Multiple time periods

Neural Networks

Dynamic Simulation & Prescriptive Analytics

Multiple Agents

Howard Hao-Chun Chuang (莊皓鈞)

Professor
College of Commerce
National Chengchi University

November, 2024
Taipei, Taiwan

A Bidding Problem (Multi-Agent)

- **God Tone Asia is bidding for a project**
  - The contract will be awarded to the lowest bid the winner will be paid the amount of his bid. **His cost to do the work (*C*) is stochastic** and
    - equally likely to be above or below 100,000 中位数,
    - probability 1/4 of being below 86,000 25% tail
    - probability 1/4 of being above 120,000 75% tail
  - Assume *C* ~ **Myerson**(q1, q2, q3, lower, upper, tailprob.)
    - for *C* in [lower, upper]
    - q1: xx%tile(xx<50, often xx=25)
    - q2: 50%tile(median)
    - q3: xx%tile(xx>50, usually xx=75)
    - lower: minimum
    - upper: maximum
    - tailprob: $P(D<q1) + P(D>q3)$
  - Thanks goes to Prof. Roger Myerson @ U of Chicago

Decision - Maker
總

$C \Rightarrow$

Decision : Bid (β)

Win $\beta < \beta_1$, $\beta < \beta_2$ $\Rightarrow \beta < A \rightarrow$ Profit $\beta - C$

lose $\beta \geq \beta_1$, $\beta < \beta_2$ $\Rightarrow \beta > A \rightarrow 0$

只要有一個
就行

competitors

Decision β_1
Decision β_2
⋮

Random
unknown
variables

Probability Distribution

$A = \text{Min}(\beta_1, \beta_2)$
Others' lowest bid

$$E(\text{profit}) = (\beta - C) \times \text{Prob}(\beta < A) + 0 \times \text{Prob}(\beta > A)$$

A Bidding Problem (Multi-Agent)


- God Tone Asia is bidding for a project
  - The contract will be awarded to the lowest bid the winner will be paid the amount of his bid. His cost to do the work (*C*) is stochastic and
    - equally likely to be above or below 100,000
    - probability 1/4 of being below 86,000
    - probability 1/4 of being above 120,000
  - Assume *C* ~ Myerson( )
  - The possible number of competitors lies in 1 to 5

# Other Bidders	1	2	3	4	5
Probability	0.2	0.3	0.3	0.1	0.1

  - Each opponent's bid is also stochastic and
    - equally likely to be above or below 140,000 50%
    - probability 1/4 of being below 120,000 25%
    - probability 1/4 of being above 180,000 75%

Run	C	Competitor	Others' Bid	$\xrightarrow{\text{Min}}$ A	$B<A$ (1: True, 0: False)	Profit
1.	C_1	2	$\{\beta_1, \beta_2\}$	β_1	1	$\beta - C_1$
2.	C_2	1	β_1	β_1	0.	0
3.	C_3	3	$\{\beta_1, \beta_2, \beta_3\}$	β_2	$\vdots$	$\vdots$
4.	C_4	5	$\{\beta_1, \vdots, \beta_5\}$	β_5	1	0
5.	$\vdots$	$\vdots$	$\vdots$	$\vdots$		
$\vdots$						
$\vdots$						
5						

Profit 近似 E(profit)

\# of time $B<A$

$\fallingdotseq$ Prob $(B<A)$

A Bidding Problem (Multi-Agent)

- God Tone Asia is bidding for a project
  - Let β be his bid & A be the lowest other opposing bid
    - β is a decision variable, not a random variable (RV)
    - Other people's decisions are, nonetheless, RVs
  - What is the expected profit from submitting bid β?
    - model uncertainty about people's information & behavior

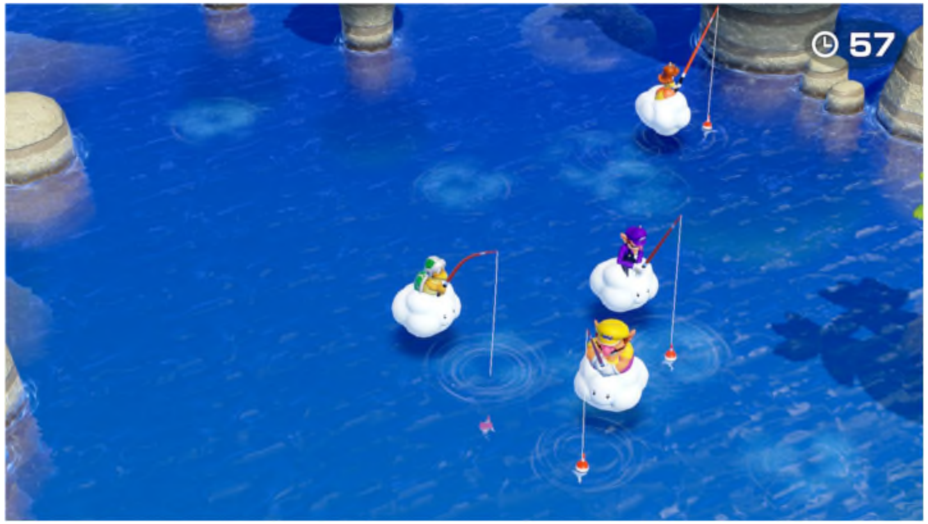
別人出的價格有隱藏資訊

ex: 價格高 → 成本高

$\text{CORR}(\beta, C) \neq 0$.

- God Tone Asia, however, over-estimates his expected profit because he falls into the winner's curse. That is,
  - $E(C|A>\beta) > E(C)$
    - what does this *inequality* imply?
  - How do we capture stochastic dependencies in the simulation model?

