Comparison between different local tests: Simes, Simes with Storey and Wilcoxon-Mann-Whitney using the natural outliers distribution

2023-07-24

The aim is to compare on real datasets the performance of three closed testing procedures, which respectively use Simes local test with and without Storey estimator for the proportion of true null hypotheses and Wilcoxon-Mann-Whitney local test. We will consider outlier population to be the set of observations tagged as "outlier" in the dataset of interest.

R functions and libraries

```
library(nout)
library(R.matlab)
library(isotree)
library(farff)
library(tictoc)
library(tidyverse)
library(doSNOW)
library(ggplot2)

compact_results = function(res){
  resT=as.data.frame(t(res))

  results = list()
  for(j in 1:length(n1s)){
    discoveries = as.data.frame(
      cbind("d_BH"=unlist(res[[j]][rownames(res[[j]])=="d_BH",]),
            "d_StoBH"=unlist(res[[j]][rownames(res[[j]])=="d_StoBH",]),
            "d_Sim"=unlist(res[[j]][rownames(res[[j]])=="d_Sim",]),
            "d_StoSimes"=unlist(res[[j]][rownames(res[[j]])=="d_StoSimes",]),
            "d_WMW"=unlist(res[[j]][rownames(res[[j]])=="d_WMW",])
            )
    )
    mean.discoveries = apply(discoveries, MARGIN = 2, FUN = mean)

    power.GlobalNull = as.data.frame(discoveries>0)
    mean.powerGlobalNull = apply(power.GlobalNull, MARGIN = 2, FUN = mean)

    out_identification = as.data.frame(
      cbind("out.identif_WMW"=
              unlist(res[[j]][rownames(res[[j]])=="outlier.identified_WMW",]),
            "out.identif_StoSimes"=
              unlist(res[[j]][rownames(res[[j]])=="outlier.identified_StoSimes",]),
            "out.identif_Simes"=
              unlist(res[[j]][rownames(res[[j]])=="outlier.identified_Simes",])
            )
    )
    mean.out_identification = apply(out_identification, MARGIN = 2, FUN = mean)
    mean.out_identification_pos = apply(out_identification>0, MARGIN = 2, FUN = mean)

    results[[j]] = list("discoveries" = discoveries,
                        "mean.discoveries" = mean.discoveries,
                        "power.GlobalNull" = power.GlobalNull,
                        "mean.powerGlobalNull" = mean.powerGlobalNull,
                        "out_identification" = out_identification,
                        "mean.out_identification" = mean.out_identification,
                        "mean.out_identification>0" = mean.out_identification_pos,
                        "pi.not" = res[[j]][rownames(res[[j]])=="pi.not",],
                        "uniques" = res[[j]][rownames(res[[j]])=="uniques",],
                        "n1" = res[[j]][rownames(res[[j]])=="n1",1],
                        "alpha" = res[[j]][rownames(res[[j]])=="alpha",1])
  }
  return(results)
}


TrainingIsoForest = function(l, dataset){

  tr_ind = sample(in_ind, size = l)
  tr = dataset[tr_ind,]
  isofo.model = isotree::isolation.forest(tr, ndim=ncol(dataset), ntrees=10, nthreads=1,
                            scoring_metric = "depth", output_score = TRUE)$model
  in_index2 = setdiff(in_ind, tr_ind)

  return(list("model"=isofo.model, "inlier_remaining" = in_index2))

}



CompareMethodNaturalOutliers = function(B, n1, n, out_ind, inlier_remaining, isofo.model, dataset){

  n0 = n-n1
  foreach(b = 1:B, .combine=cbind) %dopar% {
    if(n1==0){
      n0 = n
      N = n0 + m
      in_index3 = sample(inlier_remaining, size = N)
      cal_ind = in_index3[1:m]
      te_ind = in_index3[(m+1):N]
      cal = dataset[cal_ind,]
      te = dataset[te_ind,]
      S_cal = predict.isolation_forest(isofo.model, cal, type = "score")
      S_te = predict.isolation_forest(isofo.model, te, type = "score")

      d_WMW = nout::d_MannWhitney(S_Y = S_te, S_X = S_cal, alpha=alpha)
      d_Sim = nout::d_Simes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      StoSimes = nout::d_StoreySimes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      d_StoSimes = StoSimes$d
      pi.not = StoSimes$pi.not
      d_BH = nout::d_benjhoch(S_X = S_cal, S_Y = S_te, alpha = alpha)
      d_StoBH = nout::d_StoreyBH(S_X = S_cal, S_Y = S_te, alpha = alpha)
