Comparison between different local tests: Simes, Simes with Storey and Wilcoxon-Mann-Whitney using the natural outliers distribution

2023-07-30

The aim is to compare on real datasets the performance of three closed testing procedures, which respectively use Simes local test with and without Storey estimator for the proportion of true null hypotheses and Wilcoxon-Mann-Whitney local test. We will consider outlier population to be the set of observations tagged as "outlier" in the dataset of interest.

R functions and libraries

```
library(nout)
library(R.matlab)
library(isotree)
library(farff)
library(tictoc)
library(tidyverse)
library(doSNOW)
library(ggplot2)
library(hommel)

compact_results = function(res){
  resT=as.data.frame(t(res))

  results = list()
  for(j in 1:length(n1s)){
    lb.d = as.data.frame(
      cbind("d_BH"=unlist(res[[j]][rownames(res[[j]])=="d_BH",]),
            "d_StoBH"=unlist(res[[j]][rownames(res[[j]])=="d_StoBH",]),
            "d_Sim"=unlist(res[[j]][rownames(res[[j]])=="d_Sim",]),
            "d_StoSimes"=unlist(res[[j]][rownames(res[[j]])=="d_StoSimes",]),
            "d_WMW"=unlist(res[[j]][rownames(res[[j]])=="d_WMW",])
            )
    )
    mean.lb.d = apply(lb.d, MARGIN = 2, FUN = mean)

    power.GlobalNull = as.data.frame(lb.d>0)
    mean.powerGlobalNull = apply(power.GlobalNull, MARGIN = 2, FUN = mean)

    n.disc = as.data.frame(
      cbind("n.disc.Simes" = unlist(res[[j]][rownames(res[[j]])=="n.disc.Simes",]),
            "n.disc.Simes2" = unlist(res[[j]][rownames(res[[j]])=="n.disc.Simes2",]),
             "n.disc.StoSimes" = unlist(res[[j]][rownames(res[[j]])=="n.disc.StoSimes",]),
            "n.disc.WMW" = unlist(res[[j]][rownames(res[[j]])=="n.disc.WMW",]),
            "n.disc.WMW.cpp" = unlist(res[[j]][rownames(res[[j]])=="n.disc.WMW.cpp",])
            )
    )
```

```
    mean.n.disc = apply(n.disc, MARGIN = 2, FUN = mean)
    #mean.n.disc_pos = apply(n.disc>0, MARGIN = 2, FUN = mean)

    results[[j]] = list("lb.d" = lb.d,
                        "mean.lb.d" = mean.lb.d,
                        "power.GlobalNull" = power.GlobalNull,
                        "mean.powerGlobalNull" = mean.powerGlobalNull,
                        "n.disc" = n.disc,
                        "mean.n.disc" = mean.n.disc,
                        #"mean.n.disc>0" = mean.n.disc_pos,
                        "pi.not" = res[[j]][rownames(res[[j]])=="pi.not",],
                        "uniques" = res[[j]][rownames(res[[j]])=="uniques",],
                        "n1" = res[[j]][rownames(res[[j]])=="n1",1],
                        "alpha" = res[[j]][rownames(res[[j]])=="alpha",1])
  }
  return(results)
}


TrainingIsoForest = function(l, dataset){

  tr_ind = sample(in_ind, size = l)
  tr = dataset[tr_ind,]
  isofo.model = isotree::isolation.forest(tr, ndim=ncol(dataset), ntrees=10, nthreads=1,
                            scoring_metric = "depth", output_score = TRUE)$model
  in_index2 = setdiff(in_ind, tr_ind)

  return(list("model"=isofo.model, "inlier_remaining" = in_index2))

}



CompareMethodNaturalOutliers = function(B, n1, n, out_ind, inlier_remaining, isofo.model, dataset){

  n0 = n-n1
  foreach(b = 1:B, .combine=cbind) %dopar% {
    if(n1==0){
      N = n0 + m
      in_index3 = sample(inlier_remaining, size = N)
