Homework II (Group)

Due date: 23:59 on November (Friday), 2024

Please store the answers in a pdf file and upload the file onto WM5. Each group submits only ONE copy. Make sure names and IDs can be found on the file. Please show the written simulation program on the document too. NO late submission will be accepted.

Q1 (30%). Suppose a fraction 分數 17% of the microchips produced by a leading manufacturer is 有問題的 defective. For the manufacturer, the number of microchips delivered to its client depends on how many microchips are inspected and accepted by 50 inspectors in the factory. Historically, given that a microchip is defective, an inspector wrongly accepts the chip 10% of the time, thinking it has no defect. If a microchip is not defective, an inspector, however, wrongly rejects the chip 5% of the time. In general, each of the 50 inspectors can inspect 20 chips per hour in a working day of 8 hours (lunch break NOT included in the 8 hours).

Use the *binomial* distribution and whatever functions needed in Python to write a simulation program for the case above. In a working day, the microchips delivered to the client contain some good ones (correctly accepted) and some bad ones. In your simulation program, calculate the ratio of good ones to the sum of good and bad ones. The simulation program should be a function called *dailychips*. The function must return:

1) the daily number of delivered chips (that is *random*)

2) the daily ratio of good ones mentioned above.

Simulate the daily operations for 1,000 runs and answer the following questions.

(a) The manufacturer claims that the everyday chips delivered to its client has at least 98% good ones. What is the probability that such a claim is true?

(b) What is the probability that the manufacturer can deliver **6,400 microchips accepted by its inspectors** in a typical working day of 8 hours?

Q2 (20%) Use the *dailychips* function in Q1 to simulate *montlychips* and *quarterlychips*. The *monthlychips* is the sum of delivered chips over 30 days, whereas the *quarterlychips* is the sum of delivered chips over 90 days respectively. Ignore good ratios in the two cases and focus on the number of delivered chips over 30 and 90 days. Generate 1,000 random samples for the two numbers (*montly* and *quarterly*) of delivered chips.

Generate the hist function in *matplotlib* (set bins=20 or bigger) of the simulated *montlychips* and *quarterlychips*. Do they look like normal distributions?

Use the *shaprio* function in Scipy to examine normality of *montlychips* and *quarterlychips* (if *p*-value<=0.05, reject H0: Normality holds). Do you find any evidence for normality? If yes, can you provide theoretical explanations for that (**hint**: check chapter 3 for a theorem)?

microchip

− 17%
- 10% Accept
- 90% Reject

$\Rightarrow$ binomal(0.1, 0.9)