The Optimal Stopping Problem

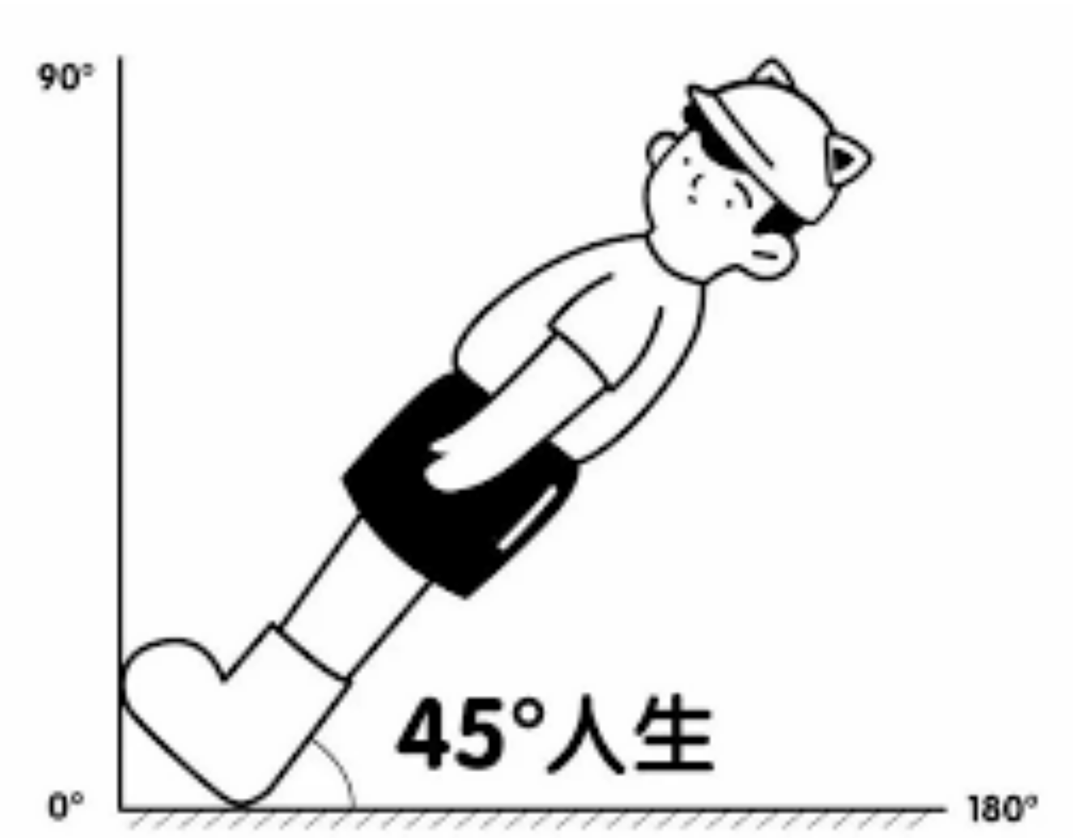


The Optimal Stopping Problem

- In search of your soul mate (or best xxx, etc.)
  - suppose you are going to date a set of candidates, each has a quality score ~ Normal(mu=0, sigma=1)
    - the optimal (soul mate) is the one w/ the highest score
  - you meet them 1 by 1. Every round, accept or reject
    - quality info is only revealed after you date someone
    - If you reject him/her, CANNOT get him/her back later
  - a "look then leap" strategy
    - you meet a pre-determined fraction (*p*) of people and record the highest score V
    - after that, you stop searching when meeting the first one with a score>V
    - If the person you accept has a score=optimal, it is a success. Otherwise, a failure
  - best *p* to maximize the chance of "success"?
  - best *p* to maximize the average score of your choice?

Retirement Planning (Multi-Period)

- Howard is hoping to retire at 60 years old with $1 million USD in savings
  - he wishes to spend $X per year and expects to increase the expenses by 3% each year
    - inflation is a monster



Saving ($)
1 million

X (spending/Year)

$Y \sim Normal(\mu, \sigma)$

投資

$(Saving(s)(t) - X(t)) * (1 + Y\%)$

$= Saving(t+1)$

$X(t+1) = X(t) * (1 + 0.03)$

inflation

Maximize X.

破產:

$$\frac{\#\ \text{of Saving} < 0}{S}$$

Scenario	Life.	Y(Return)	Remaining Savings	Saving < 0.
1	$L_1 = 5$	$\{Y_1, Y_2, \vdots, Y_5$	⋮	0.
2.	$L_2 = 3$	$\{Y_1, Y_2, Y_3$		1.
3.	$L_3 = 30.$	$\{Y_1, \vdots, Y_{30}.$		1.
⋮	⋮			⋮

Retirement Planning (Multi-Period)

- **Howard is hoping to retire at 60 years old with \$1 million USD in savings**
  - **he wishes to spend \$X per year and expects to increase the expenses by 3% each year**
  - **Return**
    - **after expenses are removed from his savings, the remaining capital increases by *Y*% each year.**
    - ***Y*~Normal(mu=8%, sigma=2%)**
  - **Life (*L*)**
    - ***L* is a gamma RV with mean=20 & stdev. =10, rounded to the nearest integer**
    - **shape= ?   scale= ?**

- **Howard wants the probability of running out of money before he dies to be less than 5%.**
  - **how big can \$X be?**

An (s, S) Model (Multi-Period)

- A 3C product dealer

  $d = 0, 1, 2, \ldots \infty.$

  - Demand (*D*) for a 65” LG TV ~ Poisson(λ=6 per day)
    - when there are NO TVs in the offline & online shop, customers will leave (i.e., lost sales) → No substitution
  - sells TVs at the price of $400 each
  - buys TVs from a supplier at the cost of $250 each
    - what could be the complications?

      ↳ Quantity discount, buy-back.
  - cost of holding inventories is $0.6 per TV per day
    - storage space, insurance, interest on the prepaid cost
  - fixed cost per delivery from supplier to dealer is $500
    - could the cost structure be different?

  How much to order?

  When to order?
  - E[daily profit]
    - sales – holding, purchase, lost sales, & delivery costs
    - what else are excluded from this model?

- What are the decisions to be made?

An (s, S) Model (Multi-Period)

- A 3C product dealer
  - Demand (*D*) for a 65” LG TV ~ Poisson(λ=6 per day)
    - when there are NO TVs in the offline & online shop, customers will leave (i.e., lost sales)
  - In the end of each day, review inventory positions (IP)
    - IP = On-hand + On-order
  - An (*s*, *S*) inventory policy is adopted
    - If IP<= *s*, order enough to bring the stock up to *S* TVs

