Power analysis of closed testing methods with Simes, Wilcoxon-Mann-Whitney and LMPI T3 as local tests considering Lehmann's alternative of order k

2023-12-01

Libraries and functions

```
library(tidyverse)
library(doSNOW)
library(nout)

gen.data <- function(m,n) {
  Z <- rnorm((m+n))
  return(Z)
}

gen.scores_Lehmann <- function(m, n, n1, k){
  if(n1==0){
    S_Z = gen.data(m,n)
    S_cal = S_Z[1:m]
    S_te = S_Z[(m+1):length(S_Z)]
  }

  if(n1==n){
    augmented.S_Z = gen.data(m,n*k)
    S_cal = augmented.S_Z[1:m]
    augmented.S_te = augmented.S_Z[(m+1):length(augmented.S_Z)]
    S_te = sapply(1:n1, FUN=function(i) max(augmented.S_te[(1+k*(i-1)):(i*k)]))
  }

  if(0<n1&n1<n){
    augmented.S_Z = gen.data(m=m,n=n-n1+n1*k)
    S_cal = augmented.S_Z[1:m]
    augmented.S_te = augmented.S_Z[(m+1):length(augmented.S_Z)]
    inlier.S_te = augmented.S_te[1:(n-n1)]
    outlier.augmented.S_te = augmented.S_te[(n-n1+1):length(augmented.S_te)]
    outlier.S_te = sapply(1:n1, FUN=function(i) max(outlier.augmented.S_te[(1+k*(i-1)):(i*k)]))
    S_te = c(inlier.S_te, outlier.S_te)
  }

  return(list("S_cal" = S_cal,
              "S_te" = S_te,
              "k" = k,
              "n1" = n1))
```

```
}



compute_lb.d = function(B, m, n, n1, k, alpha){

