Lehmann's alternative on Digits dataset

k=2, proportion of outliers from 0 to 1

2023-05-17

We consider Lehmann's alternative with $k = 2$ and we show that the theoretical result that closed testing procedure with Wilcoxon-Mann-Whitney local test is the Locally Most Powerful Invariant test for the global null is validated also by numerical simulations.

```
library(doSNOW)
library(foreach)
library(nout)
library(tictoc)
library(isotree)
library(readr)
library(R.matlab)



compact_results = function(res){
  resT=as.data.frame(t(res))
  discoveries = as.data.frame(cbind("d_BH"=unlist(resT$d_BH),
                                    "d_StoBH"=unlist(resT$d_StoBH),
                                    "d_Sim"=unlist(resT$d_Sim),
                                    "d_StoSimes"=unlist(resT$d_StoSimes),
                                    "d_WMW"=unlist(resT$d_WMW)))
  mean.discoveries = apply(discoveries, MARGIN = 2, FUN = mean)
  power.GlobalNull = as.data.frame(discoveries>0)
  mean.powerGlobalNull = apply(power.GlobalNull, MARGIN = 2, FUN = mean)
  return(list("discoveries" = discoveries,
              "mean.discoveries" = mean.discoveries,
              "power.GlobalNull" = power.GlobalNull,
              "mean.powerGlobalNull" = mean.powerGlobalNull,
              "pi.not" = unlist(resT$pi.not),
              "uniques"=unlist(resT$uniques),
              "n1"=unlist(resT$n1),
              "alpha"=unlist(resT$alpha)))
}
```

Digits dataset

```
data = readMat("~/nout/trials/RealData/Datasets/Dataset digits/pendigits.mat")

dataset = cbind(data$X, data$y); colnames(dataset)[ncol(dataset)] = "y"
in_ind = which(dataset[,ncol(dataset)]==0)
out_ind = which(dataset[,ncol(dataset)]==1)
```

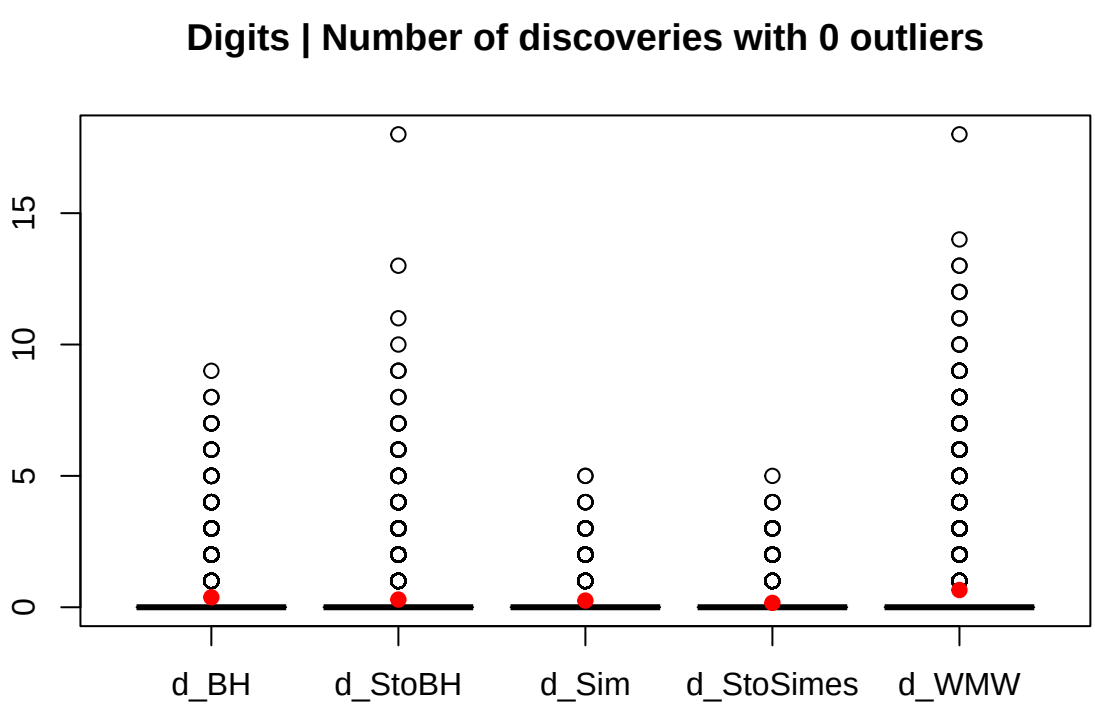
```r
# Initializing parameters
set.seed(321)

B=4000

l = 1683
m = 249
n = 50
k = 2 # exponent of Lehmann's alternative
myalpha = n/(m+1)


tr_ind = sample(in_ind, size = l)
in_ind2 = setdiff(in_ind, tr_ind)
tr = dataset[tr_ind,]
n_cpus = parallel::detectCores()
iso.fo = isotree::isolation.forest(tr, ndim = ncol(dataset), ntrees = 150, sample_size = 256,
                                   nthreads = n_cpus, scoring_metric = "depth",
                                   output_score = TRUE)
isofo.model = iso.fo$model
mycrit = nout::critWMW(m = n, n = m, alpha = myalpha)
```

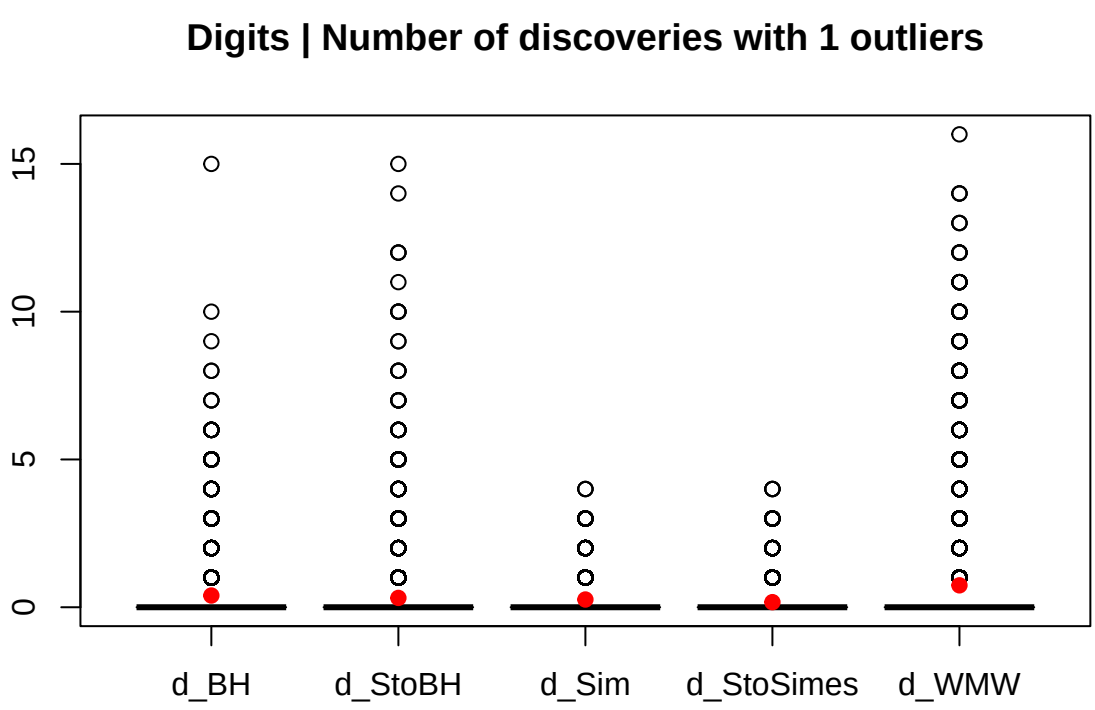
```r
prop.out = seq(0, 1, by=0.02)
n1_vec = round(prop.out*n)

cl <- makeCluster(parallel::detectCores())
clusterEvalQ(cl, {library(isotree)})
```

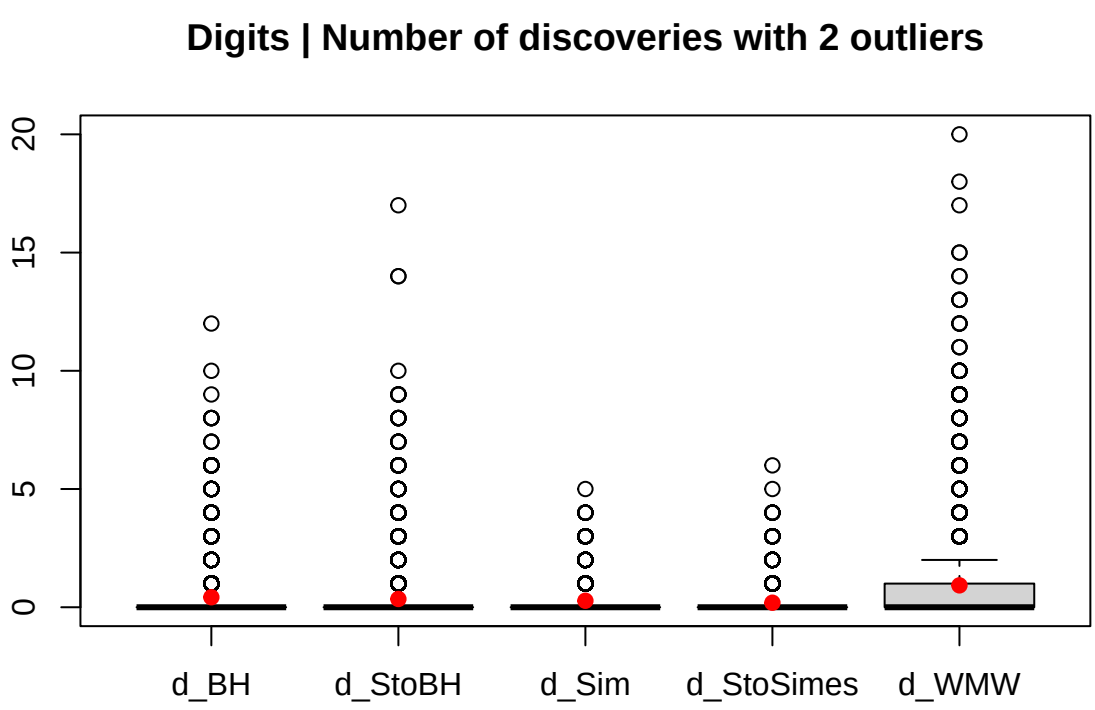
```
## [[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[2]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[3]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[4]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
```

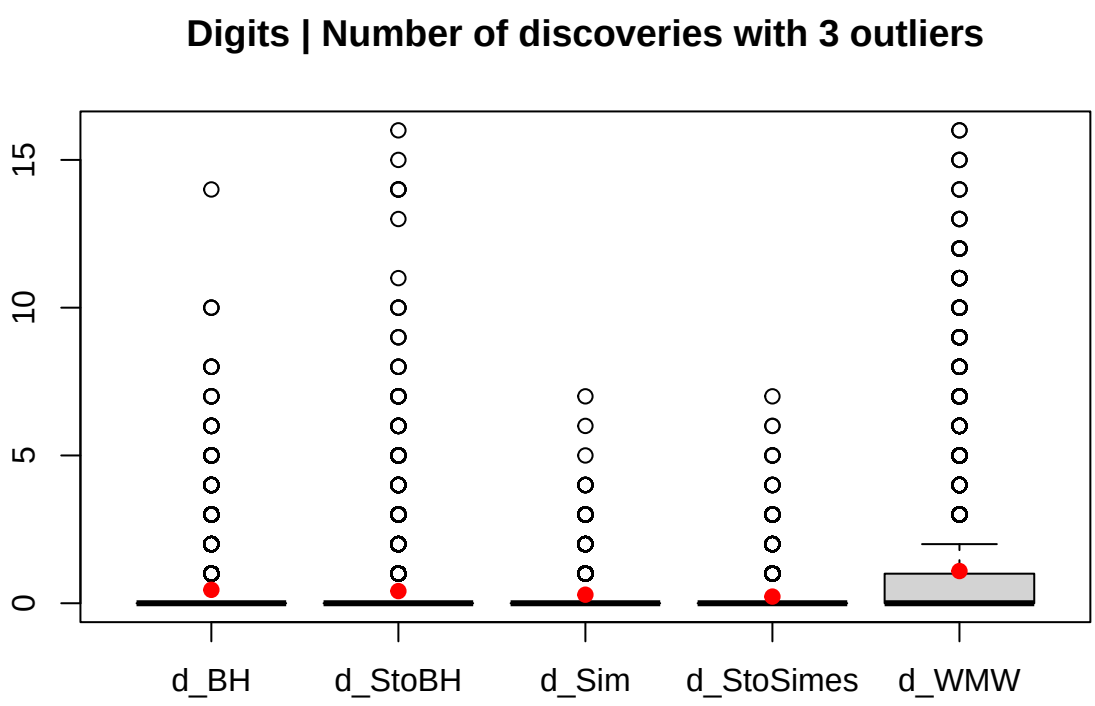
```r
registerDoSNOW(cl)

res = list()

for(n1 in n1_vec){
  i = which(n1_vec==n1)
  res[[i]] = foreach(b = 1:B, .combine=cbind) %dopar% {
      n0 = n - n1
      N = n0 + m + k*n1
      in_index3 = sample(in_ind2, size = N)
```

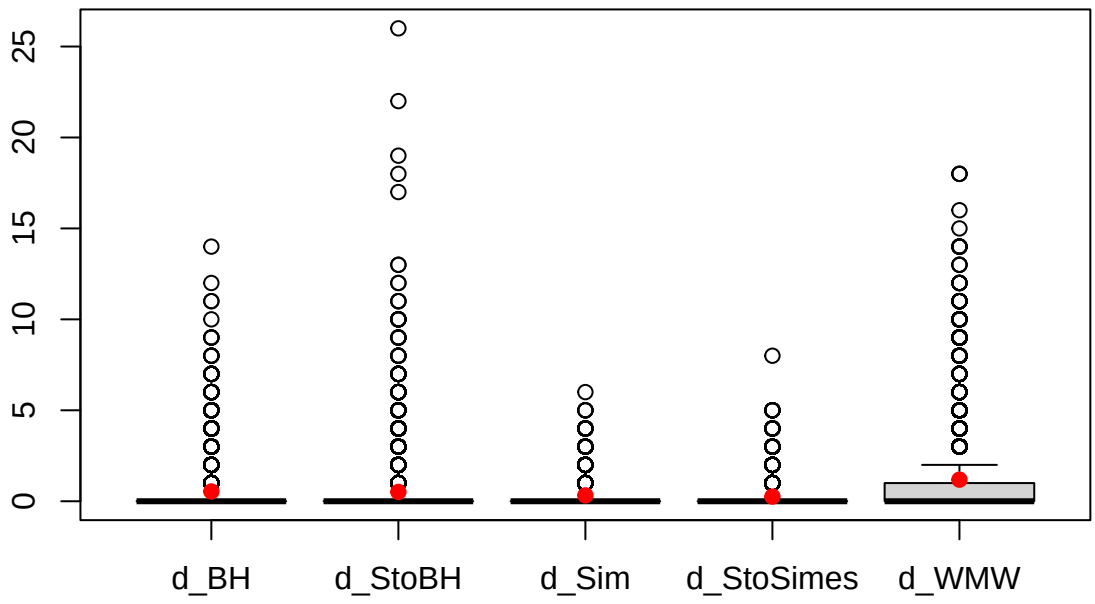
```
      cal_ind = in_index3[1:m]
      te_ind.augmented = in_index3[(m+1):N]
      cal = dataset[cal_ind,]
      te = dataset[te_ind.augmented,]
      S_cal = predict.isolation_forest(isofo.model, cal, type = "score")
      augmented.S_te = predict.isolation_forest(isofo.model, te, type = "score")
      if(n1==0)
        S_te = augmented.S_te
      if(n1==n)
        S_te = sapply(0:(n1-1), FUN=function(i) max(augmented.S_te[1+k*i], augmented.S_te[k+k*i]))
      if(0<n1&n1<n)
        S_te = c(augmented.S_te[1:n0],
                      sapply(0:(n1-1), FUN=function(i) max(augmented.S_te[1+k*i], augmented.S_te[k+k*i]))

      d_WMW = nout::d_mannwhitney(S_Y = S_te, S_X = S_cal, crit = mycrit)
      d_Sim = nout::d_Simes(S_X = S_cal, S_Y = S_te, alpha = myalpha)
      StoSimes = nout::d_StoreySimes(S_X = S_cal, S_Y = S_te, alpha = myalpha)
      d_StoSimes = StoSimes$d
      pi.not = StoSimes$pi.not
      d_BH = nout::d_benjhoch(S_X = S_cal, S_Y = S_te, alpha = myalpha)
      d_StoBH = nout::d_StoreyBH(S_X = S_cal, S_Y = S_te, alpha = myalpha)
      uniques = length(unique(c(S_cal, S_te)))
      return(list("d_BH" = d_BH,
                  "d_StoBH" = d_StoBH,
                  "d_Sim" = d_Sim,
                  "d_StoSimes" = d_StoSimes,
                  "d_WMW" = d_WMW,
                  "uniques" = uniques,
                  "n1" = n1,
                  "pi.not" = pi.not,
                  "alpha" = myalpha))

  }
}


stopCluster(cl)

results = lapply(res, compact_results)

for(i in 1:length(n1_vec)){
  boxplot(results[[i]]$discoveries, main=paste0("Digits | Number of discoveries with ", n1_vec[[i]], " c
  points(x=1:5, y=results[[i]]$mean.discoveries, pch=19, col="red")
  results[[i]]$mean.discoveries
  results[[i]]$mean.powerGlobalNull
}
```

