Comparison between different local tests: Simes, Simes with Storey and Wilcoxon-Mann-Whitney using the natural outliers distribution

2023-07-23

The aim is to compare on real datasets the performance of three closed testing procedures, which respectively use Simes local test with and without Storey estimator for the proportion of true null hypotheses and Wilcoxon-Mann-Whitney local test. We will consider outlier population to be the set of observations tagged as "outlier" in the dataset of interest.

R functions and libraries

```r
library(nout)
library(R.matlab)
library(isotree)
library(farff)
library(tictoc)
library(tidyverse)
library(doSNOW)
library(ggplot2)

compact_results = function(res){
  resT=as.data.frame(t(res))

  results = list()
  for(j in 1:length(n1s)){
    discoveries = as.data.frame(
      cbind("d_BH"=unlist(res[[j]][rownames(res[[j]])=="d_BH",]),
            "d_StoBH"=unlist(res[[j]][rownames(res[[j]])=="d_StoBH",]),
            "d_Sim"=unlist(res[[j]][rownames(res[[j]])=="d_Sim",]),
            "d_StoSimes"=unlist(res[[j]][rownames(res[[j]])=="d_StoSimes",]),
            "d_WMW"=unlist(res[[j]][rownames(res[[j]])=="d_WMW",])
            )
    )
    mean.discoveries = apply(discoveries, MARGIN = 2, FUN = mean)

    power.GlobalNull = as.data.frame(discoveries>0)
    mean.powerGlobalNull = apply(power.GlobalNull, MARGIN = 2, FUN = mean)

    out_identification = as.data.frame(
      cbind("out.identif_WMW"=
              unlist(res[[j]][rownames(res[[j]])=="outlier.identified_WMW",]),
            "out.identif_StoSimes"=
              unlist(res[[j]][rownames(res[[j]])=="outlier.identified_StoSimes",]),
            "out.identif_Simes"=
              unlist(res[[j]][rownames(res[[j]])=="outlier.identified_Simes",])
            )
    )
    mean.out_identification = apply(out_identification, MARGIN = 2, FUN = mean)
    mean.out_identification_pos = apply(out_identification>0, MARGIN = 2, FUN = mean)

    results[[j]] = list("discoveries" = discoveries,
                        "mean.discoveries" = mean.discoveries,
                        "power.GlobalNull" = power.GlobalNull,
                        "mean.powerGlobalNull" = mean.powerGlobalNull,
                        "out_identification" = out_identification,
                        "mean.out_identification" = mean.out_identification,
                        "mean.out_identification>0" = mean.out_identification_pos,
                        "pi.not" = res[[j]][rownames(res[[j]])=="pi.not",],
                        "uniques" = res[[j]][rownames(res[[j]])=="uniques",],
                        "n1" = res[[j]][rownames(res[[j]])=="n1",1],
                        "alpha" = res[[j]][rownames(res[[j]])=="alpha",1])
  }
  return(results)
}


TrainingIsoForest = function(l, dataset){

  tr_ind = sample(in_ind, size = l)
  tr = dataset[tr_ind,]
  isofo.model = isotree::isolation.forest(tr, ndim=ncol(dataset), ntrees=10, nthreads=1,
                              scoring_metric = "depth", output_score = TRUE)$model
  in_index2 = setdiff(in_ind, tr_ind)

  return(list("model"=isofo.model, "inlier_remaining" = in_index2))

}



CompareMethodNaturalOutliers = function(B, n1, n, out_ind, inlier_remaining, isofo.model, dataset){

  n0 = n-n1
  foreach(b = 1:B, .combine=cbind) %dopar% {
    if(n1==0){
      n0 = n
      N = n0 + m
      in_index3 = sample(inlier_remaining, size = N)
      cal_ind = in_index3[1:m]
      te_ind = in_index3[(m+1):N]
      cal = dataset[cal_ind,]
      te = dataset[te_ind,]
      S_cal = predict.isolation_forest(isofo.model, cal, type = "score")
      S_te = predict.isolation_forest(isofo.model, te, type = "score")

      d_WMW = nout::d_MannWhitney(S_Y = S_te, S_X = S_cal, alpha=alpha)
      d_Sim = nout::d_Simes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      StoSimes = nout::d_StoreySimes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      d_StoSimes = StoSimes$d
      pi.not = StoSimes$pi.not
      d_BH = nout::d_benjhoch(S_X = S_cal, S_Y = S_te, alpha = alpha)
      d_StoBH = nout::d_StoreyBH(S_X = S_cal, S_Y = S_te, alpha = alpha)
