Comparison between different local tests: Simes, Simes with Storey and Wilcoxon-Mann-Whitney using the natural outliers distribution

2023-07-29

The aim is to compare on real datasets the performance of three closed testing procedures, which respectively use Simes local test with and without Storey estimator for the proportion of true null hypotheses and Wilcoxon-Mann-Whitney local test. We will consider outlier population to be the set of observations tagged as "outlier" in the dataset of interest.

R functions and libraries

```
library(nout)
library(R.matlab)
library(isotree)
library(farff)
library(tictoc)
library(tidyverse)
library(doSNOW)
library(ggplot2)
library(hommel)

compact_results = function(res){
  resT=as.data.frame(t(res))

  results = list()
  for(j in 1:length(n1s)){
    lb.d = as.data.frame(
      cbind("d_BH"=unlist(res[[j]][rownames(res[[j]])=="d_BH",]),
            "d_StoBH"=unlist(res[[j]][rownames(res[[j]])=="d_StoBH",]),
            "d_Sim"=unlist(res[[j]][rownames(res[[j]])=="d_Sim",]),
            "d_StoSimes"=unlist(res[[j]][rownames(res[[j]])=="d_StoSimes",]),
            "d_WMW"=unlist(res[[j]][rownames(res[[j]])=="d_WMW",])
            )
    )
    mean.lb.d = apply(lb.d, MARGIN = 2, FUN = mean)

    power.GlobalNull = as.data.frame(lb.d>0)
    mean.powerGlobalNull = apply(power.GlobalNull, MARGIN = 2, FUN = mean)

    n.disc = as.data.frame(
      cbind("n.disc.Simes" = unlist(res[[j]][rownames(res[[j]])=="n.disc.Simes",]),
            "n.disc.Simes2" = unlist(res[[j]][rownames(res[[j]])=="n.disc.Simes2",]),
             "n.disc.StoSimes" = unlist(res[[j]][rownames(res[[j]])=="n.disc.StoSimes",]),
            "n.disc.WMW" = unlist(res[[j]][rownames(res[[j]])=="n.disc.WMW",]),
            "n.disc.WMW.cpp" = unlist(res[[j]][rownames(res[[j]])=="n.disc.WMW.cpp",])
            )
    )
```

```
    mean.n.disc = apply(n.disc, MARGIN = 2, FUN = mean)
    #mean.n.disc_pos = apply(n.disc>0, MARGIN = 2, FUN = mean)

    results[[j]] = list("lb.d" = lb.d,
                        "mean.lb.d" = mean.lb.d,
                        "power.GlobalNull" = power.GlobalNull,
                        "mean.powerGlobalNull" = mean.powerGlobalNull,
                        "n.disc" = n.disc,
                        "mean.n.disc" = mean.n.disc,
                        #"mean.n.disc>0" = mean.n.disc_pos,
                        "pi.not" = res[[j]][rownames(res[[j]])=="pi.not",],
                        "uniques" = res[[j]][rownames(res[[j]])=="uniques",],
                        "n1" = res[[j]][rownames(res[[j]])=="n1",1],
                        "alpha" = res[[j]][rownames(res[[j]])=="alpha",1])
  }
  return(results)
}


TrainingIsoForest = function(l, dataset){

  tr_ind = sample(in_ind, size = l)
  tr = dataset[tr_ind,]
  isofo.model = isotree::isolation.forest(tr, ndim=ncol(dataset), ntrees=10, nthreads=1,
                            scoring_metric = "depth", output_score = TRUE)$model
  in_index2 = setdiff(in_ind, tr_ind)

  return(list("model"=isofo.model, "inlier_remaining" = in_index2))

}



CompareMethodNaturalOutliers = function(B, n1, n, out_ind, inlier_remaining, isofo.model, dataset){

  n0 = n-n1
  foreach(b = 1:B, .combine=cbind) %dopar% {
    if(n1==0){
      N = n0 + m
      in_index3 = sample(inlier_remaining, size = N)
      cal_ind = in_index3[1:m]
      te_ind = in_index3[(m+1):N]
      cal = dataset[cal_ind,]
      te = dataset[te_ind,]
      S_cal = predict.isolation_forest(isofo.model, cal, type = "score")
      S_te = predict.isolation_forest(isofo.model, te, type = "score")

      d_WMW = nout::d_MannWhitney(S_Y = S_te, S_X = S_cal, alpha=alpha)
      d_Sim = nout::d_Simes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      StoSimes = nout::d_StoreySimes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      d_StoSimes = StoSimes$d
      pi.not = StoSimes$pi.not
      d_BH = nout::d_benjhoch(S_X = S_cal, S_Y = S_te, alpha = alpha)
```

```
  d_StoBH = nout::d_StoreyBH(S_X = S_cal, S_Y = S_te, alpha = alpha)
  uniques = length(unique(c(S_cal, S_te)))
  return(list("d_BH" = d_BH,
              "d_StoBH" = d_StoBH,