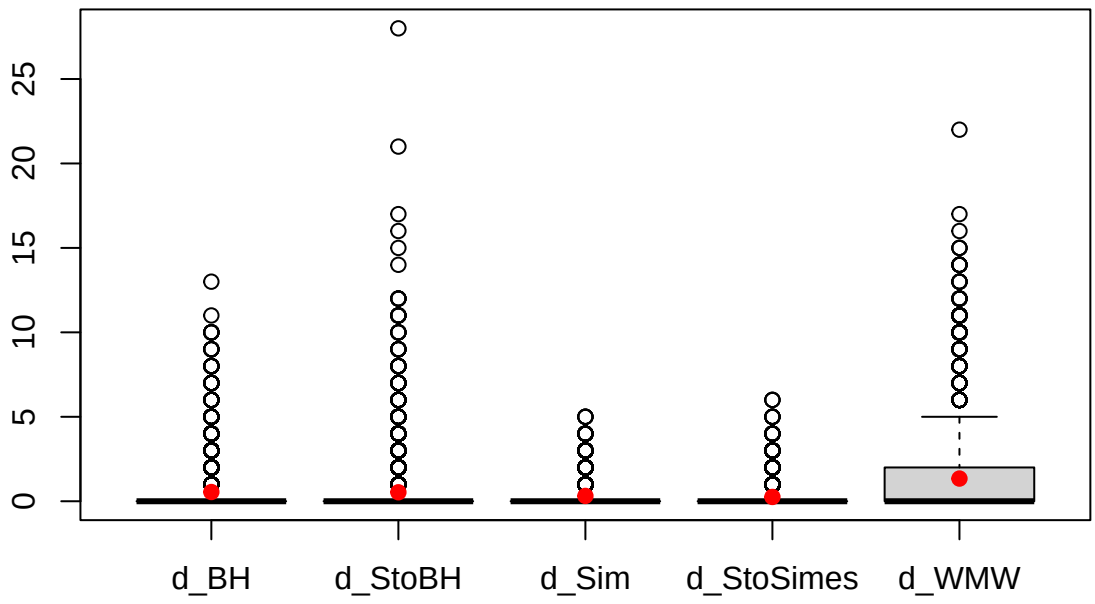

Digits | Number of discoveries with 0 outliers
0
5
10
15
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

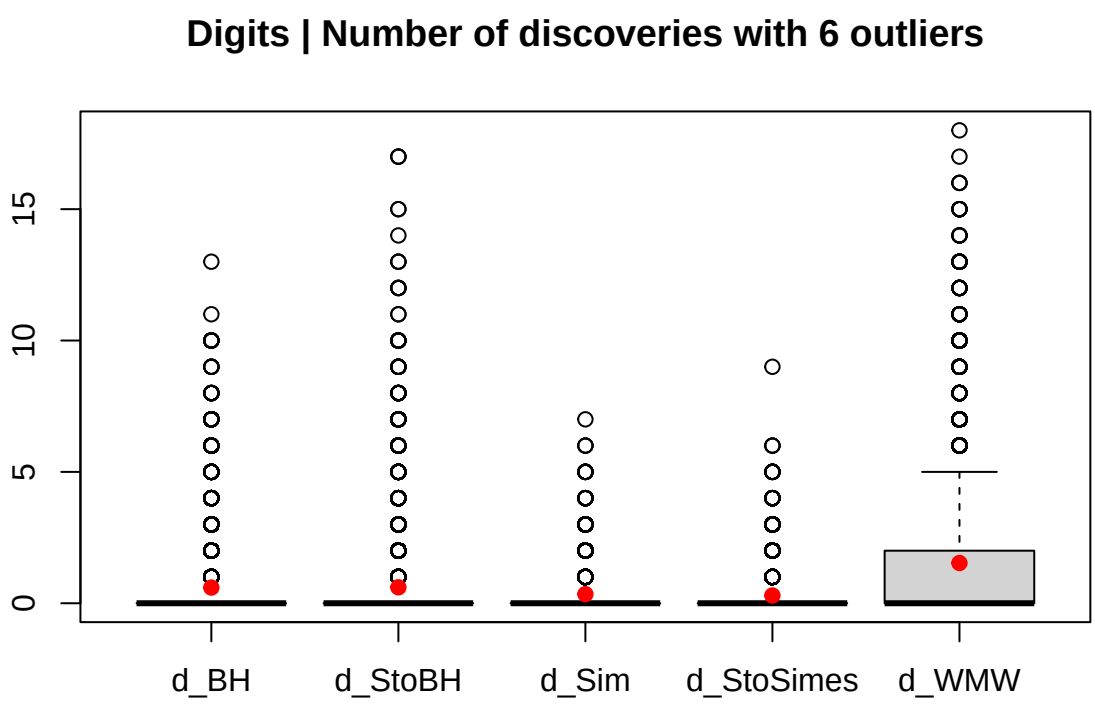

Digits | Number of discoveries with 1 outliers
0
5
10
15
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

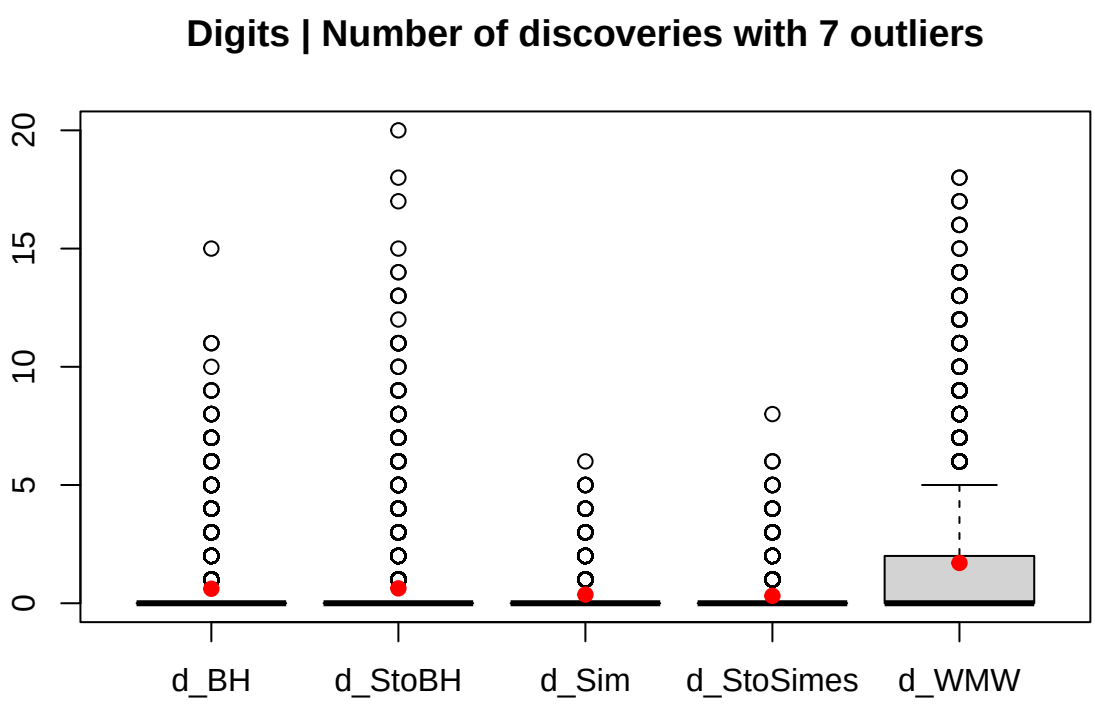

Digits | Number of discoveries with 2 outliers
0
5
10
15
20
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

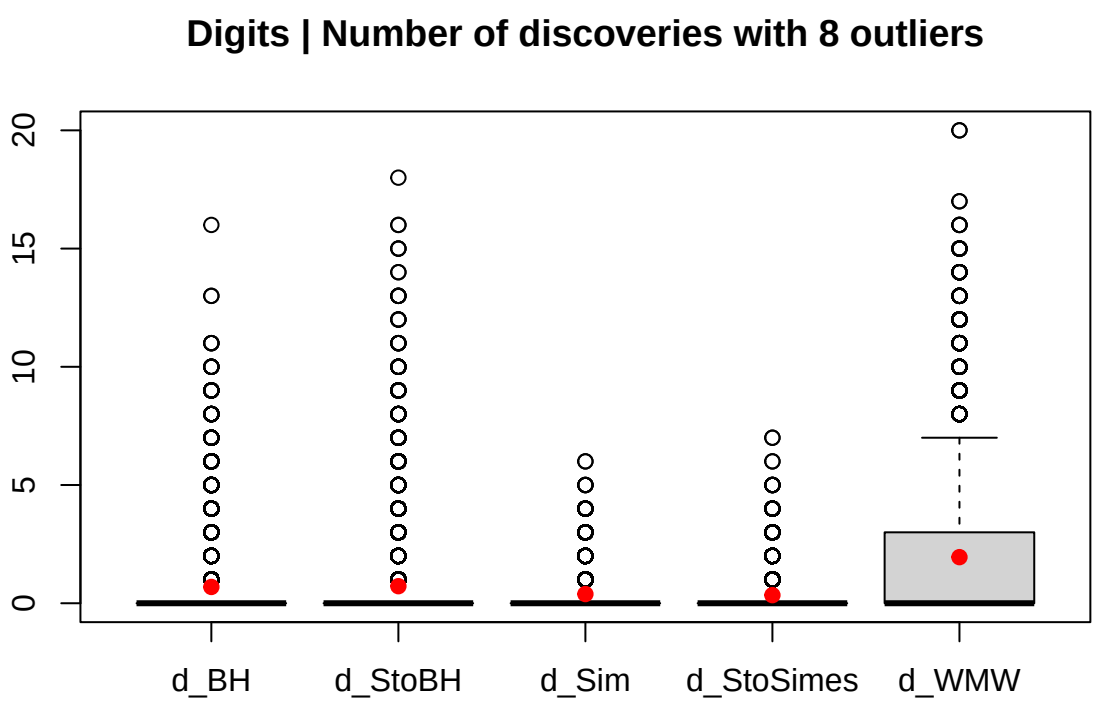

Digits | Number of discoveries with 3 outliers
0
5
10
15
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

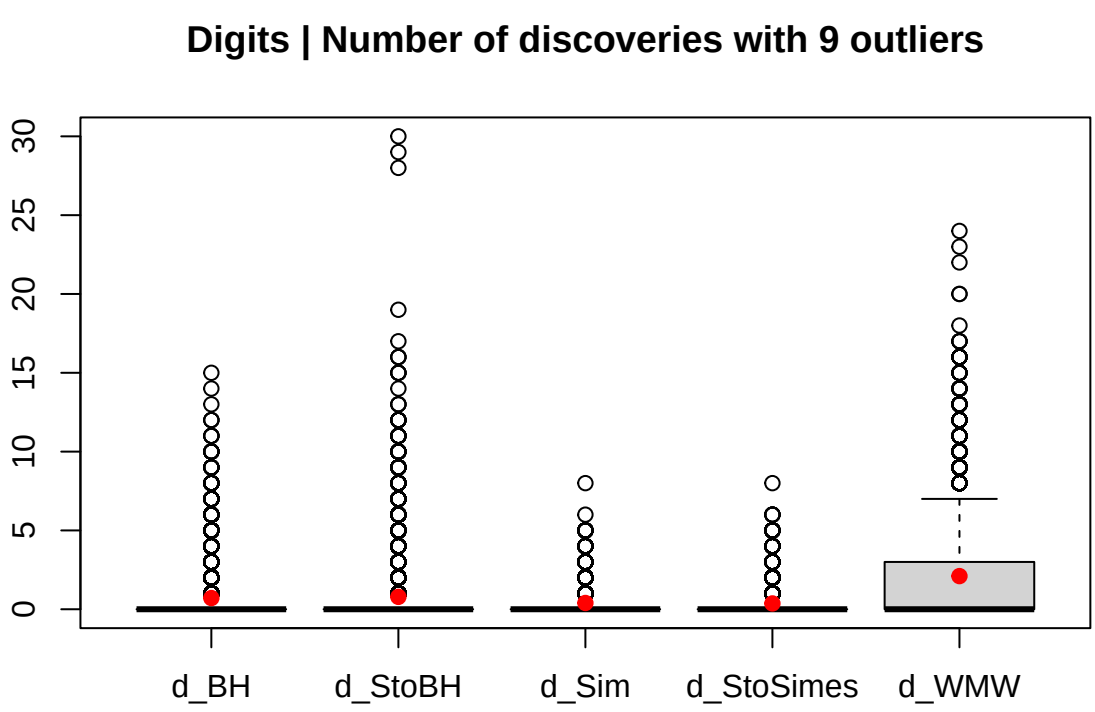

Digits | Number of discoveries with 4 outliers
0
5
10
15
20
25
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

