Power analysis of closed testing methods with Simes, Wilcoxon-Mann-Whitney and LMPI T3 as local tests considering Lehmann's alternative of order k

2023-12-04

Libraries and functions

```
library(tidyverse)
library(doSNOW)
library(nout)

gen.data <- function(m,n) {
  Z <- rnorm((m+n))
  return(Z)
}

gen.scores_Lehmann <- function(m, n, n1, k){
  if(n1==0){
    S_Z = gen.data(m,n)
    S_cal = S_Z[1:m]
    S_te = S_Z[(m+1):length(S_Z)]
  }

  if(n1==n){
    augmented.S_Z = gen.data(m,n*k)
    S_cal = augmented.S_Z[1:m]
    augmented.S_te = augmented.S_Z[(m+1):length(augmented.S_Z)]
    S_te = sapply(1:n1, FUN=function(i) max(augmented.S_te[(1+k*(i-1)):(i*k)]))
  }

  if(0<n1&n1<n){
    augmented.S_Z = gen.data(m=m,n=n-n1+n1*k)
    S_cal = augmented.S_Z[1:m]
    augmented.S_te = augmented.S_Z[(m+1):length(augmented.S_Z)]
    inlier.S_te = augmented.S_te[1:(n-n1)]
    outlier.augmented.S_te = augmented.S_te[(n-n1+1):length(augmented.S_te)]
    outlier.S_te = sapply(1:n1, FUN=function(i) max(outlier.augmented.S_te[(1+k*(i-1)):(i*k)]))
    S_te = c(inlier.S_te, outlier.S_te)
  }

  return(list("S_cal" = S_cal,
              "S_te" = S_te,
              "k" = k,
              "n1" = n1))
}
```

