Numerical Methods, 2018 Spring

Assignment 1 Answer

如有疑問請 email 聯繫助教

1. (30%) 每題過程 4 分，兩個答案各 3 分

(a) Bisection method

```
tol = 10^-5
while |b-a| > tol,
   m = (b+a)/2;
   if f(a)*f(m) < 0,
      b = m;
   else
      a = m;
   end;
end;
```

Ans: -3.6689 or -5.7591

(b) Secant method $x_2 = x_1 - f(x_1)\frac{(x0-x1)}{f(x0)-f(x1)}$

```
tol = 10^-5
while |b-a| > tol,
   m = b – f(b) * (a-b) / (f(a)-f(b));
   if f(a)*f(m) < 0,
      b = m;
   else
      a = m;
   end;
end;
```

Ans: -3.6689 or -5.7591

(c) Newton's method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

```
tol = 10^-5
g(a) = f'(a);
while |b-a| > tol,
    a = b;
    b = a – f(a) / g(a);
end;
```

Ans: -3.6689 or -5.7591

*Newton method 下若分別用-5 和-7 做 start value 則只能得到-5.7591 的解，用-3 做 start value 才能得到-3.6689 的解。

2. (25%)

(a) Converge **(9%)** (b) 2 **(9%)** (c)不能確定/不是 **(7%)**

$\mathrm{P(x)} = (x-2)^3(x-4)^2, x_0 = 3$

$\mathrm{P'(x)} = 3(x-2)^2(x-4)^2 + 2(x-2)^3(x-4)$

$\mathrm{P}'(x_0) = P(x_0)/(x_0 - x_1)$

$x_1 = x_0 - P(x_0)/P'(x_0)$

$x_1 = 3 - 1 = 2$

用 $x_0 = 3.0$ 代入此式，一次就可以得到最後的解，且 $P'(x_1) = 0$，故無法分辨是一次收斂或是二次收斂。但 x_0 代入其他的值（例如：$x_0 = 2.99$），就可以求出 $P'(x_n)$ 並得證此為一次收斂。

也可以解數學式得到: $\lim_{x\to 2} \frac{|2-\left(x-\frac{P(x)}{P'(x)}\right)|}{|2-x|} = \frac{2}{3} > 0$ ，確定為一次收斂。

每個答案的解釋各 4 分

3. (30%)

(a) (10%)

解 1: Fix point 過程 4 分，兩個解答各 2 分，

使用相同 x0 以辨認不同 g(x)可以得到不同的解 2 分

For $\mathrm{g1(x)} = \sqrt{e^x/2},\ \mathrm{g1'(x)} = \frac{e^x}{2\sqrt{2}}$

For $\mathrm{g2(x)} = -\sqrt{e^x/2},\ \mathrm{g2'(x)} = -\frac{e^x}{2\sqrt{2}}$

Fix point: $x_{n+1} = g(x_n)$

```
tol = 10^-5
b = g(a);
while |b-a| > tol,
    a = b;
    b = g(a);
end;
```

Run fix point iteration for g1 and g2 with start value = 1.5 and -0.5

We get R1=1.488 for g1 with both start value

and R2=-0.5398 for g2 with both start value.

解 2: **分別討論兩種 g'(R)各 5 分**

分別做出不同的 g'(R) < 1(會收斂)，再經由 R=g(R)的解得到收斂處(畫圖 or 找 x-g(x)=0 和 x 軸焦點)，沒寫清楚**證明** g'(R) < 1 只能拿 2 分。注意 R 是真正要

求出的根而不是帶入的 x。畫圖應有 code，不要徒手畫

(b) (10%) **兩個 g(x)各 5 分，代入 2.5,2.7 各得 2 分，代入 2.6 得 1 分**

Run fix point iteration for g1 and g2 with start value=2.5, 2.6, and 2.7

For $\mathrm{g(x)} = \sqrt{e^x/2}$, x=2.5 2.6 converge to root near 1.5, 2.7 diverge

For $\mathrm{g(x)} = -\sqrt{e^x/2}$, x=2.5 2.6 2.7 all converge to root near -0.5

(c) (10%)**答案 5 分，代回值證明可收斂至 R3 得 5 分**

$$\mathrm{x} = \ln(2x^2)$$

Run fix point iteration with start value=2.5, 2.6, and 2.7 to show it converges to R3=2.6179

4. **(25%)**

$$\begin{cases} f1 = x - 3y - z^2 + 3 \\ f2 = 2x^3 + y - 5z^2 + 2 \\ f3 = 4x^2 + y + z - 7 \end{cases}$$

(a) 9% (請解出偏微分)

$$\begin{bmatrix} \partial f1/\partial x & \partial f1/\partial y & \partial f1/\partial z \\ \partial f2/\partial x & \partial f2/\partial y & \partial f2/\partial z \\ \partial f3/\partial x & \partial f3/\partial y & \partial f3/\partial z \end{bmatrix} = \begin{bmatrix} 1 & -3 & -2z \\ 6x^2 & 1 & -10z \\ 8x & 1 & 1 \end{bmatrix}$$

(b) 4.5%

$$\begin{bmatrix} 1 & -3 & -2 \\ 6 & 1 & -10 \\ 8 & 1 & 1 \end{bmatrix}$$

(c) 11.5%

Newton Method 過程 **3.5 分**

```
tol = 10^-5
while |F(x)| > tol,
    x = x - inv(J(x))*F(x);
end;
```

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兩個答案各 2 分

1.353748, 0.925431, -1.255968

1.111408, 0.988210, 1.070878

Yes, it converges quadratically. **(2 分)**

Because of Newton method **(2 分)**