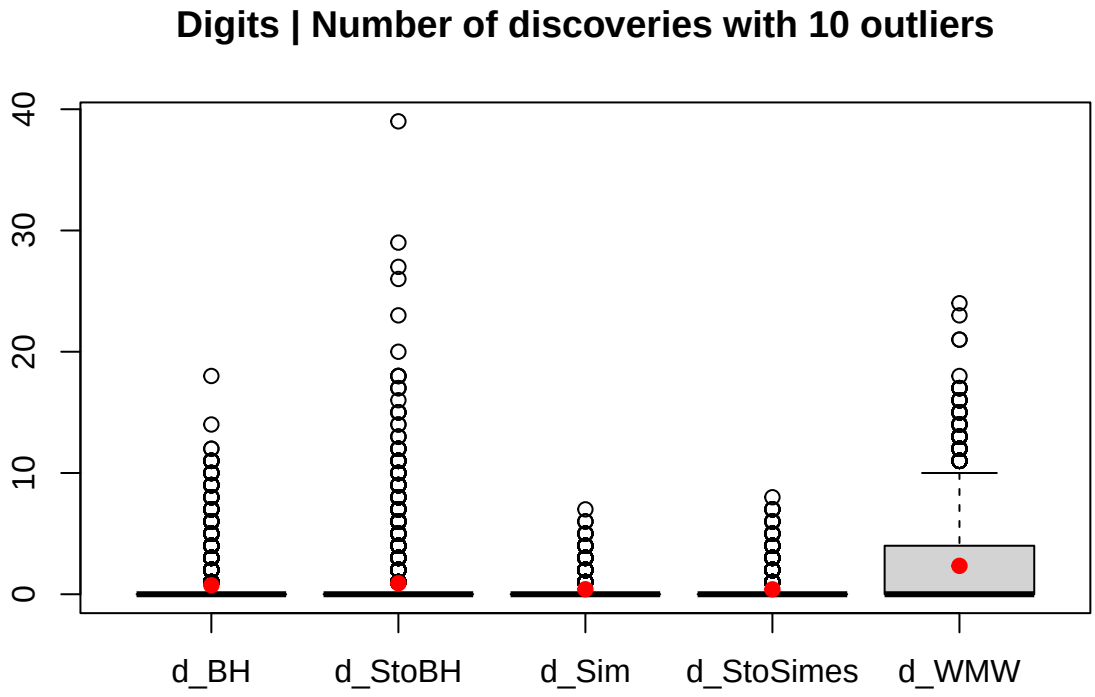

Digits | Number of discoveries with 5 outliers
0
5
10
15
20
25
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

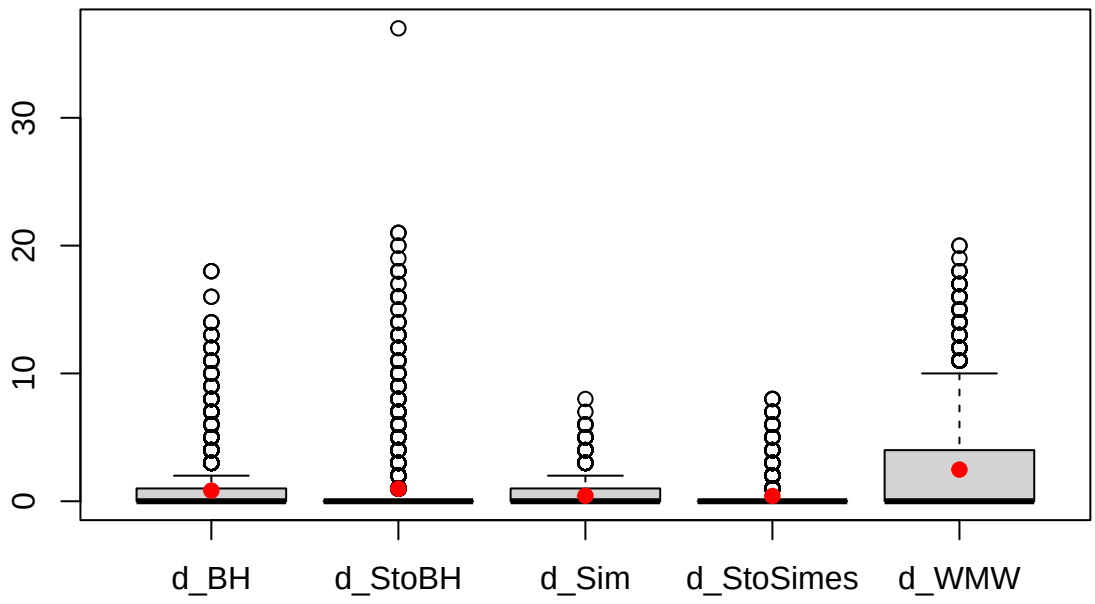

Digits | Number of discoveries with 6 outliers
0
5
10
15
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

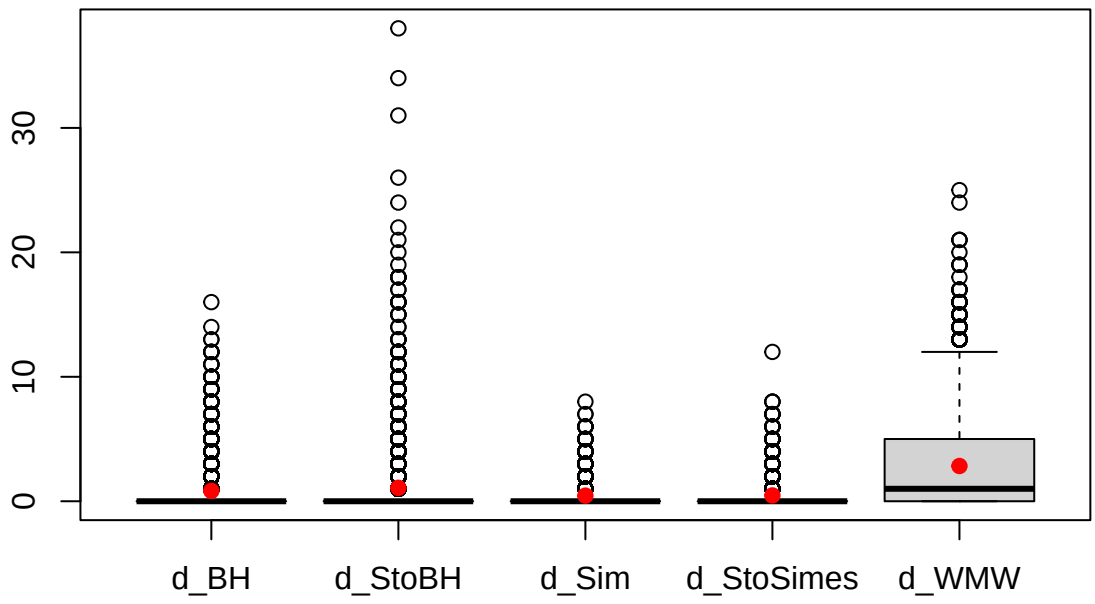

Digits | Number of discoveries with 7 outliers
0
5
10
15
20
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

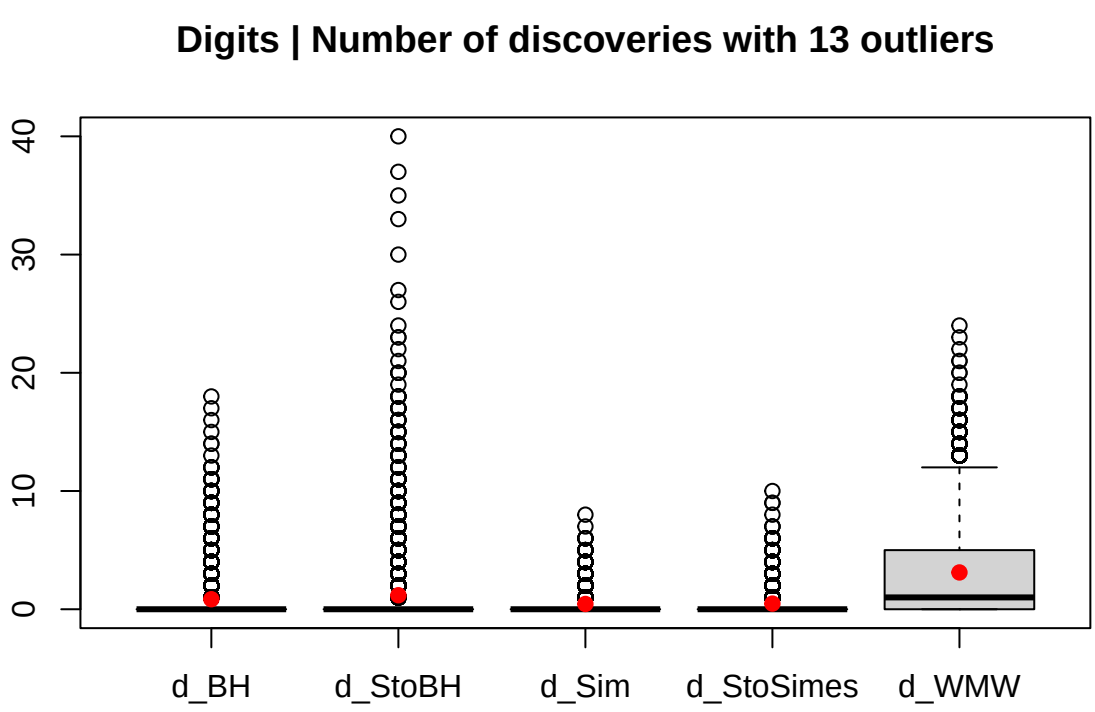

Digits | Number of discoveries with 8 outliers
0
5
10
15
20
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

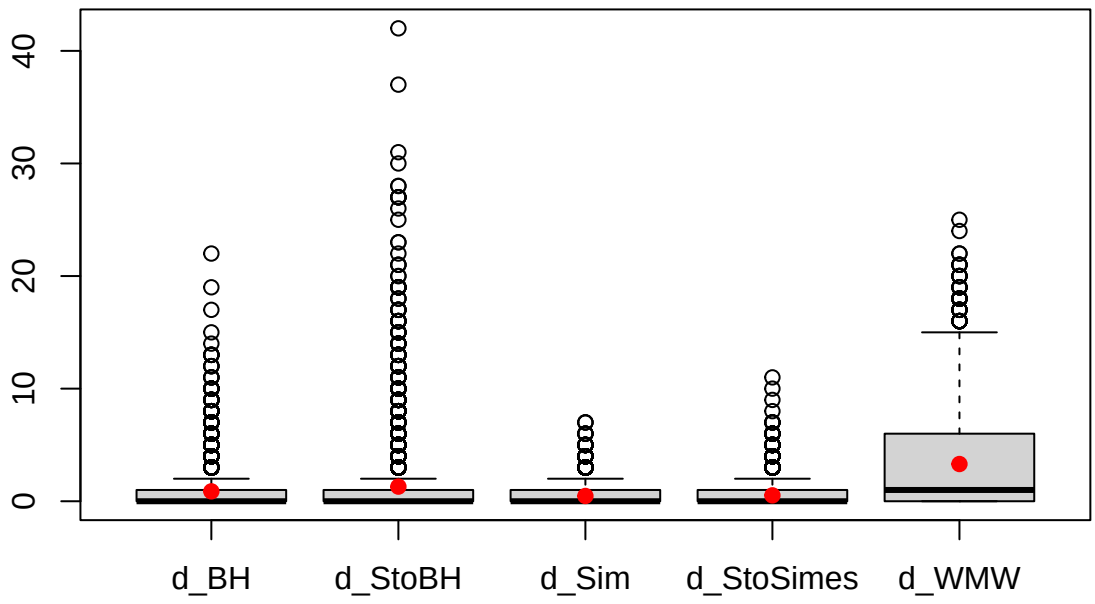

Digits | Number of discoveries with 9 outliers
0
5
10
15
20
25
30
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

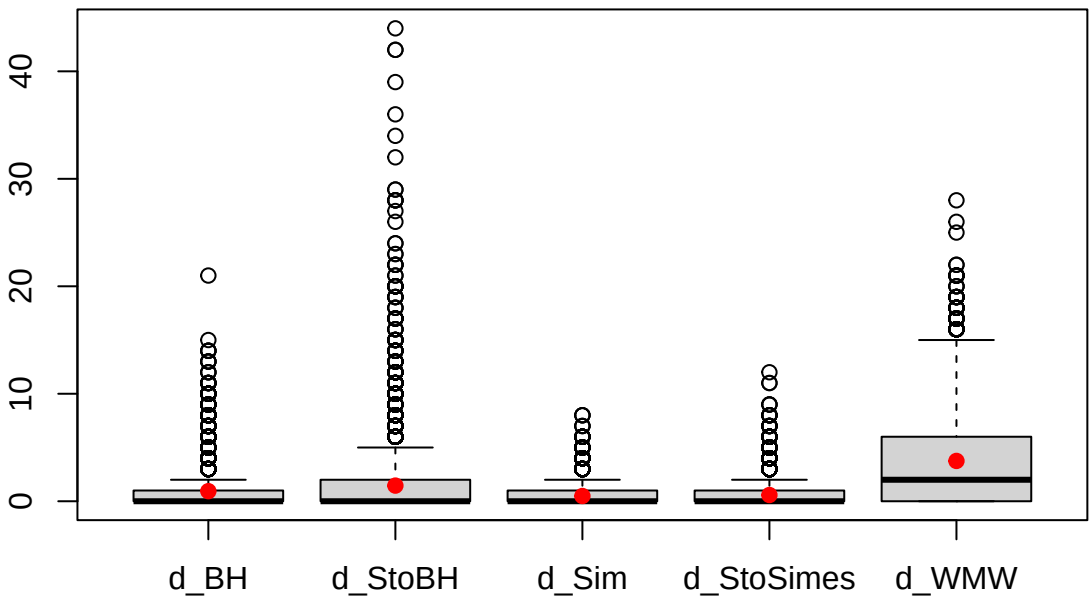

Digits | Number of discoveries with 10 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

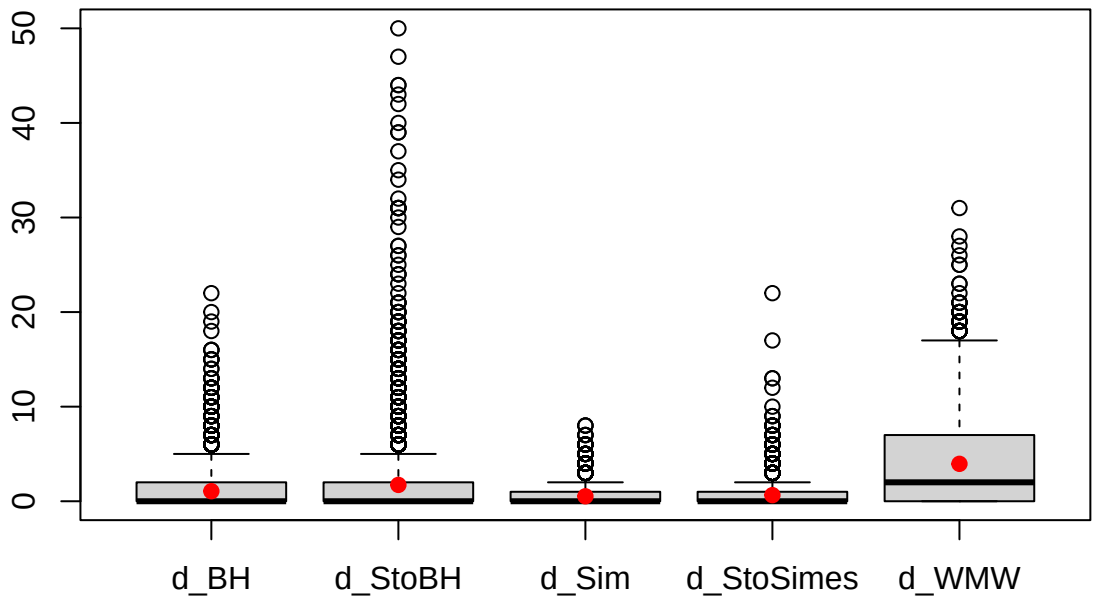

Digits | Number of discoveries with 11 outliers
0
10
20
30
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