  foreach(b = 1:B, .combine=cbind) %dopar% {

    scores = gen.scores_Lehmann(m, n, n1, k)
    S_cal = scores$S_cal
    S_te = scores$S_te

    d_WMW = nout::d_MannWhitney(S_Y = S_te, S_X = S_cal, alpha=alpha)
    d_T3 = nout::d_MannWhitneyk3(S_Y = S_te, S_X = S_cal, alpha=alpha)
    d_Sim = nout::d_Simes(S_X = S_cal, S_Y = S_te, alpha = alpha)
    # StoSimes = nout::d_StoreySimes(S_X = S_cal, S_Y = S_te, alpha = alpha)
    # d_StoSimes = StoSimes$d
    # pi.not = StoSimes$pi.not
    d_BH = nout::d_benjhoch(S_X = S_cal, S_Y = S_te, alpha = alpha)
    # d_StoBH = nout::d_StoreyBH(S_X = S_cal, S_Y = S_te, alpha = alpha)

    return(list("m" = m,
                "n" = n,
                "k" = k,
                "n1" = n1,
                "alpha" = alpha,
                "S_cal" = S_cal,
                "S_te" = S_te,
                "d_BH" = d_BH,
                #"d_StoBH" = d_StoBH,
                "d_Sim" = d_Sim,
                #"d_StoSimes" = d_StoSimes,
                "d_WMW" = d_WMW,
                "d_T3" = d_T3
                #"pi.not" = pi.not
                ))
  }
}



compact_results = function(res, ks, n1.index, n){

  mean.lb.d_n1_k =  matrix(nrow = length(ks), ncol = 4)

  rnames = vector()
  for(i in 1:length(ks)){