      uniques = length(unique(c(S_cal, S_te)))
      return(list("d_BH" = d_BH,
                  "d_StoBH" = d_StoBH,
                  "d_Sim" = d_Sim,
                  "d_StoSimes" = d_StoSimes,
                  "d_WMW" = d_WMW,
                  "outlier.identified_WMW" = 0,
                  "outlier.identified_Simes" = 0,
                  "outlier.identified_StoSimes" = 0,
                  "uniques" = uniques,
                  "n1" = n1,
                  "pi.not" = pi.not,
                  "alpha" = alpha))
    }

    else{
      N = n0 + m
      in_index3 = sample(inlier_remaining, size = N)
      cal_ind = in_index3[1:m]
      if(n0!=0)
        tein_ind = in_index3[(m+1):N]
      else
        tein_ind = NULL
      teout_ind = sample(out_ind, size = n1)
      cal = dataset[cal_ind,]
      te = dataset[c(tein_ind, teout_ind),]
      S_cal = predict.isolation_forest(isofo.model, cal, type = "score")
      S_te = predict.isolation_forest(isofo.model, te, type = "score")

      d_WMW = nout::d_MannWhitney(S_Y = S_te, S_X = S_cal, alpha=alpha)
      d_Sim = nout::d_Simes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      StoSimes = nout::d_StoreySimes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      d_StoSimes = StoSimes$d
      pi.not = StoSimes$pi.not
      d_BH = nout::d_benjhoch(S_X = S_cal, S_Y = S_te, alpha = alpha)
      d_StoBH = nout::d_StoreyBH(S_X = S_cal, S_Y = S_te, alpha = alpha)
      uniques = length(unique(c(S_cal, S_te)))

      # outlier identification with WMW
      conf.pval = sapply(1:n, function(j) (1+sum(S_cal >= S_te[j]))/(m+1))
      confvalid.pval = conf.pval<alpha
      confvalid.index = which(conf.pval<alpha)

      if(d_WMW>0){
        outlierTF = sapply(confvalid.index, function(h)
            nout::dselection_MannWhitney(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        outlier.identified_WMW = confvalid.index[as.logical(outlierTF)]
      }
      else  outlier.identified_WMW = NULL

      # outlier identification with Simes
      if(d_Sim>0){
        outlierTF = sapply(confvalid.index, function(h)
            nout::dselection_Simes(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        outlier.identified_Simes = confvalid.index[as.logical(outlierTF)]
      }
      else  outlier.identified_Simes = NULL

      # outlier identification with StoreySimes
      if(d_StoSimes>0){
        outlierTF = sapply(confvalid.index, function(h)
            nout::dselection_StoreySimes(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        outlier.identified_StoSimes = confvalid.index[as.logical(outlierTF)]
      }
      else  outlier.identified_StoSimes = NULL

      return(list("d_BH" = d_BH,
                  "d_StoBH" = d_StoBH,
                  "d_Sim" = d_Sim,
                  "d_StoSimes" = d_StoSimes,
                  "d_WMW" = d_WMW,
                  "outlier.identified_WMW" = length(outlier.identified_WMW),
                  "outlier.identified_Simes" = length(outlier.identified_Simes),
                  "outlier.identified_StoSimes" = length(outlier.identified_StoSimes),
                  "uniques" = uniques,
                  "n1" = n1,
                  "pi.not" = pi.not,
                  "alpha" = alpha))
    }
  }
}


estimatek = function(B, inlier_remaining, out_ind, isofo.model, dataset){
  ress = foreach(b = 1:B, .combine=c) %dopar% {
   inlier_ind = sample(inlier_remaining, size = 1)
   outlier_ind = sample(out_ind, size = 1)
   inlier = dataset[inlier_ind,]
   outlier = dataset[outlier_ind,]
   S_inlier = predict.isolation_forest(isofo.model, inlier, type = "score")
   S_outlier = predict.isolation_forest(isofo.model, outlier, type = "score")

   greater.logi = S_inlier<S_outlier

   return(greater.logi)
  }

  greater.prob = mean(ress)
  k=greater.prob/(1-greater.prob)
  return(k)
}
```

In the following we set the calibration set and the test set size, respectively l and m, so that the nominal level α is proportional to $\frac{m}{l+1}$. The train set size is equal to n and the number of iterations is $B = 10^4$.