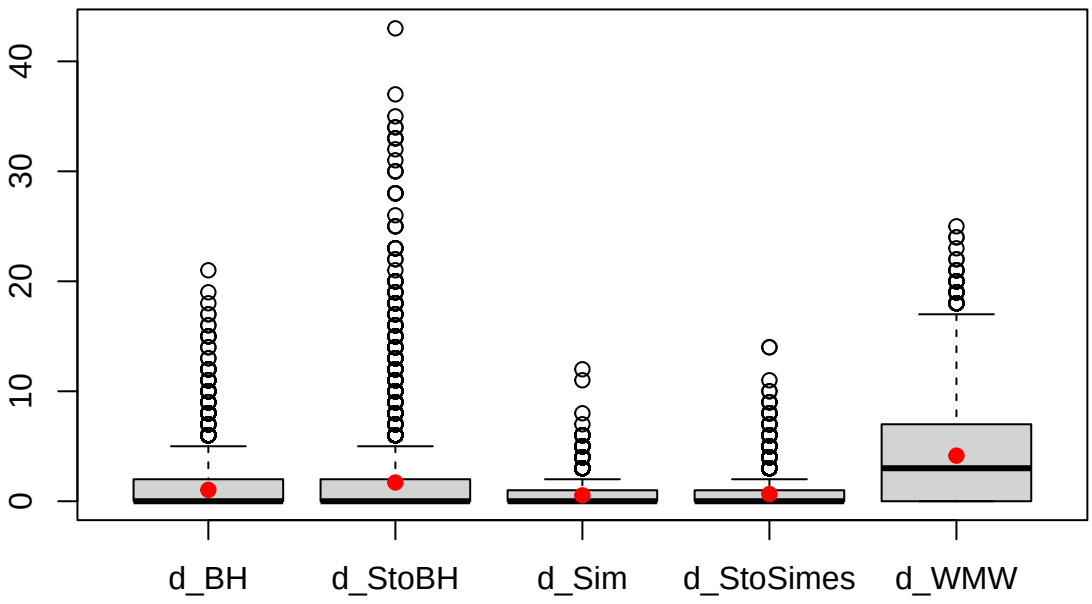

Digits | Number of discoveries with 12 outliers
0
10
20
30
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

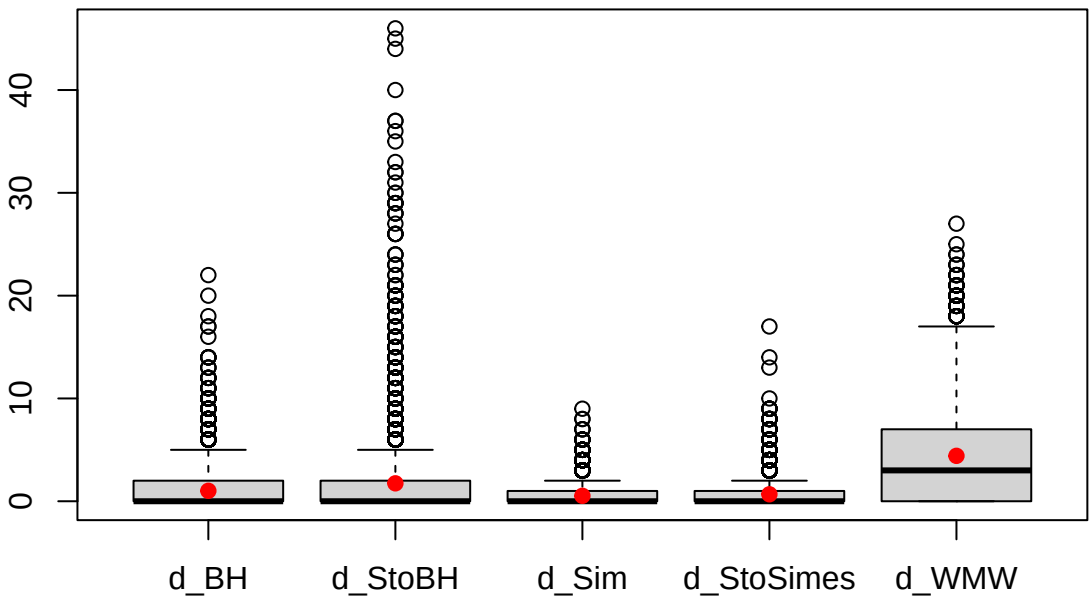

Digits | Number of discoveries with 13 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

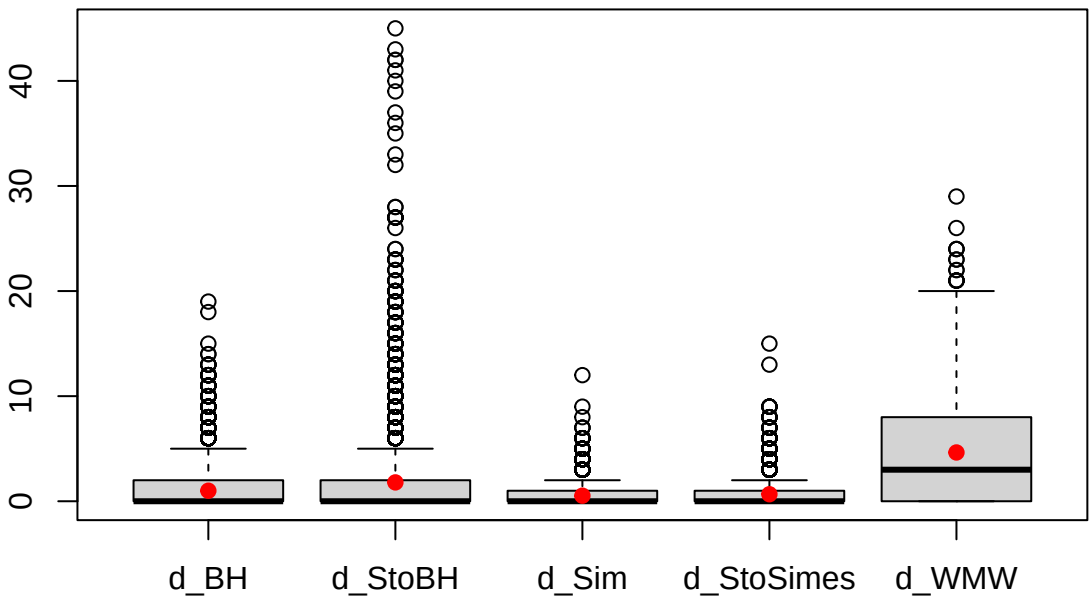

Digits | Number of discoveries with 14 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

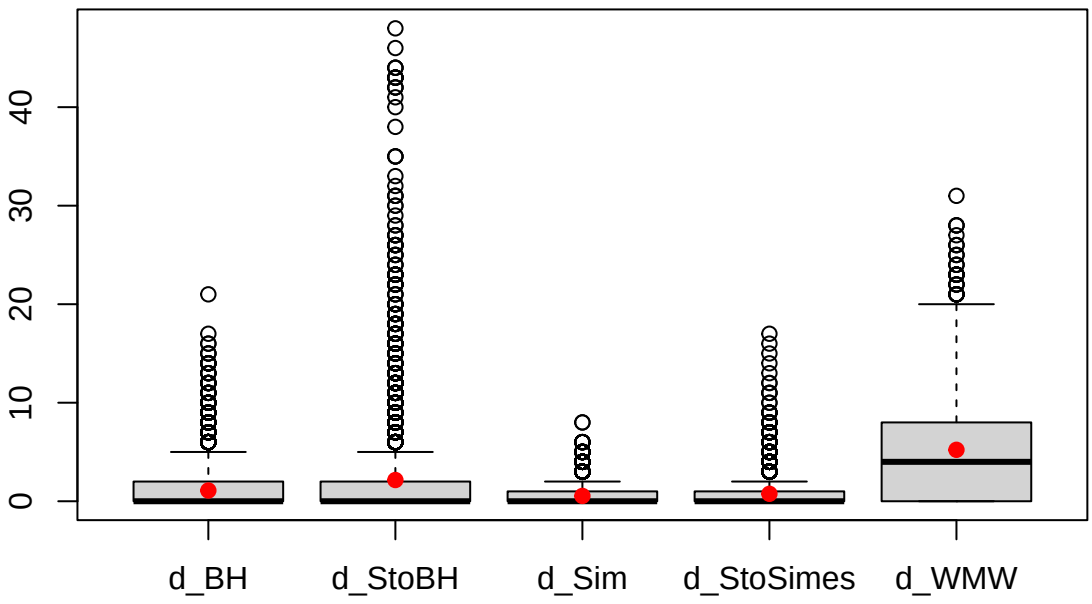

Digits | Number of discoveries with 15 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

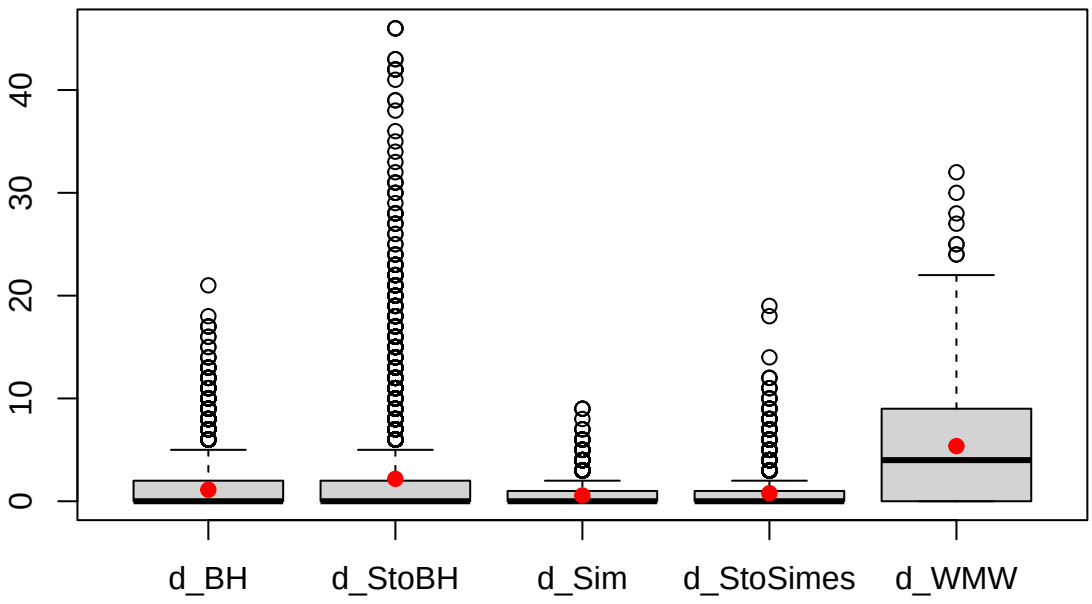

Digits | Number of discoveries with 16 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

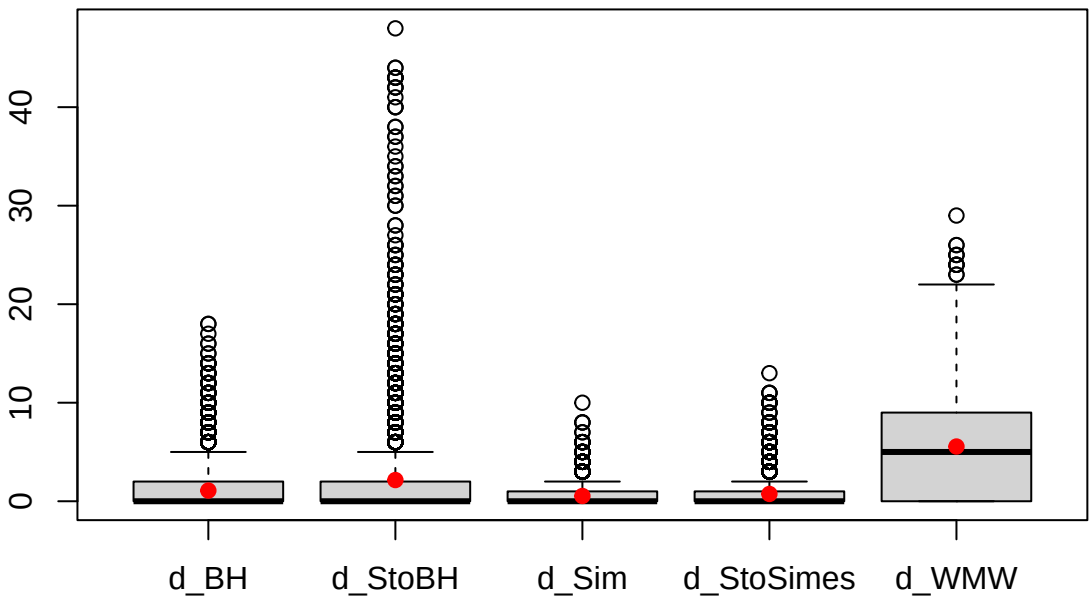

Digits | Number of discoveries with 17 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

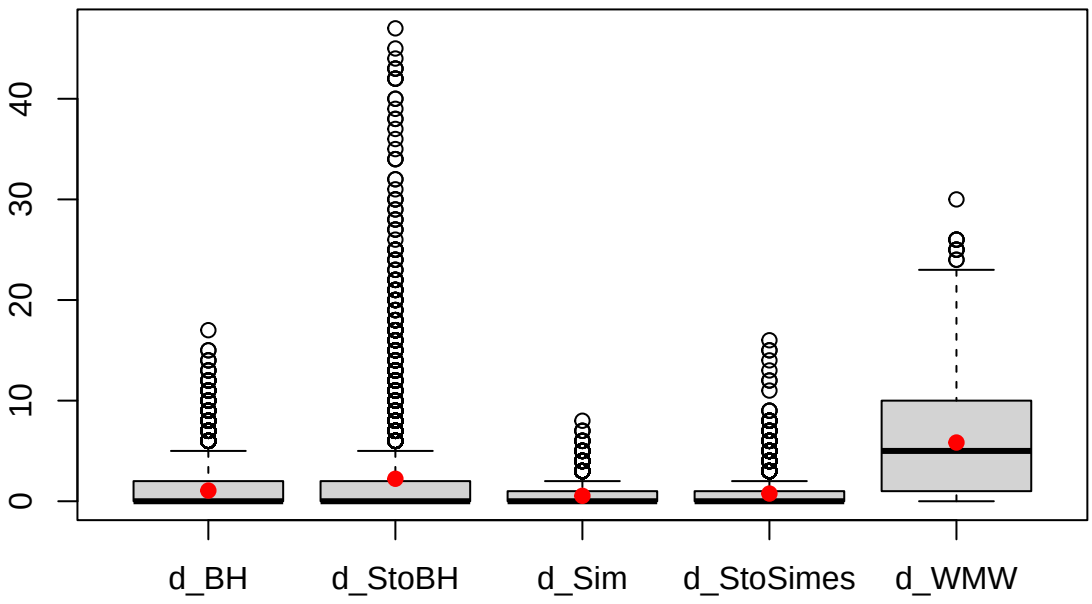

Digits | Number of discoveries with 18 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