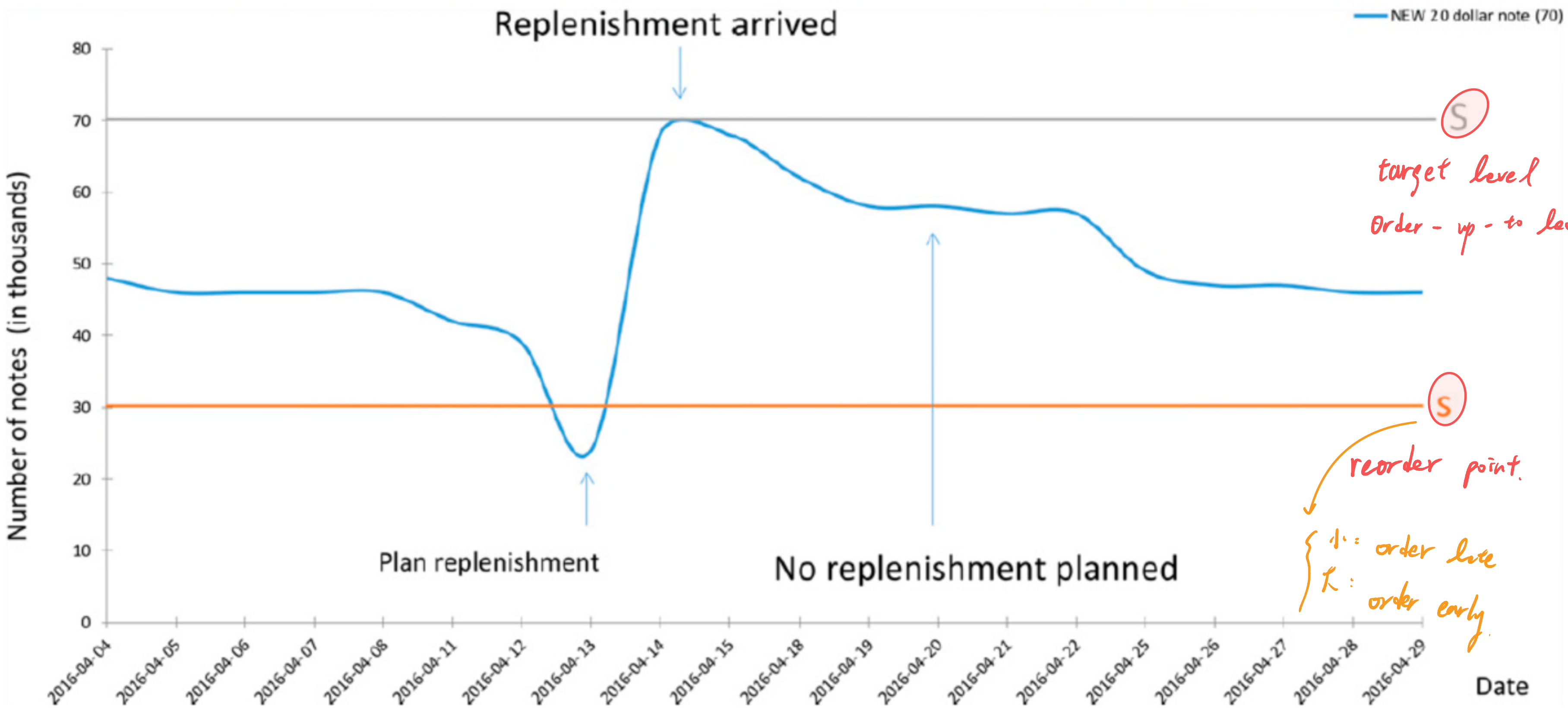
An (s, S) Model (Multi-Period)

Inventory Management Using a Weekly Review (*s*, *S*) Policy at the Bank of Canada

*: if 大S 設較大, 每次 order 多.
Delivery 下降 but inventory cost 上升.

Antoine Legrain,[a,*] Johnathan Patrick[b]

Figure 2. (Color online) A Weekly Review (*s*, *S*) Policy for the Replenishment of the New $20 Bank Notes



target level
order-up-to level.

reorder point.
小: order late
大: order early.

An (s, S) Model (Multi-Period)

- A 3C product dealer
  - Demand (*D*) for a 65” LG TV ~ Poisson(λ=6 per day)
    - When there are NO TVs in the offline & online shop, customers will leave (i.e., lost sales)
  - In the end of each day, review inventory positions (IP)
    - IP = $\underbrace{\text{On-hand}}_{\text{in the house}}$ + $\underbrace{\text{On-order}}_{\text{On the way hasn't arrive yet}}$
  - An (*s*, *S*) inventory policy is adopted
    - If IP<= *s*, order enough to bring the stock up to *S* TVs
  - the supply lead time is 4 or 5 days (50%-50%)
    - this is the key to any firm operations, why?

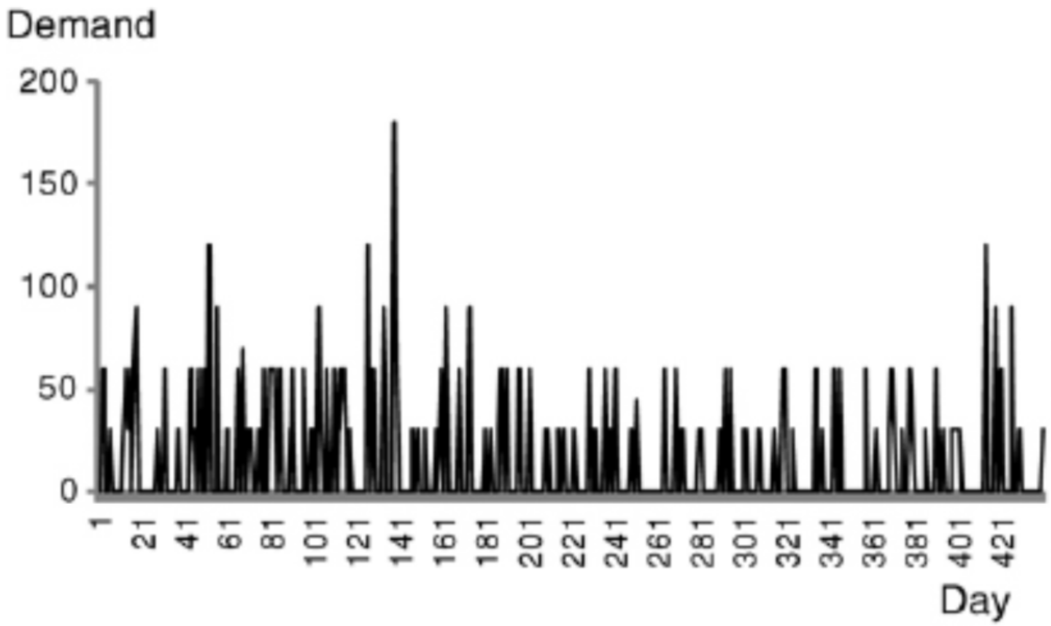
      if lead time 長 s 必須較高
  - suppose the manager sets (*s*=30, *S*=130)
    - why not set *s*=0? What is the role of *s*?
    - what are the pros & cons of setting a larger *S*?