    rnames[i] = paste0("k=", ks[i])
  }
  # cnames.lb.d = c("mean.lb.d_BH", "mean.lb.d_StoBH", "mean.lb.d_Sim",
  #                 "mean.lb.d_StoSim", "mean.lb.d_WMW", "mean.lb.d_T3")
  cnames.lb.d = c("mean.lb.d_BH", "mean.lb.d_Sim",
                  "mean.lb.d_WMW", "mean.lb.d_T3")
```

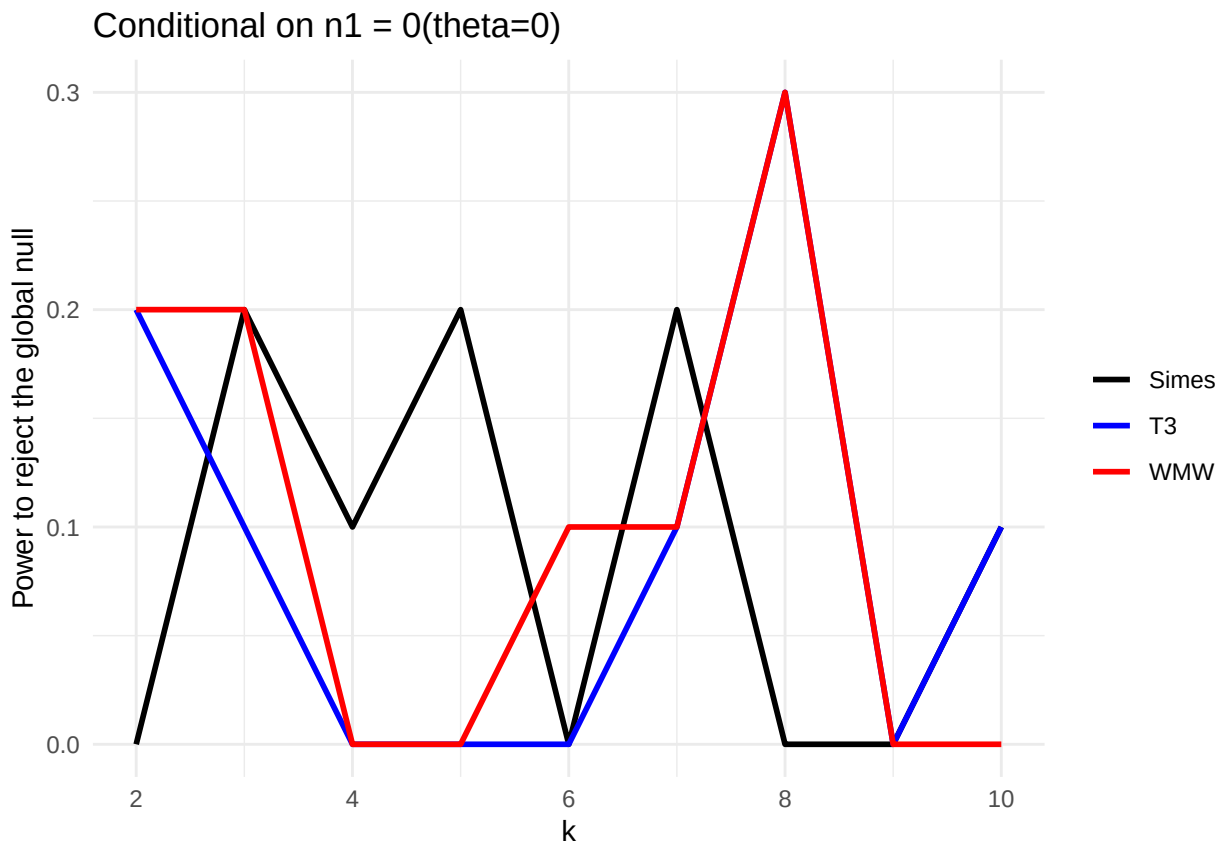
```
  rownames(mean.lb.d_n1_k) = rnames
  colnames(mean.lb.d_n1_k) = cnames.lb.d

  for(i in 1:length(ks)){
    mean.lb.d_n1_k[i,"mean.lb.d_BH"] = mean(unlist(res[[i]][[n1.index]]["d_BH",]))
    #mean.lb.d_n1_k[i,"mean.lb.d_StoBH"] = mean(unlist(res[[i]][[n1.index]]["d_StoBH",]))
    mean.lb.d_n1_k[i,"mean.lb.d_Sim"] = mean(unlist(res[[i]][[n1.index]]["d_Sim",]))
    #mean.lb.d_n1_k[i,"mean.lb.d_StoSim"] = mean(unlist(res[[i]][[n1.index]]["d_StoSimes",]))
    mean.lb.d_n1_k[i,"mean.lb.d_WMW"] = mean(unlist(res[[i]][[n1.index]]["d_WMW",]))
    mean.lb.d_n1_k[i,"mean.lb.d_T3"] = mean(unlist(res[[i]][[n1.index]]["d_T3",]))
  }


  mean.power_n1_k =  matrix(nrow = length(ks), ncol = 4)

  cnames.power = c("mean.power_BH", "mean.power_Sim",
                   "mean.power_WMW", "mean.power_T3")
  rownames(mean.power_n1_k) = rnames
  colnames(mean.power_n1_k) = cnames.power

  for(i in 1:length(ks)){
