Tailored Stories

Chiara Aina
University of Zurich

April 19th, 2022

Motivation

People often adopt very different stories to explain the same observation

- Voters do not agree on the outcome of an election

  "The election system is fair"

  "Elections are rigged"

- Consumers differ in evaluating a company

  "This company has great corporate social responsibility"

  "The company is just doing green washing"

- Feedback often misperceived in different ways

  "Grades reflect ability"

  "Grades are usually random, they don't convey much about ability"

Motivation

People often adopt very different stories to explain the same observation

- Voters do not agree on the outcome of an election

  "The election system is fair"

  "Elections are rigged"

- Consumers differ in evaluating a company

  "This company has great corporate social responsibility"

  "The company is just doing green washing"

- Feedback often misperceived in different ways

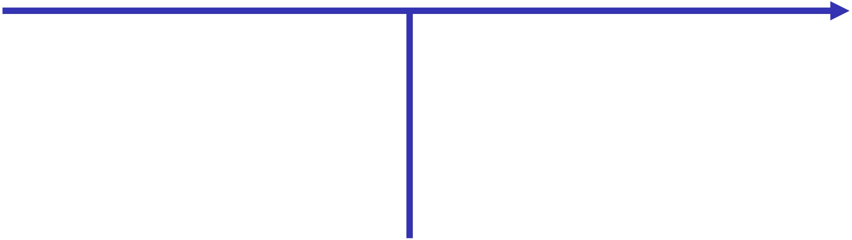
  "Grades reflect ability"

  "Grades are usually random, they don't convey much about ability"

Is it possible to persuade others only by providing interpretations of future events?

Example

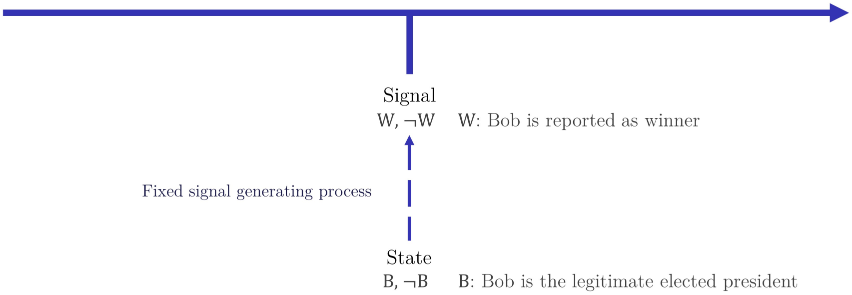
Timeline



State

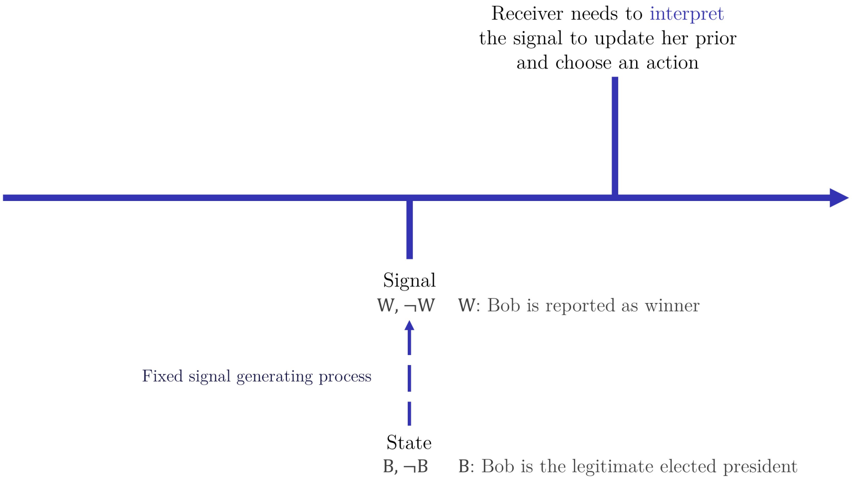
B, ¬B   B: Bob is the legitimate elected president

Timeline

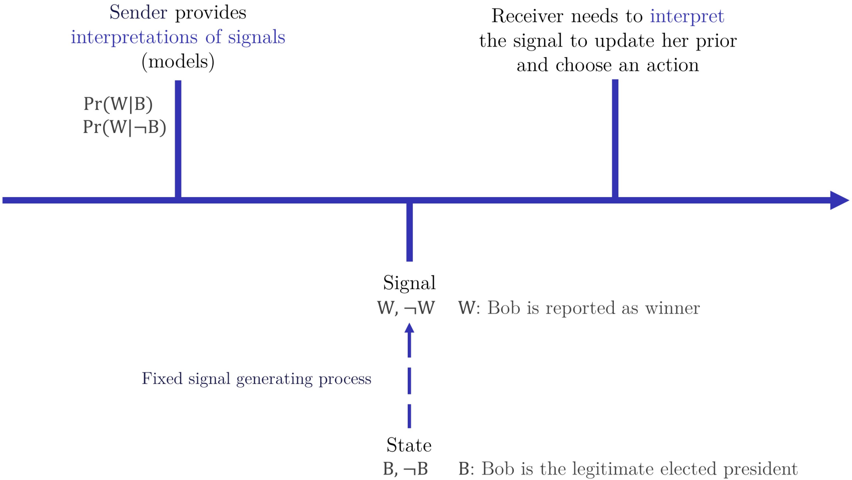


Signal
W, ¬W
W: Bob is reported as winner
Fixed signal generating process
State
B, ¬B
B: Bob is the legitimate elected president

Timeline

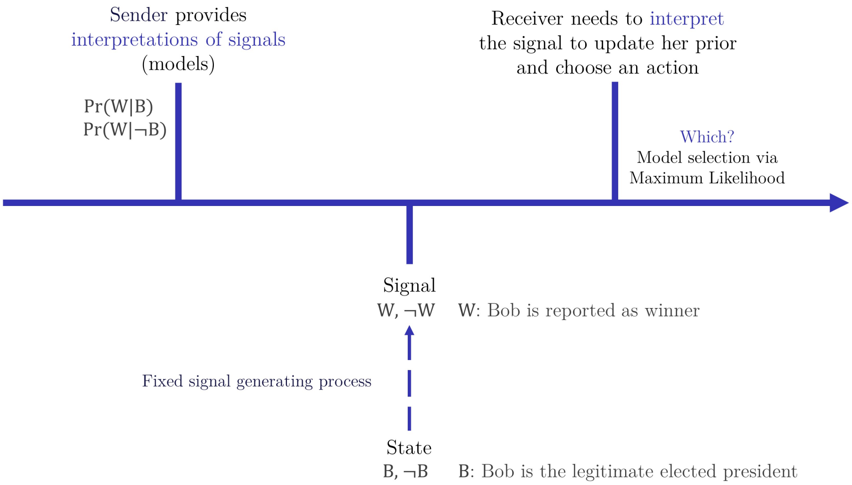

Receiver needs to interpret the signal to update her prior and choose an action
Signal
W, ¬W
W: Bob is reported as winner
Fixed signal generating process
State
B, ¬B
B: Bob is the legitimate elected president

Timeline



Sender provides
interpretations of signals
(models)
Pr(W|B)
Pr(W|¬B)
Receiver needs to interpret
the signal to update her prior
and choose an action
Signal
W, ¬W
W: Bob is reported as winner
Fixed signal generating process
State
B, ¬B
B: Bob is the legitimate elected president

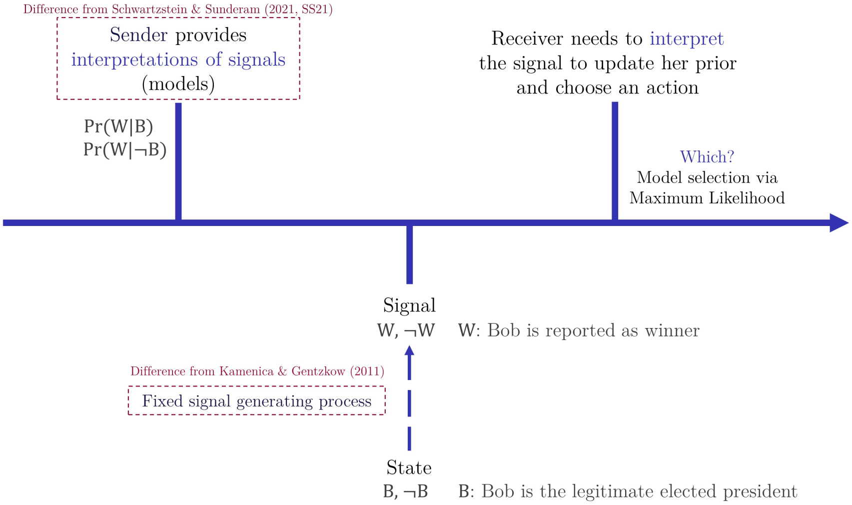
Timeline



Sender provides
interpretations of signals
(models)
Pr(W|B)
Pr(W|¬B)
Receiver needs to interpret
the signal to update her prior
and choose an action
Which?
Model selection via
Maximum Likelihood
Signal
W, ¬W
W: Bob is reported as winner
Fixed signal generating process
State
B, ¬B
B: Bob is the legitimate elected president

Timeline

Literature Review



Ex-ante Persuasion

The sender does not know the signal when he communicates the narratives

1. Temporal: the signal realizes after the sender communicates
   - A voter will observe the outcome of the election and a politician wants to be recognized as legitimate president regardless
2. Private information: the signal is only private information of the receiver
   - An investor had either good or bad experience on the financial market and a financial advisor wants to convince her to invest in an asset regardless

Model

$$\text{Model} \begin{cases} \Pr(W|B) \\ \Pr(W|\neg B) \end{cases}$$

B: Bob is the legitimate elected president
W: Bob is reported as winner

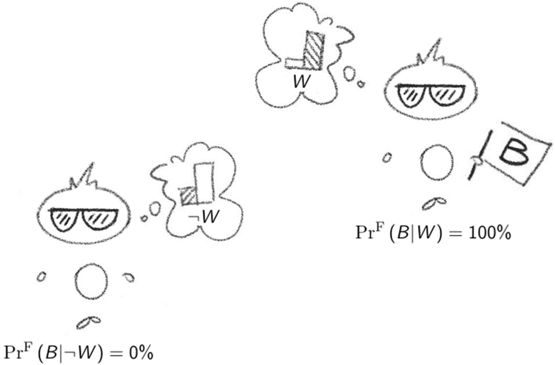
One Story

$$\Pr^{\mathrm{F}}(W|B) = 99\%$$

$$\Pr^{\mathrm{F}}(W|\neg B) = 1\%$$

B: Bob is the legitimate elected president
W: Bob is reported as winner

One Story



B: Bob is the legitimate elected president
W: Bob is reported as winner

Tailored Stories

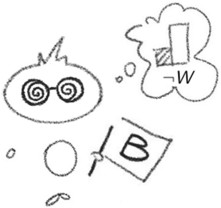
$$\Pr^{C}(W|B) = 1\%$$
$$\Pr^{C}(W|\neg B) = 50\%$$

B: Bob is the legitimate elected president
W: Bob is reported as winner

Tailored Stories

$$\Pr(B) = 70\%$$

$$\Pr^{F}(W) = 70\% > 16\% = \Pr^{C}(W)$$
$$\Rightarrow \Pr^{F}(B|W) = 100\%$$



$$\Pr^{F}(\neg W) = 30\% < 84\% = \Pr^{C}(\neg W)$$
$$\Rightarrow \Pr^{C}(B|\neg W) = 82\%$$

B: Bob is the legitimate elected president
W: Bob is reported as winner

Preview of Results

I study the problem of manipulating a boundedly rational agent by controlling her interpretation of signals she is about to receive

- I characterize the extent and limits of belief manipulability
  - The receiver can hold inconsistent beliefs across signal realizations
    Posterior beliefs across signal realizations do not average to the prior
  - Persuasion is generally limited and it depends on the initial beliefs
- When agents differing in initial beliefs are exposed to conflicting models, polarization occurs

Agenda

1. What is a story? What are its properties?
2. What can the receiver be persuaded of?
3. Which phenomena can be explained?