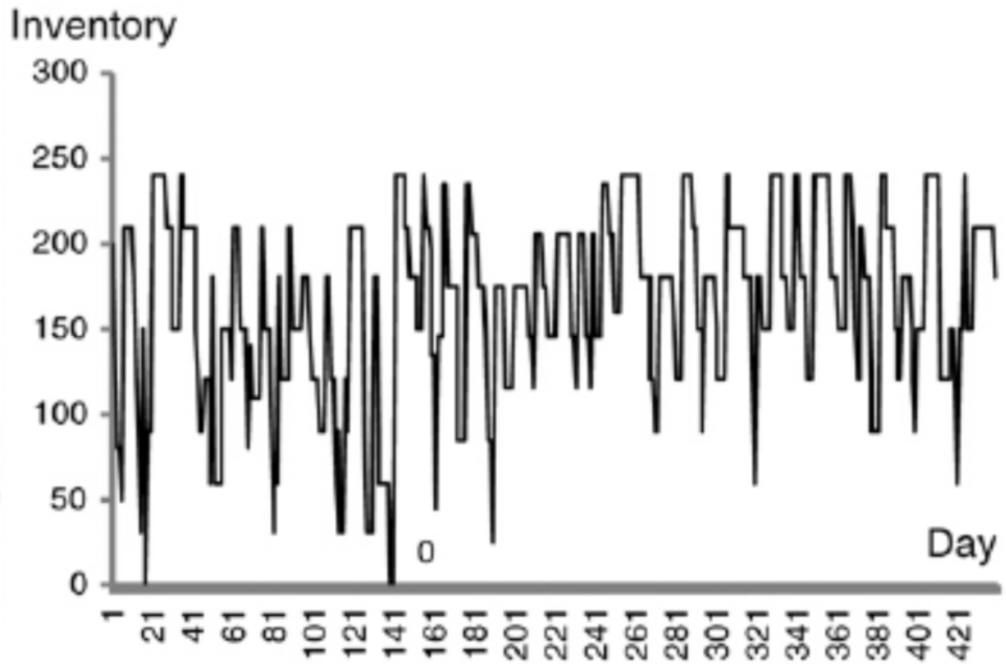
s 小 order time until inventory low

An (s, S) Model (Multi-Period)

Kroger Uses Simulation-Optimization to Improve Pharmacy Inventory Management

Xinhui Zhang, Doug Meiser, Yan Liu, Brett Bonner, Lebin Lin

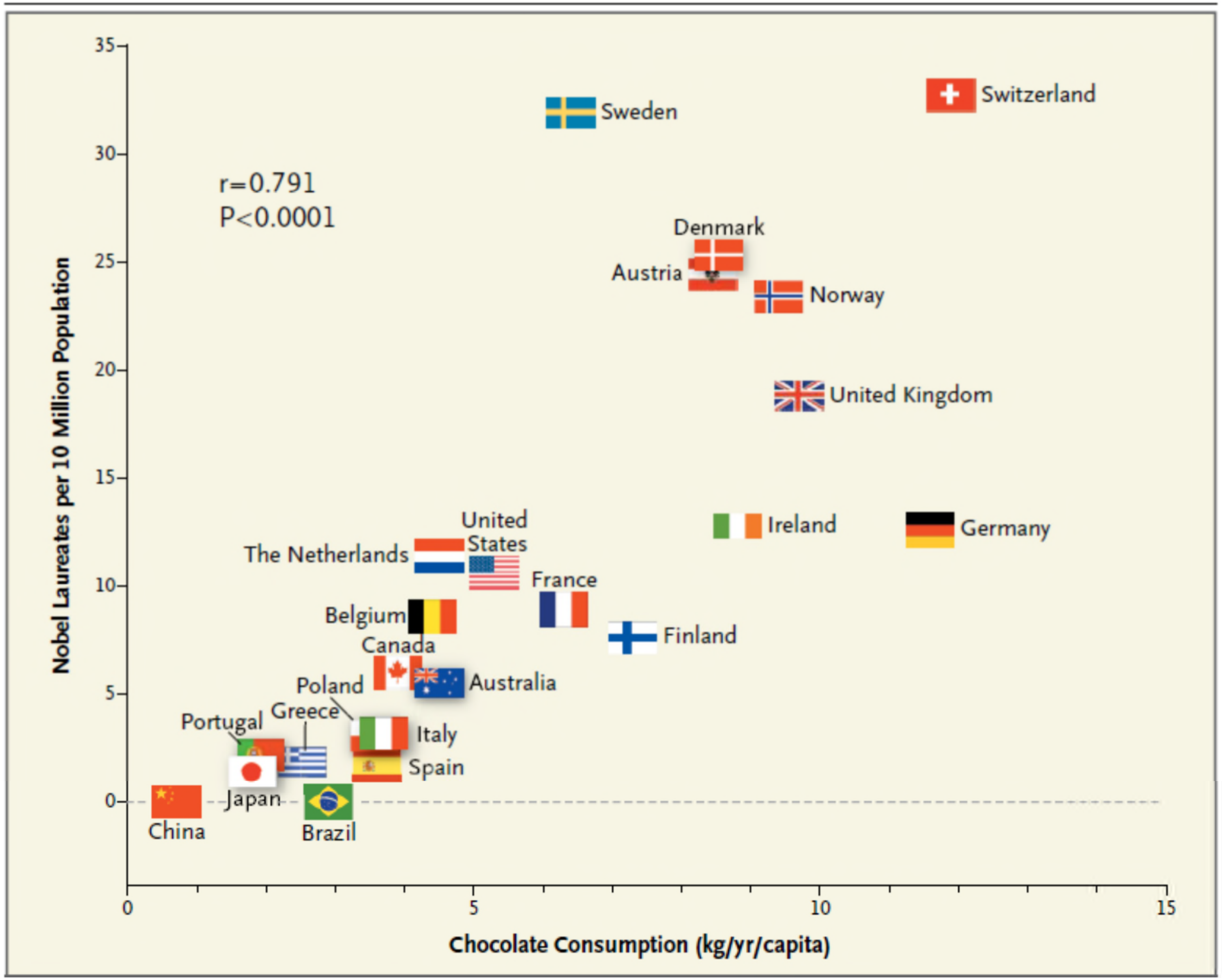




An (s, S) Model (Multi-Period)

- A dynamic (multi-period) simulation model
  - has a loop over time inside a loop over runs
  - has several state variables (that need to be initialized)
    - inventory on-hand, on-order, & on-order's time stamp
  - has several counting variables
    - sale, orderQ, delivery, lost sale, & stockout
  - find optimal (s^*, S^*) using search algorithms
- What else could be done in this case?
  - alternative settings
    - from periodic to continuous review
    - use a fixed Q instead of Q = Inventory Positions – s
    - *non-linear* cost functions
    - backlogged versus lost-sales
    - *time-varying* demand patterns