Statlog (Shuttle) dataset

The dataset is available at http://odds.cs.stonybrook.edu/shuttle-dataset

```
set.seed(321)

# Initializing parameters
B = 10^4
m = 199
l = 199
n = 20
alpha = n/(l+1)
n1s = seq(from=0, to=n, by=1)

data = readMat("~/nout/trials/RealData/Datasets/Dataset shuttle/shuttle.mat")
dataset = cbind(data$X, data$y); colnames(dataset)[ncol(dataset)] = "y"
in_ind = which(dataset[,ncol(dataset)]==0)
out_ind = which(dataset[,ncol(dataset)]==1)

cluster <- makeCluster(parallel::detectCores())
registerDoSNOW(cluster)
clusterEvalQ(cluster, {list(library(isotree), library(nout))})
```

```
## [[1]]
## [[1]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[1]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[2]]
## [[2]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[2]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[3]]
## [[3]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[3]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[4]]
## [[4]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[4]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
```

```
clusterExport(cluster, list("n", "m", "l", "in_ind", "out_ind", "dataset", "alpha"))

tic()
modeltrain = TrainingIsoForest(l=l, dataset=dataset)
kest = estimatek(B=B, inlier_remaining=modeltrain$inlier_remaining,
          out_ind=out_ind, isofo.model=modeltrain$model, dataset=dataset)
res = lapply(1:length(n1s),
             function(j) CompareMethodNaturalOutliers(B=B, n1=n1s[j], n=n, dataset=dataset,
                               isofo.model=modeltrain$model,
                               out_ind=out_ind,
                               inlier_remaining=modeltrain$inlier_remaining))
toc()
```

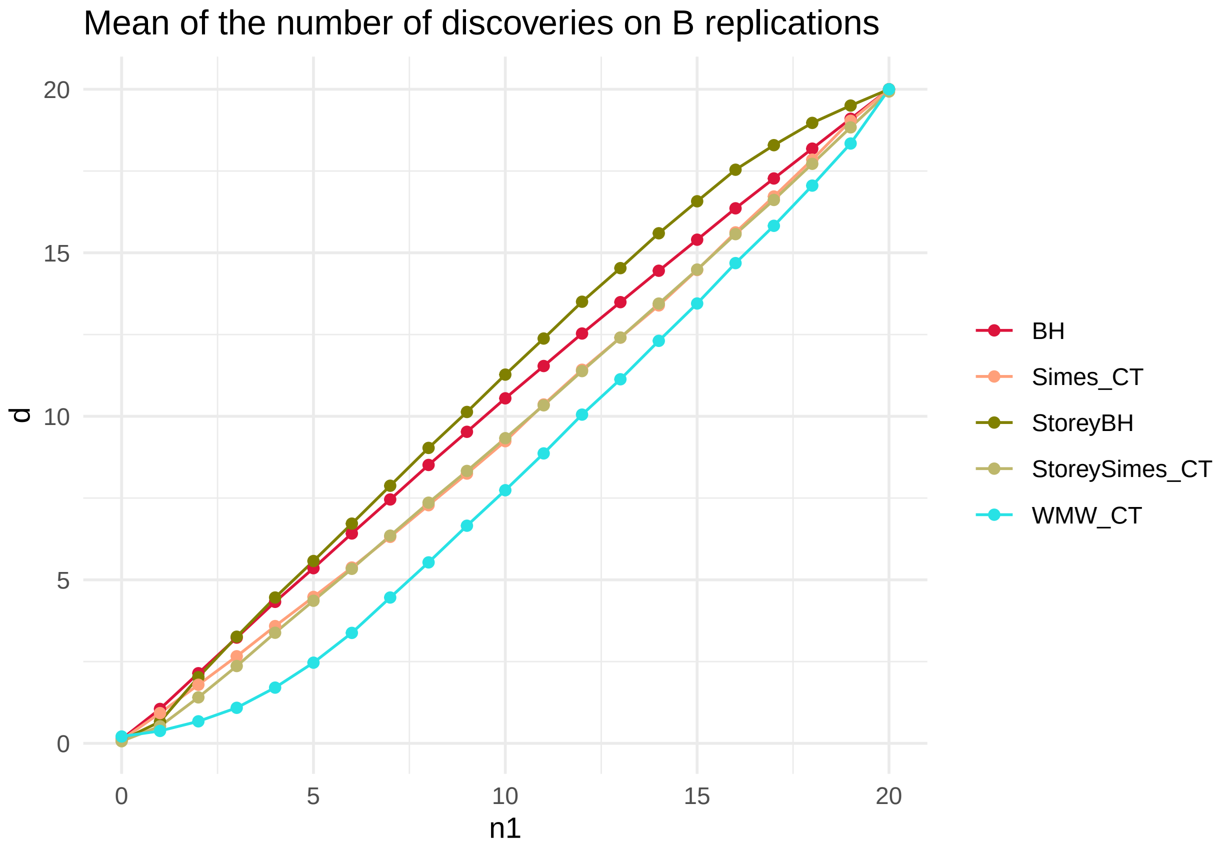
```
## 9537.89 sec elapsed
```

```
stopCluster(cluster)

kest
```

```
## [1] 525.3158
```