    mean.power_n1_k[i,"mean.power_BH"] = mean(unlist(res[[i]][[n1.index]]["d_BH",])>0)
    #mean.power_n1_k[i,"mean.power_StoBH"] = mean(unlist(res[[i]][[n1.index]]["d_StoBH",])>0)
    mean.power_n1_k[i,"mean.power_Sim"] = mean(unlist(res[[i]][[n1.index]]["d_Sim",])>0)
    #mean.power_n1_k[i,"mean.power_StoSim"] = mean(unlist(res[[i]][[n1.index]]["d_StoSimes",])>0)
    mean.power_n1_k[i,"mean.power_WMW"] = mean(unlist(res[[i]][[n1.index]]["d_WMW",])>0)
    mean.power_n1_k[i,"mean.power_T3"] = mean(unlist(res[[i]][[n1.index]]["d_T3",])>0)
  }

  results = list("mean.power_n1_k" = mean.power_n1_k,
                 "mean.lb.d_n1_k" = mean.lb.d_n1_k)

  return(results)
}
```

```
set.seed(321)

B = 10

m = 3999
n = 400
alpha = n/(m+1)

thetas = c(0, 0.005, 0.01, 0.05, 0.1)
n1s = floor(n*thetas)

# Order of the Lehmann's alternative
ks = 2:10

cluster <- makeCluster(parallel::detectCores()-1)
registerDoSNOW(cluster)
clusterEvalQ(cluster, {list(library(isotree), library(nout))})
```

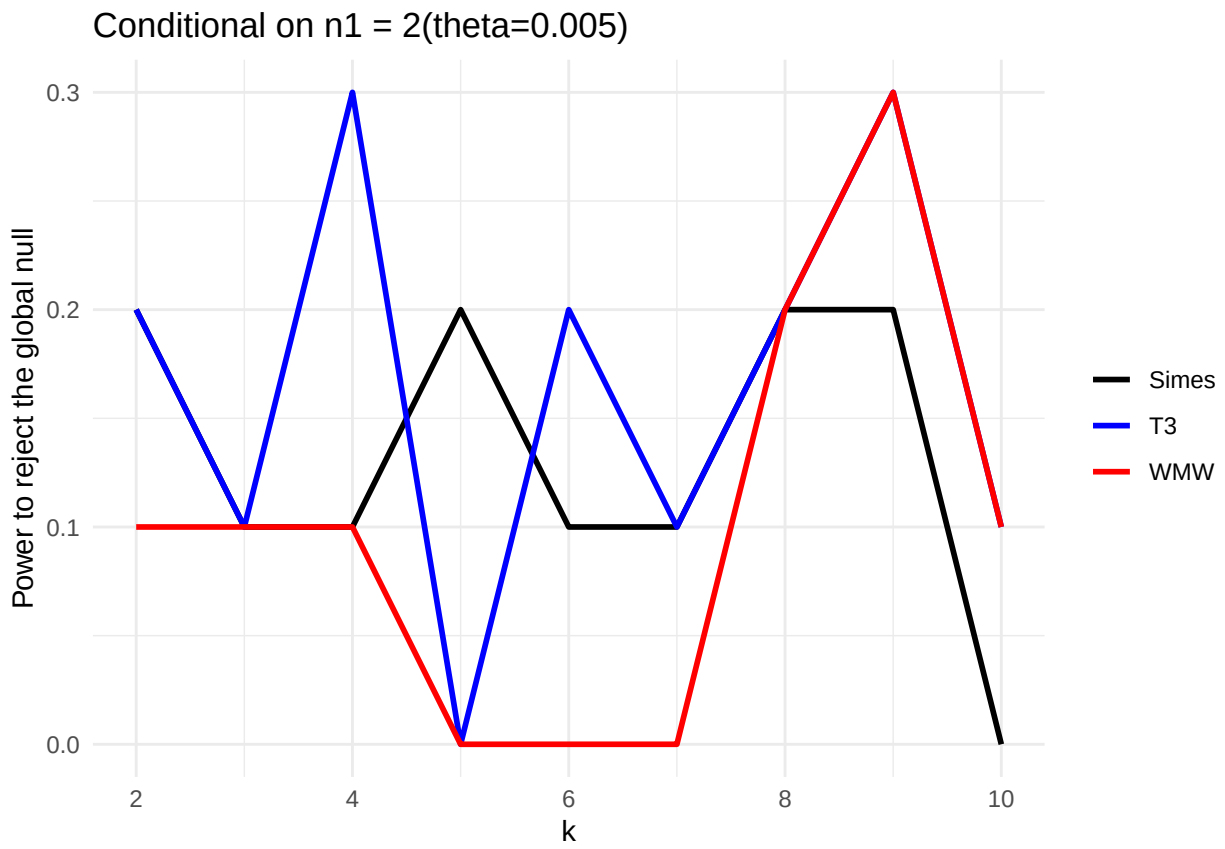
```
## [[1]]
## [[1]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[1]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[2]]
## [[2]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[2]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[3]]