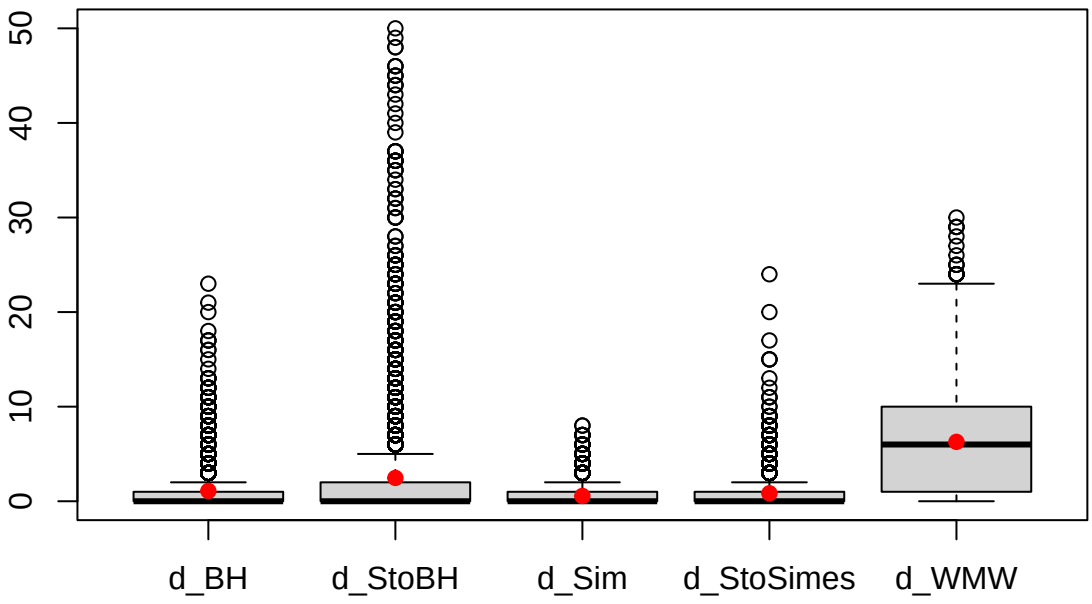

Digits | Number of discoveries with 19 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

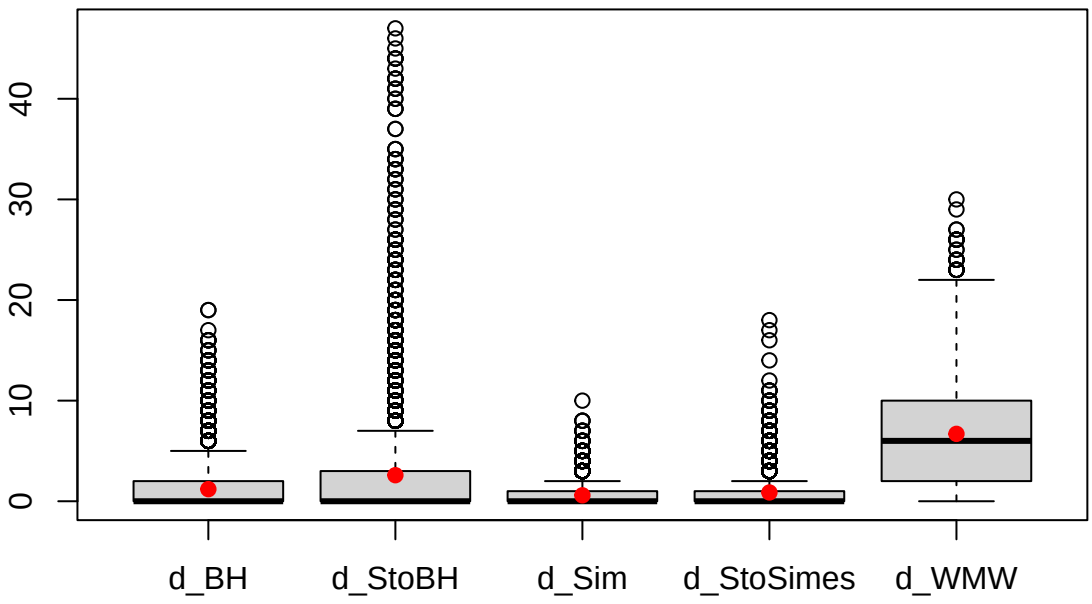

Digits | Number of discoveries with 20 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

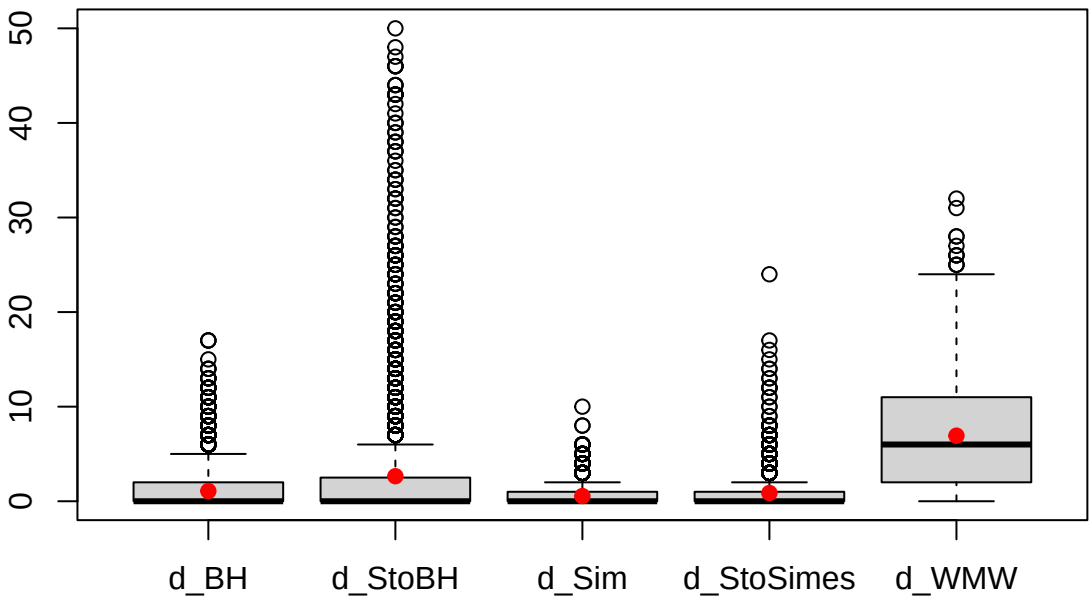

Digits | Number of discoveries with 21 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

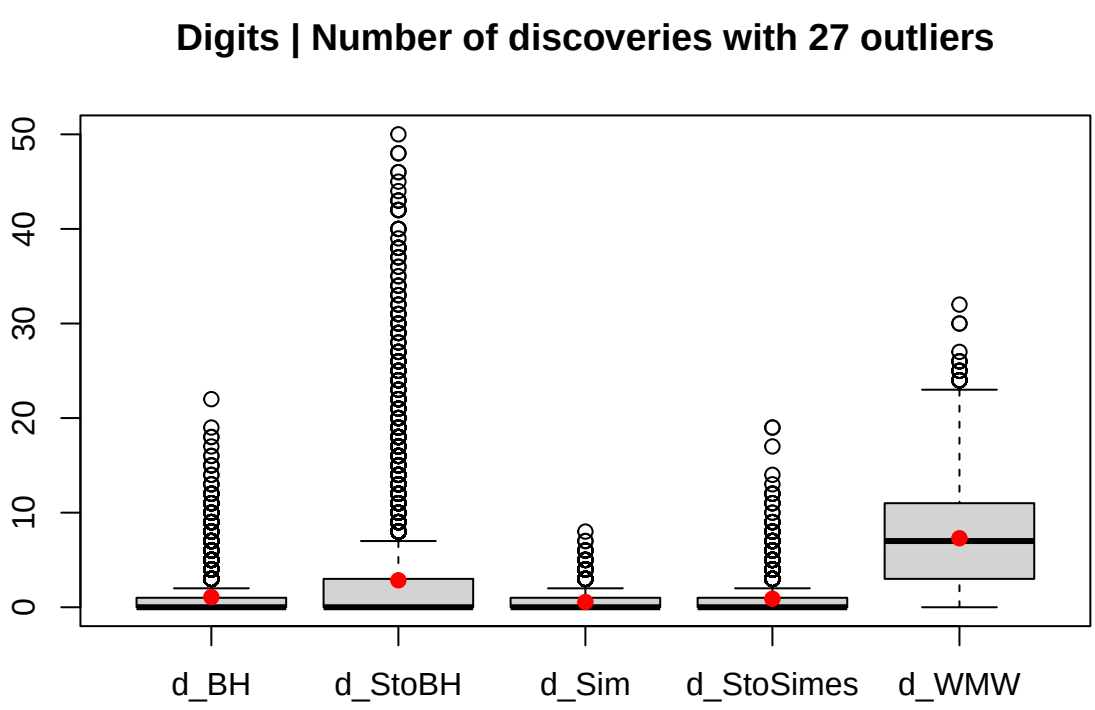

Digits | Number of discoveries with 22 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

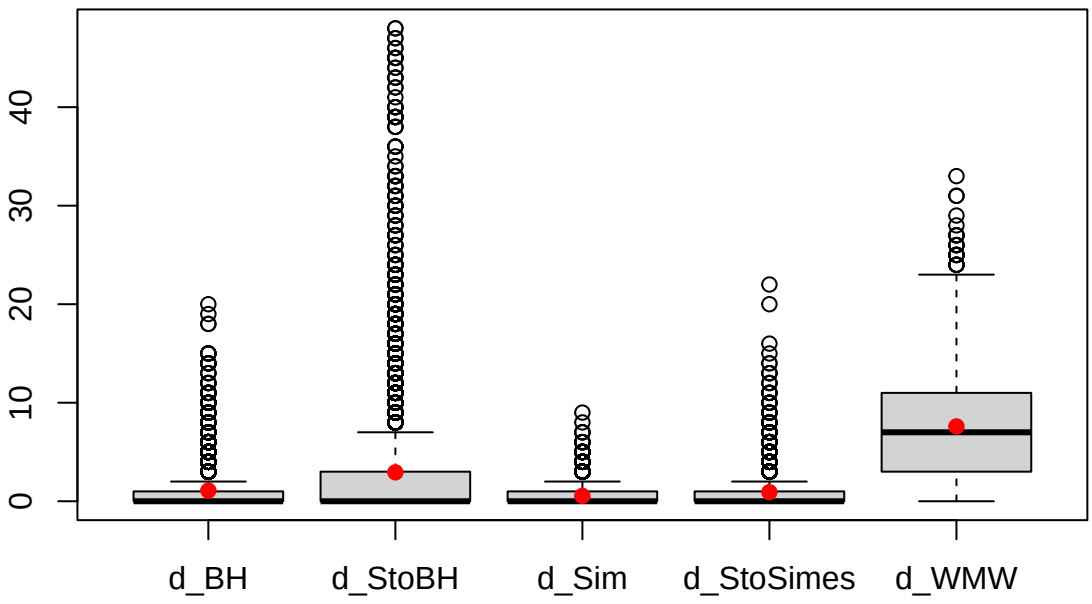

Digits | Number of discoveries with 23 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

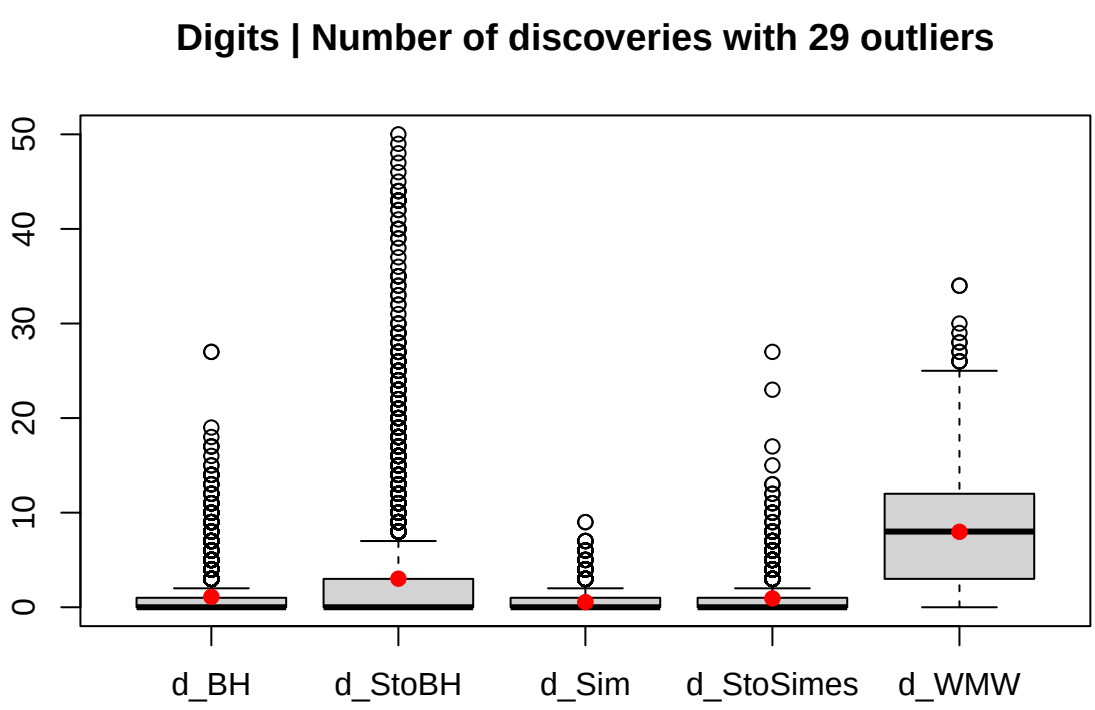

Digits | Number of discoveries with 24 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

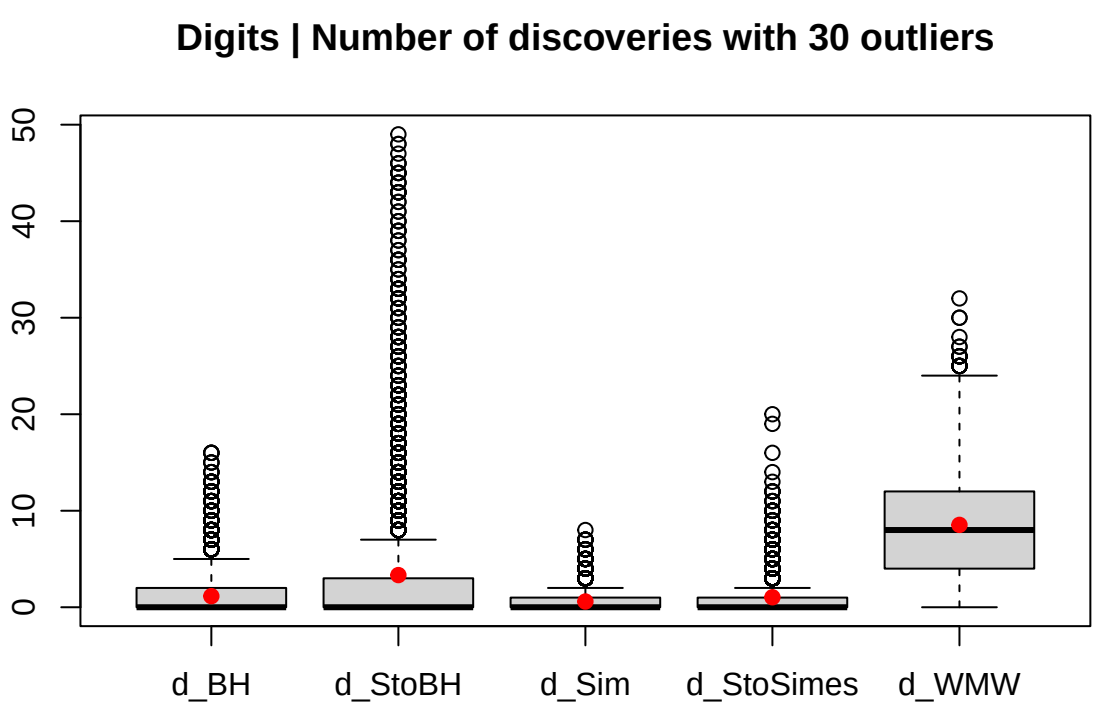

Digits | Number of discoveries with 25 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