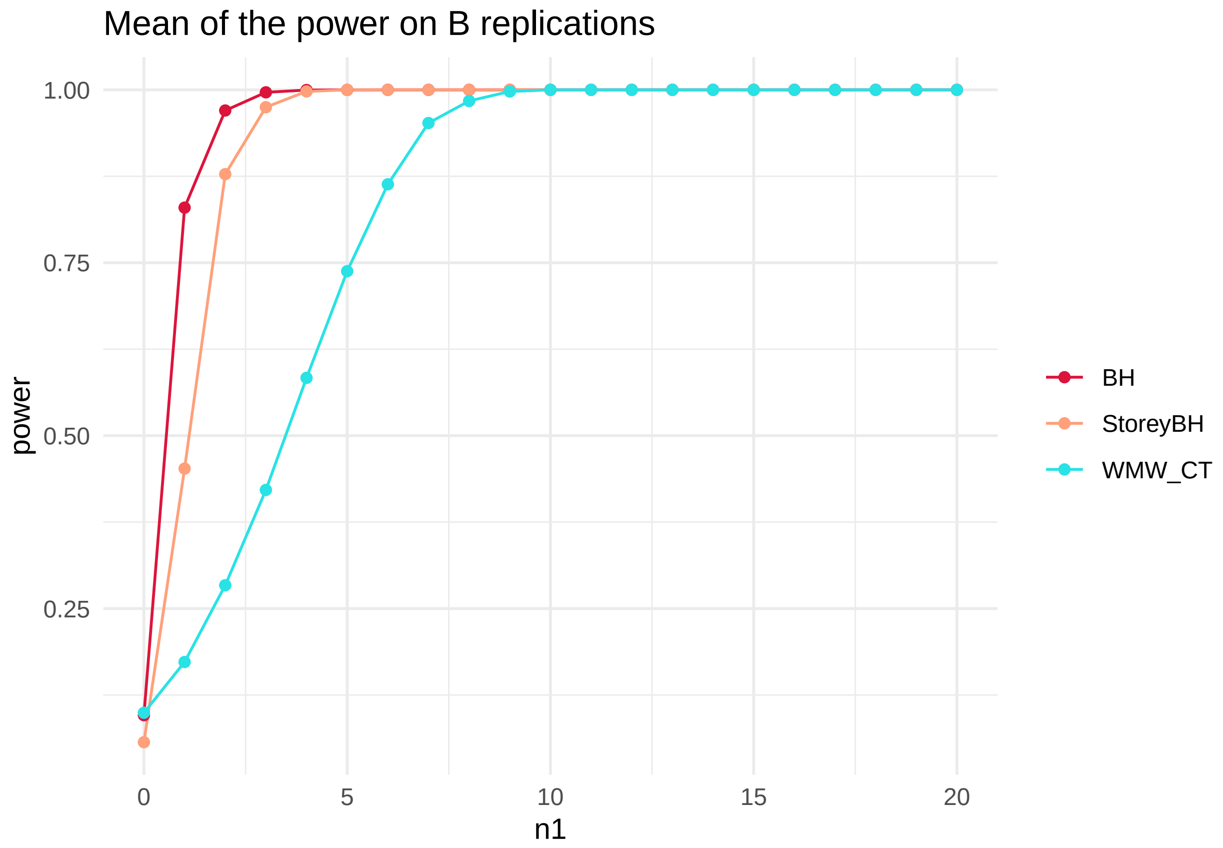
```
results = compact_results(res)

d_BH = vector()
d_StoBH = vector()
d_Sim = vector()
d_StoSimes = vector()
d_WMW = vector()

pow_BH = vector()
pow_StoBH = vector()
pow_Sim = vector()
pow_StoSimes = vector()
pow_WMW = vector()

for(j in 1:length(n1s)){
  d_BH[j] = results[[j]]$mean.discoveries[1]
  d_StoBH[j] = results[[j]]$mean.discoveries[2]
  d_Sim[j] = results[[j]]$mean.discoveries[3]
  d_StoSimes[j] = results[[j]]$mean.discoveries[4]
  d_WMW[j] = results[[j]]$mean.discoveries[5]

  pow_BH[j] = results[[j]]$mean.powerGlobalNull[1]
  pow_StoBH[j] = results[[j]]$mean.powerGlobalNull[2]
  pow_Sim[j] = results[[j]]$mean.powerGlobalNull[3]
  pow_StoSimes[j] = results[[j]]$mean.powerGlobalNull[4]
  pow_WMW[j] = results[[j]]$mean.powerGlobalNull[5]

}
```

```
# Plot discoveries
df <- data.frame(
  x = n1s,
  BH = d_BH,
  StoreyBH = d_StoBH,
  Simes_CT = d_Sim,
  StoreySimes_CT = d_StoSimes,
  WMW_CT = d_WMW
)
df_long <- tidyr::pivot_longer(df, cols = -x, names_to = "group", values_to = "y")

ggplot(df_long, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#DC143C", "#FFA07A", "#808000", "#BDB76B", 5)) +
  labs(x = "n1", y = "d", title = "Mean of the number of discoveries on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```