Regression Modeling



Time-Varying Demand Distribution

- **Let RV *Y* denote iid random demand**
  - **unconditional demand distribution $f(Y|\theta)$**
    - **iid observations rarely exist in time-series data**
  - **conditional demand distribution $f(Y|\theta, x)$**
  - **contextual variables *x***
    - **day-of-week, season, weather, etc.**

- **Generalized linear modeling (GLM)**
  - **if observations y_1, y_2, …. y_T are large & continuous, go for *linear regression***

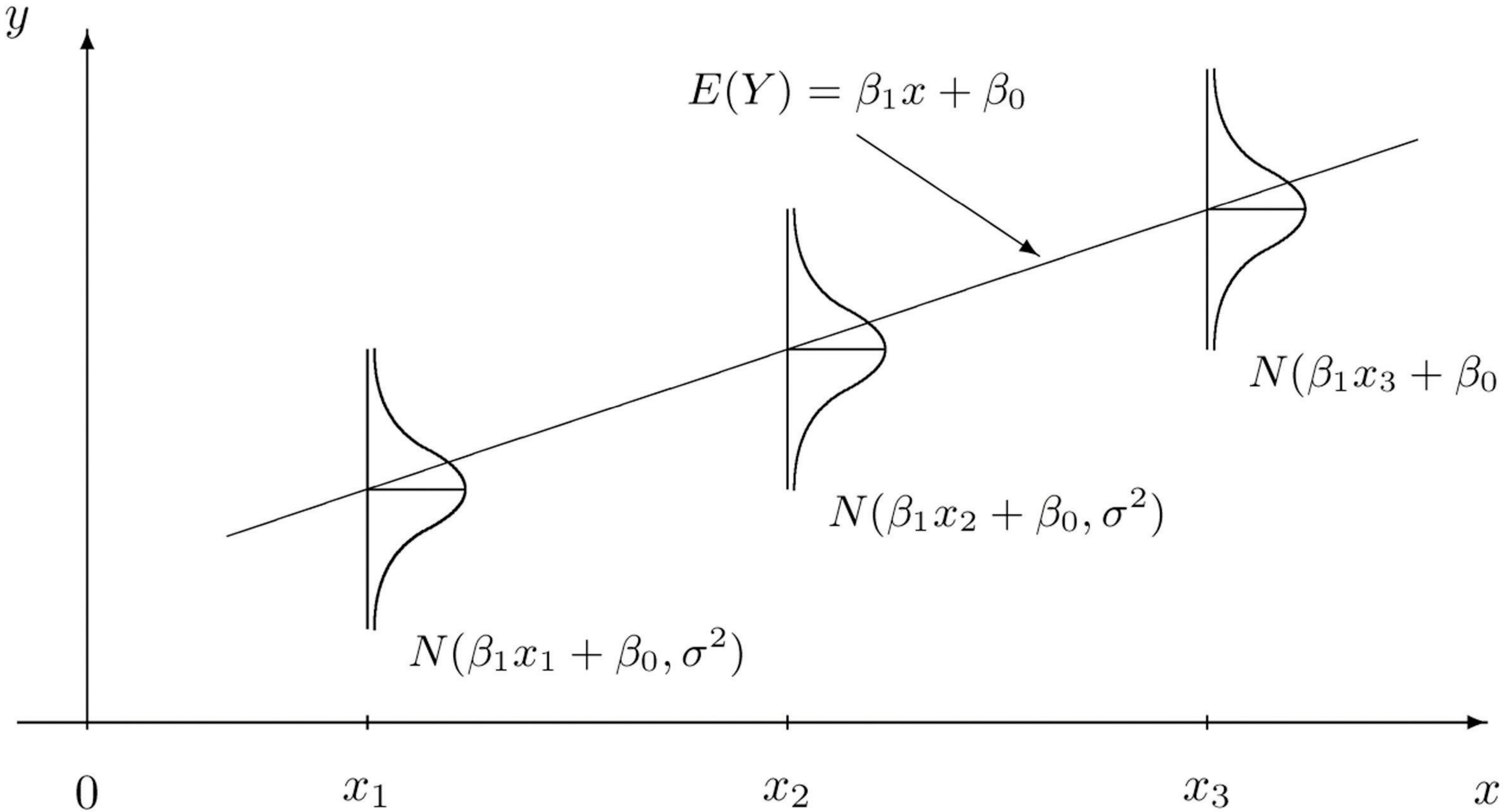
$$y_i = \beta_0 + \beta_1 x_{1i} + \ldots + \beta_k x_{ki} + \varepsilon_i, \; i = 1, 2, \ldots, n$$

$$\text{where } y_i \in [-\infty, +\infty] \;\&\; \beta_0 + \beta_1 x_{1i} + \ldots + \beta_k x_{ki} \in [-\infty, +\infty]$$

$$E(y_i \mid x_{1i}, \ldots, x_{ki}) = \beta_0 + \beta_1 x_{1i} + \ldots + \beta_k x_{ki}$$

Time-Varying Demand Distribution

$$y_i \sim N(\beta_0 + \beta_1 x_{1i} + \ldots + \beta_k x_{ki}, \sigma^2),\ i = 1, 2, \ldots, n$$



$\lambda_1 = \exp(0.037 + day0 \times 0.83 + day \times 0.4654 \ldots$

day 0 : $\lambda = e^{0.037 + 0.83}$

day 1 : $\lambda = e^{0.07 + 0.4654}$

Time-Varying Demand Distribution

- Let RV *Y* denote iid random demand
  - unconditional demand distribution $f(Y|\theta)$
    - iid observations rarely exist in time-series data
  - conditional demand distribution $f(Y|\theta, x)$
  - contextual variables x
    - day-of-week, season, weather, etc.