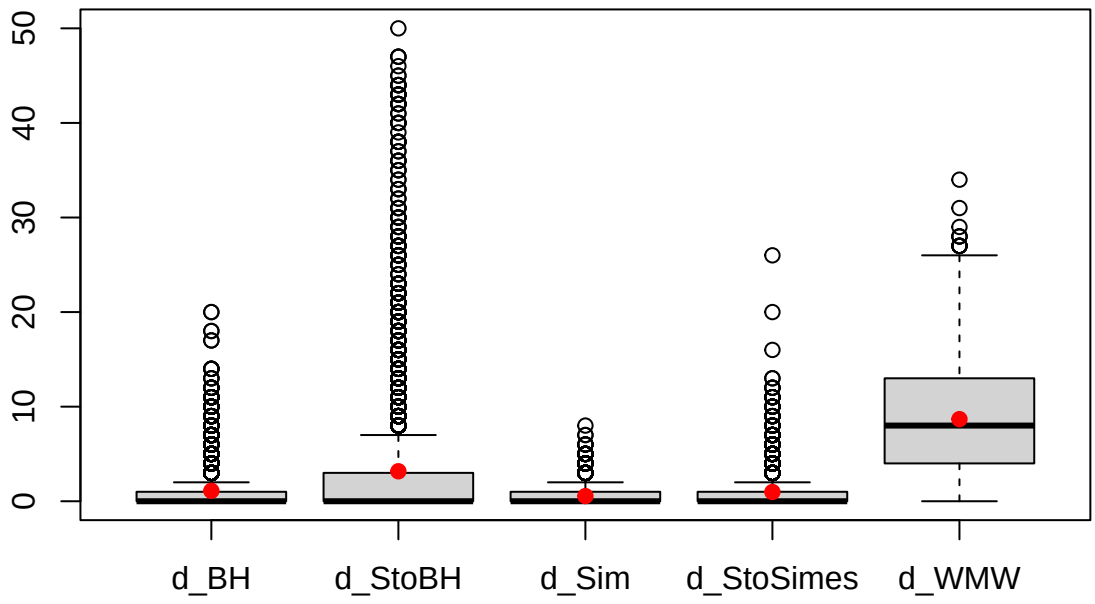

Digits | Number of discoveries with 26 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

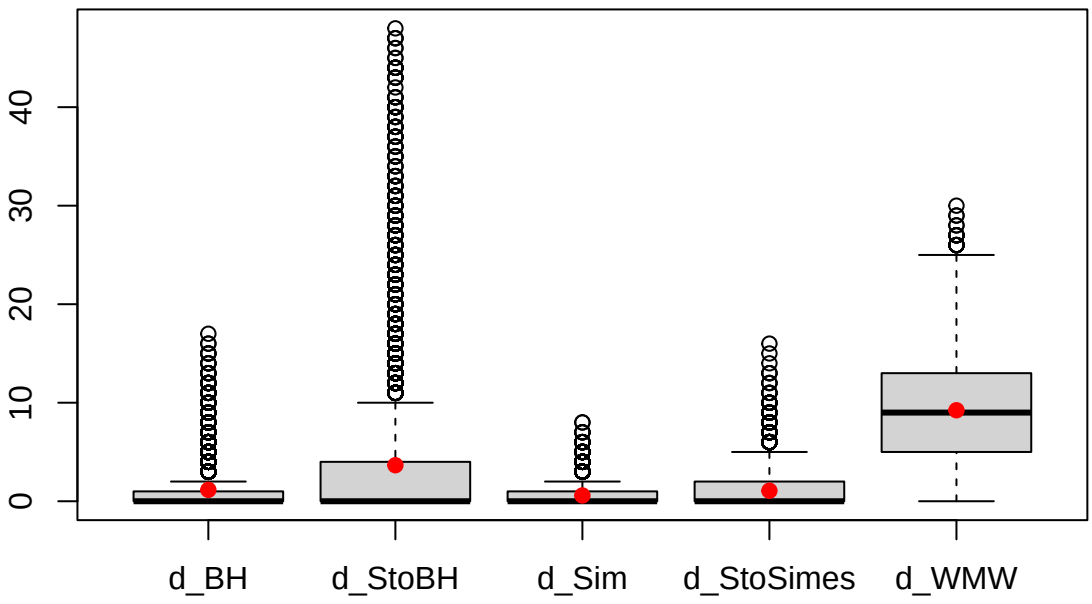

Digits | Number of discoveries with 27 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

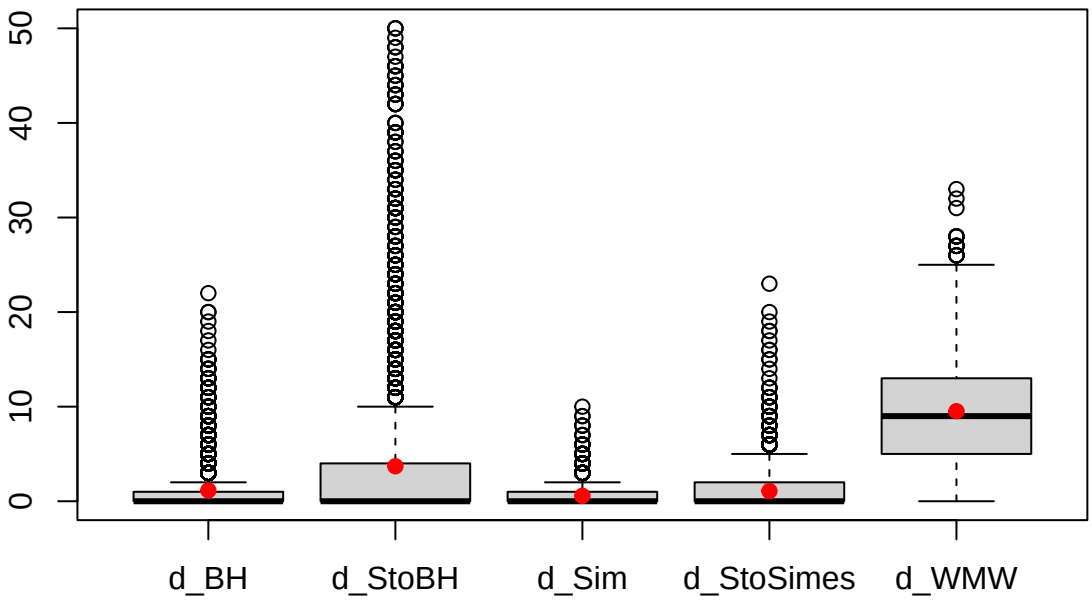

Digits | Number of discoveries with 28 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

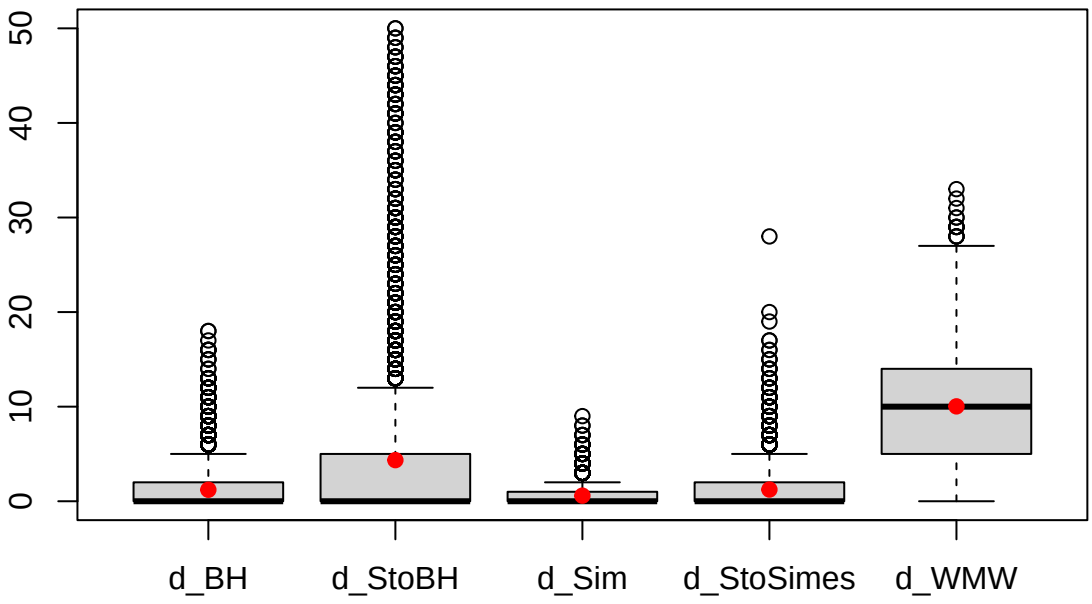

Digits | Number of discoveries with 29 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

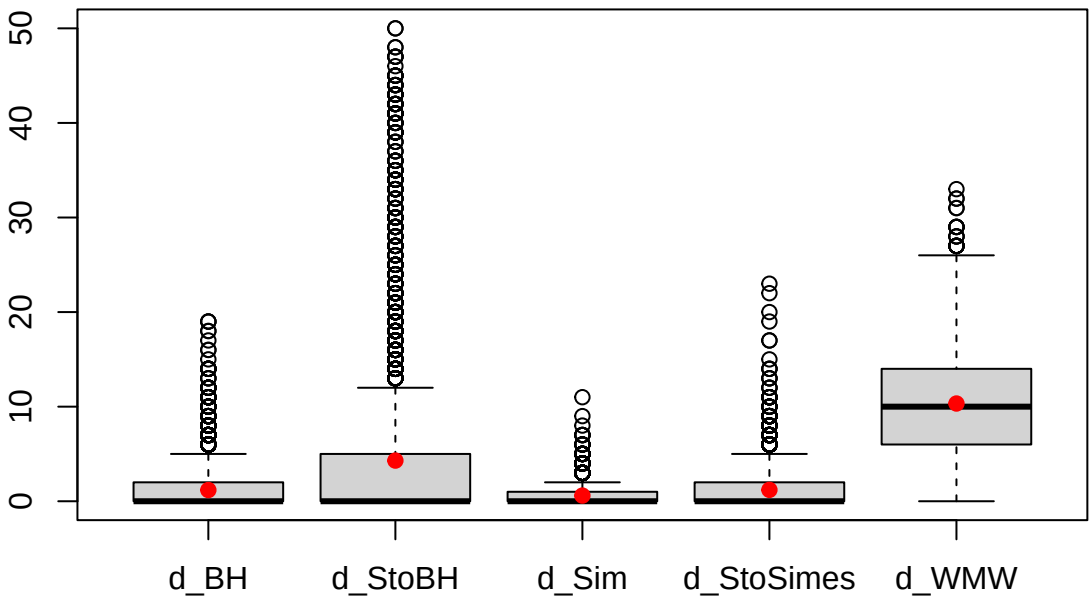

Digits | Number of discoveries with 30 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

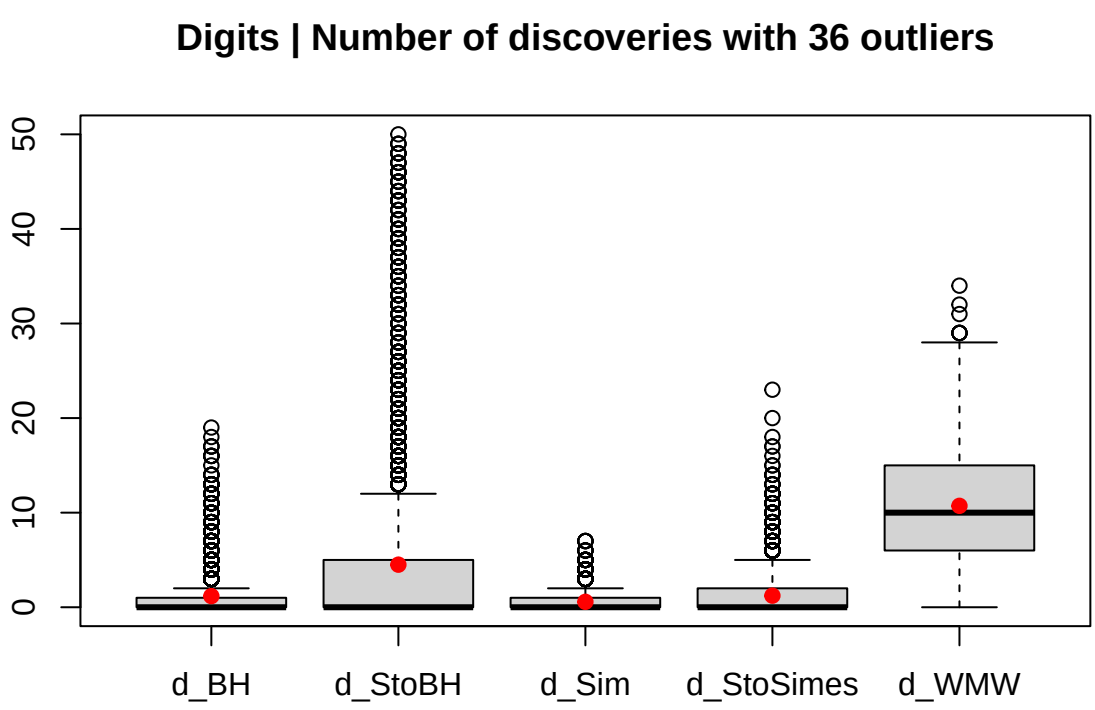

Digits | Number of discoveries with 31 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

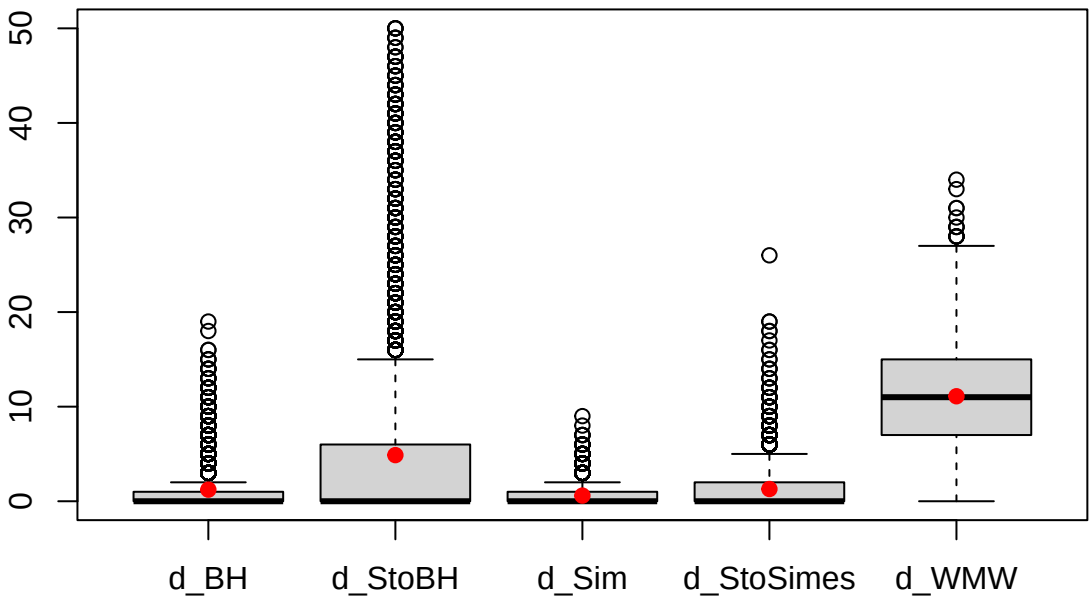

Digits | Number of discoveries with 32 outliers
0
10
20
30
40
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