```r
compute_lb.d = function(B, m, n, n1, k, alpha){

  foreach(b = 1:B, .combine=cbind) %dopar% {

    scores = gen.scores_Lehmann(m, n, n1, k)
    S_cal = scores$S_cal
    S_te = scores$S_te

    d_WMW = nout::d_MannWhitney(S_Y = S_te, S_X = S_cal, alpha=alpha)
    d_T3 = nout::d_MannWhitneyk3(S_Y = S_te, S_X = S_cal, alpha=alpha)
    d_Sim = nout::d_Simes(S_X = S_cal, S_Y = S_te, alpha = alpha)
    # StoSimes = nout::d_StoreySimes(S_X = S_cal, S_Y = S_te, alpha = alpha)
    # d_StoSimes = StoSimes$d
    # pi.not = StoSimes$pi.not
    d_BH = nout::d_benjhoch(S_X = S_cal, S_Y = S_te, alpha = alpha)
    # d_StoBH = nout::d_StoreyBH(S_X = S_cal, S_Y = S_te, alpha = alpha)

    return(list("m" = m,
                "n" = n,
                "k" = k,
                "n1" = n1,
                "alpha" = alpha,
                # "S_cal" = S_cal,
                # "S_te" = S_te,
                "d_BH" = d_BH,
                #"d_StoBH" = d_StoBH,
                "d_Sim" = d_Sim,
                #"d_StoSimes" = d_StoSimes,
                "d_WMW" = d_WMW,
                "d_T3" = d_T3
                #"pi.not" = pi.not
                ))
  }
}



compact_results = function(res, ks, n1.index, n){