              "d_Sim" = d_Sim,
              "n.disc.Simes" = 0,
              "n.disc.Simes2" = 0,
              "d_StoSimes" = d_StoSimes,
              "n.disc.StoSimes" = 0,
              "d_WMW" = d_WMW,
              "n.disc.WMW" = 0,
              "n.disc.WMW.cpp" = 0,
              "uniques" = uniques,
              "n1" = n1,
              "pi.not" = pi.not,
              "alpha" = alpha))
}

else{
  N = n0 + m
  in_index3 = sample(inlier_remaining, size = N)
  cal_ind = in_index3[1:m]
  if(n0!=0)
    tein_ind = in_index3[(m+1):N]
  else
    tein_ind = NULL
  teout_ind = sample(out_ind, size = n1)
  cal = dataset[cal_ind,]
  te = dataset[c(tein_ind, teout_ind),]
  S_cal = predict.isolation_forest(isofo.model, cal, type = "score")
  S_te = predict.isolation_forest(isofo.model, te, type = "score")

  d_WMW = nout::d_MannWhitney(S_Y = S_te, S_X = S_cal, alpha=alpha)
  d_Sim = nout::d_Simes(S_X = S_cal, S_Y = S_te, alpha = alpha)
  StoSimes = nout::d_StoreySimes(S_X = S_cal, S_Y = S_te, alpha = alpha)
  d_StoSimes = StoSimes$d
  pi.not = StoSimes$pi.not
  d_BH = nout::d_benjhoch(S_X = S_cal, S_Y = S_te, alpha = alpha)
  d_StoBH = nout::d_StoreyBH(S_X = S_cal, S_Y = S_te, alpha = alpha)
  uniques = length(unique(c(S_cal, S_te)))

  # outlier identification with WMW
  conf.pval = sapply(1:n, function(j) (1+sum(S_cal >= S_te[j]))/(m+1))
  confvalid.pval = conf.pval<alpha
  confvalid.index = which(conf.pval<alpha)

  n.disc.WMW.cpp=0
  n.disc.WMW=0
  if(d_WMW>0 & length(confvalid.index)!=0){
    outlierTF.WMW.cpp = sapply(confvalid.index, function(h)
      nout::dselection_MannWhitney(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
    #outlier.identified_MannWhitney = confvalid.index[as.logical(outlierTF)]
    n.disc.WMW.cpp = sum(outlierTF.WMW.cpp)
```

```
        outlierTF.WMW = sapply(confvalid.index, function(h)
          nout::dselection.prova_MannWhitney(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        n.disc.WMW = sum(outlierTF.WMW)
      }

      # outlier identification with Simes
      n.disc.Simes=0
      n.disc.Simes2=0
      if(d_Sim>0 & length(confvalid.index)!=0){
        outlierTF.Simes = sapply(confvalid.index, function(h)
            nout::dselection_Simes(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        #outlier.identified_Simes = confvalid.index[as.logical(outlierTF)]
        n.disc.Simes = sum(outlierTF.Simes)
        p = hommel(conf.pval)
        n.disc.Simes2 = sum(p@adjusted <= alpha)
      }

      # outlier identification with StoreySimes
      n.disc.StoSimes=0
      if(d_StoSimes>0 & length(confvalid.index)!=0){
        outlierTF.StoSim = sapply(confvalid.index, function(h)
          nout::dselection_StoreySimes(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        #outlier.identified_StoSimes = confvalid.index[as.logical(outlierTFStoSim)]
        n.disc.StoSimes = sum(outlierTF.StoSim)
      }

      return(list("d_BH" = d_BH,
                  "d_StoBH" = d_StoBH,
                  "d_Sim" = d_Sim,
                  "n.disc.Simes" = n.disc.Simes,
                  "n.disc.Simes2" = n.disc.Simes2,
                  "d_StoSimes" = d_StoSimes,
                  "n.disc.StoSimes" = n.disc.StoSimes,
                  "d_WMW" = d_WMW,
                  "n.disc.WMW" = n.disc.WMW,
                  "n.disc.WMW.cpp" = n.disc.WMW.cpp,
                  "uniques" = uniques,
                  "n1" = n1,
                  "pi.not" = pi.not,
                  "alpha" = alpha))
    }
  }
}


estimatek = function(B, inlier_remaining, out_ind, isofo.model, dataset){
  ress = foreach(b = 1:B, .combine=c) %dopar% {
   inlier_ind = sample(inlier_remaining, size = 1)
   outlier_ind = sample(out_ind, size = 1)
   inlier = dataset[inlier_ind,]
   outlier = dataset[outlier_ind,]
   S_inlier = predict.isolation_forest(isofo.model, inlier, type = "score")
   S_outlier = predict.isolation_forest(isofo.model, outlier, type = "score")
```

```
   greater.logi = S_inlier<S_outlier

   return(greater.logi)
  }

  greater.prob = mean(ress)
  k=greater.prob/(1-greater.prob)
  return(k)
}
```