What is a story? What are its properties?

Set Up

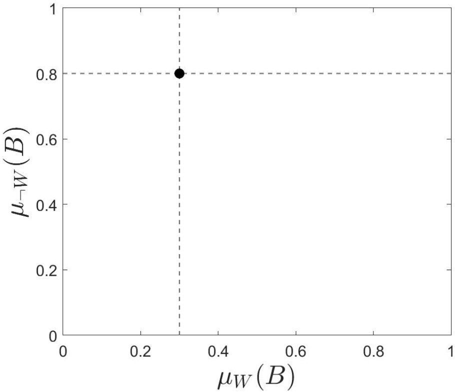
- States: $\omega \in \Omega$ Binary state: $\{B, \neg B\}$
- Signals: $s \in S$ Binary signal: $\{W, \neg W\}$
- Common prior over states: $\mu_0 \in int(\Delta(\Omega))$ $\Pr(B)$
- **Model** m: $(\pi^m(s|\omega))_{s\in S,\omega\in\Omega} \in [\Delta(S)]^\Omega$ $\Pr(W|B),\ \Pr(W|\neg B)$
  map assigning to each state a distribution of signals conditional on that state
- Adopting model m, an agent forms beliefs conditional on signal s via Bayes rule

$$\mu_s^m = (\mu_s^m(\omega))_{\omega\in\Omega} \in \Delta(\Omega)$$

Main Object of the Analysis

- Vector of posterior beliefs: $\boldsymbol{\mu}^m = (\mu_s^m)_{s \in S} \in [\Delta(\Omega)]^S$
  array of posterior distributions conditional on each signal realization

- Graphical representation for the binary case
  - Fix a state: B
- Axis: posterior of B conditional on each signal
  - x-axis: posterior probability of B given W
  - y-axis: posterior probability of B given $\neg W$
- Every point is a vector of posterior beliefs

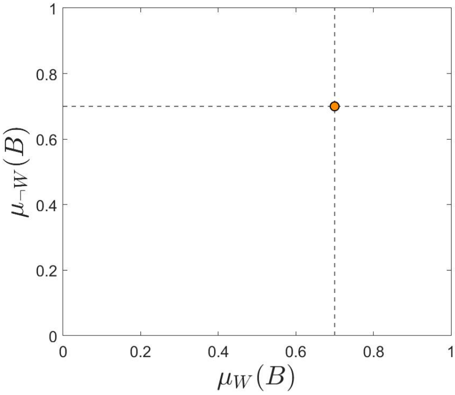


Equivalent Representation

A vector of posterior beliefs $\boldsymbol{\mu}$ is **Bayes-consistent**
if the prior μ_0 is a convex combination of the posterior across signals $(\mu_s)_{s \in S}$

- Equivalent representation between models and Bayes-consistent vectors of posteriors
  Details

- Orange point: prior on $B = 0.7$

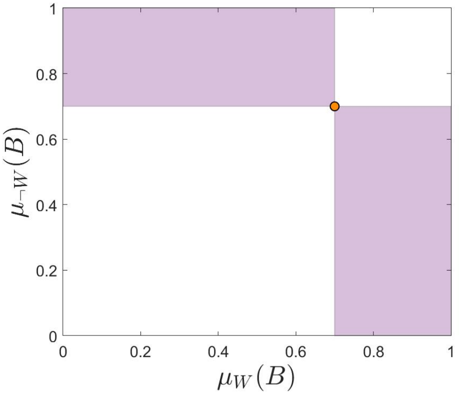


Equivalent Representation

A vector of posterior beliefs $\boldsymbol{\mu}$ is **Bayes-consistent**
if the prior μ_0 is a convex combination of the posterior across signals $(\mu_s)_{s \in S}$

- Equivalent representation between models and Bayes-consistent vectors of posteriors
  Details

- Orange point: prior on $B = 0.7$
- Every point in the purple area corresponds to a model

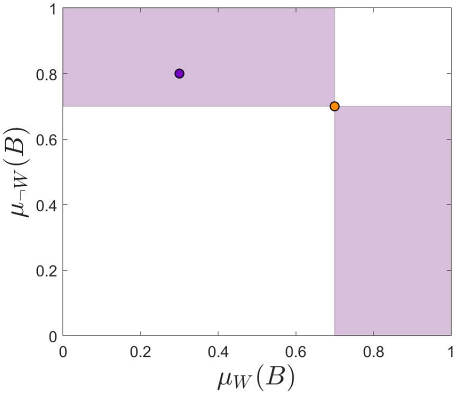


Properties: Fit

Fit of a model m conditional on the signal s: $\Pr^m(s) = \sum_{\omega \in \Omega} \mu_0(\omega)\ \pi^m(s|\omega)$

- It measures how likely a model fits the observed data

- *How to see the fit in the graph?*



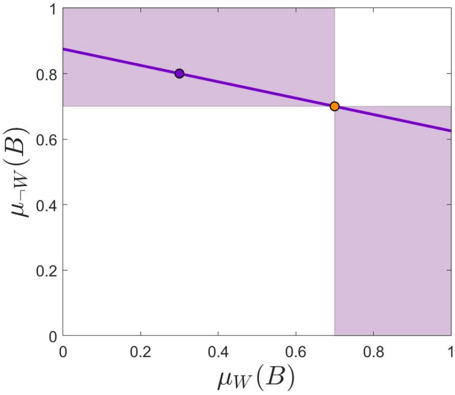
Properties: Fit

Fit of a model *m* conditional on the signal *s*: $\Pr^m(s) = \sum_{\omega \in \Omega} \mu_0(\omega)\ \pi^m(s|\omega)$

- It measures how likely a model fits the observed data

- *How to see the fit in the graph?*
  - Isofit line: all the points correspond to models that have the same fit (except the prior) More

  What if the slope changes?
  - The steeper, the higher fit given W
  - The flatter, the higher fit given $\neg W$

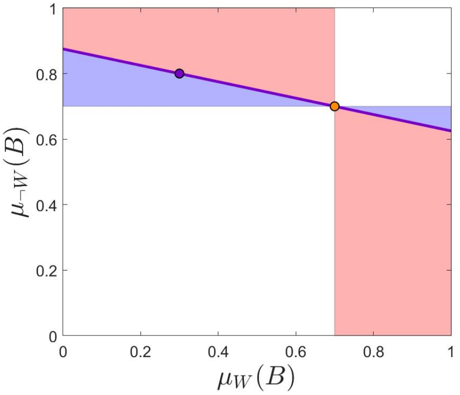


Properties: Fit

Fit of a model *m* conditional on the signal *s*: $\Pr^m(s) = \sum_{\omega \in \Omega} \mu_0(\omega)\ \pi^m(s|\omega)$

- It measures how likely a model fits the observed data

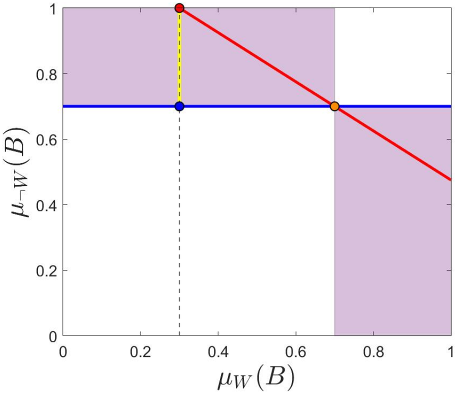
- *How to see the fit in the graph?*
    - Isofit line: all the points correspond to models that have the same fit (except the prior) More
- *What if the slope changes?*
    - The steeper, the higher fit given W
    - The flatter, the higher fit given $\neg W$
- The isofit partions the set of Bayes-consistent vectors of posterior beliefs
    - Line: same fit
    - Red area: higher fit given W
    - Blue area: higher fit given $\neg W$



Properties: Fit

- There is a multiplicity of models that induce the same posterior distribution conditional on a signal with different levels of fit

- Dotted line: target posterior given W
- Yellow line: all models inducing the target
- Red point: model inducing the target with highest fit given W $\rightarrow$ steepest isofit
- Blue point: model inducing the target with highest fit given $\neg W$ $\rightarrow$ flattest isofit



Properties: Movement

Movement for μ_s in state ω: $\delta(\omega;\mu_s) = \dfrac{\mu_s(\omega)}{\mu_0(\omega)}$

- It is a measure of how much the target posterior is far from the prior in a state
- Maximal movement for μ_s: $\bar{\delta}(\mu_s) = \max_{\omega\in\Omega} \delta(\omega;\mu_s)$

Lemma Proof

A model induces μ_s conditional on s if and only if $\Pr^m(s) \in \left[0, \bar{\delta}(\mu_s)^{-1}\right]$

- SS21 characterizes the upper bound: the maximal fit for a target posterior coincides with the reciprocal of the maximal movement
  - Any model that leads beliefs to react a lot given a signal (higher movement) cannot fit the data well (lower fit)

What can the receiver be persuaded of?

Receiver's Problem

- The receiver does not know the state but she has observed a signal
- She needs a model to interpret the signal and update her prior
- The sender communicates a set of models $M \subseteq \mathcal{M}$
  $|M|$ is not greater than the number of models that the receiver is willing to consider

Model Adoption

$$\tilde{m}_s \in \arg\max_{m \in M} \mathrm{Pr}^m(s)$$

- Maximum likelihood selection

Action Choice

$$a^*(\mu_s) \in \arg\max_{a \in A} \mathbb{E}_{\mu_s^{\tilde{m}_s}}[U^R(a, \omega)]$$

Tie breaking rule: if indifferent, adopt the model/action maximizing the sender's expected utility

Model Adoption

The receiver adopts the model that fits best the observed signal among the models provided by the sender and uses only that to update her beliefs

- Inference to the Best Explanation (Harman, 1965): only the best hypothesis to make inference, but agnostic on how to judge the best
    - Hypotheses are supported by the very observations they are supposed to explain (Lipton, 2003; Keil, 2006; Douven et al., 2015);
    - The better an hypothesis explains the data, the more confidence in it (Koehler, 1991; Pennington and Hastie, 1992; Lombrozo and Carey, 2006)

Bayesian Posterior: average of the posteriors given each model, weighted by the probability of each model given the observed signal (based on priors over models)

- Computationally intense given our finite cognitive resources More

Sender's Problem

What does the sender know?