      uniques = length(unique(c(S_cal, S_te)))
      return(list("d_BH" = d_BH,
                  "d_StoBH" = d_StoBH,
                  "d_Sim" = d_Sim,
                  "d_StoSimes" = d_StoSimes,
                  "d_WMW" = d_WMW,
                  "outlier.identified_WMW" = 0,
                  "outlier.identified_Simes" = 0,
                  "outlier.identified_StoSimes" = 0,
                  "uniques" = uniques,
                  "n1" = n1,
                  "pi.not" = pi.not,
                  "alpha" = alpha))
    }

    else{
      N = n0 + m
      in_index3 = sample(inlier_remaining, size = N)
      cal_ind = in_index3[1:m]
      if(n0!=0)
        tein_ind = in_index3[(m+1):N]
      else
        tein_ind = NULL
      teout_ind = sample(out_ind, size = n1)
      cal = dataset[cal_ind,]
      te = dataset[c(tein_ind, teout_ind),]
      S_cal = predict.isolation_forest(isofo.model, cal, type = "score")
      S_te = predict.isolation_forest(isofo.model, te, type = "score")

      d_WMW = nout::d_MannWhitney(S_Y = S_te, S_X = S_cal, alpha=alpha)
      d_Sim = nout::d_Simes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      StoSimes = nout::d_StoreySimes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      d_StoSimes = StoSimes$d
      pi.not = StoSimes$pi.not
      d_BH = nout::d_benjhoch(S_X = S_cal, S_Y = S_te, alpha = alpha)
      d_StoBH = nout::d_StoreyBH(S_X = S_cal, S_Y = S_te, alpha = alpha)
      uniques = length(unique(c(S_cal, S_te)))

      # outlier identification with WMW
      conf.pval = sapply(1:n, function(j) (1+sum(S_cal >= S_te[j]))/(m+1))
      confvalid.pval = conf.pval<alpha
      confvalid.index = which(conf.pval<alpha)

      if(d_WMW>0){
        outlierTF = sapply(confvalid.index, function(h)
            nout::dselection_MannWhitney(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        outlier.identified_WMW = confvalid.index[as.logical(outlierTF)]
      }
      else  outlier.identified_WMW = NULL

      # outlier identification with Simes
```

```
      if(d_Sim>0){
        outlierTF = sapply(confvalid.index, function(h)
            nout::dselection_Simes(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        outlier.identified_Simes = confvalid.index[as.logical(outlierTF)]
      }
      else  outlier.identified_Simes = NULL

      # outlier identification with StoreySimes
      if(d_StoSimes>0){
        outlierTF = sapply(confvalid.index, function(h)
            nout::dselection_StoreySimes(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        outlier.identified_StoSimes = confvalid.index[as.logical(outlierTF)]
      }
      else  outlier.identified_StoSimes = NULL

      return(list("d_BH" = d_BH,
                  "d_StoBH" = d_StoBH,
                  "d_Sim" = d_Sim,
                  "d_StoSimes" = d_StoSimes,
                  "d_WMW" = d_WMW,
                  "outlier.identified_WMW" = length(outlier.identified_WMW),
                  "outlier.identified_Simes" = length(outlier.identified_Simes),
                  "outlier.identified_StoSimes" = length(outlier.identified_StoSimes),
                  "uniques" = uniques,
                  "n1" = n1,
                  "pi.not" = pi.not,
                  "alpha" = alpha))
    }
  }
}


estimatek = function(B, inlier_remaining, out_ind, isofo.model, dataset){
  ress = foreach(b = 1:B, .combine=c) %dopar% {
   inlier_ind = sample(inlier_remaining, size = 1)
   outlier_ind = sample(out_ind, size = 1)
   inlier = dataset[inlier_ind,]
   outlier = dataset[outlier_ind,]
   S_inlier = predict.isolation_forest(isofo.model, inlier, type = "score")
   S_outlier = predict.isolation_forest(isofo.model, outlier, type = "score")

   greater.logi = S_inlier<S_outlier

   return(greater.logi)
  }

  greater.prob = mean(ress)
  k=greater.prob/(1-greater.prob)
  return(k)
}
```

In the following we set the calibration set and the test set size, respectively l and m, so that the nominal level α is proportional to $\frac{m}{l+1}$. The train set size is equal to n and the number of iterations is $B = 10^4$.