In the following we set the calibration set and the test set size, respectively l and m, so that the nominal level α is proportional to $\frac{m}{l+1}$. The train set size is equal to n and the number of iterations is $B = 10^4$.

Credit Card Fraud Detection dataset

The dataset is available at https://www.kaggle.com/mlg-ulb/creditcardfraud.

```
set.seed(321)

# Initializing parameters
B = 10^4
m = 199
l = 199
n = 20
alpha = n/(l+1)
n1s = seq(from=0, to=n, by=1)

dataset = read_csv("~/nout/trials/RealData/Datasets/Dataset creditcard/creditcard.csv")
in_ind = which(dataset[,ncol(dataset)]==0)
out_ind = which(dataset[,ncol(dataset)]==1)

cluster <- makeCluster(parallel::detectCores())
registerDoSNOW(cluster)
clusterEvalQ(cluster, {list(library(isotree), library(nout), library(hommel))})
```

```
## [[1]]
## [[1]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[1]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[1]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
##
##
## [[2]]
## [[2]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
```

```
## [[2]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[2]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
##
##
## [[3]]
## [[3]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[3]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[3]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
##
##
## [[4]]
## [[4]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[4]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[4]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
```

```
clusterExport(cluster, list("n", "m", "l", "in_ind", "out_ind", "dataset", "alpha"))

tic()
modeltrain = TrainingIsoForest(l=l, dataset=dataset)
kest = estimatek(B=B, inlier_remaining=modeltrain$inlier_remaining,
          out_ind=out_ind, isofo.model=modeltrain$model, dataset=dataset)
res = lapply(1:length(n1s),
             function(j) CompareMethodNaturalOutliers(B=B, n1=n1s[j], n=n,
                                                      dataset=dataset,
                                                      isofo.model=modeltrain$model,
                                                      out_ind=out_ind,
                                                      inlier_remaining=modeltrain$inlier_remaining))
toc()
```