- The receiver's preferences, the (common) prior, and the number of models that the receiver is willing to consider
- The sender does not know the state, but he knows the true model t
  - Predictive probabilities of each signal realization $\mathrm{Pr}^t(s)$
  - Posterior induced by t conditional on each signal realization μ_s^t

Sender's Value of μ, calculated over signal and state realizations using model t

$$V(\mu) = \mathbb{E}^t[U^S(a^*(\mu_s), \omega)] = \sum_{s \in S} \mathrm{Pr}^t(s)\ \mathbb{E}_{\mu^t}\left[U^S(a^*(\mu_s), \omega) | s\right]$$

Sender's Problem

Many Models

The sender chooses the set of models M^* that maximizes his value at $\mu^M = \left(\mu_s^{\tilde{m}_s}\right)_{s \in S}$

$$M^* \in \arg\max_{M \subseteq \mathcal{M}} V(\mu^M) \quad \text{such that} \quad \tilde{m}_s \in \arg\max_{m \in M} \mathrm{Pr}^m(s)$$

One Model

If the receiver considers only one model from the sender, the problem is

$$m^* \in \arg\max_{m \in \mathcal{M}} V(\mu^m)$$

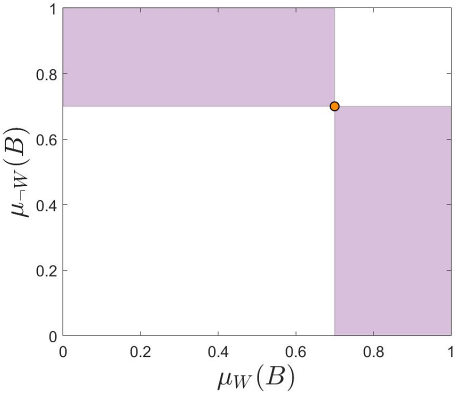
To solve these, it is enough to characterize the set of feasible vectors of posteriors

- Why? From the perspective of the sender, there is a fixed distribution over the signals induced by model t: $\left(\mathrm{Pr}^t(s)\right)_{s \in S}$

Set of Feasible Vectors of Posteriors: One Model

With a model, the sender can only induce Bayes-consistent vectors of posteriors

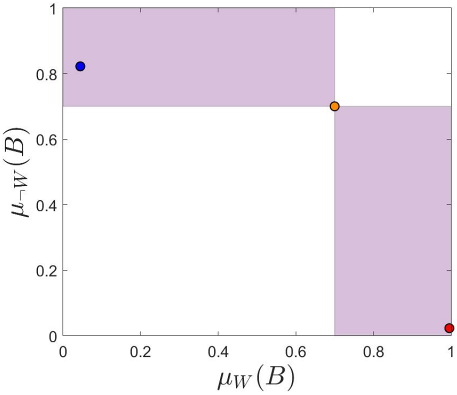
- Comparable characterizing condition to Kamenica & Gentzkow (2011): Bayes-plausibility



Many Models

What is the resulting vector of posteriors if the receiver is exposed to many models?

1. Fair election system (red):
   $\pi^F(W|B) = 99\%$ and $\pi^F(W|\neg B) = 1\%$
2. Conspiracy theory, rigged elections (blue):
   $\pi^C(W|B) = 1\%$ and $\pi^{m_2}(W|\neg B) = 50\%$
   If legitimate, the votes count is reversed;
   otherwise, the votes are counted randomly

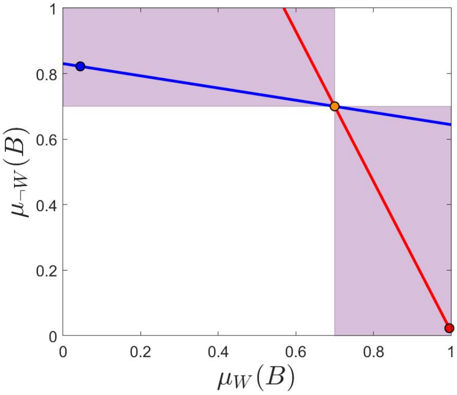


Many Models

What is the resulting vector of posteriors if the receiver is exposed to many models?

- Enough to anticipate the model adopted conditional on each signal

1. Fair election system (red):
   $\pi^F(W|B) = 99\%$ and $\pi^F(W|\neg B) = 1\%$
2. Conspiracy theory, rigged elections (blue):
   $\pi^C(W|B) = 1\%$ and $\pi^{m_2}(W|\neg B) = 50\%$
   If legitimate, the votes count is reversed;
   otherwise, the votes are counted randomly

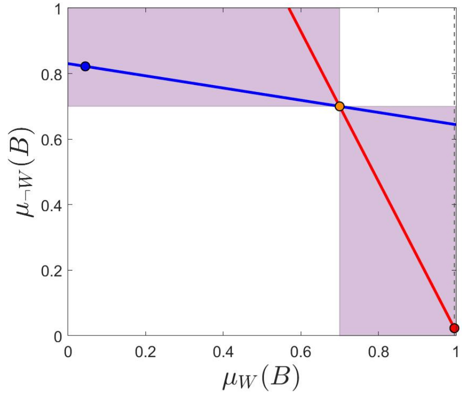


Many Models

What is the resulting vector of posteriors if the receiver is exposed to many models?

- Enough to anticipate the model adopted conditional on each signal

- *Which model is adopted given W?*
  The model on the steepest isofit line



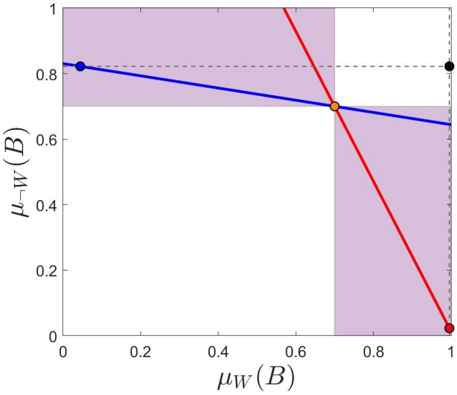
Many Models

What is the resulting vector of posteriors if the receiver is exposed to many models?

- Enough to anticipate the model adopted conditional on each signal

- *Which model is adopted given W?*
  The model on the steepest isofit line
- *Which model is adopted given $\neg W$?*
  The model on the flattest isofit line

Full Bayesian   More models



Set of Feasible Vectors of Posteriors: Many Models

With more models, the sender can also induce Bayes-inconsistent vectors of posteriors

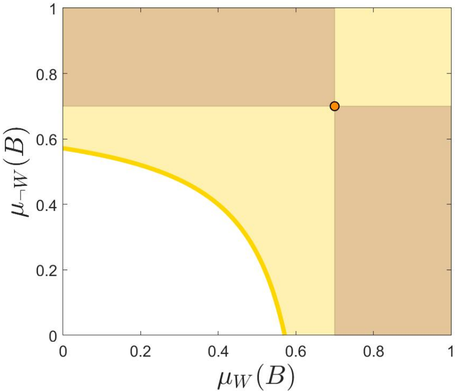
Theorem $|M| \geq |S|$ Proof

The set of feasible vectors of posterior beliefs is

$$\mathcal{F} = \left\{ \boldsymbol{\mu} \in [\Delta(\Omega)]^S : \sum_{s \in S} \bar{\delta}(\mu_s)^{-1} \geq 1 \right\}$$

To be feasible, the sum of the maximal fit levels associated with each signal realization has to exceed one

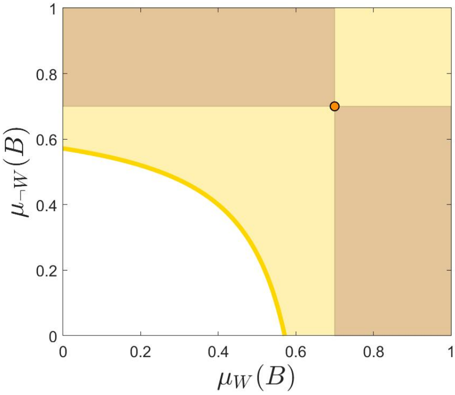
More Graphical Intuition



Set of Feasible Vectors of Posteriors: Many Models

To simplify, there are three main take-aways:

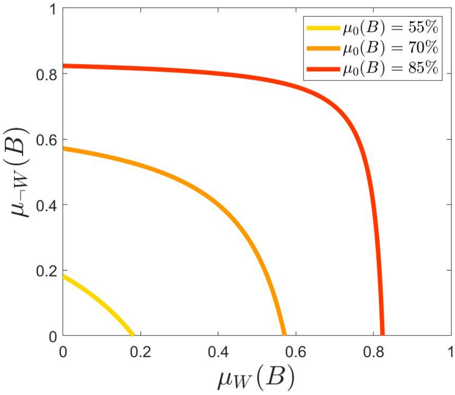
1. Bayes-inconsistent vectors of posteriors could be induced
2. Not all vectors of posteriors are feasible
3. This results is more general, not necessarily about strategic persuasion



Comparative Statics with Respect to the Priors

Generally not all vectors of posteriors are feasible, but there are exceptions

- The more uniform prior or the more signals, the more belief manipulability Details
- Binary case: the closer the prior is to 50-50, the more belief manipulability Details
  - Full manipulability at 50-50



Which phenomena can be explained?

Applications

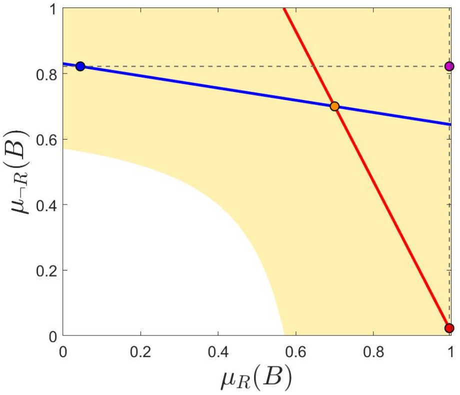
Firehose of Falsehood: model of Russian propaganda based on a large number of possibly contradictory and mutually inconsistent messages (Paul & Matthew, 2016)

- Effective disinformation campaign
- Coordinated operations led by official or unofficial sources

Firehose of Falsehood

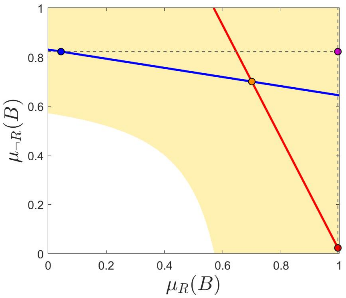
- Voters are exposed to multiple models before elections

1. Fair election system (red):
   $\pi^F(W|B) = 99\%$ and $\pi^F(W|\neg B) = 1\%$
2. Conspiracy theory, rigged elections (blue):
   $\pi^C(W|B) = 1\%$ and $\pi^C(W|\neg B) = 50\%$
   If legitimate, the votes count is reversed;
   otherwise, the votes are counted randomly

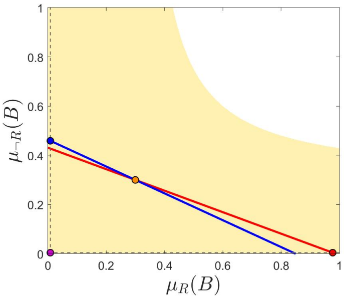


Firehose of Falsehood

Inevitable Polarization



(a) $\mu_0(B) = 70\%$

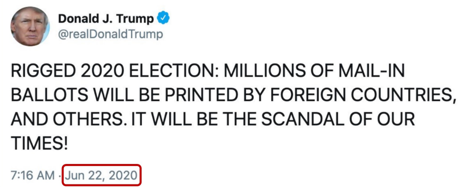


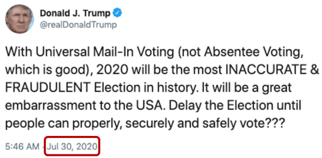
(b) $\mu_0(B) = 30\%$

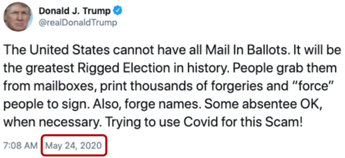
- With conflicting narratives, belief polarization occurs
  - There is a threshold in prior such that voters with prior higher (lower) than the threshold would hold extreme high (low) posteriors regardless of the election outcome

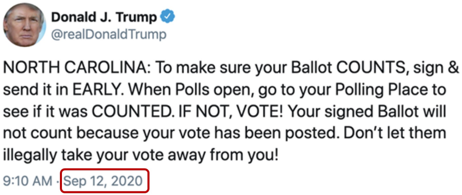
More Details

Firehose of Falsehood: 2020 US Presidential Elections









Firehose of Falsehood: 2020 US Presidential Elections

Predictions: when exposed to conflicting stories...