```
# Plot power
dfpower <- data.frame(
  x = n1s,
  BH = pow_BH,
  StoreyBH = pow_StoBH,
  WMW_CT = pow_WMW
)
```

```
df_long_power <- tidyr::pivot_longer(dfpower, cols = -x, names_to = "group", values_to = "y")

# Plot the lines with different colors and legends
ggplot(df_long_power, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#DC143C","#FFA07A",5)) +
  labs(x = "n1", y = "power", title = "Mean of the power on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```



```
outlier.identification = list()
for(i in 1:length(n1s)){
  outlier.identification[[i]] = matrix(nrow = 3, ncol = 4)
  rownames(outlier.identification[[i]]) = c("WMW", "Simes", "StoSimes")
  colnames(outlier.identification[[i]]) = c("mean.out.identif", "%successful.identification",
                                            "mean.d", "mean.d>0(power)")
  outlier.identification[[i]][,1] = apply(
    results[[i]][["out_identification"]], MARGIN = 2, FUN = mean)
  outlier.identification[[i]][,2] = apply(
    results[[i]][["out_identification"]]>0, MARGIN = 2, FUN = mean)
  outlier.identification[[i]][,3] = results[[i]]$mean.discoveries[c(3,4,5)]
  outlier.identification[[i]][,4] = results[[i]]$mean.powerGlobalNull[c(3,4,5)]
}

for(i in 1:length(n1s)){
```

```r
  cat("\n")
  cat(paste("n1=", n1s[i]))
  print(outlier.identification[[i]])
}
```

```
##
## n1= 0         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                         0                          0 0.1065          0.0958
## Simes                       0                          0 0.0652          0.0568
## StoSimes                    0                          0 0.2097          0.0993
##
## n1= 1         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    0.6525                     0.1725 0.9257          0.8297
## Simes                  0.0000                     0.0000 0.5210          0.4524
## StoSimes               0.9023                     0.8159 0.3799          0.1727
##
## n1= 2         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    1.2524                     0.2837 1.7915          0.9701
## Simes                  0.0000                     0.0000 1.4060          0.8781
## StoSimes               1.6944                     0.9034 0.6733          0.2837
##
## n1= 3         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    2.1374                     0.4215 2.6631          0.9964
## Simes                  0.0000                     0.0000 2.3645          0.9749
## StoSimes               2.4905                     0.9274 1.0872          0.4215
##
## n1= 4         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    3.3822                     0.5835 3.5858          0.9997
## Simes                  0.0000                     0.0000 3.3820          0.9977
## StoSimes               3.3135                     0.9328 1.7041          0.5835
##
## n1= 5         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    4.8110                     0.7378 4.4760          0.9999
## Simes                  0.0000                     0.0000 4.3607          0.9999
## StoSimes               4.1029                     0.9378 2.4665          0.7378
##
## n1= 6         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    6.3001                     0.8633 5.3765          1.0000
## Simes                  0.0000                     0.0000 5.3363          1.0000
## StoSimes               4.9089                     0.9422 3.3776          0.8634
##
## n1= 7         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    7.7612                     0.9520 6.3137           1.000
## Simes                  0.0000                     0.0000 6.3508           1.000
## StoSimes               5.7161                     0.9399 4.4594           0.952
##
## n1= 8         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    8.8883                     0.9839 7.2821          1.0000
## Simes                  0.0000                     0.0000 7.3589          1.0000
## StoSimes               6.5359                     