  mean.lb.d_n1_k =  matrix(nrow = length(ks), ncol = 4)

  rnames = vector()
  for(i in 1:length(ks)){
    rnames[i] = paste0("k=", ks[i])
  }
  # cnames.lb.d = c("mean.lb.d_BH", "mean.lb.d_StoBH", "mean.lb.d_Sim",
  #                 "mean.lb.d_StoSim", "mean.lb.d_WMW", "mean.lb.d_T3")
  cnames.lb.d = c("mean.lb.d_BH", "mean.lb.d_Sim",
                  "mean.lb.d_WMW", "mean.lb.d_T3")
  rownames(mean.lb.d_n1_k) = rnames
  colnames(mean.lb.d_n1_k) = cnames.lb.d
```

```
  for(i in 1:length(ks)){
    mean.lb.d_n1_k[i,"mean.lb.d_BH"] = mean(unlist(res[[i]][[n1.index]]["d_BH",]))
    #mean.lb.d_n1_k[i,"mean.lb.d_StoBH"] = mean(unlist(res[[i]][[n1.index]]["d_StoBH",]))
    mean.lb.d_n1_k[i,"mean.lb.d_Sim"] = mean(unlist(res[[i]][[n1.index]]["d_Sim",]))
    #mean.lb.d_n1_k[i,"mean.lb.d_StoSim"] = mean(unlist(res[[i]][[n1.index]]["d_StoSimes",]))
    mean.lb.d_n1_k[i,"mean.lb.d_WMW"] = mean(unlist(res[[i]][[n1.index]]["d_WMW",]))
    mean.lb.d_n1_k[i,"mean.lb.d_T3"] = mean(unlist(res[[i]][[n1.index]]["d_T3",]))
  }


  mean.power_n1_k =  matrix(nrow = length(ks), ncol = 4)

  cnames.power = c("mean.power_BH", "mean.power_Sim",
                   "mean.power_WMW", "mean.power_T3")
