[1]

(1) $f_X(x) = \binom{10}{x}\left(\frac{1}{10}\right)^x\left(\frac{9}{10}\right)^{10-x}$, $x = 0, 1, 2, \ldots 10$

x	0	1	2	3	4	5	6	7	8	9	10
$f_X(x)$	$\left(\frac{9}{10}\right)^{10}$	$C^{10}_1\left(\frac{9}{10}\right)^9\left(\frac{1}{10}\right)$	$C^{10}_2\left(\frac{8}{10}\right)^8\left(\frac{1}{10}\right)^2$	$C^{10}_3\left(\frac{9}{10}\right)^7\left(\frac{1}{10}\right)^3$	$C^{10}_4\left(\frac{9}{10}\right)^6\left(\frac{1}{10}\right)^4$	$C^{10}_5\left(\frac{9}{10}\right)^5\left(\frac{1}{10}\right)^5$	$C^{10}_6\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)^6$	$C^{10}_7\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right)^7$	$C^{10}_8\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right)^8$	$C^{10}_9\left(\frac{9}{10}\right)^1\left(\frac{1}{10}\right)^9$	$C^{10}_{10}\left(\frac{1}{10}\right)^{10}$

#

(2) $\mu = np = 10 \times \frac{1}{10} = \underline{1}$ #

(3) $\sqrt{\sigma^2} = \sqrt{npq} = \sqrt{10 \cdot \frac{1}{10} \cdot \frac{9}{10}} = \underline{0.9487}$ #

(4) $f_Y(y) = \frac{C^{10}_y C^{40}_{10-y}}{C^{50}_{10}}$, $y = 0, 1, 2, \ldots 10$

y	0	1	2	3	4	5	6	7	8	9	10

[2] $\lambda = 1$ 、 $n = 100$ $\mu = \lambda t = 1 \cdot 100 = 100$

(1) $f_W(w) = P(W \le 100) = \sum_{x=0}^{100} p(x; 100) \approx \underline{0.5267}$ #

(2) $E[W] + std(W) = \lambda t + \lambda t = \underline{200}$ #

(4) $P(W > 120) = 1 - P(W \le 120) = 1 - \sum_{w=0}^{120} p(w; 100) \approx 1 - 0.9773\ldots \approx \underline{0.0227}$ #

(5) ∵發生机率很小

∴在"W>120"的情況下，"火災發生之平均頻率為每天一件"不會發生

A ~B

但B確實發生 ∴由反證法可知，"W>120"不會發生 #

B ~A

(3) $P(\text{"}|W - E[W]| \le 2 \cdot std(W)\text{"}) = P(\text{"}|W - 100| \le 200\text{"})$

$= 1 - \sum_{x=0}^{200} p(|W-100|, 100)$

$1 - 0.5267$

$= 1 - \sum_{x=100}^{200} p(w, 100)$

$= 1 - \left(\sum_{x=0}^{200} p(w, 100) - \sum_{x=0}^{100} (w, 100)\right) \approx 1 - 0.4733 \approx \underline{0.5267}$ #

[3] (1) $p(\text{"}x>=10\text{"} \mid \text{"}p=0.05\text{"}) = \underline{0.0282}$ #

(2) 利用反證法

∵ 0.0282 發生機率很小

∴ 在 $p=0.05$ 的情況下，"$x \geq 10$ 且 $n=100$" 應不會發生

但 "$x \geq 10$ 且 $n=100$" 確實發生、

可知 "若假設為真" → "不該發生證據"

$\qquad\qquad p \qquad\qquad\qquad\qquad \sim q$

∴ "證據發生時" → "假設不為真"

$\qquad\qquad \sim q \qquad\qquad\qquad\qquad \sim p$

可推論 $p=0.05$ 的決策不可行 #

[4] $b(x;n,p) = \dfrac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

$= \dfrac{n(n-1)(n-2)\cdots(n-x+1)}{x!} \left(\dfrac{\mu}{n}\right)^x \left(1-\dfrac{\mu}{n}\right)^{n-x}$

$= (1-\frac{1}{n})(1-\frac{2}{n})\cdots(1-\frac{x-1}{n}) \frac{1}{x!}\mu^x \left(\left(1-\frac{\mu}{n}\right)^{-\frac{n}{\mu}}\right)^{-\mu} \left(1-\frac{\mu}{n}\right)^{-x}$

當 t 保持不變 (μ 不變)、let $n \to \infty$ ($\mu t \to 0$)，則.

$(1-\frac{1}{n})(1-\frac{2}{n})\cdots(1-\frac{x-1}{n}) \to 1$

$(1-\frac{\mu}{n})^{-\frac{n}{\mu}} \to e$

$(1-\frac{\mu}{n})^{-x} \to 1$

$\sum_{x=0}^{\infty} p(x;\mu) = e^{-\mu}\sum_{x=0}^{\infty}\frac{\mu^x}{x!} = e^{-\mu}e^{\mu} = 1$

$\Rightarrow b(x;n,p) \to \dfrac{\mu^x e^{-\mu}}{x!}$

$\Rightarrow b(x;n,p) \to p(x,\mu)$ 得證 #