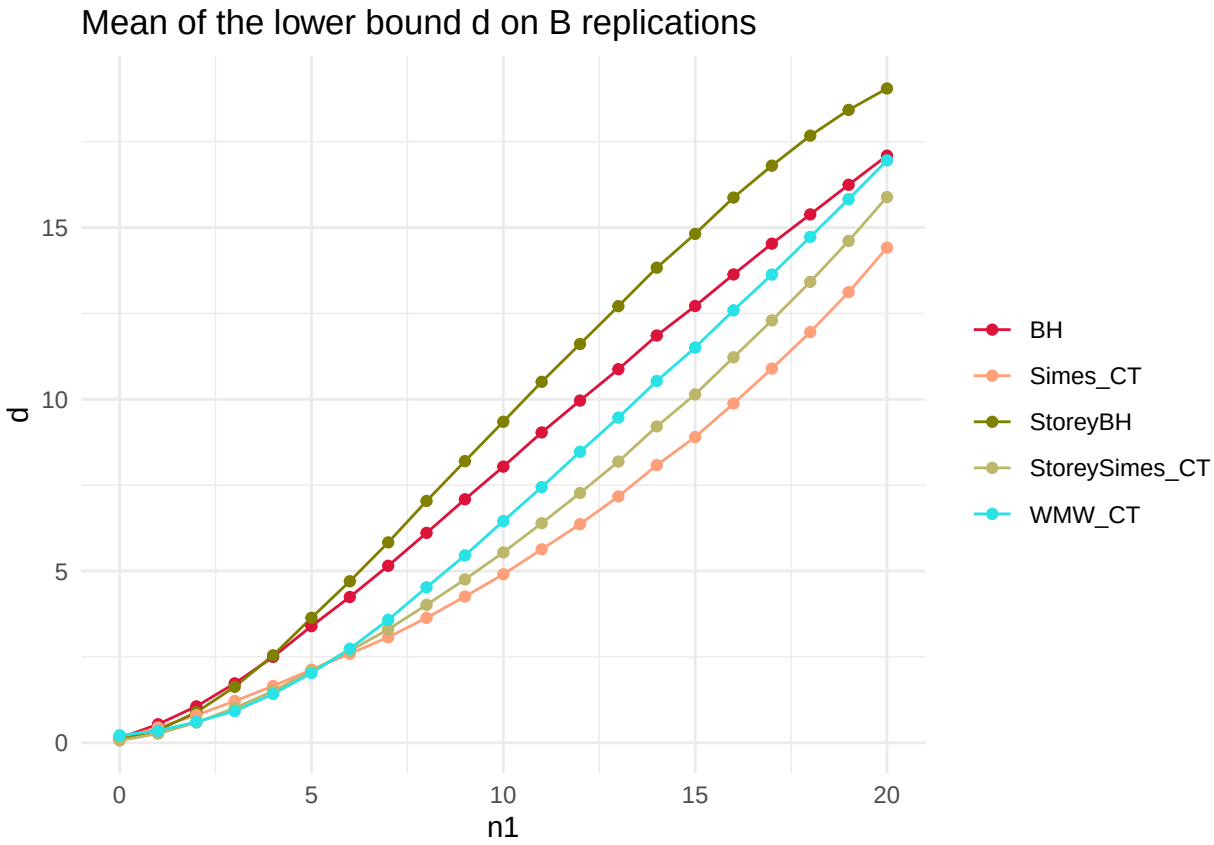
```
## 81414.99 sec elapsed
```

```
stopCluster(cluster)

kest
```