  rownames(mean.power_n1_k) = rnames
  colnames(mean.power_n1_k) = cnames.power

  for(i in 1:length(ks)){
    mean.power_n1_k[i,"mean.power_BH"] = mean(unlist(res[[i]][[n1.index]]["d_BH",])>0)
    #mean.power_n1_k[i,"mean.power_StoBH"] = mean(unlist(res[[i]][[n1.index]]["d_StoBH",])>0)
    mean.power_n1_k[i,"mean.power_Sim"] = mean(unlist(res[[i]][[n1.index]]["d_Sim",])>0)
    #mean.power_n1_k[i,"mean.power_StoSim"] = mean(unlist(res[[i]][[n1.index]]["d_StoSimes",])>0)
    mean.power_n1_k[i,"mean.power_WMW"] = mean(unlist(res[[i]][[n1.index]]["d_WMW",])>0)
    mean.power_n1_k[i,"mean.power_T3"] = mean(unlist(res[[i]][[n1.index]]["d_T3",])>0)
  }

  results = list("mean.power_n1_k" = mean.power_n1_k,
                 "mean.lb.d_n1_k" = mean.lb.d_n1_k)

  return(results)
}
```

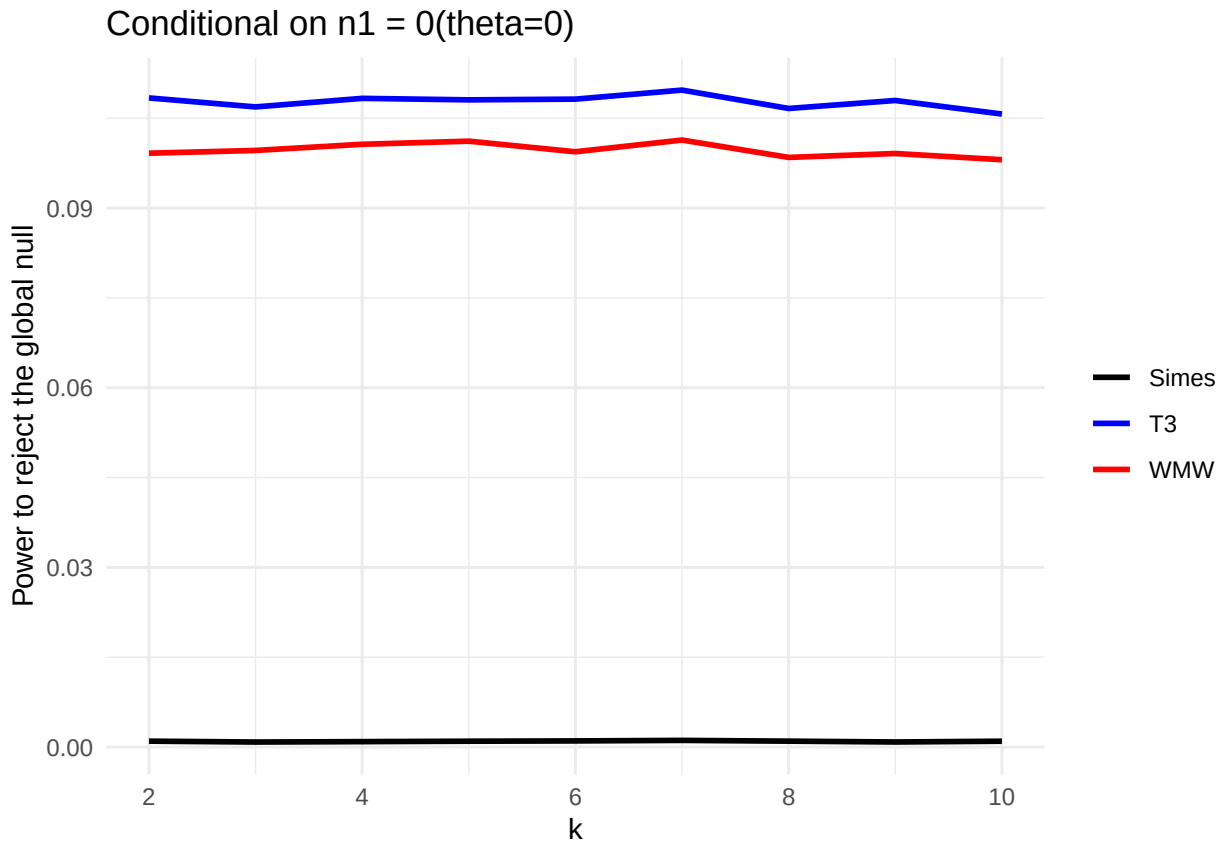
```
set.seed(321)

B = 10^5

m = 3000
n = 3000
# alpha = n/(m+1)
alpha = 0.1

thetas = c(0, 0.005, 0.01, 0.05, 0.1)
n1s = floor(n*thetas)

# Order of the Lehmann's alternative
ks = 2:10

cluster <- makeCluster(parallel::detectCores()-1)
registerDoSNOW(cluster)
clusterEvalQ(cluster, {list(library(isotree), library(nout))})
```

```
## [[1]]
## [[1]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
```