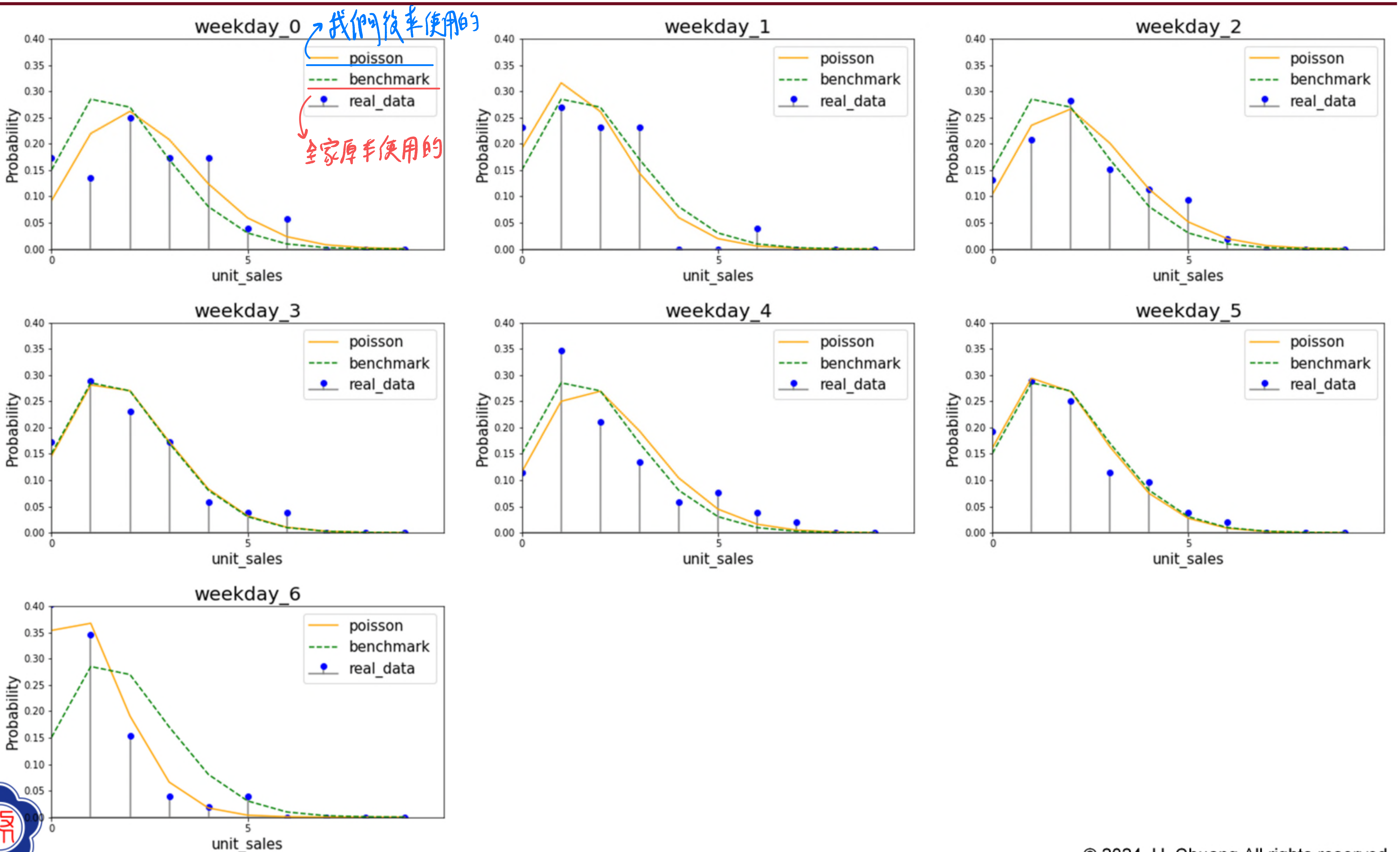
- Generalized linear modeling (GLM)
  - if observations $y_1, y_2, \ldots. y_T$ are small & discrete, go for *Poisson regression*

$$f(y|\lambda) = \frac{\lambda^y e^{-\lambda}}{y!}, \ y = 0, 1, \ldots, +\infty \ \text{ where } \lambda \in (0, +\infty]$$

$$E(y_i \mid x_{1i}, \ldots, x_{ki}) = \lambda_i = g(\beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki}), \ i = 1, 2, \ldots, n$$

$$\lambda_i = \exp(\beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki})$$