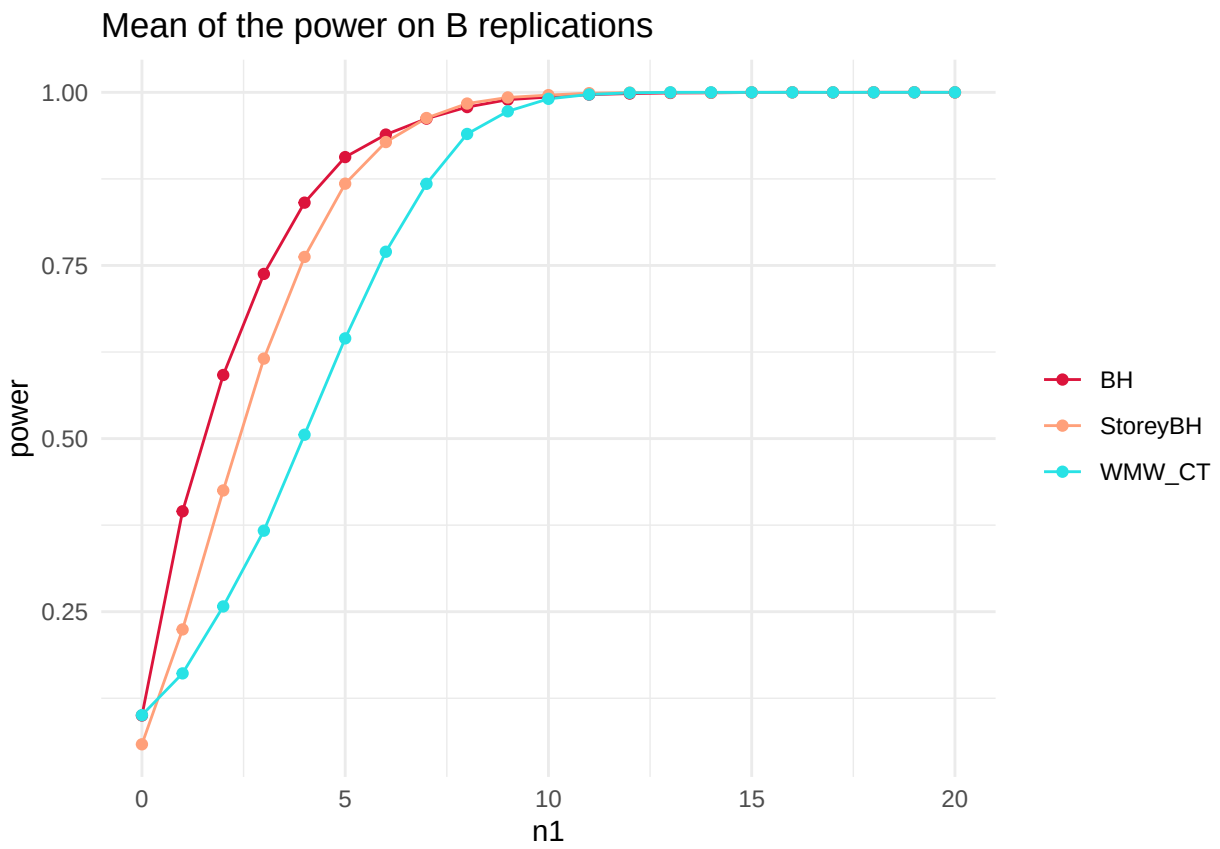
```
## [1] 14.03759
```

```
results = compact_results(res)

d_BH = vector()
d_StoBH = vector()
d_Sim = vector()
d_StoSimes = vector()
d_WMW = vector()

pow_BH = vector()
pow_StoBH = vector()
pow_Sim = vector()
pow_StoSimes = vector()
pow_WMW = vector()

disc_Sim = vector()
disc_StoSimes = vector()
disc_WMW = vector()
disc_WMW.cpp = vector()

for(j in 1:length(n1s)){
  d_BH[j] = results[[j]]$mean.lb.d[1]
  d_StoBH[j] = results[[j]]$mean.lb.d[2]
  d_Sim[j] = results[[j]]$mean.lb.d[3]
  d_StoSimes[j] = results[[j]]$mean.lb.d[4]
  d_WMW[j] = results[[j]]$mean.lb.d[5]

  pow_BH[j] = results[[j]]$mean.powerGlobalNull[1]
  pow_StoBH[j] = results[[j]]$mean.powerGlobalNull[2]
  pow_Sim[j] = results[[j]]$mean.powerGlobalNull[3]
  pow_StoSimes[j] = results[[j]]$mean.powerGlobalNull[4]
  pow_WMW[j] = results[[j]]$mean.powerGlobalNull[5]

  disc_Sim[j] = results[[j]]$mean.n.disc[1]
  disc_StoSimes[j] = results[[j]]$mean.n.disc[3]
  disc_WMW[j] = results[[j]]$mean.n.disc[4]
  disc_WMW.cpp[j] = results[[j]]$mean.n.disc[5]
}

# Plot lower bound d
df <- data.frame(
  x = n1s,
  BH = d_BH,
  StoreyBH = d_StoBH,
  Simes_CT = d_Sim,
  StoreySimes_CT = d_StoSimes,
  WMW_CT = d_WMW
)
df_long <- tidyr::pivot_longer(df, cols = -x, names_to = "group", values_to = "y")

ggplot(df_long, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
```