1. Voters with different priors adopt different narratives once the signal realizes
2. Voters with similar priors adopt different narratives once observed different signals

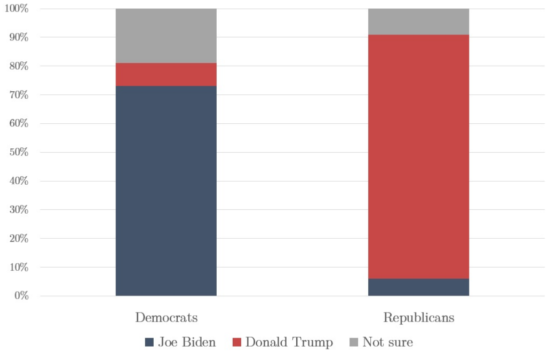
Firehose of Falsehood: 2020 US Presidential Elections

Predictions: when exposed to conflicting stories...

1. Voters with different priors adopt different narratives once the signal realizes
2. Voters with similar priors adopt different narratives once observed different signals

Assumption: each voter expects his partisan candidate to win

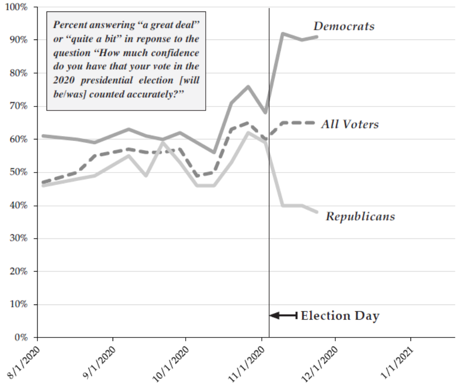
- Republicans expect Trump to win
- Democrats expect Biden to win



Firehose of Falsehood: 2020 US Presidential Elections

Clark and Stewart (2021) More

Voters with different priors adopt different narratives once the signal realizes

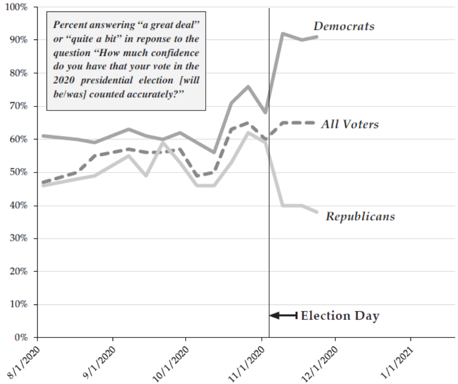


Voters with similar priors adopt different narratives once observed different signals

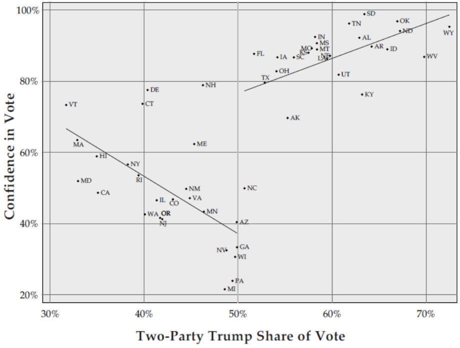
Firehose of Falsehood: 2020 US Presidential Elections

Clark and Stewart (2021) More

Voters with different priors adopt different narratives once the signal realizes



Voters with similar priors adopt different narratives once observed different signals



Other Applications

Financial Advice: a financial advisor wants to persuade investors to make an investment (e.g., cryptocurrency, real estate, start-up, etc.); however, he knows that investors' past financial experience — either good or bad — influences their beliefs on the quality of the new investment but he does not know it

Other Applications

Financial Advice: a financial advisor wants to persuade investors to make an investment (e.g., cryptocurrency, real estate, start-up, etc.); however, he knows that investors' past financial experience — either good or bad — influences their beliefs on the quality of the new investment but he does not know it

1. *"Early success predicts LR success"* Hot-hand bias
2. *"What goes down comes up"* Gambler's fallacy

- Inconsistent ways of looking at signals, but consistent with the idea of LR success
  "Going forward, this has so much promise"
- Optimistic investors fully invest regardless of new information or past experience

More Pessimistic

Other Applications

- Merchants of Doubt: strategy to challenge a well-established way of looking at scientific evidence and to manufacture uncertainty (e.g., Michaels, 2008; Oreskes & Conway, 2011)
  - Given a shared default model (trust in science), it is still possible to insinuate doubt
- Nudging: proposing ad-hoc narraives can be seen as a soft intervention to influence choices of an agent with the purpose of increase her welfare
  - Confidence manipulation by a paternalistic planner could be optimal to influence the agent's behavior in a risky task
- Intrapersonal Phenomena: an individual may keep facts open to interpretation in order to adopt the interpretation that makes her feel better about herself (belief-based utilty for the sender)
  - In a multi-selves model, an agent has incentive to distort her self-confidence in order to offset her time inconsistent preferences (Bénabou & Tirole, 2002)

Merchants of Doubt Nudging Commitment

Conclusion

- It is possible to persuade others by providing interpretations of events not yet known, either future or private information
- My results show that not only it is possible, but it can lead the receiver to hold incoherent beliefs across signal realizations (relaxation of Bayes-plausibility constraint)
  - For maximal belief manipulability, each contingency has to trigger the adoption of a tailored story to manipulate belief conditional on that contingency
- Extension: receiver is endowed with a default model
- This form of persuasion sheds light on a mechanism common to inter-personal (conflict of interest in finance, polarization, lobbying, and nudge) or intra-personal phenomena (commitment)

Thank you!

chiara.aina@econ.uzh.ch

Sender Applications Default Model

Appendix

Literature Review

Back

Literature on Narratives

- Schwartzstein & Sunderam (2021, hereafter SS21): building block of this project
- Eliaz and Spiegler (2020): formalization of narratives as causal models (directed acyclical graphs) to understand public-policy debates
- Benabou, Falk, & Tirole (2018): investigate the role of narratives and imperatives in moral reasoning
- Eliaz, Spiegler, & Thysen (2019): sender-receiver model in which persuaders seek to influence receivers' understanding of messages
- Barron and Powell (2018): theoretical analysis of markets for rhetorical services

Literature on Persuasion

- Kamenica & Gentzkow (2011): Bayesian persuasion model
- Alonso & Camara (2016), Galperti (2019): generalization of Bayesian Persuasion
  - These models are about providing information fixing a signal generating process
- Levy & Razin (2021): Persuasion with Correlation Neglect

Appendix: Equivalent Representation

Back

Lemma

(i) For each vector of posterior beliefs $\boldsymbol{\mu} \in \mathcal{B}$, there exists a model that induces $\boldsymbol{\mu}$

(ii) Each model m induces a vector of posterior beliefs $\boldsymbol{\mu}^m \in \mathcal{B}$

Appendix: Equivalent Representation Back

(i) For each $\boldsymbol{\mu} \in \mathcal{B}$, there exists a model that induce $\boldsymbol{\mu}$

- Consider $\boldsymbol{\mu} \in \mathcal{B}$. Then, there exists a distribution $\sigma \in \Delta(S)$ such that $\sum_s \mu_s(\omega)\ \sigma(s) = \mu_0(\omega)$.
- For each σ, define a model such that, for each s and ω,

$$\pi^\sigma(s|\omega) = \frac{\mu_s(\omega)\ \sigma(s)}{\sum_{s'} \mu_{s'}(\omega)\ \sigma(s')}.$$

- Notice that the fit of such a model is, for each signal, $\Pr^\sigma(s) = \sigma(s)$

$$\Pr^\sigma(s) = \sum_\omega \mu_0(\omega)\ \pi^\sigma(s|\omega) = \sum_\omega \left(\sum_{s'} \mu_{s'}(\omega)\ \sigma(s')\right)\left(\frac{\mu_s(\omega)\ \sigma(s)}{\sum_{s'} \mu_{s'}(\omega)\ \sigma(s')}\right) = \sigma(s)\sum_\omega \mu_s(\omega) = \sigma(s)$$

- The posterior attached to state ω conditional on signal s induced by the model σ is

$$\mu_s^\sigma(\omega) = \frac{\mu_0(\omega)\pi^\sigma(s|\omega)}{\Pr^m(s)} = \frac{\mu_0(\omega)}{\sigma(s)}\left(\frac{\mu_s(\omega)\ \sigma(s)}{\sum_{s'} \mu_{s'}(\omega)\ \sigma(s')}\right) = \frac{\sum_{s'} \mu_{s'}(\omega)\ \sigma(s')}{\sigma(s)}\frac{\mu_s(\omega)\ \sigma(s)}{\sum_{s'} \mu_{s'}(\omega)\ \sigma(s')} = \mu_s(\omega)$$

Appendix: Equivalent Representation

Back

(ii) Each model m induces a vector of posterior that is Bayes-consistent: $\boldsymbol{\mu}^m \in \mathcal{B}$

- It is enough to show that there exists a signal distribution such that the Bayes-consistency constraint holds
- Consider as the distribution of signals the fits of the model m conditional on each signal: given that $m \in [\Delta(S)]^\Omega$, it holds that it is a proper distribution with $\sum_s \mathrm{Pr}^m(s) = 1$
- Then, for every $\omega \in \Omega$,

$$\begin{aligned}\sum_{s \in S} \mathrm{Pr}^m(s)\ \mu_s^m(\omega) &= \sum_{s \in S} \mathrm{Pr}^m(s)\ \frac{\mu_0(\omega)\pi^m(s|\omega)}{\mathrm{Pr}^m(s)} \\ &= \sum_{s \in S} \mu_0(\omega)\pi^m(s|\omega) \\ &= \mu_0(\omega) \sum_{s \in S} \pi^m(s|\omega) = \mu_0(\omega)\end{aligned}$$

- Every vector of posterior beliefs induced by a model satisfies Bayes-consistency

Equivalent Representation: Binary Case

Corollary

In the binary signal and binary state, for each vector of posterior beliefs $\boldsymbol{\mu} \in \mathcal{B} \backslash \{\boldsymbol{\mu}^{\varnothing}\}$ with $\boldsymbol{\mu}^{\varnothing} = (\mu_0, \mu_0)$, there exists a unique model m that induces $\boldsymbol{\mu}$

Proof

- To show the uniqueness of a model associated to a Bayes-consistent vector of posterior in the binary signal and binary state case, it is enough to show that there exists only a distribution over the signal space such that a vectors of posterior is Bayes-consistent

- Let $(\sigma_{s_1}, \sigma_{s_2}) = (\sigma, 1 - \sigma)$. For each state ω, the Bayes-consistency condition implies that

$$\mu_0(\omega) = \sigma \mu_{s_1}(\omega) + (1 - \sigma)\mu_{s_2}(\omega).$$

  Then, it holds that $\sigma = \frac{\mu_0(\omega) - \mu_{s_2}(\omega)}{\mu_{s_1}(\omega) - \mu_{s_2}(\omega)}$.