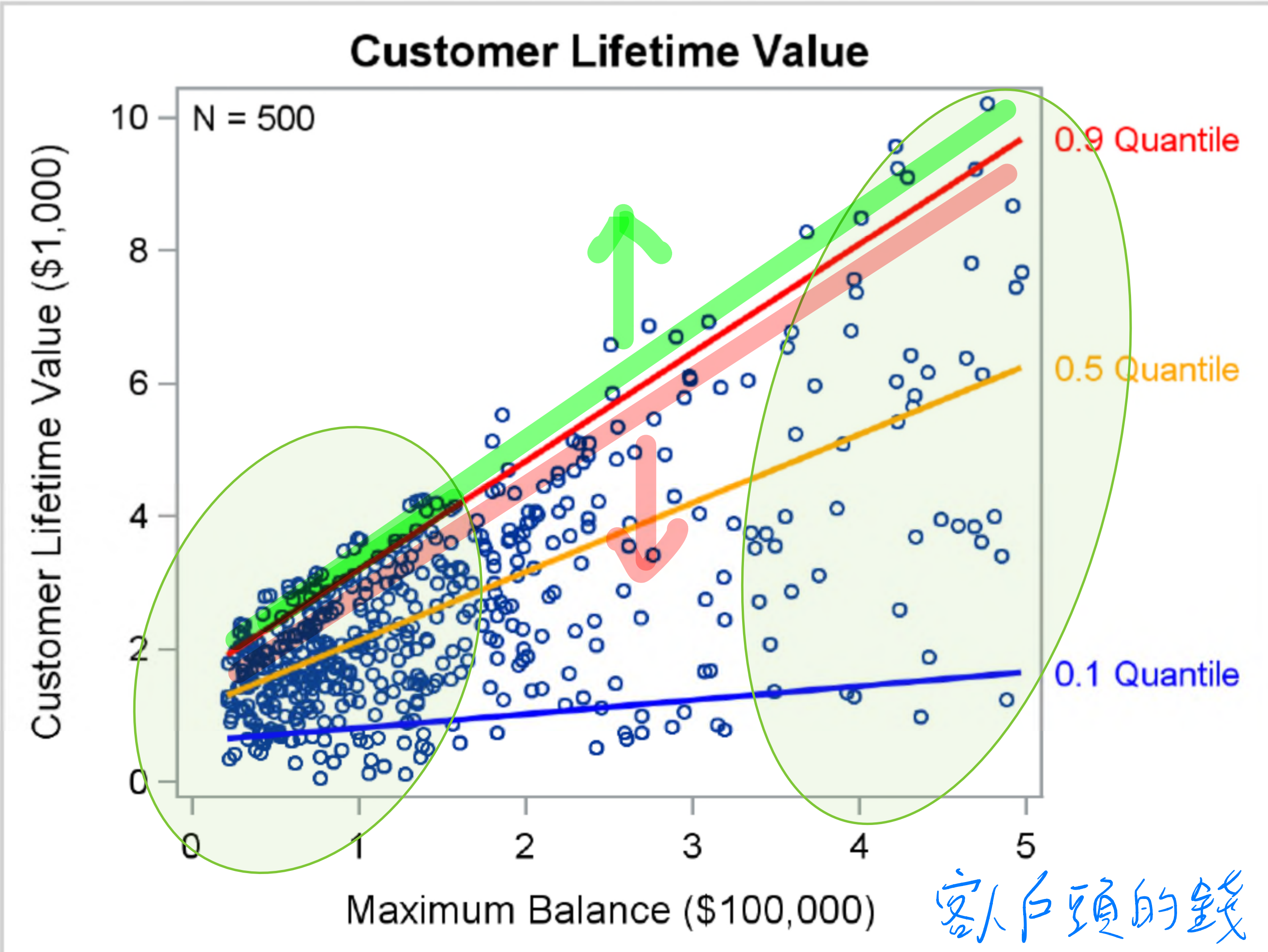
Time-Varying Demand Distribution

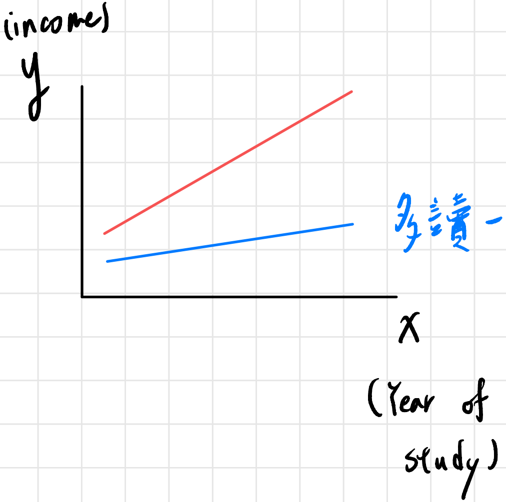


Quantile Regression

- Oftentimes, we care more than E[*y*|*x*]

顧客終身價值





〃 高

〃

多讀一年書，對低收入的
有什麼影響

Quantile Regression

- **Let τ denote quantile ($0<\tau<1$)**

$$y_i = \beta_0^\tau + \beta_1^\tau x_{1i} + \ldots \beta_k^\tau x_{ki} + \varepsilon_i^\tau \quad i = 1,2,\ldots,n$$

$$Q^\tau(y_i \mid \boldsymbol{x}_i) = \beta_0^\tau + \beta_1^\tau x_{1i} + \ldots \beta_k^\tau x_{ki}$$

where $Q^\tau(y \mid \boldsymbol{x})$ denotes the τth quantile of y conditional on $\boldsymbol{x}$

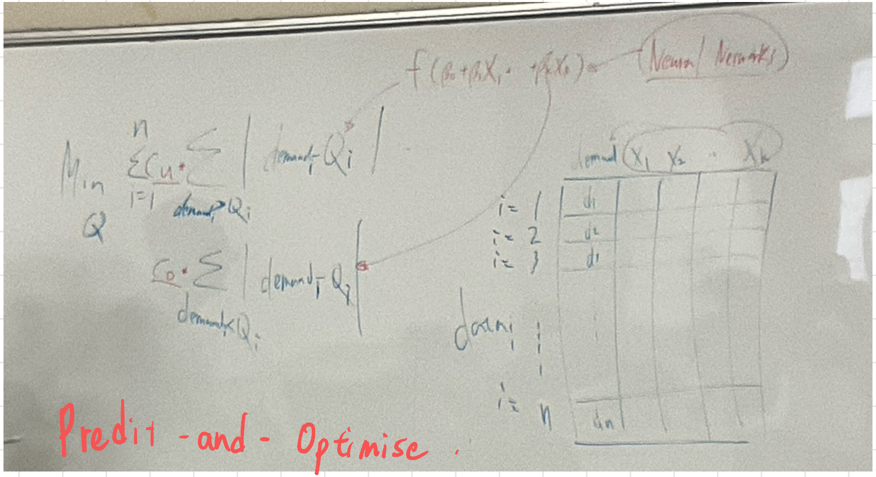
- **Does this objective function look familiar?**
  - **remember C_u=p-c & C_o=c-s** Demand < Q

0.9

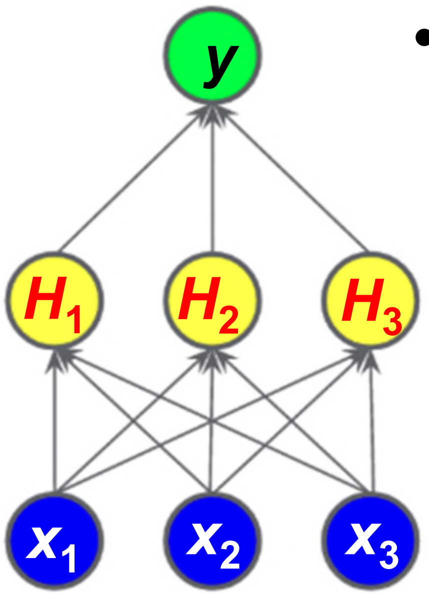
Demand > Q

$$Min \; \tau \sum_{y_i \geq (\hat{\beta}_0^\tau + \hat{\beta}_1^\tau x_{1i} + \cdots + \hat{\beta}_k^\tau x_{ki})} \left| y_i - (\hat{\beta}_0^\tau + \hat{\beta}_1^\tau x_{1i} + \ldots + \hat{\beta}_k^\tau x_{ki}) \right| +$$

$$(1\text{-}\tau) \sum_{y_i < (\hat{\beta}_0^\tau + \hat{\beta}_1^\tau x_{1i} + \cdots + \hat{\beta}_k^\tau x_{ki})} \left| y_i - (\hat{\beta}_0^\tau + \hat{\beta}_1^\tau x_{1i} + \ldots + \hat{\beta}_k^\tau x_{ki}) \right|$$