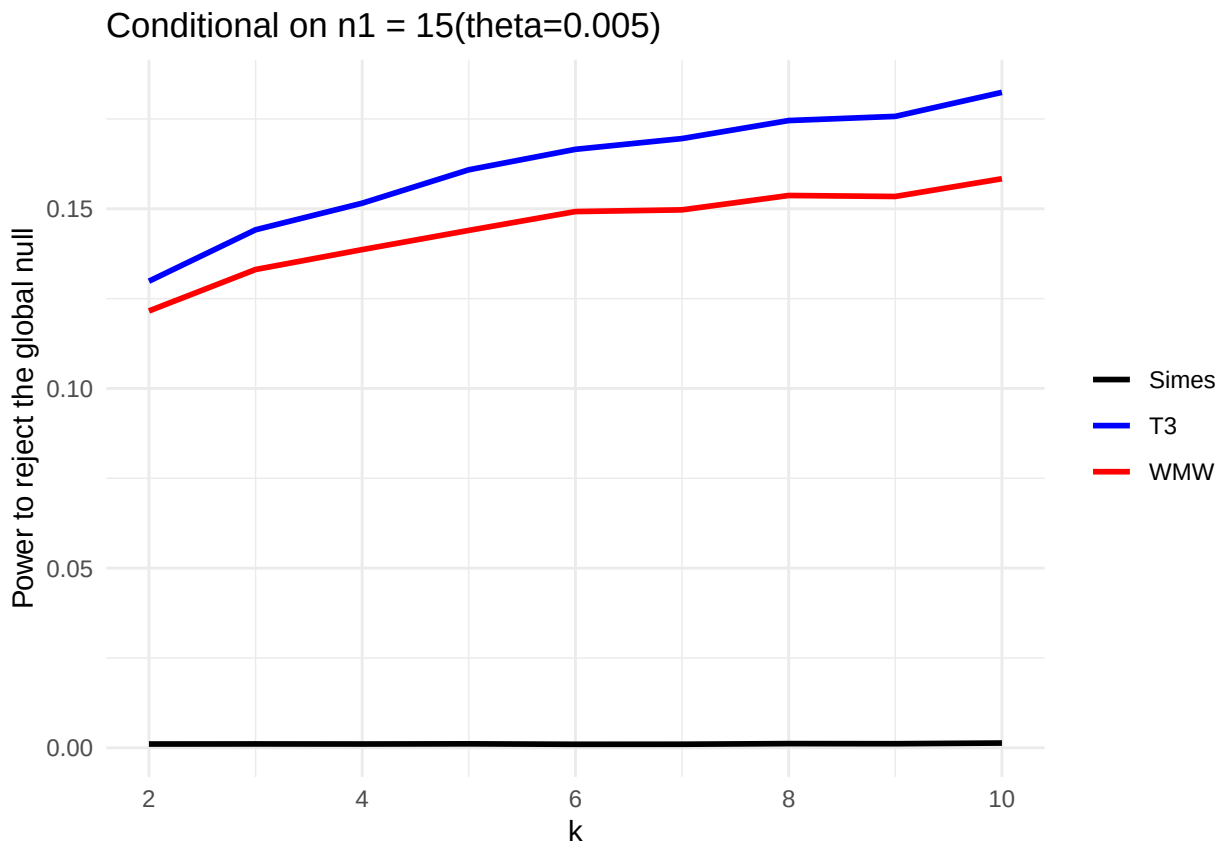
```
## [7] "datasets"  "methods"   "base"
##
## [[1]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[2]]
## [[2]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[2]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[3]]
## [[3]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[3]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
```

```
clusterExport(cluster, list("n", "m", "ks", "n1s", "alpha", "gen.data", "gen.scores_Lehmann"))

res <- lapply(1:length(ks), function(i){
  lapply( 1:length(n1s), function(j) compute_lb.d(B=B, m=m, n=n,
                                                  n1=n1s[j], k=ks[i], alpha=alpha))
  }
)

stopCluster(cluster)

results = lapply(1:length(n1s),
                 function(j) compact_results(res=res, ks=ks, n=n, n1.index=j) )

pp = list()
for(i in 1:length(n1s)){
  pow_BH = results[[i]]$mean.power_n1_k[,"mean.power_BH"]
  pow_Sim = results[[i]]$mean.power_n1_k[,"mean.power_Sim"]
  pow_WMW = results[[i]]$mean.power_n1_k[,"mean.power_WMW"]
  pow_T3 = results[[i]]$mean.power_n1_k[,"mean.power_T3"]

  dfpower <- data.frame(
    x = ks,
    Simes = pow_BH,
    WMW = pow_WMW,
    T3 = pow_T3