## [[3]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[3]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[4]]
## [[4]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[4]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[5]]
## [[5]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[5]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[6]]
## [[6]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
```

```
##
## [[6]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[7]]
## [[7]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[7]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
```

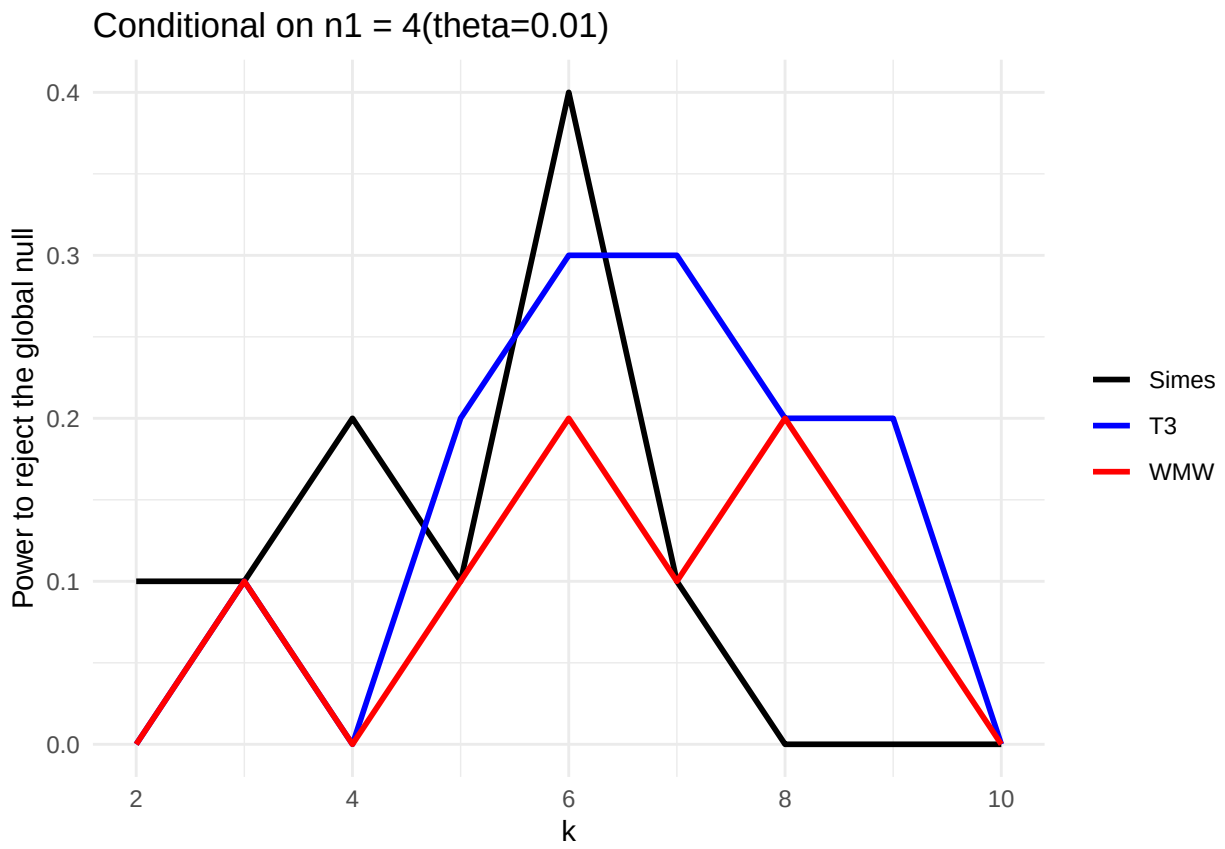
```
clusterExport(cluster, list("n", "m", "ks", "n1s", "alpha", "gen.data", "gen.scores_Lehmann"))

res <- lapply(1:length(ks), function(i){
  lapply( 1:length(n1s), function(j) compute_lb.d(B=B, m=m, n=n,
                                                  n1=n1s[j], k=ks[i], alpha=alpha))
  }
)

stopCluster(cluster)

results = lapply(1:length(n1s),
                 function(j) compact_results(res=res, ks=ks, n=n, n1.index=j) )

pp = list()
for(i in 1:length(n1s)){
  pow_BH = results[[i]]$mean.power_n1_k[,"mean.power_BH"]
  pow_Sim = results[[i]]$mean.power_n1_k[,"mean.power_Sim"]