Credit Card Fraud Detection dataset

The dataset is available at https://www.kaggle.com/mlg-ulb/creditcardfraud.

```
set.seed(321)

# Initializing parameters
B = 10^4
m = 199
l = 199
n = 20
alpha = n/(l+1)
n1s = seq(from=0, to=n, by=1)

dataset = read_csv("~/nout/trials/RealData/Datasets/Dataset creditcard/creditcard.csv")
in_ind = which(dataset[,ncol(dataset)]==0)
out_ind = which(dataset[,ncol(dataset)]==1)

cluster <- makeCluster(parallel::detectCores())
registerDoSNOW(cluster)
clusterEvalQ(cluster, {list(library(isotree), library(nout))})
```

```
## [[1]]
## [[1]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[1]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[2]]
## [[2]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[2]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[3]]
## [[3]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[3]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[4]]
## [[4]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[4]][[2]]
##  [1] "nout"       "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"      "datasets"  "methods"   "base"
```

```
clusterExport(cluster, list("n", "m", "l", "in_ind", "out_ind", "dataset", "alpha"))

tic()
modeltrain = TrainingIsoForest(l=l, dataset=dataset)
kest = estimatek(B=B, inlier_remaining=modeltrain$inlier_remaining,
          out_ind=out_ind, isofo.model=modeltrain$model, dataset=dataset)
res = lapply(1:length(n1s),
             function(j) CompareMethodNaturalOutliers(B=B, n1=n1s[j], n=n, dataset=dataset,
                                  isofo.model=modeltrain$model,
                                  out_ind=out_ind,
                                  inlier_remaining=modeltrain$inlier_remaining))
toc()
```

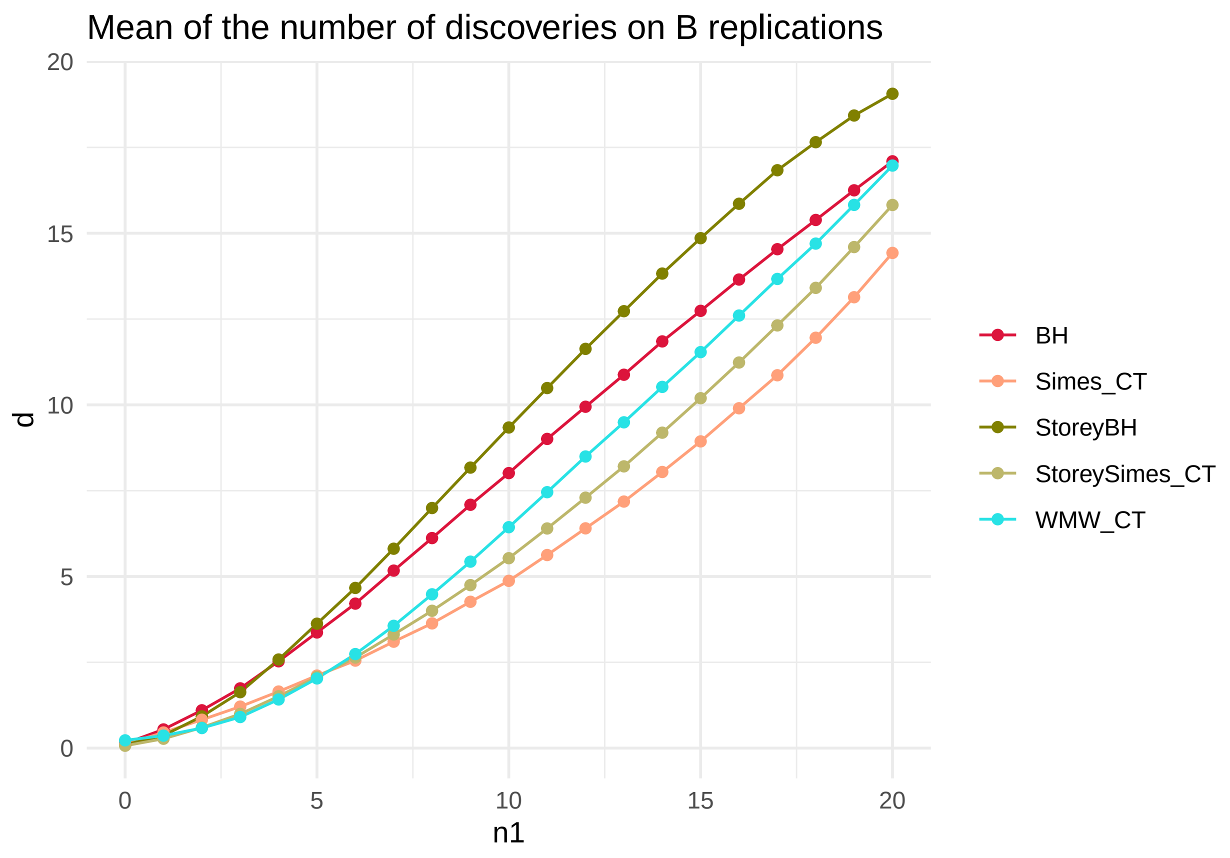
```
## 9989.38 sec elapsed
```

```
stopCluster(cluster)

kest
```

```
## [1] 15.20746
```