- Hence, $(\sigma_{s_1}, \sigma_{s_2})$ is a signal distribution over signals if either (i) $\mu_{s_1}(\omega) > \mu_0(\omega) > \mu_{s_2}(\omega)$, or (ii) $\mu_{s_1}(\omega) < \mu_0(\omega) < \mu_{s_2}(\omega)$. These two conditions are equivalent to $\mu \in F \backslash \{\mu^{\varnothing}\}$ in the binary case.

Back

Appendix: Isofit Back

Isofit: set of vectors of posterior beliefs that are induced by models that have the same fit conditional on every signal realization

For each $\varphi \in \Delta(S)$,

$$I(\varphi) = \left\{ \boldsymbol{\mu} \in [\Delta(\Omega)]^S : \exists m \in \mathcal{M} \text{ such that } \boldsymbol{\mu}^m = \boldsymbol{\mu} \ \text{ and } \ \forall s \in S,\ \mathrm{Pr}^m(s) = \varphi_s \right\}$$
$$= \left\{ \boldsymbol{\mu} \in [\Delta(\Omega)]^S :\ \forall \omega \in \Omega,\ \mu_0(\omega) = \sum_{s \in S} \varphi_s \mu_s(\omega) \right\}$$

Binary case

- Consider the Bayes-consistency constraint for ω_1 for $\boldsymbol{\mu}^m$:
$$\mu_0(\omega_1) = \mathrm{Pr}^m(s_1)\ \mu^m_{s_1}(\omega_1) + \mathrm{Pr}^m(s_2)\ \mu^m_{s_2}(\omega_1)$$
- Re-arrange:
$$\mu^m_{s_2}(\omega_1) = \frac{\mu_0(\omega_1)}{\mathrm{Pr}^m(s_2)} - \frac{\mathrm{Pr}^m(s_1)}{\mathrm{Pr}^m(s_2)} \mu^m_{s_1}(\omega_1)$$
- All the models with the same fit $(\mathrm{Pr}^m(s_1), \mathrm{Pr}^m(s_2))$ corresponds to points on this line
- Slope $-\frac{\mathrm{Pr}^m(s_1)}{1-\mathrm{Pr}^m(s_1)}$: the higher $\mathrm{Pr}^m(s_1)$, the steeper the line

Proof Lemma Back

(i) A model induces μ_s conditional on s if $\mathrm{Pr}^m(s) \in \left[0, \bar{\delta}(\mu_s)^{-1}\right]$

- Show that for every $p \in \left[0, \bar{\delta}_s(\mu_s)^{-1}\right]$, there exists a model inducing μ_s with $\mathrm{Pr}^m(s) = p$
- Construct $\boldsymbol{\mu}$ such that (i) μ_s is induced conditional on s, and (ii) for each state ω, there exists $\sigma(s') \in \Delta(S)$ with the additional property $\sigma(s) = p$ such that Bayes-consistency holds:

$$\sum_{s'} \mu_{s'}(\omega)\ \sigma(s') = \mu_s(\omega)\ \sigma(s) + \sum_{s' \neq s} \mu_{s'}(\omega)\ \sigma(s') = \mu_0(\omega). \tag{a}$$

- By Lemma 1, there exists a model that induce this Bayes-consistent vector of posteriors with fit p
- There exists multiple vectors of posteriors that satisfy condition (a) as long as, for each ω,

$$\mu_0(\omega) - \mu_s(\omega)\ p = \sum_{s' \neq s} \mu_{s'}(\omega)\ \sigma(s') \geq 0. \tag{b}$$

Proof Lemma Back

(i) A model induces μ_s conditional on s if $\Pr^m(s) \in \left[0, \bar{\delta}(\mu_s)^{-1}\right]$

- For instance, fix a signal $s'' \neq s$ and, for each ω, let $\mu_{s''}(\omega) = \frac{\mu_0(\omega) - p\ \mu_s(\omega)}{1-p}$
  - Condition (a) is satisfied for the distribution $\sigma(s')$ such that $\sigma(s) = p$, $\sigma(s'') = 1 - p$, and $\sigma(s') = 0$ for all the other signals
  - Condition (b) is implied by $p \in \left[0, \bar{\delta}_s(\mu_s)^{-1}\right]$: holding for every state, then

$$p \leq \frac{\mu_0(\omega)}{\mu_s(\omega)} \leq \left(\frac{1}{\max_{\omega} \frac{\mu_s(\omega)}{\mu_0(\omega)}} \right)^{-1} = \bar{\delta}_s(\mu_s)^{-1}$$

Proof Lemma Back

(ii) If a model induces μ_s conditional on s then $\Pr^m(s) \in \left[0, \bar{\delta}(\mu_s)^{-1}\right]$

- Consider an arbitrary model inducing μ_s conditional on s
- It follows from Bayes rule that the fit of any m inducing the target $\mu_s^m = \mu_s$ conditional on s must be such that, for every ω

$$\Pr^m(s) = \frac{\mu_0(\omega)}{\mu_s(\omega)} \pi^m(s|\omega)$$

- Notice that if $\pi^m(s|\omega) = 0$ the fit equals 0 (minimal fit). Instead, if $\pi^m(s|\omega) = 1$, it follows that

$$\Pr^m(s) \leq \frac{\mu_0(\omega)}{\mu_s(\omega)}$$

- Because this holds for every state, the maximal fit for μ_s is the minimum of the ratio across states, which equals the reciprocal of the maximal movement for μ_s:

$$\min_{\omega} \frac{\mu_0(\omega)}{\mu_s(\omega)} = \frac{1}{\max_{\omega} \frac{\mu_s(\omega)}{\mu_0(\omega)}} = \bar{\delta}_s(\mu_s)^{-1}$$

- The fit of a model that induces the target posterior can only take values in $\left[0, \bar{\delta}_s(\mu_s)^{-1}\right]$

Appendix: Bayesian Benchmark

Back

Bayesian Posterior: average of the posteriors given each model weighted by the probability of each model given the observed signal (based on priors over models)

- Priors over models: $\rho \in \Delta(M)$ with ρ^m is the prior over model m
- Probability of each model m once signal s is observed:

$$\rho_s^m = \frac{\rho^m \, \mathrm{Pr}^m(s)}{\sum_{m' \in M} \rho^{m'} \, \mathrm{Pr}^{m'}(s)}$$

- Bayesian posterior of state ω given signal s is calculated as

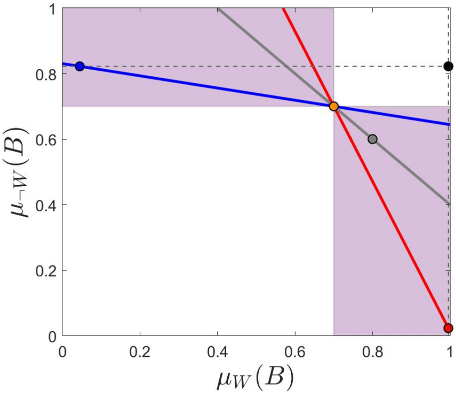
$$\mu_s^B(\omega) = \sum_{m \in M} \rho_s^m \, \mu_s^m(\omega)$$

Many Models Back

What is the resulting vector of posteriors if the receiver is exposed to many models?

- Enough to anticipate the model adopted conditional on each signal

- *Which model is adopted given W?*
  The model on the steepest isofit line
- *Which model is adopted given $\neg W$?*
  The model on the flattest isofit line
- *What if there are more than two models?*

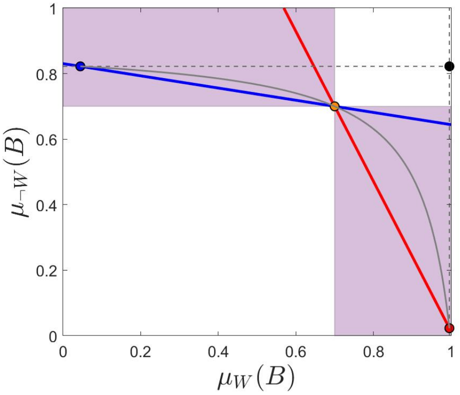


Many Models Back

What is the resulting vector of posteriors if the receiver is exposed to many models?

- Enough to anticipate the model adopted conditional on each signal

- *Which model is adopted given W?*
  The model on the steepest isofit line
- *Which model is adopted given $\neg W$?*
  The model on the flattest isofit line
- *What if fully Bayesian?*



Set of Feasible Vectors of Posteriors: Many Models

Back

With more models, the sender can also induce Bayes-inconsistent vectors of posteriors

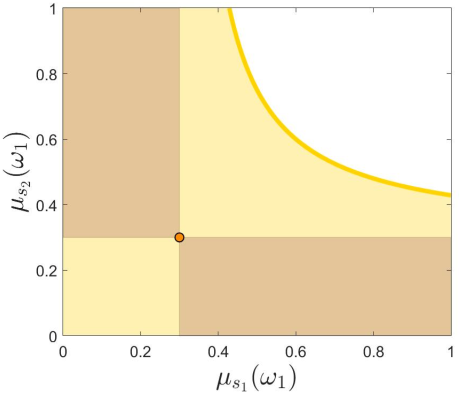
Theorem $|M| \geq |S|$

Proof

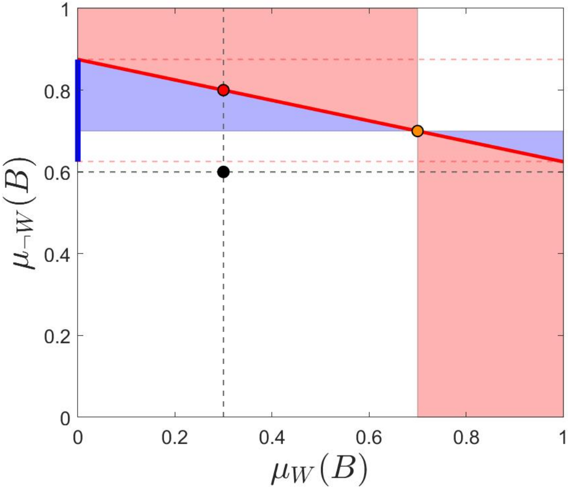
The set of feasible vectors of posterior beliefs is

$$\mathcal{F} = \left\{ \boldsymbol{\mu} \in [\Delta(\Omega)]^S : \sum_{s \in S} \bar{\delta}(\mu_s)^{-1} \geq 1 \right\}$$