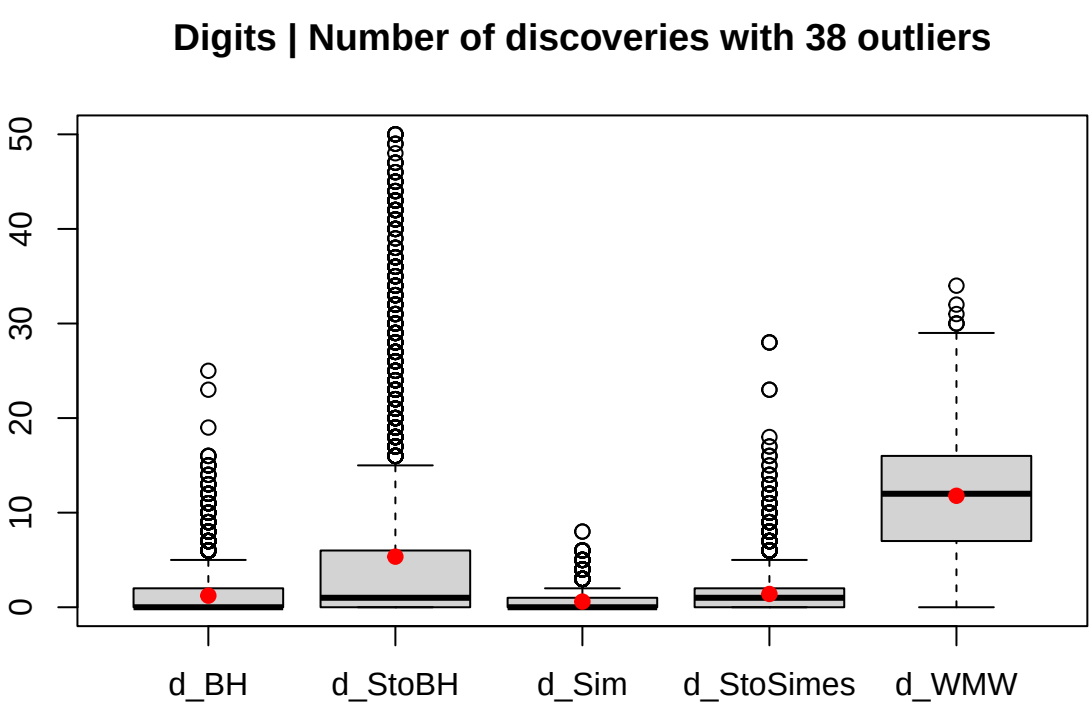

Digits | Number of discoveries with 33 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

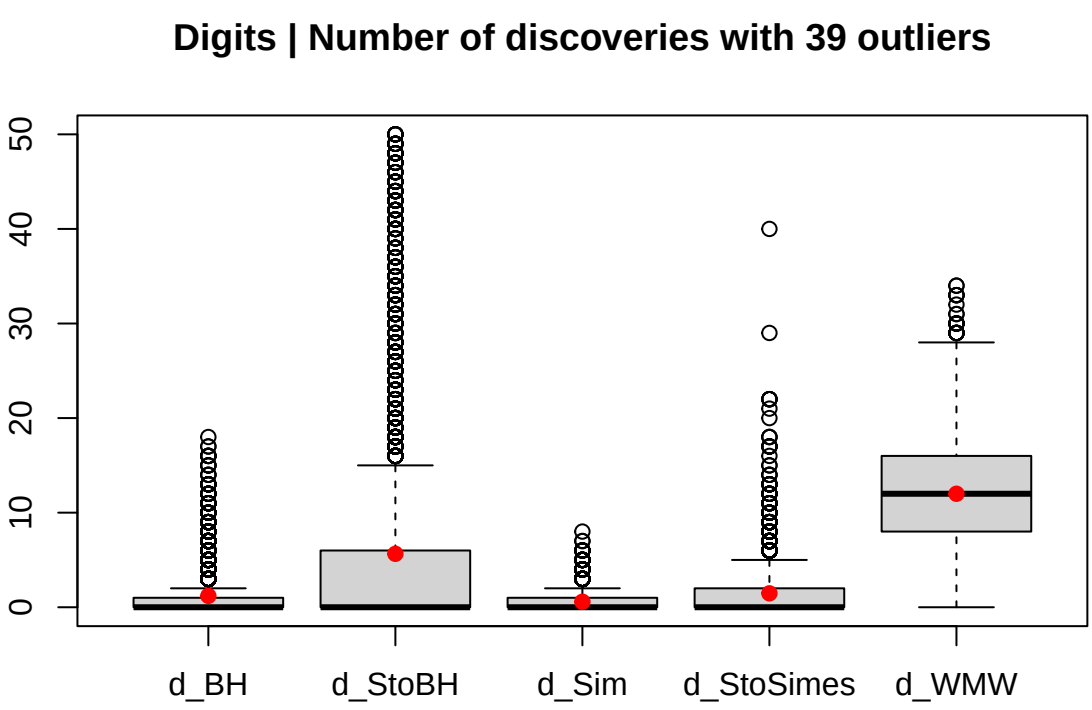

Digits | Number of discoveries with 34 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

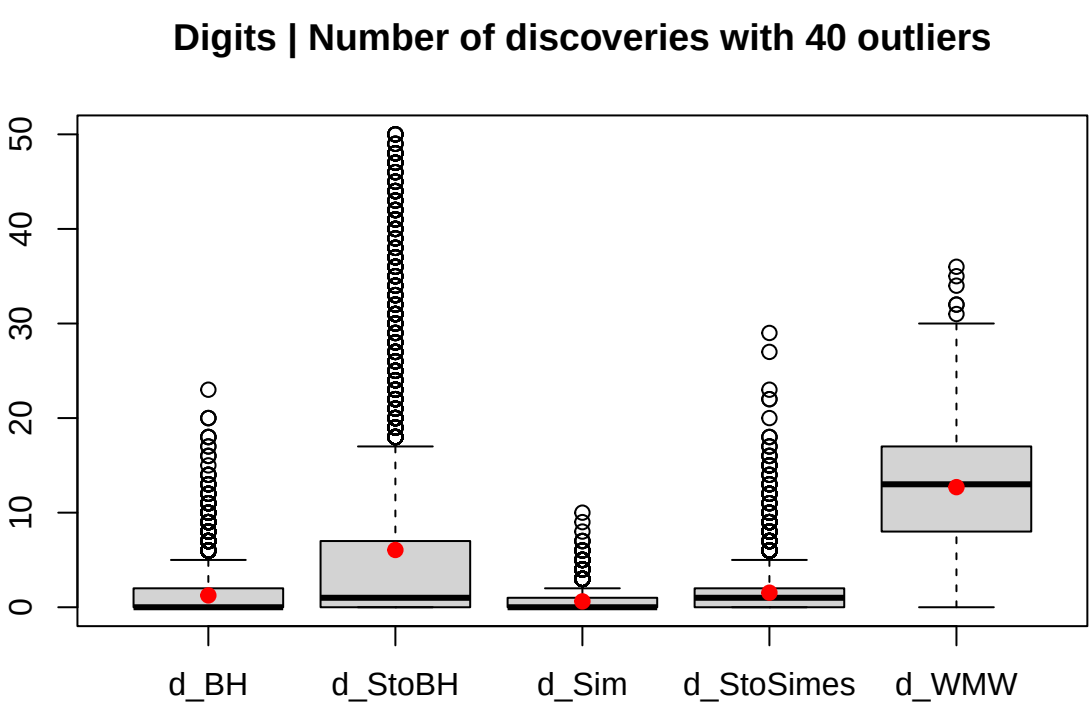

Digits | Number of discoveries with 35 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

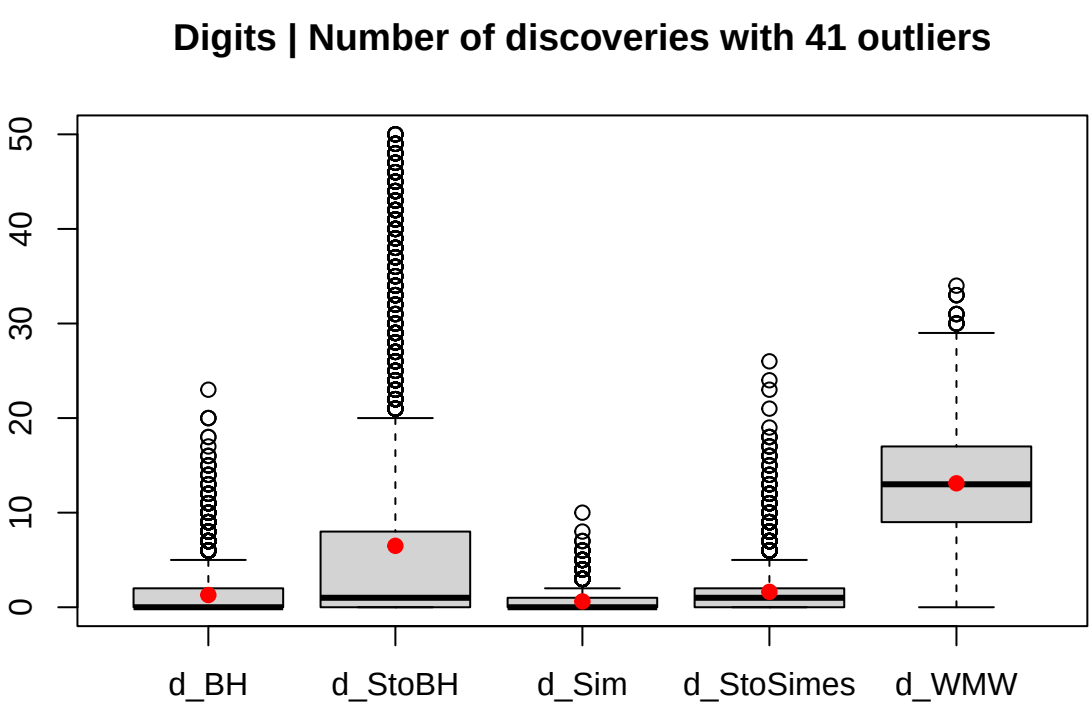

Digits | Number of discoveries with 36 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

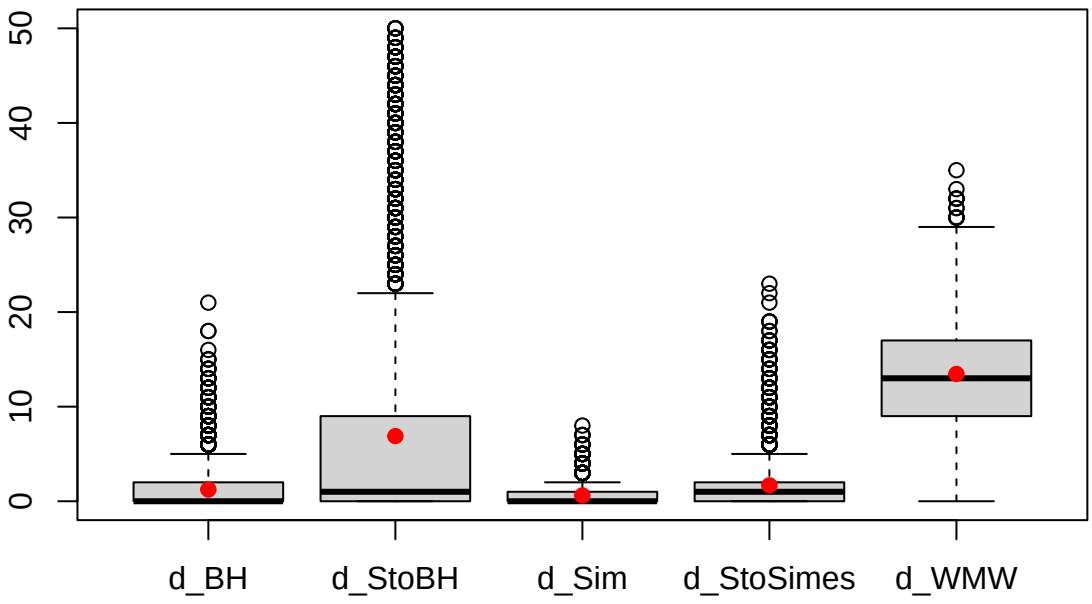

Digits | Number of discoveries with 37 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

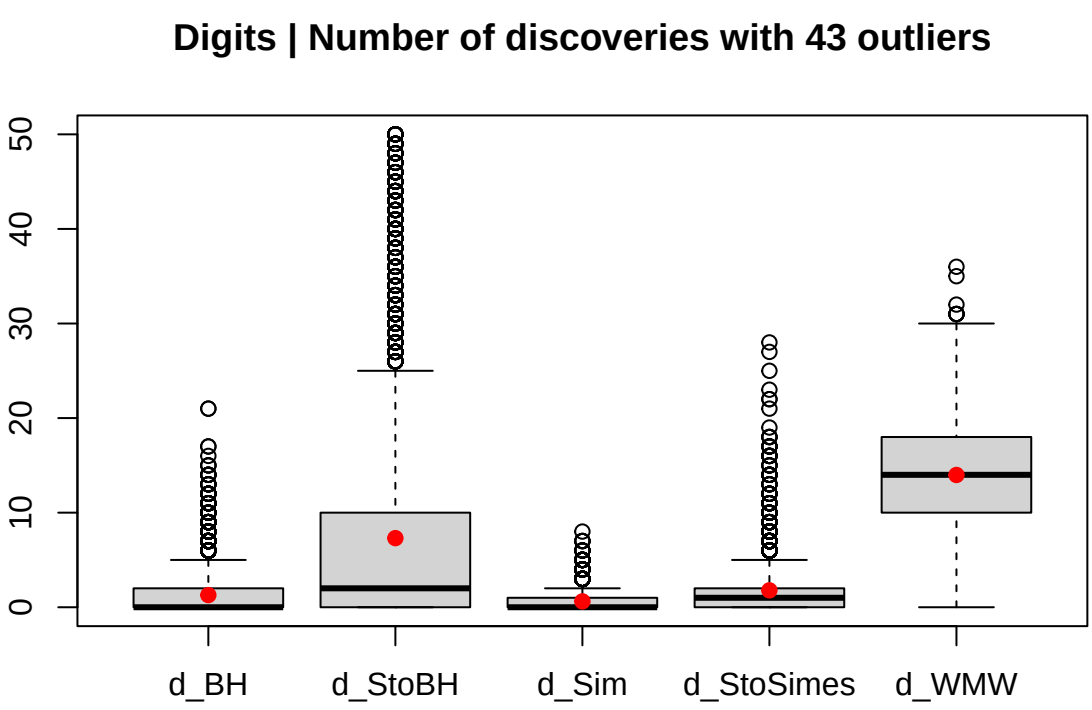

Digits | Number of discoveries with 38 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