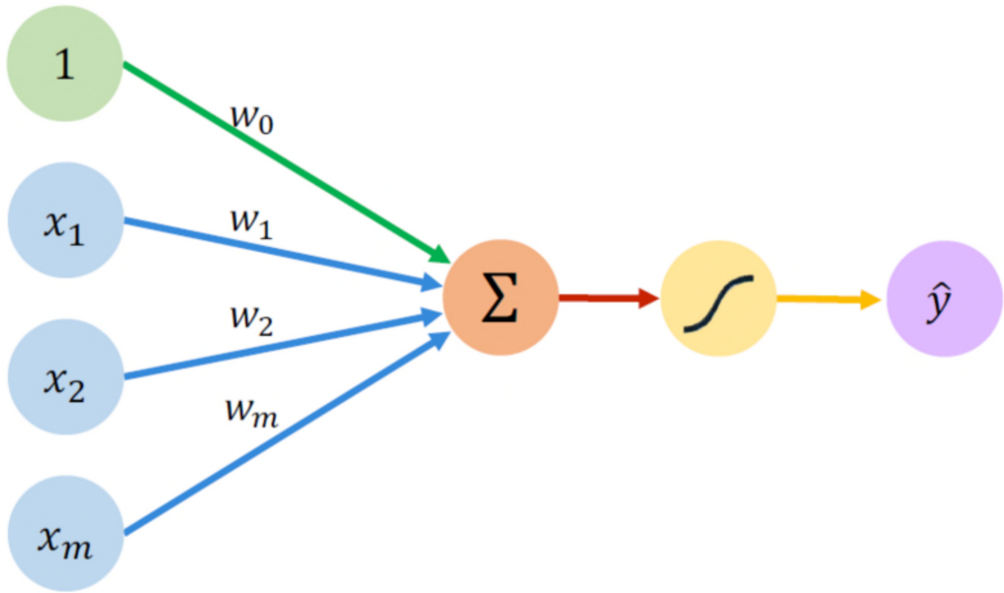

Predit - and - Optimise .

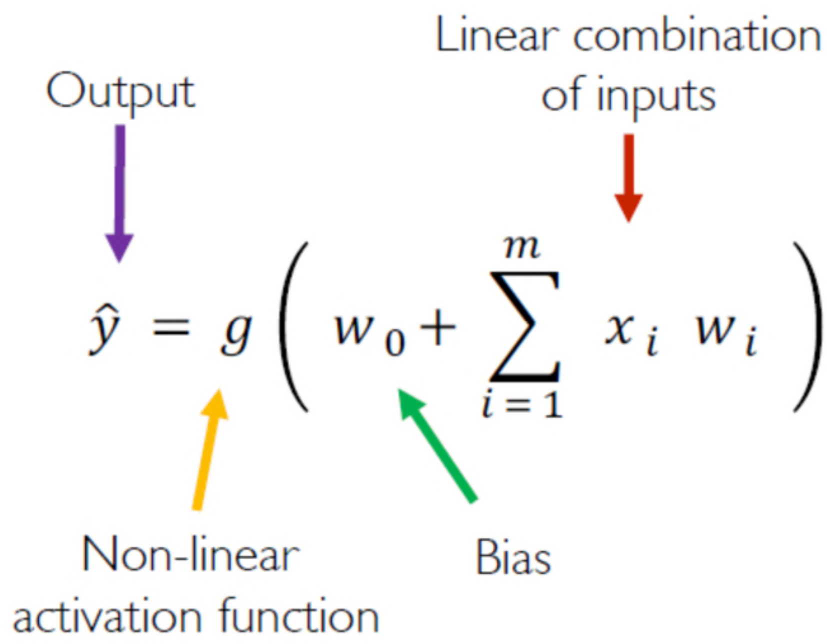
Deep Learning Newsvendor



Hidden Layer

- A **neural network**
  - with **ReLU activation**, $g(x) = \text{Max}(0, x)$
    - $H_1 = \text{Max}(0, w_{10} + w_{11}x_1 + w_{12}x_2 + w_{23}x_3)$
    - $H_2 = \text{Max}(0, w_{20} + w_{21}x_1 + w_{22}x_2 + w_{23}x_3)$
    - $H_3 = \text{Max}(0, w_{30} + w_{31}x_1 + w_{32}x_2 + w_{33}x_3)$
    - $y = w_{40} + w_{41}H_1 + w_{42}H_2 + w_{43}H_3$
    - How many parameters (參數) to be learnt?





Deep Learning Newsvendor

- **Let RV *D* denote non-iid random demand**
  - **$E[D|x]= mu_d$=5*temperature+50**
  - **$Var[D|x]$ depends on weather**
    - **if weather = 1, $sigma_d=0.6*mu_d$**
    - **if weather = 0, $sigma_d=0.5*mu_d$**
  - **we have *n* demand observations ($d_1, d_2, \ldots, d_n$) as well as data on temperature & weather**
  - **can we train a neural network *f*(*xW*) that will output a decision *Q* given *x*?**

***Loss* (損失)= distance(y^{actual} & $y^{predict}$)**

- **gradient descent (梯度下降)**

$$W_{new} = W_{old} - \eta \left(\frac{\partial Loss}{\partial W_{old}}\right)$$

- **η: learning rate (>0)**

Data driven
Newsvendor
Problem

Deep Learning Newsvendor

- Let RV D denote non-iid random demand
  - $E[D|x]= mu_d$ =5*temperature+50
  - Var[D|x] depends on weather
    - if weather = 1, $sigma_d$ =0.6*mu_d
    - if weather = 0, $sigma_d$ =0.5*mu_d
  - we have n demand observations (d_1, d_2, …, d_n) as well as data on temperature & weather
  - can we train a neural network $f(xW)$ that will output a decision Q given x?
    - see the Python code
- ***Loss (Q, d)***
  - if $Q \geq d$, loss = C_o*|$Q – d$|
  - if $Q < d$, loss = C_u*|$Q – d$|
  - you could use XGBoost, Random Forest, Lasso, etc.
    - features x must be available in advance!

Simulation for Service Innovation

- ibon power bank rental service
  - I was asked to design an optimal re-stocking policy
    - https://chat.openai.com/share/aaab96f3-3580-4c39-895d-3f7a2f22f70b

  - inter-arrival time distributions are the key
  - customer APP & in-store employee workload

- 7-11 vs Family Mart, difference?

Recap: Decision Science in Action

- **For key performance outcome *Y=f*(*X* , *Z*; *c*)**
  - **analyze how two sets of variables – controllable *X* & non-controllable *Z* – and parameters *c* affect *Y***
    - **low-dim or high-dim *X*? deterministic or stochastic *Z*?**
    - **need to estimate *c* from subjective/objective data**
    - ***f*( ) may be 1 or >1 equations for a problem or system**
  - **probability distributions for stochasticity/uncertainty**
    - **well-defined or unknown *f*( )?**
    - **machine learning/deep learning for unknown *f*( )**

Predictive modelling

- Estimate the target for new observations

$$y = f(x)$$

Explanatory modelling

- Describe the effect that a change of certain inputs has on the target

$$y = f(x)$$

Optimisation

- Find the inputs that give optimal performance
- f is known

$$y = f(x)$$