  pow_WMW = results[[i]]$mean.power_n1_k[,"mean.power_WMW"]
  pow_T3 = results[[i]]$mean.power_n1_k[,"mean.power_T3"]

  dfpower <- data.frame(
    x = ks,
    Simes = pow_BH,
    WMW = pow_WMW,
    T3 = pow_T3
  )
  df_long_power <- tidyr::pivot_longer(dfpower, cols = -x, names_to = "group", values_to = "y")

  pp[[i]] = ggplot(df_long_power, aes(x = x, y = y, color = group)) +
                  geom_line(size=1) +
                  scale_color_manual(values = c("black","blue","red")) +
                  ggtitle(paste0("Conditional on n1 = ", n1s[i], "(theta=", thetas[i], ")")) +
                  labs(x = "k", y = "Power to reject the global null") +
                  theme_minimal() +
                  theme(legend.title = element_blank())
  print(pp[[i]])
  print(cbind(pow_BH, pow_WMW, pow_T3))
}
```



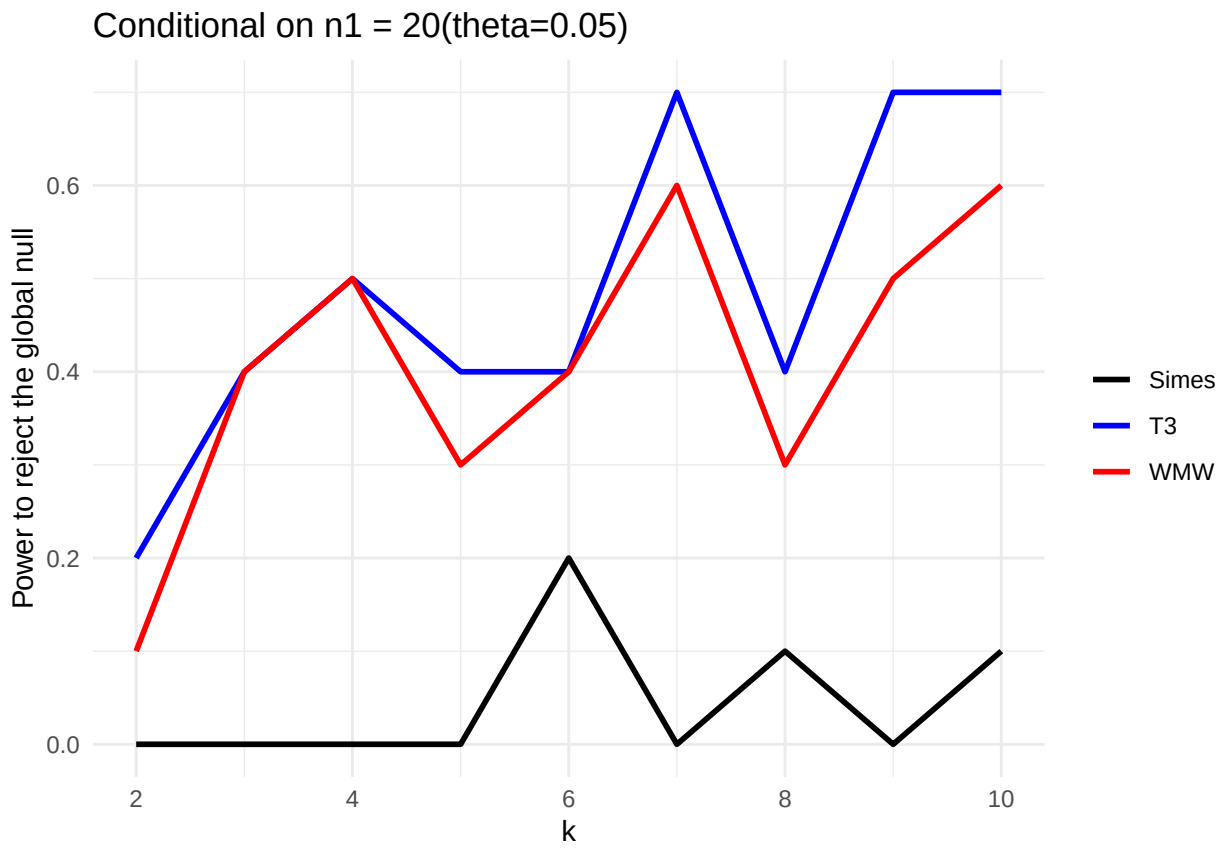

```
##       pow_BH pow_WMW pow_T3
## k=2      0.0     0.2    0.2
## k=3      0.2     0.2    0.1
## k=4      0.1     0.0    0.0
## k=5      0.2     0.0    0.0
## k=6      0.0     0.1    0.0
## k=7      0.2     0.1    0.1
## k=8      0.0     0.3    0.3
## k=9      0.0     0.0    0.0
## k=10     0.1     0.0    0.1
```



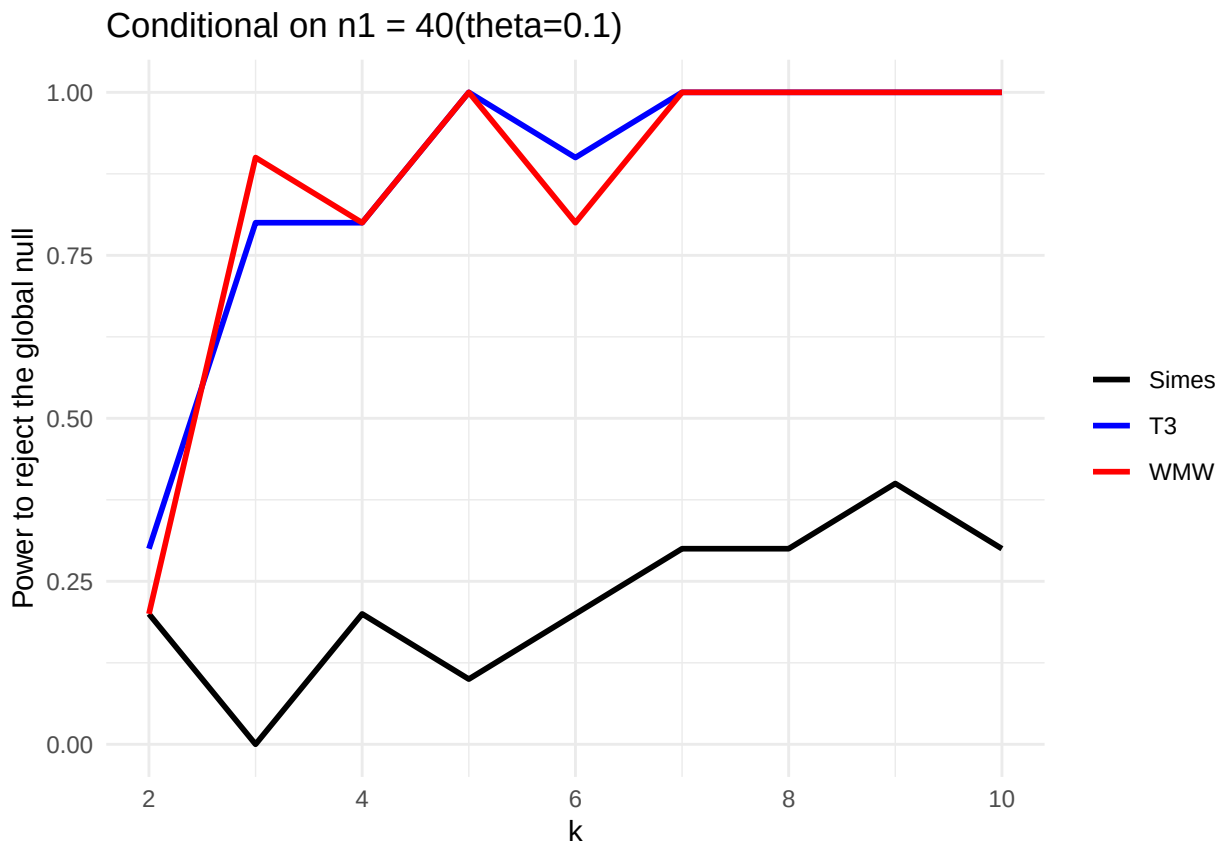

```
