Homework I (Group)
Due date: 23:59 on October 11 (Friday), 2024

Q1(20%). Exercise 1.4 in Bertsimas and Freund (2004) Chapter 1

Q2(20%). Exercise 1.5 in Bertsimas and Freund (2004) Chapter 1

Q3(15%) Three dices are to be rolled. Suppose two of the three are fair dies, but one die is unfair with probability of 0.25 to be 2 and equal probabilities for (1, 3, 4, 5, 6). Please write a simulation program (use np.random.choice) that calculates the expected value of the smallest number of rolling the three dies (三顆骰子中最小的值).

Q4(30%). You are a manager of SAP, and you just hire a salesperson from Oracle. Suppose the salesperson has to visit 20 potential clients this month. Before he/she makes these visits, you think high skill or low skill is equally likely. If there is high skill, then the probability of making a sale is 2/3 in each of the 20 visits. If there is low skill, then the probability of making a sale is 1/3 in each of the 20 visits. Use np.random.choice( ) and whatever functions needed in *Python* to write a simulation program for the case described above. Simulate the scenario for 1,000 runs and answer the following questions based on simulation results.

(a) Suppose you have a policy of promoting the salesperson if there are at least 9 sales in these 20 visits. What is the probability this salesperson will be promoted this month?
(b) What is the conditional probability that this person will be promoted if there is high skill?
(c) What is the conditional probability that there is high skill given that the salesperson is promoted under this policy?
(d) Make a table showing how the three probabilities in (a), (b), and (c) would change, if the policy instead promotes the salesperson when there are at least *n* sales in the 20 visits for *n* in 1, 2, 3,…, 19, 20.

Q5(15%) Suppose you go to the college of commerce library (商圖) on Monday evening and would like to borrow a book. You are told that the book has been checked out the previous Thursday. Assume no one else is waiting for the book. The library staff tells you that borrowers return books after 4, 5, 6, or 7 days, with probabilities of 0.1, 0.2, 0.3, 0.4. Note that the library is open 7 days a week.
Hint: The book was checked out on previous Thursday and not available on Monday evening. So, the borrower already held the book for 4 days (i.e., Fri, Sat, Sun, and Mon).

As before, 50% of students return their books to a "foreign" library (社圖/總圖), resulting in an extra 2-day delay before the book arrives back to the home library. You decide to check the status of the book every evening. What is the probability you will need to wait until Wednesday evening to get the book (i.e., the borrow returns the book after 6 days)?
Write a Monte-Carlo simulation program to compute the probability (Hint: the probability is conditional should be close to 4/19).

Please store the answers in a pdf file and upload the file onto WM5. Each group submits only ONE copy. Make sure names and IDs of students within each group can be found on the file. For Q3, Q4 & Q5, please show the written simulation program on the document too. NO late submission will be accepted.

P.S.: While this is a group assignment, everyone in the group must have a solid grasp of what is going on. Free riders would be in trouble in the midterm exam.

EXERCISE 1.4 Anders and Michael were classmates in college. In their spare time while undergraduates, they developed a software product that regulates traffic on internet sites. Their product uses very imaginative and original ideas, and they have applied for a patent. They estimate that there is an 80% chance their patent will be approved by the US Patent Office.

Anders and Michael have also formed a start-up company called ITNET, and they have started to market their software product. Last month, they presented some of their ideas tof Singular, Inc., the dominant player in this growing market, after Singular had signed a confidentiality agreement with ITNET that ITNET's lawyer had prepared.

Yesterday, Singular announced a new software product that seemed suspiciously similar to the one that Anders and Michael have developed. Anders' first reaction was to plan to sue Singular immediately. However, Michael felt that they should wait until they have received notification of their patent, which is still pending before the U.S. Patent Office. Michael reasoned that their case would be much stronger if they had a patent for their product.

Suppose that Anders and Michael have a 90% chance of winning a lawsuit against Singular if their patent application is approved, and that they still have a 60% chance of winning such a lawsuit even while their patent application is pending (because Singular had signed the confidentiality agreement). However, if their patent application is not approved, suppose that the chance of winning the lawsuit would drop to 40%.

Anders feels that if they sue Singular immediately, there is a 70% chance that Singular would settle out of court for $400,000, and a 30% chance that Singular would not settle out of court. If they win the lawsuit, their settlement would be $1 million. However, they estimate that the legal costs of going to court would be $100,000.

(a) Structure ITNET's problem of whether or not to sue Singular as a decision tree.

(b) Solve for the optimal decision strategy.

EXERCISE 1.5 Javier Peña has always been interested in financing artistic projects. He has recently been offered two financing opportunities in the fashion industry: financing a new line of avant-garde youth fashions designed by Jorge Vera, and financing a line of business attire designed by Paolo Ricci. Javier has had a lot of past experience with these two designers, and has observed that 20% of Vera's fashion lines are "hits" and 80% of them are "misses." Furthermore, Ricci's fashion lines are "hits" 30% of the time, and are "misses" 70% of the time.