  )
  df_long_power <- tidyr::pivot_longer(dfpower, cols = -x, names_to = "group", values_to = "y")
```

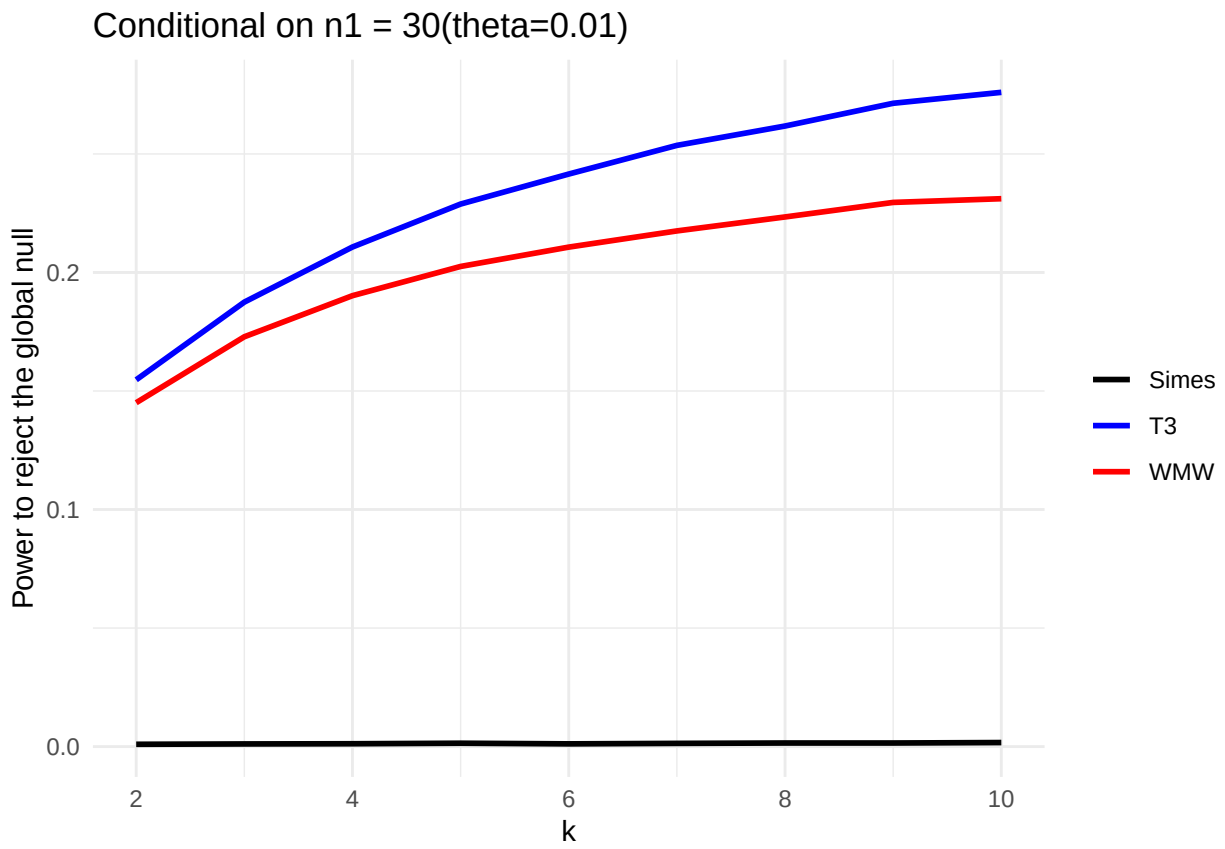
```
  pp[[i]] = ggplot(df_long_power, aes(x = x, y = y, color = group)) +
                   geom_line(size=1) +
                   scale_color_manual(values = c("black","blue","red")) +
                   ggtitle(paste0("Conditional on n1 = ", n1s[i], "(theta=", thetas[i], ")")) +
                   labs(x = "k", y = "Power to reject the global null") +
                   theme_minimal() +
                   theme(legend.title = element_blank())
  print(pp[[i]])
  print(cbind(pow_BH, pow_WMW, pow_T3))
}
```



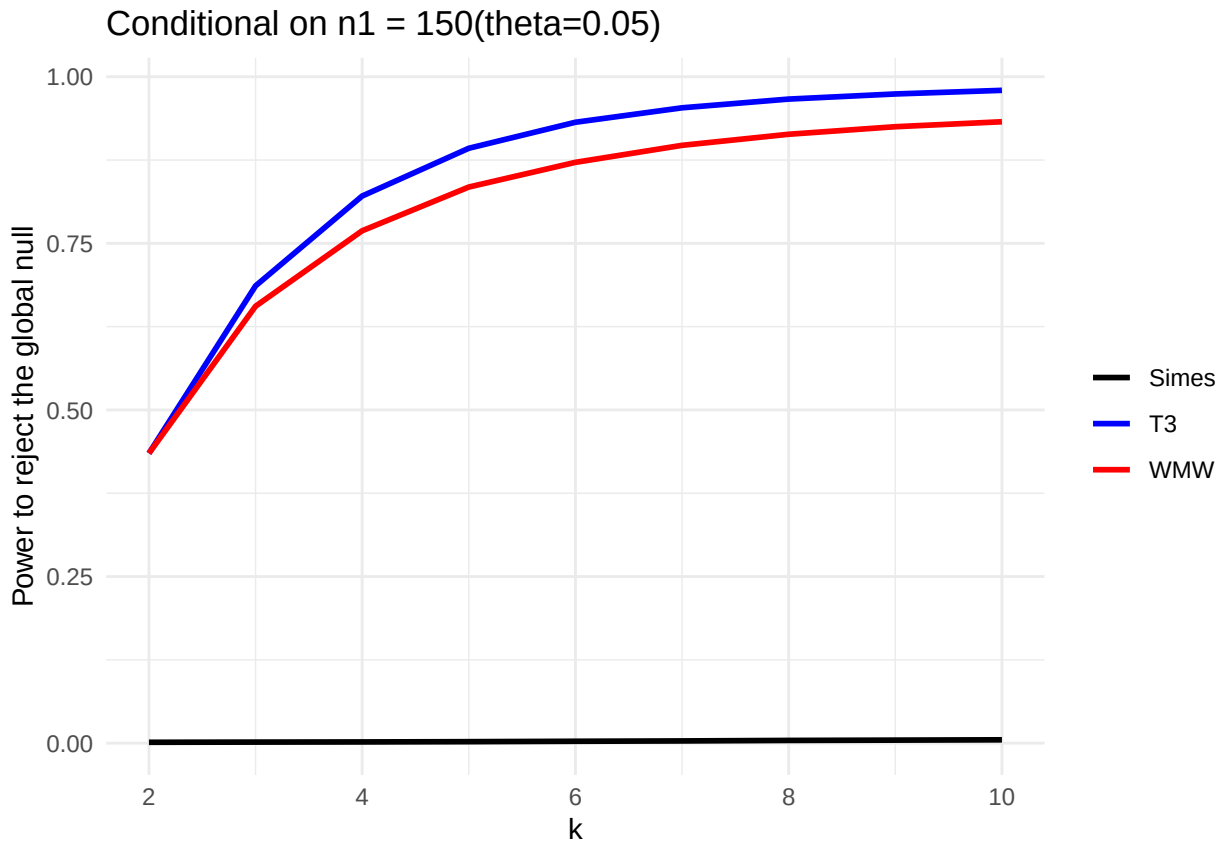

```
##        pow_BH pow_WMW  pow_T3
## k=2  0.00098 0.09916 0.10839
## k=3  0.00083 0.09962 0.10689
## k=4  0.00090 0.10065 0.10832
## k=5  0.00097 0.10117 0.10807
## k=6  0.00102 0.09940 0.10819
## k=7  0.00111 0.10136 0.10970
## k=8  0.00098 0.09846 0.10663
## k=9  0.00084 0.09911 0.10796
## k=10 0.00097 0.09807 0.10570
```



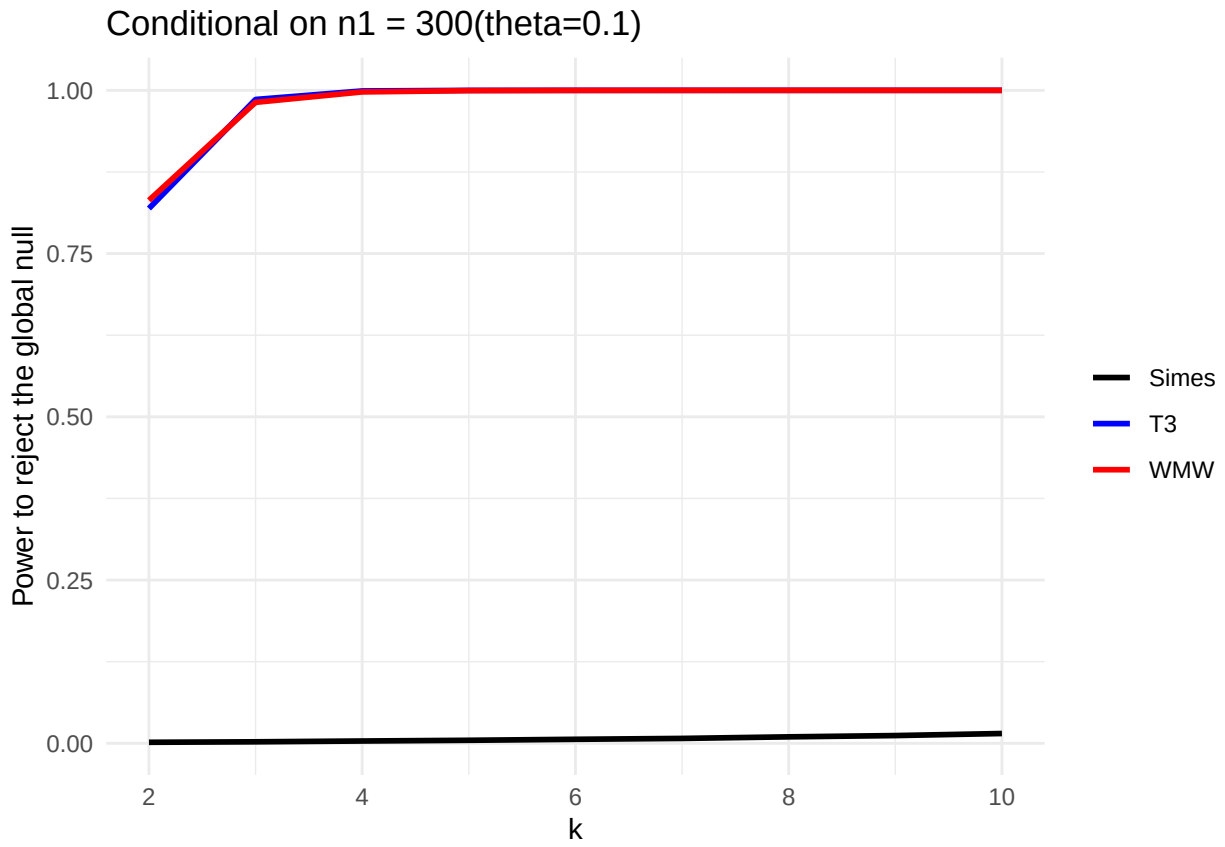

```