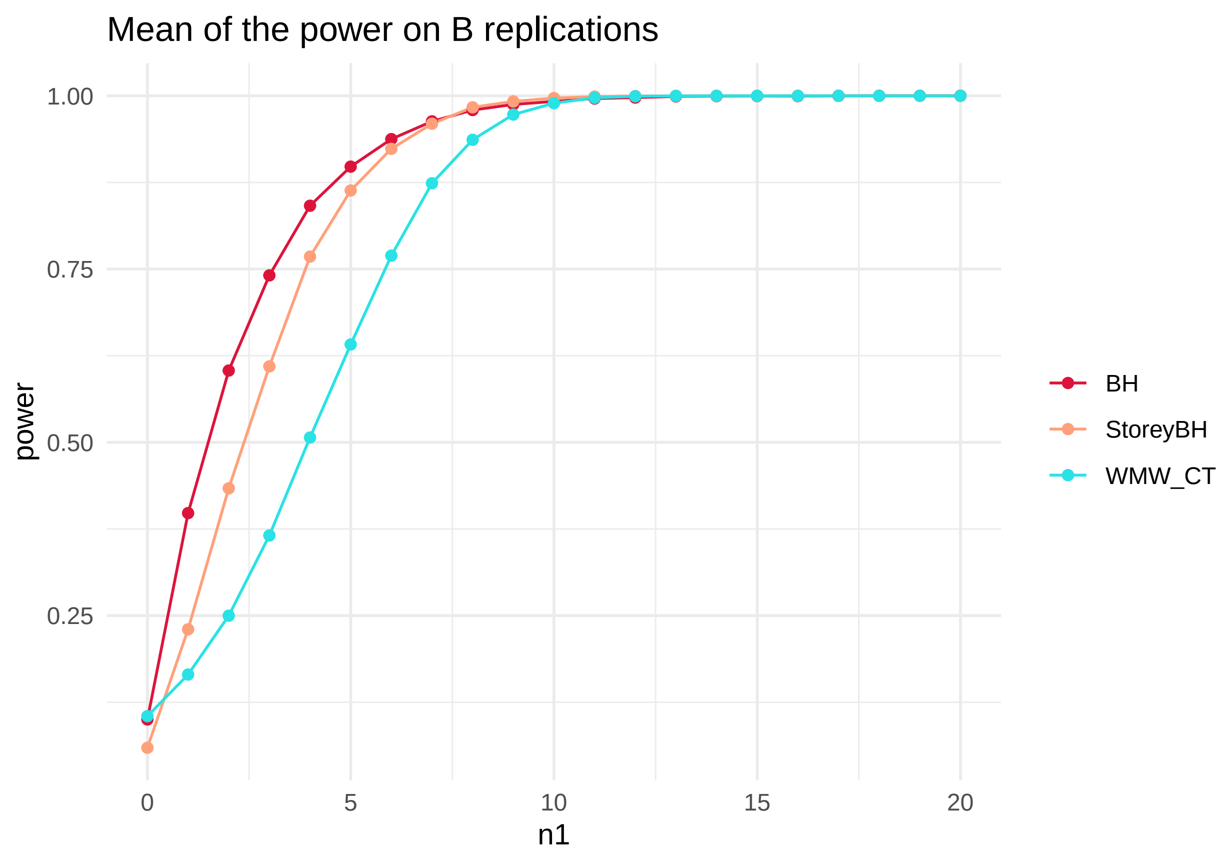
```
results = compact_results(res)

d_BH = vector()
d_StoBH = vector()
d_Sim = vector()
d_StoSimes = vector()
d_WMW = vector()

pow_BH = vector()
pow_StoBH = vector()
pow_Sim = vector()
pow_StoSimes = vector()
pow_WMW = vector()

for(j in 1:length(n1s)){
  d_BH[j] = results[[j]]$mean.discoveries[1]
  d_StoBH[j] = results[[j]]$mean.discoveries[2]
  d_Sim[j] = results[[j]]$mean.discoveries[3]
  d_StoSimes[j] = results[[j]]$mean.discoveries[4]
  d_WMW[j] = results[[j]]$mean.discoveries[5]

  pow_BH[j] = results[[j]]$mean.powerGlobalNull[1]
  pow_StoBH[j] = results[[j]]$mean.powerGlobalNull[2]
  pow_Sim[j] = results[[j]]$mean.powerGlobalNull[3]
  pow_StoSimes[j] = results[[j]]$mean.powerGlobalNull[4]
  pow_WMW[j] = results[[j]]$mean.powerGlobalNull[5]

}
```

```r
# Plot discoveries
df <- data.frame(
  x = n1s,
  BH = d_BH,
  StoreyBH = d_StoBH,
  Simes_CT = d_Sim,
  StoreySimes_CT = d_StoSimes,
  WMW_CT = d_WMW
)
df_long <- tidyr::pivot_longer(df, cols = -x, names_to = "group", values_to = "y")

ggplot(df_long, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#DC143C", "#FFA07A", "#808000", "#BDB76B", 5)) +
  labs(x = "n1", y = "d", title = "Mean of the number of discoveries on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```