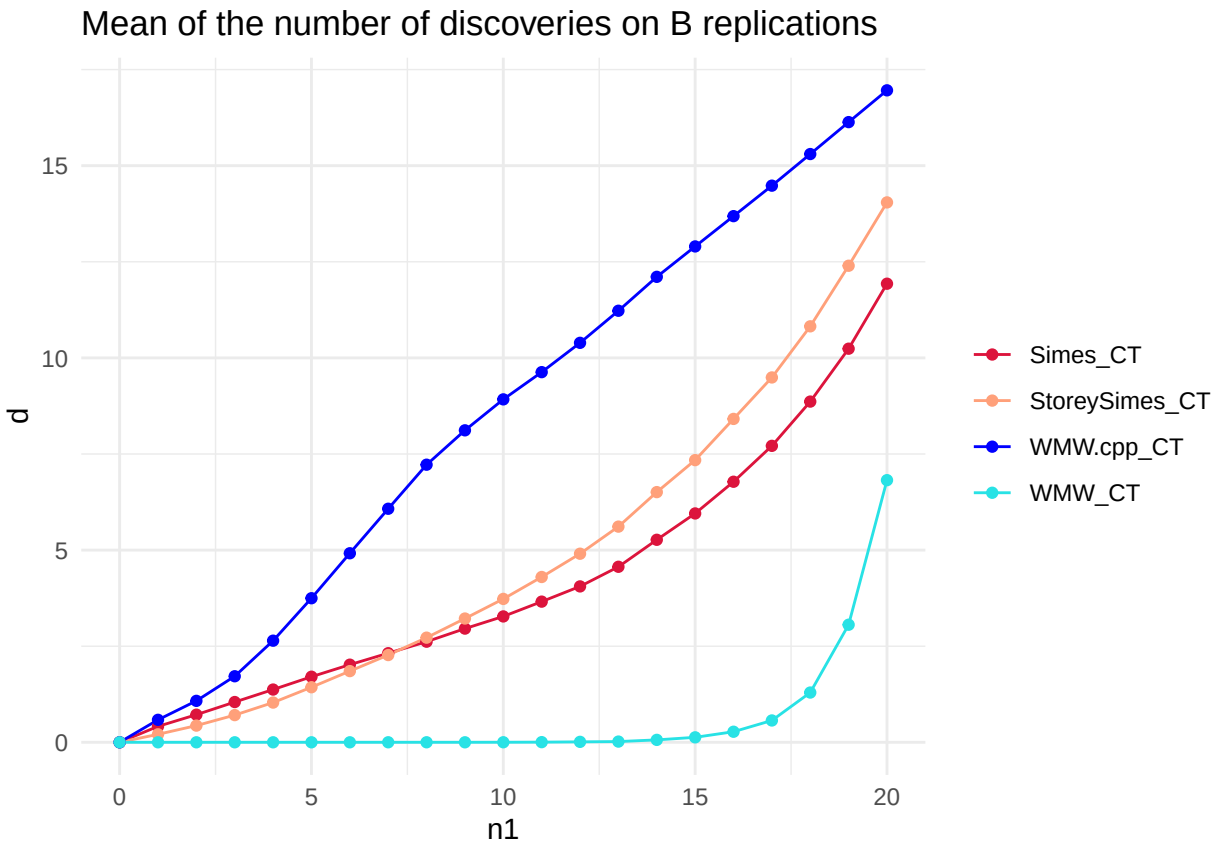
```
  scale_color_manual(values = c("#DC143C", "#FFA07A", "#808000", "#BDB76B", 5)) +
  labs(x = "n1", y = "d", title = "Mean of the lower bound d on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```




```
# Plot power
dfpower <- data.frame(
  x = n1s,
  BH = pow_BH,
  StoreyBH = pow_StoBH,
  WMW_CT = pow_WMW
)
df_long_power <- tidyr::pivot_longer(dfpower, cols = -x, names_to = "group", values_to = "y")

ggplot(df_long_power, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#DC143C","#FFA07A",5)) +
  labs(x = "n1", y = "power", title = "Mean of the power on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```




```r
# Plot discoveries
df <- data.frame(
  x = n1s,
  Simes_CT = disc_Sim,
  StoreySimes_CT = disc_StoSimes,
  WMW.cpp_CT = disc_WMW.cpp,
  WMW_CT = disc_WMW
)
df_long <- tidyr::pivot_longer(df, cols = -x, names_to = "group", values_to = "y")

ggplot(df_long, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#DC143C", "#FFA07A", "blue", 5)) +
  labs(x = "n1", y = "d", title = "Mean of the number of discoveries on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```




```
n.disc.tablelist = list()
for(i in 1:length(n1s)){
  n.disc.tablelist[[i]] = matrix(ncol = 5, nrow = 2)
  colnames(n.disc.tablelist[[i]]) = c("Simes", "Simes2", "StoSimes", "WMW", "WMW.cpp")
  rownames(n.disc.tablelist[[i]]) = c("mean.n.disc", "mean.d")
  n.disc.tablelist[[i]][1,] = apply(results[[i]][["n.disc"]], MARGIN = 2, FUN = mean)
  n.disc.tablelist[[i]][2,] = results[[i]]$mean.lb.d[c(3,3,4,5,5)]
}

for(i in 1:length(n1s)){
  cat("\n")
  cat(paste("n1=", n1s[i]))
  cat("\n")
  print(n.disc.tablelist[[i]])
}
```