0.9426 5.5334          0.9839
##
## n1= 9         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    9.8759                     0.9976 8.2481          1.0000
## Simes                  0.0000                     0.0000 8.3289          1.0000
## StoSimes           7.3425                     0.9437 6.6546          0.9976
##
## n1= 10         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW               10.8646                     1.0000 9.2396               1
## Simes              0.0000                     0.0000 9.3314               1
## StoSimes           8.2677                     0.9463 7.7402               1
##
## n1= 11         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               11.7098                     1.0000 10.3615               1
## Simes              0.0000                     0.0000 10.3368               1
## StoSimes           9.8222                     0.9749  8.8658               1
##
## n1= 12         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               12.6266                     1.0000 11.4238               1
## Simes              0.0000                     0.0000 11.3823               1
## StoSimes          11.1099                     0.9921 10.0516               1
##
## n1= 13         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               13.5580                      1.000 12.4061               1
## Simes              0.0000                      0.000 12.4074               1
## StoSimes          12.1398                      0.998 11.1325               1
##
## n1= 14         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               14.4388                      1.000 13.3907               1
## Simes              0.0000                      0.000 13.4468               1
## StoSimes          13.1088                      0.999 12.3069               1
##
## n1= 15         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               15.2866                     1.0000 14.4738               1
## Simes              0.0000                     0.0000 14.4865               1
## StoSimes          14.2920                     0.9991 13.4496               1
##
## n1= 16         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               16.3072                     1.0000 15.6237               1
## Simes              0.0000                     0.0000 15.5705               1
## StoSimes          15.5282                     0.9999 14.6841               1
##
## n1= 17         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               17.1804                          1 16.7219               1
## Simes              0.0000                          0 16.6158               1
## StoSimes          16.6464                          1 15.8248               1
##
## n1= 18         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               18.0949                          1 17.8474               1
## Simes              0.0000                          0 17.7200               1
## StoSimes          17.8007                          1 17.0546               1
##
## n1= 19         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               18.9585                          1 19.0344               1
## Simes              0.0000                          0 18.8346               1
## StoSimes          19.0252                          1 18.3410               1
##
## n1= 20         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               19.8324                          1 19.9997               1
## Simes              0.0000                          0 19.9308               1
## StoSimes          19.9995                          1 20.0000               1
```

```
resShuttle0.1 = list("raw.res"=res,
                    "k.est" = kest,
                    "compact.results" = results,
                    "outlier.identification" = outlier.identification)
save(resShuttle0.1, file="~/nout/trials/RealData/PowerStudy/FinalSimu/Shuttle/resShuttle0.1")
```