```r
# Plot power
dfpower <- data.frame(
  x = n1s,
  BH = pow_BH,
  StoreyBH = pow_StoBH,
  WMW_CT = pow_WMW
)
```

```r
df_long_power <- tidyr::pivot_longer(dfpower, cols = -x, names_to = "group", values_to = "y")

# Plot the lines with different colors and legends
ggplot(df_long_power, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#DC143C","#FFA07A",5)) +
  labs(x = "n1", y = "power", title = "Mean of the power on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```



```r
outlier.identification = list()
for(i in 1:length(n1s)){
  outlier.identification[[i]] = matrix(nrow = 3, ncol = 4)
  rownames(outlier.identification[[i]]) = c("WMW", "Simes", "StoSimes")
  colnames(outlier.identification[[i]]) = c("mean.out.identif", "%successful.identification",
                                  "mean.d", "mean.d>0(power)")
  outlier.identification[[i]][,1] = apply(
    results[[i]][["out_identification"]], MARGIN = 2, FUN = mean)
  outlier.identification[[i]][,2] = apply(
    results[[i]][["out_identification"]]>0, MARGIN = 2, FUN = mean)
  outlier.identification[[i]][,3] = results[[i]]$mean.discoveries[c(3,4,5)]
  outlier.identification[[i]][,4] = results[[i]]$mean.powerGlobalNull[c(3,4,5)]
}

for(i in 1:length(n1s)){
  cat("\n")
  cat(paste("n1=", n1s[i]))
  print(outlier.identification[[i]])
}
```

```
##
## n1= 0        mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                         0                          0 0.1118          0.1002
## Simes                       0                          0 0.0686          0.0594
## StoSimes                    0                          0 0.2261          0.1050
##
## n1= 1        mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    0.6008                     0.1635 0.4550          0.3979
## Simes                  0.0000                     0.0000 0.2726          0.2303
## StoSimes               0.4228                     0.3735 0.3634          0.1650
##
## n1= 2        mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    1.0468                     0.2497 0.8265          0.6036
## Simes                  0.0000                     0.0000 0.5987          0.4337
## StoSimes               0.7350                     0.5452 0.5842          0.2499
##
## n1= 3        mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    1.7417                     0.3655 1.2091          0.7409
## Simes                  0.0000                     0.0000 0.9959          0.6096
## StoSimes               1.0408                     0.6606 0.9049          0.3657
##
## n1= 4        mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    2.7107                     0.5070 1.6479          0.8414
## Simes                  0.0000                     0.0000 1.5089          0.7680
## StoSimes               1.3663                     0.7401 1.4169          0.5070
##
## n1= 5        mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    3.8090                     0.6412 2.1136          0.8978
## Simes                  0.0000                     0.0000 2.0737          0.8634
## StoSimes               1.6967                     0.7935 2.0300          0.6412
##
## n1= 6        mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                     5.048                     0.7694 2.5489          0.9375
## Simes                   0.000                     0.0000 2.6387          0.9235
## StoSimes                1.978                     0.8320 2.7393          0.7694
##
## n1= 7        mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    6.2756                     0.8738 3.1000          0.9631
## Simes                  0.0000                     0.0000 3.3097          0.9597
## StoSimes               2.3225                     0.8636 3.5625          0.8738
##
## n1= 8        mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    7.3779                     0.9365 3.6340          0.9794
## Simes                  0.0000                     0.0000 4.0006          0.9834
## StoSimes               2.6194                     0.8877 4.4801          