##       pow_BH pow_WMW pow_T3
## k=2      0.2     0.1    0.2
## k=3      0.1     0.1    0.1
## k=4      0.1     0.1    0.3
## k=5      0.2     0.0    0.0
## k=6      0.1     0.0    0.2
## k=7      0.1     0.0    0.1
## k=8      0.2     0.2    0.2
## k=9      0.2     0.3    0.3
## k=10     0.0     0.1    0.1
```




```
##       pow_BH pow_WMW pow_T3
## k=2      0.1     0.0    0.0
## k=3      0.1     0.1    0.1
## k=4      0.2     0.0    0.0
## k=5      0.1     0.1    0.2
## k=6      0.4     0.2    0.3
## k=7      0.1     0.1    0.3
## k=8      0.0     0.2    0.2
## k=9      0.0     0.1    0.2
## k=10     0.0     0.0    0.0
```




```
##       pow_BH pow_WMW pow_T3
## k=2      0.0     0.1    0.2
## k=3      0.0     0.4    0.4
## k=4      0.0     0.5    0.5
## k=5      0.0     0.3    0.4
## k=6      0.2     0.4    0.4
## k=7      0.0     0.6    0.7
## k=8      0.1     0.3    0.4
## k=9      0.0     0.5    0.7
## k=10     0.1     0.6    0.7
```




```
##       pow_BH pow_WMW pow_T3
## k=2      0.2     0.2    0.3
## k=3      0.0     0.9    0.8
## k=4      0.2     0.8    0.8
## k=5      0.1     1.0    1.0
## k=6      0.2     0.8    0.9
## k=7      0.3     1.0    1.0
## k=8      0.3     1.0    1.0
## k=9      0.4     1.0    1.0
## k=10     0.3     1.0    1.0
```