```
##
## n1= 0
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 0.0000 0.0000   0.0000 0.0000  0.0000
## mean.d      0.1093 0.1093   0.0665 0.2151  0.2151
##
## n1= 1
##               Simes Simes2 StoSimes   WMW WMW.cpp
## mean.n.disc 0.4143 0.4143   0.2079 0.000  0.5849
## mean.d      0.4463 0.4463   0.2633 0.347  0.3470
```

```
##
## n1= 2
##               Simes Simes2 StoSimes   WMW WMW.cpp
## mean.n.disc 0.7185 0.7185   0.4344 0.000  1.0783
## mean.d      0.8037 0.8037   0.5890 0.607  0.6070
##
## n1= 3
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 1.0473 1.0473   0.7077 0.0000  1.7185
## mean.d      1.2137 1.2137   1.0099 0.9226  0.9226
##
## n1= 4
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 1.3723 1.3723   1.0339 0.0000  2.6434
## mean.d      1.6524 1.6524   1.5040 1.4203  1.4203
##
## n1= 5
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 1.7057 1.7057   1.4327 0.0000  3.7481
## mean.d      2.1251 2.1251   2.0972 2.0283  2.0283
##
## n1= 6
##               Simes Simes2 StoSimes   WMW WMW.cpp
## mean.n.disc 2.0193 2.0193   1.8518 0.000  4.9178
## mean.d      2.5908 2.5908   2.6767 2.738  2.7380
##
## n1= 7
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 2.3179 2.3179   2.2699 0.0006  6.0751
## mean.d      3.0706 3.0706   3.3020 3.5794  3.5794
##
## n1= 8
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 2.6173 2.6173   2.7223 0.0006  7.2209
## mean.d      3.6353 3.6353   4.0150 4.5264  4.5264
##
## n1= 9
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 2.9578 2.9578   3.2197 0.0000  8.1156
## mean.d      4.2569 4.2569   4.7564 5.4556  5.4556
##
## n1= 10
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 3.2742 3.2742   3.7298 0.0012  8.9209
## mean.d      4.9042 4.9042   5.5414 6.4529  6.4529
##
## n1= 11
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 3.6592 3.6592   4.3004 0.0040  9.6292
## mean.d      5.6325 5.6325   6.3929 7.4432  7.4432
##
## n1= 12
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 4.0554 4.0554   4.9060 0.0117 10.3916
```

```
## mean.d      6.3644 6.3644   7.2709 8.4716  8.4716
##
## n1= 13
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 4.5656 4.5656   5.6108 0.0189 11.2257
## mean.d      7.1723 7.1723   8.1863 9.4676  9.4676
##
## n1= 14
##               Simes Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 5.2680 5.2680   6.5082  0.0630 12.1080
## mean.d      8.0829 8.0829   9.2096 10.5334 10.5334
##
## n1= 15
##               Simes Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 5.9531 5.9531   7.3408  0.1292 12.8988
## mean.d      8.9012 8.9012  10.1421 11.5067 11.5067
##
## n1= 16
##               Simes Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 6.7779 6.7779   8.4123  0.2747 13.6871
## mean.d      9.8759 9.8759  11.2234 12.5910 12.5910
##
## n1= 17
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc  7.7115  7.7115   9.4907  0.5681 14.4797
## mean.d      10.8940 10.8940  12.2953 13.6323 13.6323
##
## n1= 18
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc  8.8631  8.8641  10.8208  1.2943 15.3022
## mean.d      11.9571 11.9571  13.4191 14.7299 14.7299
##
## n1= 19
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 10.2387 10.2412  12.3968  3.0587 16.1327
## mean.d      13.1179 13.1179  14.6094 15.8267 15.8267
##
## n1= 20
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 11.9294 11.9373  14.0441  6.8191 16.9596
## mean.d      14.4142 14.4142  15.8884 16.9574 16.9574
```

```
resCreditCard0.1v2 = list("raw.res"=res,
                          "k.est" = kest,
                          "compact.results" = results,
                          "n.disc.tablelist" = n.disc.tablelist)
save(resCreditCard0.1v2,
     file="~/nout/trials/RealData/PowerStudy/FinalSimu/CreditCard/resCreditCard0.1v2")
```