 16.14:

A two-pole 60 Hz induction motor produces an output power of 3 hp at a speed of 1700 rpm. With no load, the speed is 1798 rpm. Assume that the rotational torque loss is independent of speed. Find the rotational power loss at 1700 rpm.

Solution:
From the problem discription we have: $P = 2, f = 60Hz, P_{out} = 5Hp, n_{m1} = 3500rpm, n_{n0-load} = 3598rpm, T_{loss} = constant.$
The output power in Watt is:

$$P_{out} = 5.746 = 3730$$

The output power:

$$P_{out} = T_{out}.\omega.m_1 = T_{out}.n.m_1.\frac{2\pi}{60}$$

The output torque is:

$$T_{out} = \frac{P_{out}}{nm_1}\frac{60}{2\pi} = \frac{3730}{3500}.\frac{60}{2\pi} = 10,177$$

Furthermore, we know that T_{out} is equal to:

$$T_{out} = T_{dev} - T_{rot} \Rightarrow T_{dev} = T_{out} + T_{rot}$$

From the previous we know that the developed torque is proportional to slip for the small values of slip. In our case the slip is:

$$s = \frac{n_s - n_{m1}}{n_s}.100\% \Rightarrow n_s = \frac{120f}{p} = 3600rpm$$

$$s = \frac{3600 - 3500}{3600}.100\% = 2,78\% = 0,0278$$

We can conclude that the previous claim is correct. Then we can write:

$$T_{dev} = Ks$$

$$T_{out} + T_{rot} = 0,0278K$$

$$10,177 + T_{rot} = 0,0278K$$

Under no load condition, we have:

$$n_s = 3600rpm, n_{n0-load} = 3598rpm$$

$$T_{out} = T_{dev} - T_{rot} = 0 \Rightarrow T_{dev} = T_{rot}$$

The developed torque is proportional to slip, so we can write again:

$$T_{dev} = T_{rot} = Ks = K\frac{n_s - n_{n0} - load}{n_s} = K.\frac{3600 - 3598}{3600} = \frac{2K}{3600}$$

Substituting T_{dev} value, we have:

$$10.177 + \frac{2K}{3600} = 0,0278K \Rightarrow K = 373,55$$

Then, the torque loss is:

$$T_{rot} = \frac{2K}{3600} = 0,2075$$

Finally, the rotational power is:

$$P_{rot} = T_{rot}.\omega m_1 = T_{rot}.nm_1\frac{2\pi}{60} = 0,2075.3500.\frac{2\pi}{60} = 76,05W$$

Problem 16.15:

A certain four-pole $230 - V - rms$ 60 Hz delta connected three-phase induction motor has $R_s = 1\Omega, X_s = 1,5\Omega, X_m = 40\Omega, R_r^{'} = 0,5\Omega, X_r^{'} = 0,8\Omega$. Under load, the machine operates at 1740 rpm and has rotational losses of 300 W. Neglecting the rotational losses, find the no-load speed, line current, and power factor for the motor.

Solution: Synchronous speed (no-load speed):

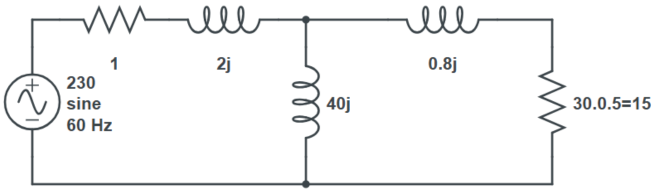

Figure 1: Circuit for problem 16.15

$$n_s = \frac{120.60}{4} = 1800$$

The slip:

$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1740}{1800} = \frac{1}{30}$$

$$Z_s = 1 + 2j + \frac{(0,8j + 15)40j}{0,8j + 15 + 40j} = 13,7 + 7.45j = 15.6\angle 28,55^\circ$$

Power factor $= \cos 28.55^\circ = 87.84\%$ lagging

$$I_s = \frac{V_s}{Z_s} = \frac{230\angle 0^\circ}{15,6\angle 28,55^\circ} = 14,74\angle - 28,55^\circ$$

Line current:

$$I_{line} = \sqrt{3}I_s = 14,74\sqrt{3} = 25,53(Arms)$$

Problem 16.21:

A 3 hp six-pole 60 Hz delta-connected three-phase induction motor is rated for 1140 rpm, 220 V rms, and $8,58$ A rms (line current) at an 80 percent lagging power factor. Find the full-load efficiency.