+ 83%
- 95% Accept
- 5% Reject

$\Rightarrow$ binomal(

20 chips / hour
8 hours
50 inspectors

$\Rightarrow$ 1天

20×8 chips / 人

$\Rightarrow \underline{160 \times 50 \text{ chips}}$

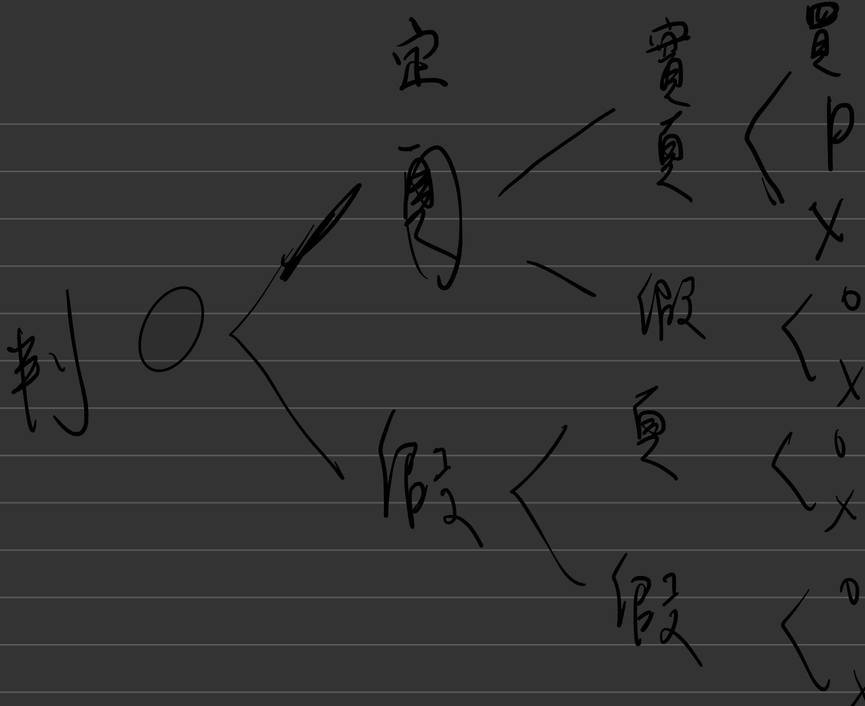
$= 8000$

6 定中心

R
- 真中 220
- 假中 1

老

- 0.25 在真
  - 0.7 假
  - 0.3 真
- 0.75 在假
  - 0.7 真
  - 0.3 假



think

$P(假|真) = \dfrac{P(假) \times P(真)}{}$

$= 0.7$ p

$P(定假|實真) = 0.7$

$p = 0.25$ x

Q3 (30%) Let's revisit the Conley Fisheries in Lecture 5. Define stochastic demand for cold fish as D_{Glou}~triangular(*min*=4000, *max*=8000, *mode*=6000) and D_{Rock}~triangular (*min*=4500, *max*=7500, *mode*=6000) as. The prices are random $Price_{Glou}$~Normal(μ=3.5/lb, σ=0.5/lb) and $Price_{Rock}$~Normal(μ=3.65/lb, σ=0.25/lb). Also, corr($Price_{Glou}$, $Price_{Rock}$) is not zero.

In addition, the Conley Fisheries has two boats (each with a full load of 3800 lbs) driven by captain Rick and captain Morty. The daily fraction captured by Risk is a uniform(0.5, 1.0) random variable whereas the daily fraction captured by Morty is a triangular(min=0.5, max=1, mode=0.75) random variable. The day-to-day operating costs for both boats are about \$7,200. Being the business analyst of Conley Fisheries, you lay out four possible selling strategies:

a) Rick goes to Glou and Morty goes to Rock; b) Rick goes to Rock and Morty goes to Glou;
c) Rick and Morty go to Glou; d) Rick and Morty go to Rock.

To assess which strategy is more profitable or less risky, write a simulation program that allows one to compute expected profit and CVAR(10%) for the four selling strategies above. After that, please finish the following tasks.

(1) Show a figure where the y-axis is CVAR(10%) and x-axis is corr($\text{Price}_{\text{Glou}}$, $\text{Price}_{\text{Rock}}$). For each of the following values in the x-axis: -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, and 0.8, simulate 10,000 runs respectively. Compute the payoff of the four strategies and plot their CVAR(10%) values. The figure should exhibit four lines (in different colors and types). Provide a succinct and logical discussion about findings in the figure above. Also, explain what kind of selling strategies you would recommend the CEO to adopt.

(2) Consider a more sophisticated case where *D* and *Price* are negatively correlated. Try to standardize D_{Glou} and D_{Rock} into standard normal variables (Lecture 4). Assume your own correlation values in a 4 by 4 correlation matrix for D_{Glou}, D_{Rock}, $Price_{Rock}$, and $Price_{Glou}$. Then simulate the four random variables using a multi-variate normal distribution (Hint: you need to transform (de-standardize) D_{Glou} and D_{Rock} back to their original scales). Then evaluate the expected profit and CVAR(10%) for the four selling strategies.

Q4 (20%) Suppose you have successfully built a start-up and are looking to sell your business for early retirement. The maximum value each potential acquirer (*a*) is willing to pay is an independently and normally distributed RV X_a~Normal(μ=\$3,000,000, σ=350,000).

(1) If you get two acquirers bidding against one another, how much money do you expect to receive by selling the firm? How much more would you expect to get if you find a third?

(2) How much money do you expect to get if you can find 10 competitors to bid for your firm? What have you observed from simulation results? Can you provide logical explanations for the observed outcomes/patterns?

$S=1$

Demand
- Glou
- Rock

⇒ trian

Price
- Glou
- Rock

corr

3,800,

Boat
- Risk ⇒ uniform (0.5, 1.0)
- Morty ⇒ trian

\$ 7,200 cost