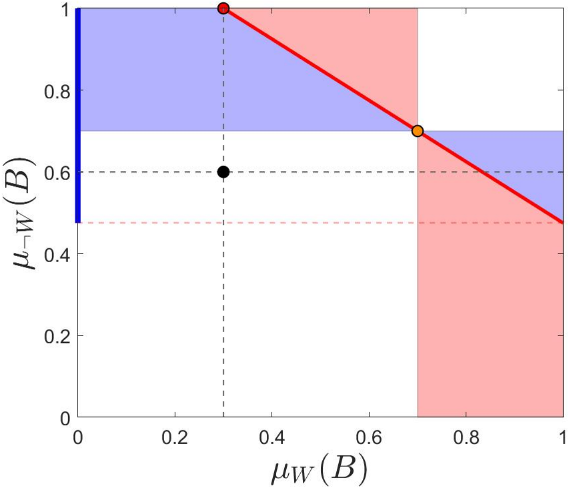
To be feasible, the sum of the maximal fit levels associated with each signal realization has to exceed one



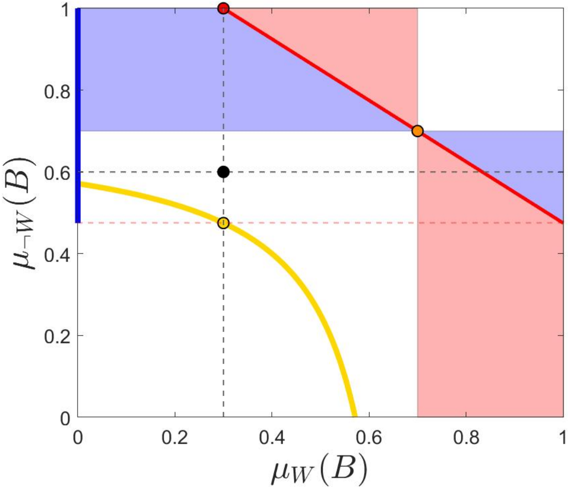
Graphical Intuition



Graphical Intuition

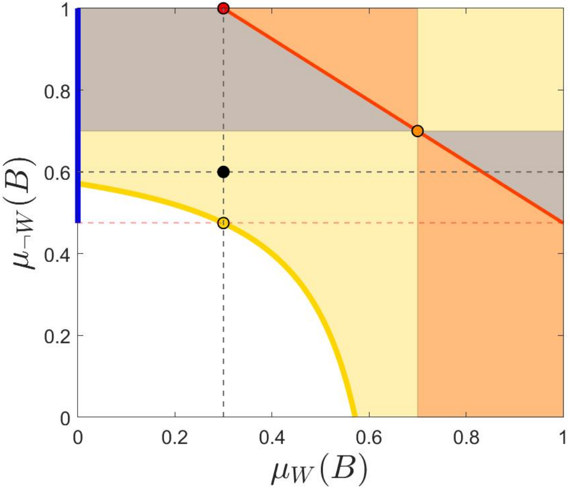


Graphical Intuition



Graphical Intuition

Back



Proof Theorem

Back

- Take an arbitrary vector of posterior beliefs $\boldsymbol{\mu}$
  - To induce $\boldsymbol{\mu}$, construct a set of $K = |S|$ models $(m_k)_{k=1}^K$ such that each model m_k is tailored to induce the target distribution μ_{s_k} conditional on the signal s_k. This implies two conditions on each m_k: (i) $\mu_{s_k}^{m_k} = \mu_{s_k}$, and (ii) $\Pr^{m_k}(s_k) \geq \Pr^{m_j}(s_k)$ for each $k \neq j$

- Assume $\boldsymbol{\mu} \in \mathcal{F}$: I show that there exists a set of models inducing $\boldsymbol{\mu}$
  - For each model m_k, I specify the vector of posteriors $\boldsymbol{\mu}^{m_k}$ and the induced fit levels $(\Pr^{m_k}(s))_{s\in S}$: the corresponding distribution of posteriors corresponds to a unique model
  - For each model m_k, specify the following posteriors and fit levels: if $s = s_k$, set $\mu_s^{m_k} = \mu_{s_k}$ and $\Pr^{m_k}(s) = \bar{\delta}(\mu_{s_k})^{-1}$; otherwise , for each signal and state, set
$$\mu_s^{m_k}(\omega) = \frac{\mu_0(\omega) - \bar{\delta}(\mu_{s_k})^{-1}\mu_{s_k}(\omega)}{1-\bar{\delta}(\mu_{s_k})^{-1}}, \qquad \Pr^{m_k}(s) = \left(\frac{1-\bar{\delta}(\mu_{s_k})^{-1}}{\sum_{s\neq s_k}\bar{\delta}(\mu_s)^{-1}}\right)\bar{\delta}(\mu_{s_k})^{-1}$$
    Equivalent to an information structure with binary signal s_k and s_{-k}
  - Each tailored model is chosen conditional on the signal is tailored to because
$$\Pr^{m_k}(s_k) = \bar{\delta}(\mu_{s_k})^{-1} \geq \underbrace{\left(\frac{1-\bar{\delta}(\mu_{s_k})^{-1}}{\sum_{s\neq s_k}\bar{\delta}(\mu_s)^{-1}}\right)}_{\leq 1 \text{ since } \boldsymbol{\mu}\in\mathcal{F}}\bar{\delta}(\mu_{s_k})^{-1} = \Pr^{m_j}(s_k)$$

Proof Theorem Back

- Assume $\boldsymbol{\mu} \notin \mathcal{F}$: $\sum_{s \in S} \bar{\delta}(\mu_s)^{-1} < 1$, equivalent to $\bar{\delta}(\mu_{s_k})^{-1} < 1 - \sum_{i \neq k} \bar{\delta}(\mu_{s_i})^{-1}, \forall k$
  - If it were to exist a set of models inducing the target $\boldsymbol{\mu}$, each tailored model m_k inducing the posterior μ_{s_k} has to be adopted conditional on s_k
  - Thus, it must hold that $\mathrm{Pr}^{m_k}(s_k) \geq \mathrm{Pr}^{m_j}(s_k)$ for each $j \neq k$
  - Notice that
$$\mathrm{Pr}^{m_j}(s_k) = 1 - \sum_{i \neq k} \mathrm{Pr}^{m_j}(s_i) \geq 1 - \sum_{i \neq k} \bar{\delta}(\mu_{s_i})^{-1},$$
    since for every other signal the fit must be lower than the maximal fit associated to the target posterior conditional on that signal, i.e. $\mathrm{Pr}^{m_j}(s_i) \leq \mathrm{Pr}^{m_i}(s_i) \leq \bar{\delta}(\mu_{s_i})^{-1}$ for every i
  - Contradiction:
$$1 - \sum_{i \neq k} \bar{\delta}(\mu_{s_i})^{-1} > \bar{\delta}(\mu_s)^{-1} \geq \mathrm{Pr}^{m_k}(s_k) \geq \mathrm{Pr}^{m_j}(s_k) \geq 1 - \sum_{i \neq k} \bar{\delta}(\mu_{s_i})^{-1}$$
  - It is not possible to construct a set of models to induce $\boldsymbol{\mu} \notin \mathcal{F}$

Comparative Statics with Respect to the Priors

Back

Proposition

If $\min_{\omega \in \Omega} \mu_0(\omega) \geq \frac{1}{|S|}$, all vectors of posterior beliefs are feasible

- The more signals, the more belief manipulability
- The more uniform the prior, the more belief manipulability
  The minimal prior across states is the lower bound for the maximal fit for any updated posteriors starting from given prior, i.e., $\bar{\delta}(\mu_s)^{-1} \leq \min_{\omega \in \Omega} \mu_0(\omega)$ for any μ_s

Corollary

If $|S| \geq |\Omega|$ and $\mu_0(\omega) = \frac{1}{|\Omega|}$ for every $\omega \in \Omega$, all vectors of posterior beliefs are feasible

Comparative Statics with Respect to the Priors: Binary Case

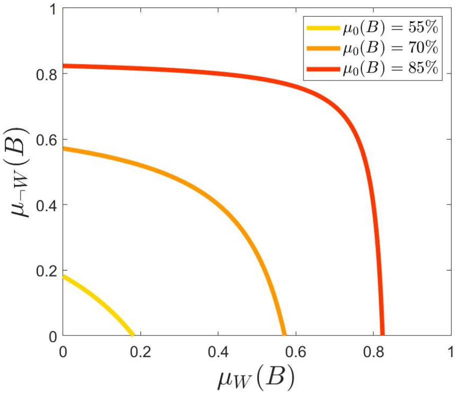
Back

Let $\mu_{0,\varepsilon} = \left(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right)$ and $\mathcal{F}_\varepsilon$ the set of the feasible vectors of posteriors with respect to the prior $\mu_{0,\varepsilon}$

Proposition

For $\varepsilon' < \varepsilon''$, it holds that $\mathcal{F}_{\varepsilon''} \subseteq \mathcal{F}_{\varepsilon'}$

- The closer the prior is to 50-50, the more belief manipulability



Solving the Sender's Problem Back

- All the information the sender needs to learn how to maximize his value through stories is the receiver's prior and the number of models she accepts
- Persuasion is beneficial if there exists a feasible vector of posterior beliefs $\mu \in \mathcal{F}$ such that its value is higher than the value of the prior $V(\mu) \geq V(\mu^{\varnothing})$
  - if the sender does not communicate any model to the receiver, she does not update her beliefs, discarding the realized signal: for each s, $\mu_s = \mu_0$
- Adding a dummy signal the sender can persuade the receiver to hold any beliefs
  - Here the signal space is assumed to be fixed and the sender cannot manipulate it; otherwise, he could leverage on not-realizable signals to manipulate beliefs further
- Value of Information is the sender's willingness to pay to learn the signal realization before choosing models to communicate

$$\Delta = \max_{\boldsymbol{\mu}} V(\boldsymbol{\mu}) - \max_{\boldsymbol{\mu} \in \mathcal{F}} V(\boldsymbol{\mu}) \geq 0$$

  - If all vectors of posteriors are feasible, $\Delta = 0$ (see comparative statics)

Appendix: Firehose of Falsehood

Back

- These stories are able to shift any receiver that has prior higher than 33%
- How? The conspiracy theory is adopted when the reported majority is not for Bob and the just narrative is adopted when the reported majority is in favor of Bob

$$\Pr^F(W) > \Pr^C(W)$$

- To see this, calculate for which prior p this is the case

$$p\ 99\% + (1-p)\ 1\% \geq p\ 1\% + (1-p)\ 50\%$$

$$p \geq 33\%$$

- Depending on how prior is distributed in the population, Bob might be able to leverage on this bundle of truth and conspiracy theory to be elected

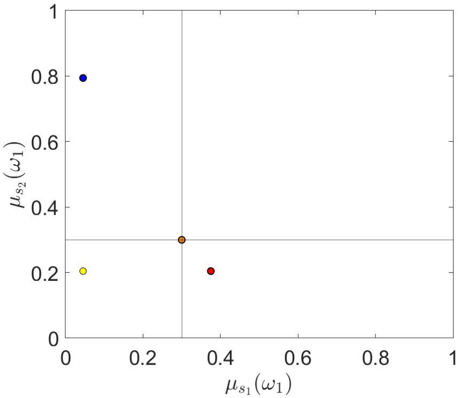
Conflicting Narratives

Back

Binary Case

Conflicting Narratives m, m' if $\pi^m(s_1|\omega_1) > \pi^m(s_1|\omega_2)$ and $\pi^{m'}(s_1|\omega_2) > \pi^{m'}(s_1|\omega_1)$