0.9365
##
## n1= 9        mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                    8.3839                     0.9729 4.2630          0.9875
## Simes                  0.0000                     0.0000 4.7485          0.9919
## StoSimes           2.9468                     0.8970 5.4321          0.9729
##
## n1= 10         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW                9.2758                     0.9893 4.8725          0.9919
## Simes              0.0000                     0.0000 5.5327          0.9967
## StoSimes           3.2619                     0.9174 6.4358          0.9893
##
## n1= 11         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW               10.1275                     0.9970 5.6249          0.9960
## Simes              0.0000                     0.0000 6.3989          0.9988
## StoSimes           3.6517                     0.9377 7.4561          0.9970
##
## n1= 12         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW               10.9083                     0.9993 6.4024          0.9974
## Simes              0.0000                     0.0000 7.2955          0.9996
## StoSimes           4.0725                     0.9408 8.4952          0.9993
##
## n1= 13         mean.out.identif %successful.identification mean.d mean.d>0(power)
## WMW               11.6307                     0.9998 7.1821          0.9991
## Simes              0.0000                     0.0000 8.2080          0.9999
## StoSimes           4.6013                     0.9473 9.4916          0.9998
##
## n1= 14         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               12.3938                     1.0000  8.0441          0.9995
## Simes              0.0000                     0.0000  9.1889          1.0000
## StoSimes           5.2490                     0.9564 10.5227          1.0000
##
## n1= 15         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               13.1314                      1.000  8.9357          0.9996
## Simes              0.0000                      0.000 10.1937          0.9999
## StoSimes           5.9413                      0.968 11.5370          1.0000
##
## n1= 16         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               13.9310                     1.0000  9.9004          0.9995
## Simes              0.0000                     0.0000 11.2331          1.0000
## StoSimes           6.7619                     0.9735 12.6016          1.0000
##
## n1= 17         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               14.7280                     1.0000 10.8625          0.9999
## Simes              0.0000                     0.0000 12.3147          1.0000
## StoSimes           7.6772                     0.9772 13.6678          1.0000
##
## n1= 18         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               15.5362                     1.0000 11.9551               1
## Simes              0.0000                     0.0000 13.4082               1
## StoSimes           8.8449                     0.9876 14.6975               1
##
## n1= 19         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               16.3624                     1.0000 13.1357               1
## Simes              0.0000                     0.0000 14.5971               1
## StoSimes          10.2652                     0.9916 15.8253               1
##
## n1= 20         mean.out.identif %successful.identification  mean.d mean.d>0(power)
## WMW               17.1417                     1.0000 14.4262               1
## Simes               0.0000                      0.0000 15.8224              1
## StoSimes           11.9576                      0.9958 16.9711              1
```

```
resCreditCard0.1 = list("raw.res"=res,
                    "k.est" = kest,
                    "compact.results" = results,
                    "outlier.identification" = outlier.identification)
save(resCreditCard0.1, file="~/nout/trials/RealData/PowerStudy/FinalSimu/CreditCard/resCreditCard0.1")
```