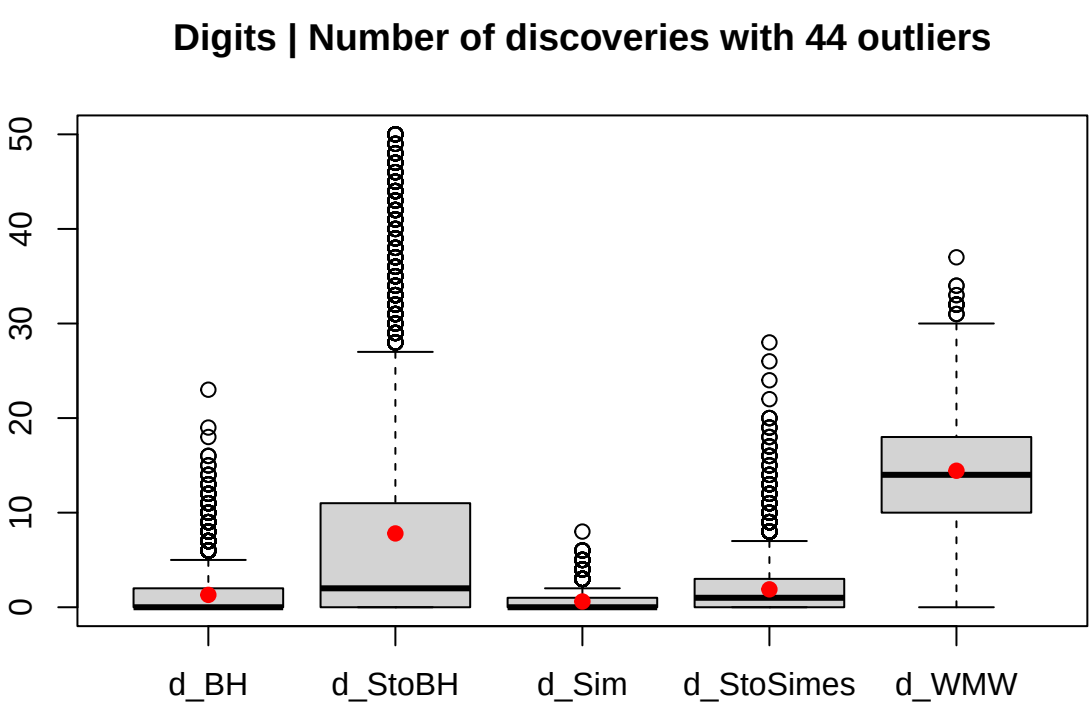

Digits | Number of discoveries with 39 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

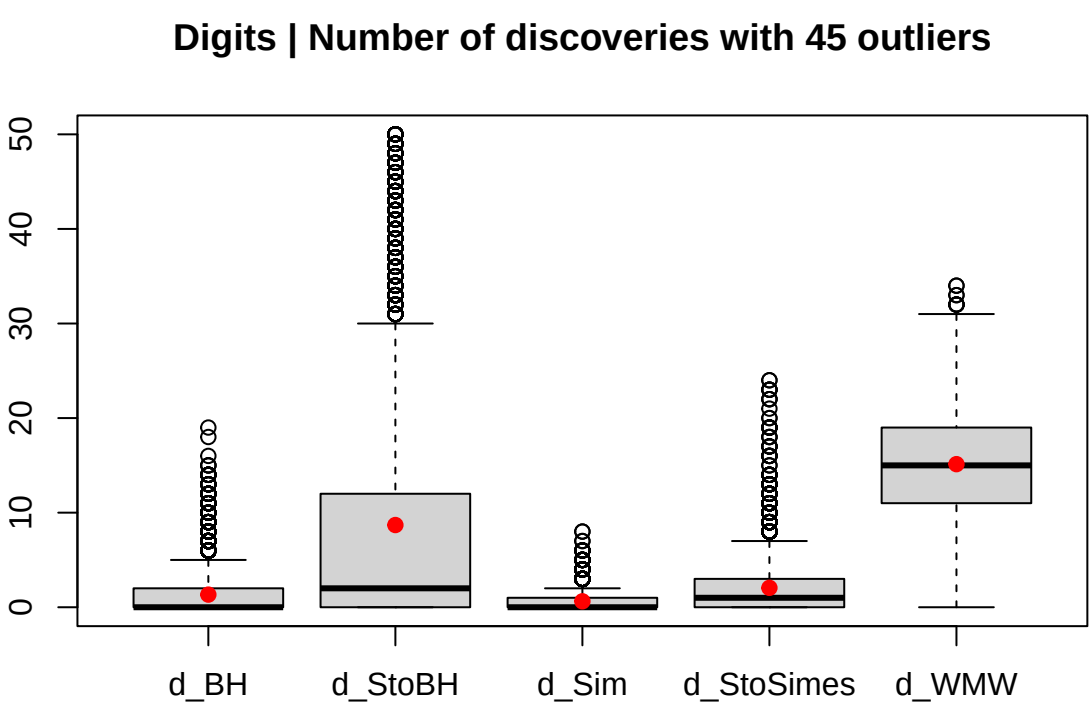

Digits | Number of discoveries with 40 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

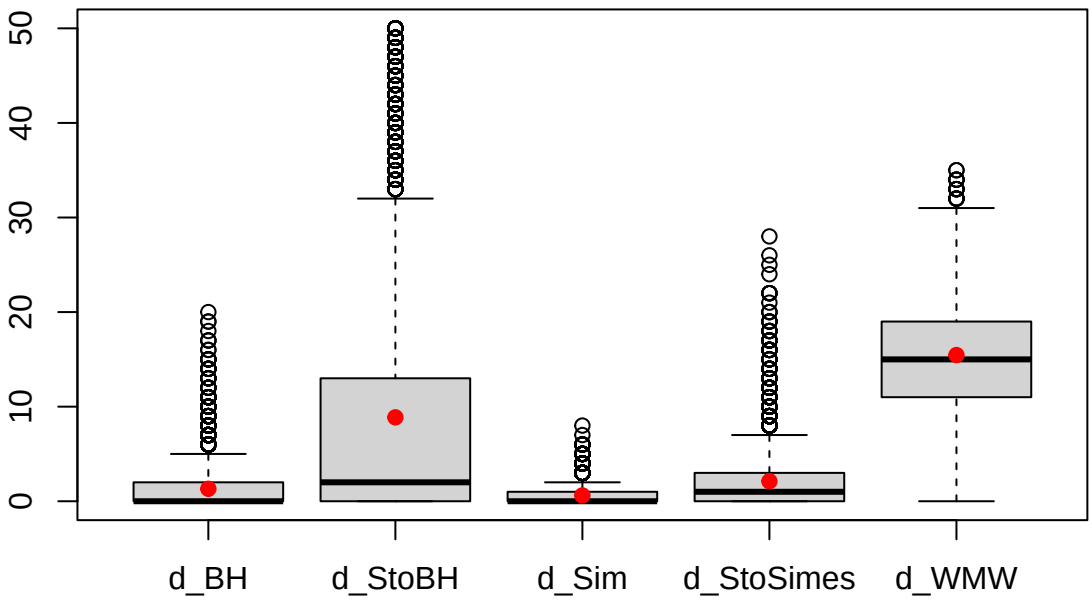

Digits | Number of discoveries with 41 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

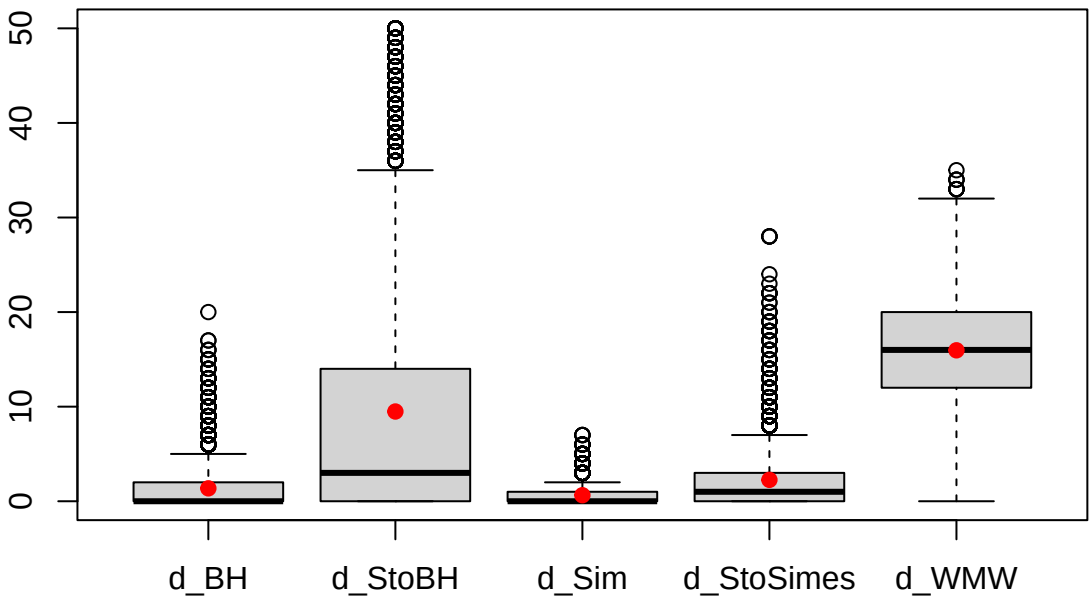

Digits | Number of discoveries with 42 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

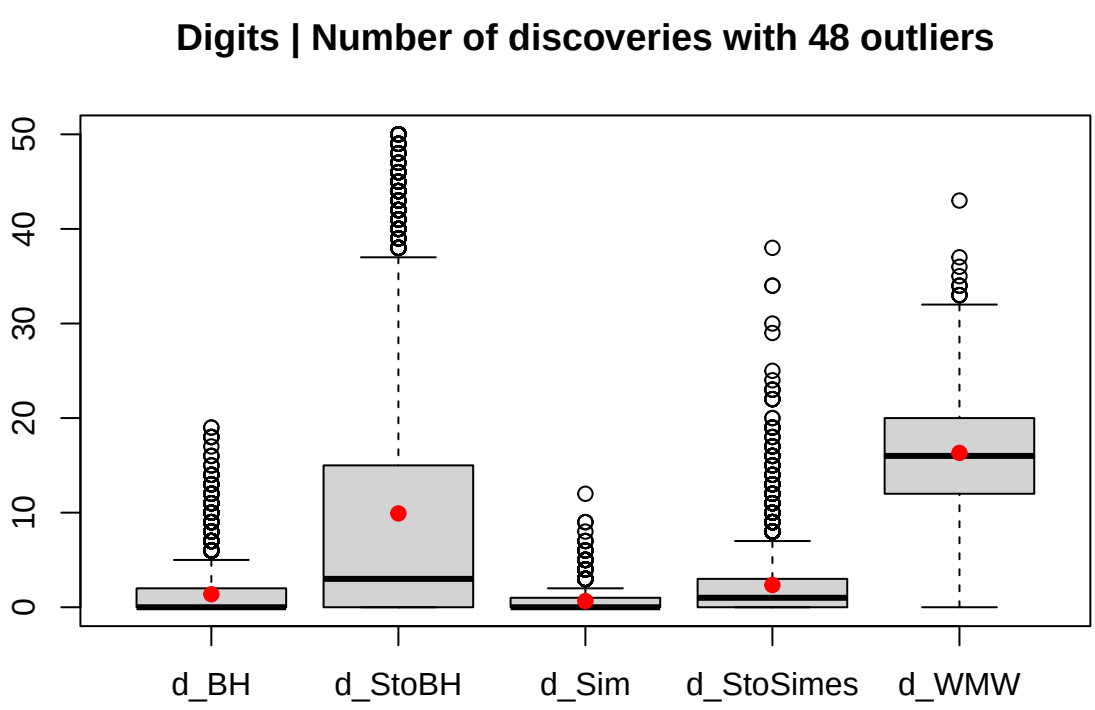

Digits | Number of discoveries with 43 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

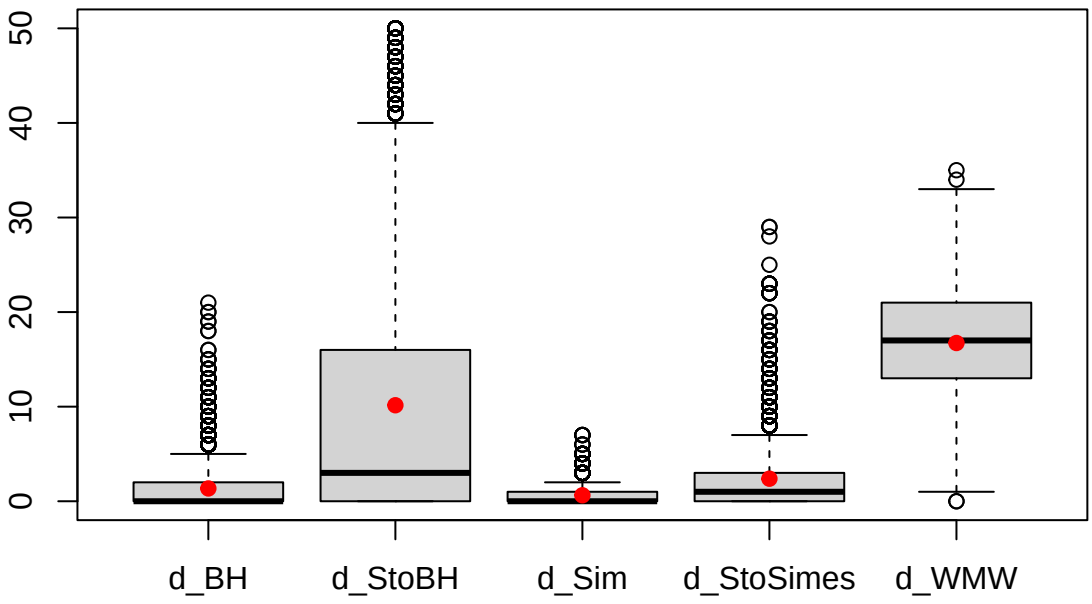

Digits | Number of discoveries with 44 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW

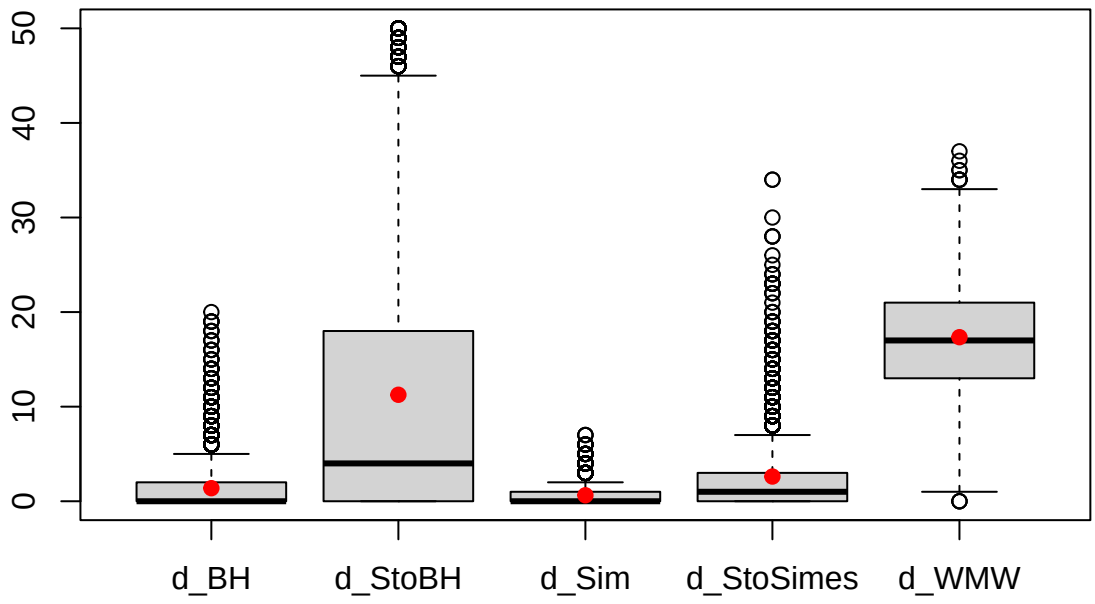

Digits | Number of discoveries with 45 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW


Digits | Number of discoveries with 46 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW


Digits | Number of discoveries with 47 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW


Digits | Number of discoveries with 48 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW


Digits | Number of discoveries with 49 outliers
0
10
20
30
40
50
d_BH
d_StoBH
d_Sim
d_StoSimes
d_WMW




```
resDigits_alln1_alpha02_k4 = results
save(resDigits_alln1_alpha02_k4,
     file="~/nout/trials/RealData/PowerStudy/New!/alpha0.2/DigitsOnly0.2/Lehmann2/resDigits_CORRECT_alln
```