- m implies that $\mu^m_{s_1}(\omega_1) > \mu_0(\omega_1)$ and $\mu^m_{s_2}(\omega_1) < \mu_0(\omega_1)$
  - South-East quadrant
- m' implies that $\mu^{m'}_{s_1}(\omega_1) < \mu_0(\omega_1)$ and $\mu^{m'}_{s_2}(\omega_1) > \mu_0(\omega_1)$
  - North-West quadrant
- Together induce always a vector of beliefs that is Bayes-inconsistent



Belief Polarization

Back

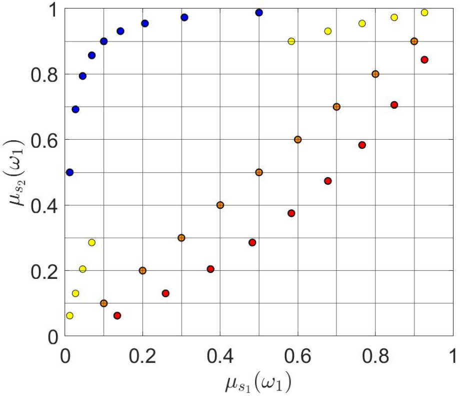
Binary Case

- Whenever two conflicting narratives are communicated, belief polarization occurs
- Depending on the prior, Bayes-consistency is violated differently

Proposition

For each pair of conflicting stories, there exists a threshold in prior p such that for every signal s, it holds that

1. $\mu_s(\omega_1) < \mu_0(\omega_1)$ if $\mu_0(\omega_1) < p$
2. $\mu_s(\omega_1) > \mu_0(\omega_1)$ if $\mu_0(\omega_1) > p$

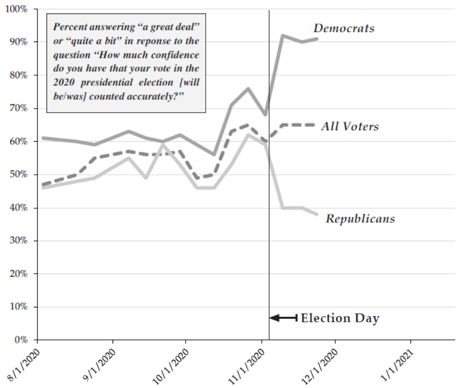


Firehose of Falsehood: 2020 US Presidential Elections

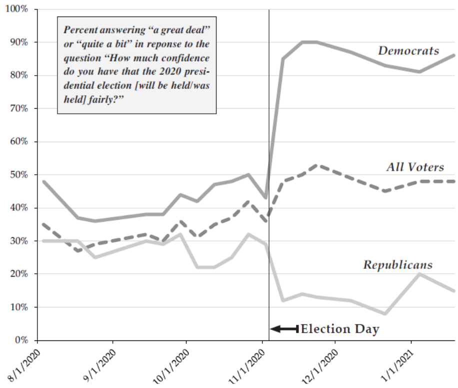
Clark and Stewart (2021)

Back

Accuracy of Vote Count



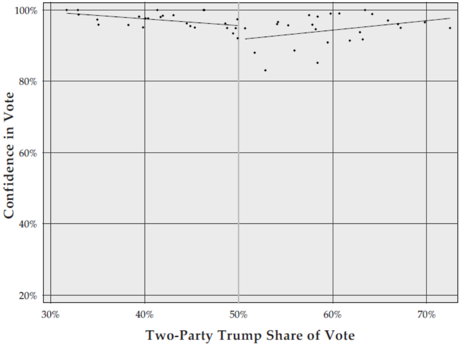
Confidence in Fair Elections



Firehose of Falsehood: 2020 US Presidential Elections

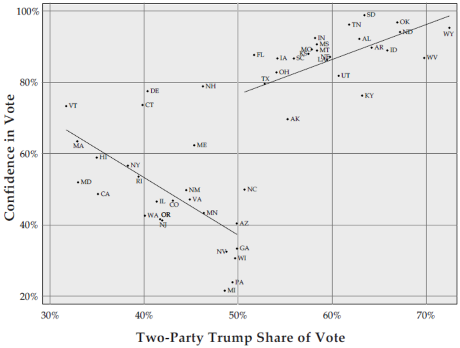
Clark and Stewart (2021) Back

Not exposed to multiple models



Confidence in Vote Count by State, Democrats

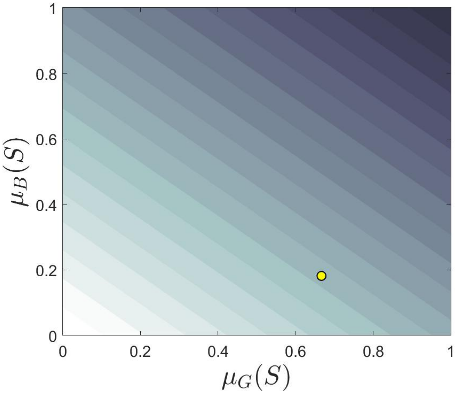
Exposed to multiple models



Confidence in Vote Count by State, Republicans

Financial Advice Back

- States: $\{S, \neg S\}$, where S is the event in which the investment succeeds
- Signals: $\{G, B\}$, where if G the investor had a good experience
- Investor chooses investment allocation and advisor gets commission More



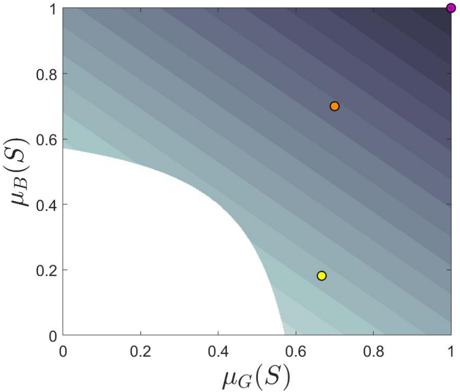
- Yellow point: sender's vector of posteriors by t
- The darker, the higher the sender's value

Financial Advice: Optimistic Investors

Back

- States: $\{S, \neg S\}$, where S is the event in which the investment succeeds
- Signals: $\{G, B\}$, where if G the investor had a good experience
- Investor chooses investment allocation and advisor gets commission More

- Yellow point: sender's vector of posteriors by t
- The darker, the higher the sender's value



Financial Advice: Optimistic Investors Back

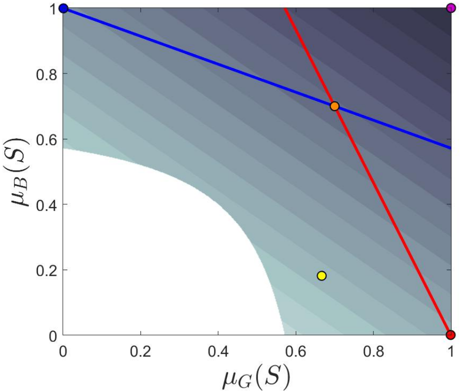
- States: $\{S, \neg S\}$, where S is the event in which the investment succeeds
- Signals: $\{G, B\}$, where if G the investor had a good experience
- Investor chooses investment allocation and advisor gets commission More

1. *"Early success predicts LR success"* (red)
   - Hot-hand bias
2. *"What goes down comes up"* (blue)
   - Gambler's fallacy

- Inconsistent ways of looking at signals, but consistent with the idea of LR success

  "Going forward, this has so much promise"

- Optimistic investors fully invest regardless of new information or past experience More



Financial Advice: Details

Back

Investor

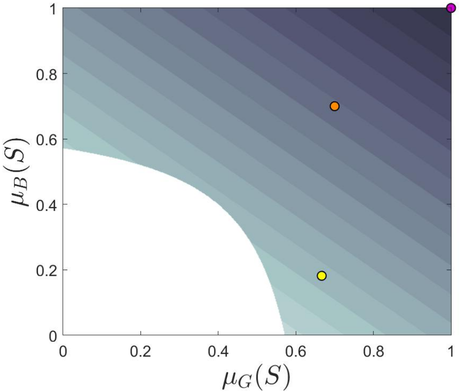
- Allocate her 1 unit over two outcomes $\Omega = \{S, \neg S\}$ resulting in $\alpha = (\alpha_S, \alpha_{\neg S})$
  S is the event in which the investment succeeds
- Expected utility: $\mathbb{E}[U^R(\alpha)] = \sum_{\omega \in \Omega} \mu(\omega) \log(\alpha_\omega)$ with $\alpha^*_\omega = \mu_s(\omega)$
- Private info is good or bad $S = \{G, B\}$

Advisor

- Commission proportional to α_S

$$V(\mu) = \sum_s r \; \mu_s(S)$$

- Expect info to be informative:
  $\pi^t(G|S) = \pi^t(B|\neg S) = 75\%$

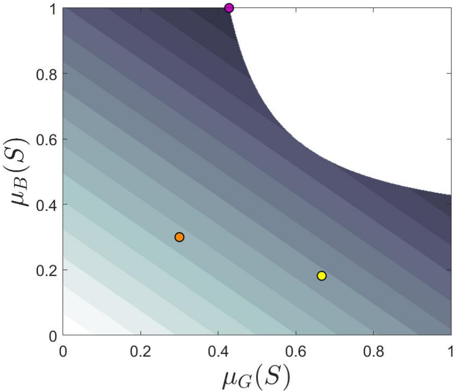


- Yellow point: sender's vector of posteriors by t
- The darker, the higher the sender's value

Financial Advice: Pessimistic Investors

Back

- States: $\{S, \neg S\}$, where S is the event in which the investment succeeds
- Signals: $\{G, B\}$, where if G the investor had a good experience
- Investor chooses investment allocation and advisor gets commission More



Financial Advice: Pessimistic Investors

Back

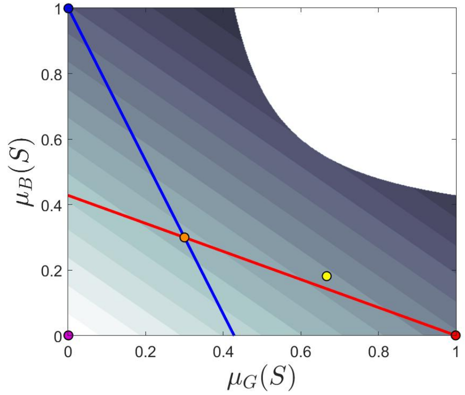
- States: $\{S, \neg S\}$, where S is the event in which the investment succeeds
- Signals: $\{G, B\}$, where if G the investor had a good experience
- Investor chooses investment allocation and advisor gets commission More

1. *Early success predicts LR success* (red)
   - Hot-hand bias
2. *What goes down comes up* (blue)
   - Gambler's fallacy

- Inconsistent ways of looking at signals, but consistent with the idea of LR success

  Going forward, this has so much promise

- Pessimistic investors disinvest regardless of new information or past experience



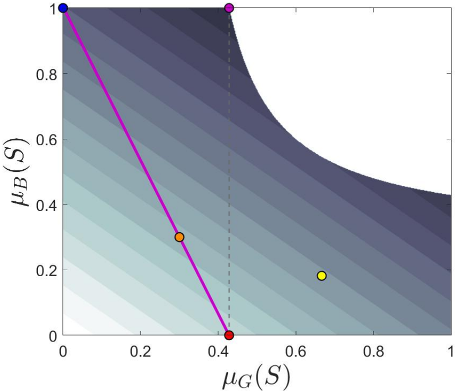
Financial Advice: Pessimistic Investors

Back

- Focus on the largest group: investors with negative experience
- Choose the narrative adopted by the other group without being counterproductive

1. *Bad experience occurs only if small stock is bad, but good experience can happen in both cases* (red)
2. *What goes down comes up* (blue)
   - Gambler's fallacy

- Pessimistic investors with bad experience fully invest, optimistic investors invest more than given their priors



Applications

Back

Merchants of Doubt: strategy to challenge a well-established way of looking at scientific evidence and to manufacture uncertainty (e.g., Michaels, 2008; Oreskes & Conway, 2011)

- Tobacco industry against health effect of smoking
- Oil companies against climate change

Merchants of Doubt

"Doubt is our product, since it is the best means of competing with the 'body of fact' that exists in the minds of the general public. It is also the means of establishing a controversy."