Solution:

$$P_{out} = 2.746 = 1492$$

We have:

$$V_s = V_{line}$$

$$I_s = \frac{I_{line}}{\sqrt{3}}$$

$$P_{in} = 3I_sV_s\cos\theta = \frac{3I_{line}}{\sqrt{3}}V_{line}\cos\theta = \sqrt{3}I_{line}V_{line}\cos\theta$$
$$= \sqrt{3}.5,72.220.0,8 = 1743,69(W)$$

The efficiency is:

$$\eta = \frac{1492}{1743,69}.100\% = 85,57\%$$

Problem 16.31:

A certain four-pole 440 $V\ rms$ 60 Hz three phase delta-connected induction motor has $R_s = 0,12\Omega, X_s = 0,30\Omega, X_m = 7,5\Omega, R_r^{'} = 0,10\Omega, X_r^{'} = 0,20\Omega$. Under load, the machine operates with a slip of 4 percent and has rotational losses of 2 kW. Determine the power factor, output power, copper losses, output torque, and efficiency.

Solution:

$$P_{out} = 2.746 = 1492W$$

Synchronous speed:

$$n_s = \frac{120f}{p} = \frac{120.60}{8} = 900(rpm)$$

Slip:

$$s = \frac{n_s - n_m}{n_s} = \frac{900 - 850}{900} = 0.0556$$

Frequency of motor currents:

$$f_r = s.f_s = 0.0556.60 = 3.33(Hz)$$

Developed power:

$$P_{dev} = 3\frac{1-s}{s}.R_r^{'}I_r^2 = \frac{1-s}{s}P_r$$

Also: $P_{dev} = P_{out} + P_{rot} = 1492 + 100 = 1592(W)$

$$\Rightarrow P_r = \frac{s}{1-s}P_{dev} = \frac{0.0556}{1-0.0556}.1592 = 93,726(W)$$

Problem 16.42:

A six-pole 60 Hz synchronous motor is operating with a developed power of 5 hp and a torque angle of $5°$. Find the speed and developed torque. Suppose that the load increases such that the developed torque doubles. Find the new torque angle. Find the pull-out torque and maximum developed power for this machine.

Solution:

From the problem description we have:

$$P = 6W, f = 60Hz, P_{dev1} = 5hp, \delta_1 = 5^\circ$$

The developed power in watts is:

$$P_{dev1} = 5.746 = 3730$$

The synchronous speed is:

$$n_s = \frac{120f}{p} = \frac{120.60}{6} = 1200rpm$$

The developed torque is:

$$T_{dev1} = \frac{P_{dev1}}{\omega s} = \frac{P_{dev1}}{ns}.\frac{60}{2\pi} = \frac{3730.1200}{1200}.\frac{60}{2\pi} = 29,682Nm$$

The developed torque is doubled, so we can write:

$$T_{dev2} = 2T_{dev1} = 2.29,682 = 59,635Nm$$

Recall that developed torque is given by following equation:

$$T_{dev} = K.B_r.B_{total}.\sin\delta$$

We can use the previous equation to determine new torque angle δ_2. But first, we have to calculate the value of constants $K.B_r.B_{total}$. For the T_{dev1} and δ_1, we have:

$$T_{dev1} = K.B_r.B_{total}.\sin\delta_1 \Rightarrow K.B_r.B_{total} = \frac{T_{dev1}}{\sin\delta_1} = \frac{29,682}{\sin 5^\circ} = 340,563$$

For the T_{dev2} and δ_2, we can write:

$$T_{dev2} = K.B_r.B_{total}.\sin\delta_2 \Rightarrow \sin\delta_2 = \frac{59,635}{340,563} = 0,175$$

Finally, the torque angle is:

$$\delta_2 = \arcsin 0,175 = 10,08^\circ$$

The pull out torque occurs for torque angle $\delta = 90^\circ$

$$T_{pull-out} = K.B_r.B_{total}.\sin 90^\circ = 340,563$$

The maximum developed power is defined as follows:

$$P_{max} = T_{pull-out}\omega.s = T_{pull-out}.ns\frac{2\pi}{60} = 42976,41W$$