##       pow_BH pow_WMW  pow_T3
## k=2  0.00105 0.12157 0.12985
## k=3  0.00107 0.13310 0.14416
## k=4  0.00105 0.13864 0.15156
## k=5  0.00110 0.14399 0.16086
## k=6  0.00097 0.14923 0.16655
## k=7  0.00099 0.14970 0.16955
## k=8  0.00116 0.15370 0.17456
## k=9  0.00113 0.15343 0.17572
## k=10 0.00132 0.15836 0.18241
```




```
##        pow_BH pow_WMW  pow_T3
## k=2  0.00096 0.14506 0.15472
## k=3  0.00111 0.17288 0.18750
## k=4  0.00119 0.19021 0.21074
## k=5  0.00141 0.20254 0.22886
## k=6  0.00114 0.21070 0.24152
## k=7  0.00135 0.21752 0.25361
## k=8  0.00151 0.22342 0.26178
## k=9  0.00150 0.22955 0.27138
## k=10 0.00170 0.23111 0.27599
```




```
##        pow_BH pow_WMW  pow_T3
## k=2  0.00128 0.43495 0.43492
## k=3  0.00156 0.65561 0.68620
## k=4  0.00182 0.76887 0.82099
## k=5  0.00223 0.83448 0.89267
## k=6  0.00282 0.87155 0.93168
## k=7  0.00333 0.89708 0.95338
## k=8  0.00409 0.91367 0.96647
## k=9  0.00454 0.92501 0.97414
## k=10 0.00504 0.93247 0.97960
```




```
##        pow_BH pow_WMW  pow_T3
## k=2  0.00153 0.83154 0.81905
## k=3  0.00229 0.98160 0.98602
## k=4  0.00352 0.99758 0.99907
## k=5  0.00462 0.99956 0.99995
## k=6  0.00611 0.99990 1.00000
## k=7  0.00746 0.99993 1.00000
## k=8  0.00989 0.99997 0.99999
## k=9  0.01184 0.99999 1.00000
## k=10 0.01498 0.99998 1.00000
```