— Cigarette Executive (1969)

"Victory will be achieved when average citizens understand uncertainties in climate science."

— Internal memo by The American Petroleum Institute (1998)

Aim: delay regulations, defeat delegations, and insinuate doubt in the population

- Provide an alternative way of interpreting the emerging scientific evidence
- Establish a trustworthy presence in academia and media to discredit peer-reviewed articles and confusing public's understanding

More

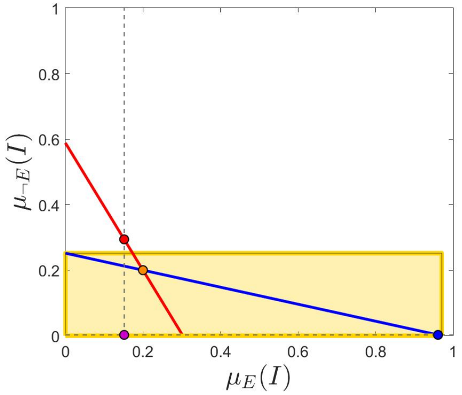
Merchants of Doubt

Back

- States: $\{I, \neg I\}$, where I is the event that the issue is real
- Signals: $\{E, \neg E\}$, where E is that there are evidence on the issue
- Trust in science by default

Default Model

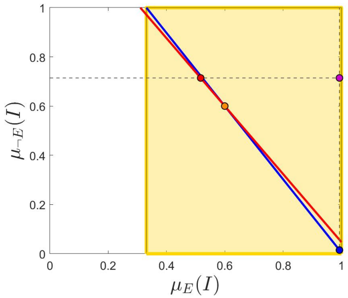
1. Default (red):
   $\pi^d(E|I) = \pi^d(\neg E|\neg I) = 99\%$
2. Strategic response (blue):
   $\pi^m(E|I) = 50\%$ and $\pi^m(E|\neg I) = 70\%$
   There is no link between evidence and issue, there are only false positive



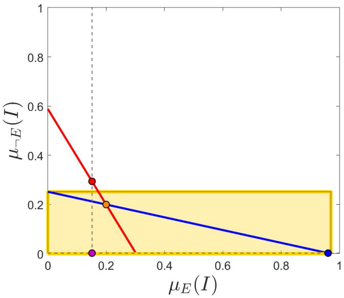
Merchants of Doubt

Back

Manufacturing Doubt



(a) $\mu_0(I) = 60\%$



(b) $\mu_0(I) = 20\%$

- Polarization can occur even when there is a shared narrative by default

Applications

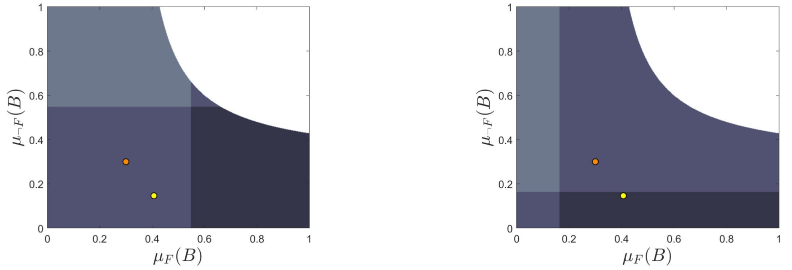
Back

Nudging: proposing ad-hoc narratives can be seen as a soft intervention to influence in a not coercive manner choices of an agent with the purpose of increase her welfare

- Proposing ad-hoc narratives can be seen as a soft intervention adopted by a paternalistic planner to influence in a not coercive manner choices of an agent with the purpose of increase her welfare
- Confidence manipulation by a paternalistic planner, via distorting the interpretation of signals, is optimal to influence the agent's behavior in a risky task

Nudging Risk Attitude

- States: $\{B, \neg B\}$, where B is the event that Arthur is brave enough
- Signals: $\{F, \neg F\}$, where F is the event that Ford gives Arthur a positive feedback
- Arthur chooses whether to go on adventure (high reward if B, low if $\neg B$) or not (medium reward)
- Marvin wants Arthur to undertake the adventure only conditional on a positive feedback
- Marvin believes Ford would bias his advice optimistically:
  low false negative $\pi^t(F|B) = 0.8$ but high false positive $\pi^t(F|\neg B) = 0.5$



(a) Risk adverse (b) Risk seeking

- It is beneficial to distort the agent's beliefs to be moderately overconfident (underconfident) if risk averse (risk seeking)

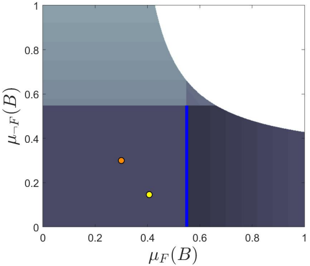
Refinment Back

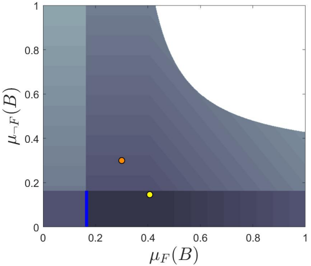
Appendix: Nudging Risk Attitude

- Assume that belief distortion bears some psychological costs for the sender, such as disappointment aversion
- Disappointment = the positive gap between the expected payoff calculated with the induced beliefs and the expected payoff with the true model
- The resulting sender's value function

$$V(\mu) = \sum_{s\in\{F,\neg F\}} \Pr^t(s) \left[\mathbb{E}_{\mu^t}\left[U^S(a(\mu))\right] - k \cdot \underbrace{\max\left\{0, \mathbb{E}_{\mu^t}\left[\pi^R(a(\mu))\right] - \mathbb{E}_{\mu}\left[\pi^R(a(\mu))\right]\right\}}_{\textit{disappointment}} \right]$$

where k is a sensitivity parameter to disappointment, and $\pi_R : \{a, \neg a\} \to \mathbb{R}$ is the receiver's payoff





Applications

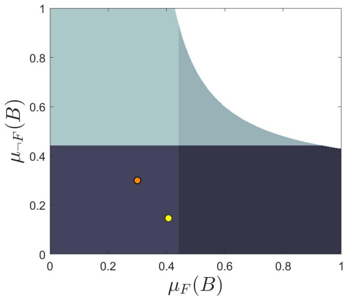
Back

Intra-personal phenomena: a mechanism through which the individual may distort his beliefs without assuming exogenous parameter of memory loss, inattention, first-impression, etc.

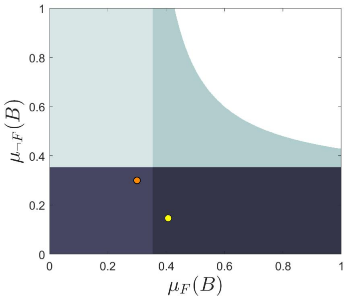
- Multiple stories allow motivated reasoning to take root
- In a multi-selves model, an agent has incentives to distort his self-confidence in order to offset his time inconsistent preferences

Commitment (Bénabou & Tirole, 2002) Model

- Arthur has to decide whether to go on an adventure or not
- Arthur is risk-neutral with quasi-hyperbolic discounting preferences
- Arthur knows that at the moment of the decision the imminent cost of undertaking the adventure c (t = 1) will be more salient than the future reward of a success v if B (t = 2)
- Deep down he believes Ford to be optimistic: $\pi^t(F|B) = 0.8$ and $\pi^t(F|\neg B) = 0.5$



(a) Present Bias



(b) No Present Bias

- At date 0, Arthur might have incentives to distort his interpretations of Ford's advice only to overcome his present bias

Appendix: Bénabou & Tirole (2002)

Timing: At $t = 0$, the individual can take an action that potentially affects his information at $t = 1$ with some utility flow. At $t = 1$, he decides whether to take an action with disutility c that, if successful, would yield benefit v at $t = 2$

- Consider a risk-neutral individual with quasi-hyperbolic discounting
- No action a leads to zero utility, hence $U^1(\neg a) = 0$ and $U^0(\neg a) = u_0$
- Utility at $t = 1$ when taking the action conditional on s:

$$U^1(a) = u_1 + \beta\delta \, \mathbb{E}_{\mu_s}[u_2] = -c + \mu_s(success)\, \beta\delta v,$$

  where $\delta \leq 1$ is his discount factor and $\beta > 0$ is his present bias.
- At $t = 1$, the action is optimal if $\mu_s(success) \geq \frac{c}{\beta\delta v}$.
- Utility at date 0 when taking the action:

$$U^0(a) = u_0 + \beta \, \mathbb{E}_{\mu^t}\left[\delta \; u_1 + \delta^2 \; u_2\right] = u_0 + \beta\delta\big(-c + \mu_s^t(success)\; \delta v\big),$$

- At date 0, the action is optimal if $\mu_s^t(success) \geq \frac{c}{\delta v}$, lower if he suffers from present bias $\beta < 1$
- The probability of success, discussed as his self-confidence by the authors, may depend on either new information received or forgotten

Back

Set of Feasible Vectors of Posteriors: Receiver's Default Model

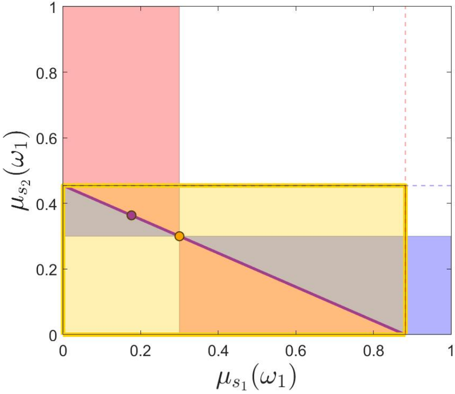
Back

- The receiver has endowed with a default model d, known by the sender
- More challenging for the sender to induce posteriors:
  conditional on the signal, the receiver adopts the posterior distribution induced by the sender's model only if the latter has higher fit than the default model

Proposition $|M| \geq |S|$

The set of feasible vectors of posterior beliefs is

$$\mathcal{F}^d = \Big\{ \mu \in [\Delta(\omega)]^S : \forall \omega \in \Omega, s \in S, \\ \delta_s(\omega; \mu_s)\mathrm{Pr}^d(s) \leq 1 \Big\}$$



Receiver's Default Model: With and Without Back

The set of the feasible vectors of posteriors without the default model is the union of all sets of the feasible vectors of posteriors with default model for every default model

Proposition

The union of $\mathcal{F}^d$ for all possible default model is $\mathcal{F}$, i.e.,

$$\bigcup_{d \in \mathcal{M}} \mathcal{F}^d = \mathcal{F}.$$