Javier's net liquid assets amount to $750,000. As a result, he can afford to finance at most one of the two fashion lines. However, he does have the option of pre-testing at most one of the fashion lines at the upcoming design show in San Francisco, before deciding which, if any, fashion line he would like to finance for the entire U.S. market for the fall fashion season. The costs and revenue associated with the two fashion lines are given in Table 1.6.

Javier has observed, based on previous years, that of the avant-garde fashion lines that were hits nationwide, 80% were hits in the San Francisco pre-test; of the avant-garde fashion lines that were misses nationwide, 40% were hits in the San Francisco pre-test. Of the business attire fashion lines that were hits nationwide, 90% were hits in the San Francisco pre-test; of the business attire fashion lines that were misses nationwide, 60% were hits in the San Francisco pre-test. While Javier may find pre-test results useful, he knows the accuracy of this kind of test is not high enough to compel him in all cases to act in accordance with the pre-test results. In any event, Javier is willing to act on the basis of expected monetary values.

Fashion Line:	*Jorge Vera (avant-garde)*	*Paolo Ricci (business attire)*
Net cost of San Francisco pre-test	$200,000	$75,000
Additional cost of U.S. production of line after a San Francisco pre-test	$500,000	$275,000
Cost of U.S. production if not pre-tested in San Francisco	$600,000	$325,000
Revenue if fashion line is a "hit"	$4,000,000	$1,000,000
Revenue if fashion line is a "miss"	$300,000	$100,000

Tumor	*Remove Tumor*	*Leave Tumor*
Benign	5	8
Malignant	5	1

(a) Develop a decision tree to assist Javier in deciding what to do.

(b) What probabilities need to be computed in order to solve the decision tree?

(c) After reading Chapter 2, compute the necessary probabilities, and solve for Javier's optimal decision strategy.

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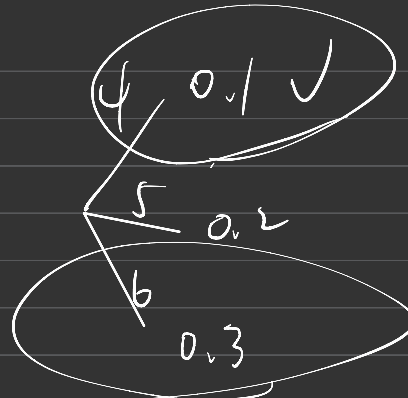
4.

(0.5) high $\Rightarrow \frac{2}{3} \rightarrow$ 20人

(0.5) low $\Rightarrow \frac{1}{3} \rightarrow$ 20人

$n=20$

1000x



1.

$P(X \geq 9) = ?$

2.

$P(X \geq 9 \mid high) = \frac{P(X \geq 9 \cap high)}{P(high)} =$

✓ ✓ ✓ ✓

4. 5. 6. 7

0.1, 0.2, 0.3, 0.4

Mon $\frac{2}{4}$ $\frac{3}{4}$ $\frac{1}{4}$

0.5 ⇒ extra

一 二 三 四 五 六 日

X ✓ ✓ ✓

90

?

$P(6天 \mid 4x) = \frac{0.3}{0.9}$

$0.1 \times 0.5 + 0.3 \times 0.5$

$= 0.4 \times 0.5$

$= 0.20$

0.2×0.5

$= 0.1$

$$\frac{0.3\times0.5+0.1\times0.5}{0.9+0.1\times0.5}$$

$$=\frac{0.2}{0.95}$$

$$=\frac{4}{19}$$

一 二 三 四 五 六 日

X、1、2、3、4、5、6、7、

$$\frac{2}{9},\ \frac{3}{9},\ \frac{4}{9}$$

$$\frac{3}{9}\times0.5+0.1\times0.5$$

$$=\frac{30+9}{90}=\frac{13}{30}\times\frac{1}{2}=\frac{13}{60}$$

P(下雨 | 收衣服)

= 5天 ∩ 收衣

$$\frac{0.4}{0.55}$$

$$\frac{0.5}{0.95}$$

$$\frac{0.2}{0.95}$$

P(小偷 | 狗沒在過瑤)

$$=\frac{0.1\times0.5}{0.9+0.1\times0.5}=\frac{0.05}{0.95}$$

After 4. 5. 6. 7.

0.1 0.2 0.3 0.4

∵ 題目 ⇒ "not availiable on Monday"

條件：4天後沒在總圖還書 → 已知 $= 1 - 0.1 \times 0.5 = 0.95$ → 還在商圖

Q: Until Wed get the book

$P(\text{Wed get} \mid \quad)$

$= \dfrac{\text{Wed} \wedge}{\quad} = \dfrac{P(\text{第六天還}) + P(\text{第四天商})}{0.95}$ (總圖)

$= \dfrac{0.3 \times 0.5 + 0.1 \times 0.5}{0.95}$

$= \dfrac{0.2}{0.95} = \dfrac{4}{19}$ ⇒ Hint

第四天 ⇒ P(第四天還 | 已走2)

$$= \frac{0.1 \times 0.5}{0.95} = \frac{0.05}{0.95}$$