

[1]

$$(1) f_X(x) = \binom{10}{x} \left(\frac{1}{10}\right)^x \left(\frac{9}{10}\right)^{10-x}, \quad x=0, 1, 2, \dots, 10$$

x	0	1	2	3	4	5	6	7	8	9	10
$f_X(x)$	$\left(\frac{9}{10}\right)^{10}$	$C_1^{10} \left(\frac{9}{10}\right)^9 \left(\frac{1}{10}\right)$	$C_2^{10} \left(\frac{9}{10}\right)^8 \left(\frac{1}{10}\right)^2$	$C_3^{10} \left(\frac{9}{10}\right)^7 \left(\frac{1}{10}\right)^3$	$C_4^{10} \left(\frac{9}{10}\right)^6 \left(\frac{1}{10}\right)^4$	$C_5^{10} \left(\frac{9}{10}\right)^5 \left(\frac{1}{10}\right)^5$	$C_6^{10} \left(\frac{9}{10}\right)^4 \left(\frac{1}{10}\right)^6$	$C_7^{10} \left(\frac{9}{10}\right)^3 \left(\frac{1}{10}\right)^7$	$C_8^{10} \left(\frac{9}{10}\right)^2 \left(\frac{1}{10}\right)^8$	$C_9^{10} \left(\frac{9}{10}\right)^1 \left(\frac{1}{10}\right)^9$	$C_{10}^{10} \left(\frac{1}{10}\right)^{10}$

$$(2) \mu = np = 10 \times \frac{1}{10} = 1$$

$$(3) \sqrt{\sigma^2} = \sqrt{npq} = \sqrt{10 \times \frac{1}{10} \times \frac{9}{10}} = 0.9487$$

$$(4) f_Y(y) = \frac{C_1^{10} C_{10}^{100}}{C_{10}^{100}}, \quad y=0, 1, 2, \dots, 10$$

y	0	1	2	3	4	5	6	7	8	9	10
$f_Y(y)$	$\frac{C_0^{10} C_0^{100}}{C_{10}^{100}}$	$\frac{C_1^{10} C_1^{100}}{C_{10}^{100}}$	$\frac{C_2^{10} C_2^{100}}{C_{10}^{100}}$	$\frac{C_3^{10} C_3^{100}}{C_{10}^{100}}$	$\frac{C_4^{10} C_4^{100}}{C_{10}^{100}}$	$\frac{C_5^{10} C_5^{100}}{C_{10}^{100}}$	$\frac{C_6^{10} C_6^{100}}{C_{10}^{100}}$	$\frac{C_7^{10} C_7^{100}}{C_{10}^{100}}$	$\frac{C_8^{10} C_8^{100}}{C_{10}^{100}}$	$\frac{C_9^{10} C_9^{100}}{C_{10}^{100}}$	$\frac{C_{10}^{10} C_{10}^{100}}{C_{10}^{100}}$

$$(5) E(Y) + \text{std}[Y] = 1.9487$$

$$(6) f_Z(z) = b^* \left(z; 5, \frac{1}{10}\right) = C_5^{z-1} \left(\frac{1}{10}\right)^5 \left(\frac{9}{10}\right)^{z-5}$$

$$[2] \quad \lambda = 1, \quad n = 100 \quad \mu = \lambda t = 1 \cdot 100 = 100$$

$$(1) f_W(w) = p(W \leq 100) = \sum_{x=0}^{100} p(x; 100) \approx 0.5267$$

$$(2) E[W] + \text{std}(W) = \lambda t + \lambda t = 200$$

$$(4) p(W > 120) = 1 - p(W \leq 120) = 1 - \sum_{w=0}^{120} p(w; 100) \approx 1 - 0.9773 \approx 0.0227$$

(5) 發生機率很小

在 $W > 120$ 的情況下，"A 發生之平均頻率為每天一件" 不會發生

但 B 確實發生 \rightarrow 由反證法可知，" $W > 120$ " 不會發生

$$(3) p(|W - E[W]| \leq 2 \cdot \text{std}(W)) = p(|W - 100| \leq 200)$$

$$= 1 - \sum_{x=0}^{200} p(x; 100)$$

$$= 1 - \sum_{x=100}^{200} p(x; 100)$$

$$= 1 - \left(\sum_{x=0}^{200} p(x; 100) - \sum_{x=0}^{100} p(x; 100) \right) \approx 1 - 0.4733 \approx 0.5267$$

$$[3]^{(1)} p("x \geq 10" | "p = 0.05") = 0.0282 \quad \#$$

(2) 利用反證法

$\because 0.0282$ 發生機率很小

\therefore 在 $p=0.05$ 的情況下, " $x \geq 10$ 且 $n=100$ " 應不會發生

但 " $x \geq 10$ 且 $n=100$ " 確實發生、

可知 "若假設為真" \rightarrow "不該發生證據"

\therefore "證據發生時" \rightarrow "假設不為真"

可推論 $p=0.05$ 的決策不可行 $\#$

$$[4] \quad b(x; n, p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \frac{n(n-1)(n-2)\dots(n-x+1)}{x!} \left(\frac{\mu}{n}\right)^x \left(1-\frac{\mu}{n}\right)^{n-x}$$

$$= \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\dots\left(1-\frac{x-1}{n}\right) \frac{1}{x!} \mu^x \left(1-\frac{\mu}{n}\right)^{-\frac{\mu}{n}} \left(1-\frac{\mu}{n}\right)^{-x}$$

當 μ 保持不變 (μ 不變), let $n \rightarrow \infty$ ($\mu/n \rightarrow 0$), 則:

$$\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\dots\left(1-\frac{x-1}{n}\right) \rightarrow 1$$

$$\left(1-\frac{\mu}{n}\right)^{-\frac{\mu}{n}} \rightarrow e$$

$$\left(1-\frac{\mu}{n}\right)^{-x} \rightarrow 1$$

$$\Rightarrow b(x; n, p) \rightarrow \frac{\mu^x e^{-\mu}}{x!}$$

$$\Rightarrow b(x; n, p) \rightarrow p(x; \mu) \text{ 得證 } \#$$

$$\sum_{x=0}^{\infty} p(x; \mu) = e^{-\mu} \sum_{x=0}^{\infty} \frac{\mu^x}{x!} = e^{-\mu} e^{\mu} = 1$$