      cal_ind = in_index3[1:m]
      te_ind = in_index3[(m+1):N]
      cal = dataset[cal_ind,]
      te = dataset[te_ind,]
      S_cal = predict.isolation_forest(isofo.model, cal, type = "score")
      S_te = predict.isolation_forest(isofo.model, te, type = "score")

      d_WMW = nout::d_MannWhitney(S_Y = S_te, S_X = S_cal, alpha=alpha)
      d_Sim = nout::d_Simes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      StoSimes = nout::d_StoreySimes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      d_StoSimes = StoSimes$d
      pi.not = StoSimes$pi.not
      d_BH = nout::d_benjhoch(S_X = S_cal, S_Y = S_te, alpha = alpha)
```

```
  d_StoBH = nout::d_StoreyBH(S_X = S_cal, S_Y = S_te, alpha = alpha)
  uniques = length(unique(c(S_cal, S_te)))
  return(list("d_BH" = d_BH,
              "d_StoBH" = d_StoBH,
              "d_Sim" = d_Sim,
              "n.disc.Simes" = 0,
              "n.disc.Simes2" = 0,
              "d_StoSimes" = d_StoSimes,
              "n.disc.StoSimes" = 0,
              "d_WMW" = d_WMW,
              "n.disc.WMW" = 0,
              "n.disc.WMW.cpp" = 0,
              "uniques" = uniques,
              "n1" = n1,
              "pi.not" = pi.not,
              "alpha" = alpha))
}

else{
  N = n0 + m
  in_index3 = sample(inlier_remaining, size = N)
  cal_ind = in_index3[1:m]
  if(n0!=0)
    tein_ind = in_index3[(m+1):N]
  else
    tein_ind = NULL
  teout_ind = sample(out_ind, size = n1)
  cal = dataset[cal_ind,]
  te = dataset[c(tein_ind, teout_ind),]
  S_cal = predict.isolation_forest(isofo.model, cal, type = "score")
  S_te = predict.isolation_forest(isofo.model, te, type = "score")

  d_WMW = nout::d_MannWhitney(S_Y = S_te, S_X = S_cal, alpha=alpha)
  d_Sim = nout::d_Simes(S_X = S_cal, S_Y = S_te, alpha = alpha)
  StoSimes = nout::d_StoreySimes(S_X = S_cal, S_Y = S_te, alpha = alpha)
  d_StoSimes = StoSimes$d
  pi.not = StoSimes$pi.not
  d_BH = nout::d_benjhoch(S_X = S_cal, S_Y = S_te, alpha = alpha)
  d_StoBH = nout::d_StoreyBH(S_X = S_cal, S_Y = S_te, alpha = alpha)
  uniques = length(unique(c(S_cal, S_te)))

  # outlier identification with WMW
  conf.pval = sapply(1:n, function(j) (1+sum(S_cal >= S_te[j]))/(m+1))
  confvalid.pval = conf.pval<alpha
  confvalid.index = which(conf.pval<alpha)

  n.disc.WMW.cpp=0
  n.disc.WMW=0
  if(d_WMW>0 & length(confvalid.index)!=0){
    outlierTF.WMW.cpp = sapply(confvalid.index, function(h)
      nout::dselection_MannWhitney(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
    #outlier.identified_MannWhitney = confvalid.index[as.logical(outlierTF)]
    n.disc.WMW.cpp = sum(outlierTF.WMW.cpp)
```

```
        outlierTF.WMW = sapply(confvalid.index, function(h)
          nout::dselection.prova_MannWhitney(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        n.disc.WMW = sum(outlierTF.WMW)
      }

      # outlier identification with Simes
      n.disc.Simes=0
      n.disc.Simes2=0
      if(d_Sim>0 & length(confvalid.index)!=0){
        outlierTF.Simes = sapply(confvalid.index, function(h)
            nout::dselection_Simes(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        #outlier.identified_Simes = confvalid.index[as.logical(outlierTF)]
        n.disc.Simes = sum(outlierTF.Simes)
        p = hommel(conf.pval)
        n.disc.Simes2 = sum(p@adjusted <= alpha)
      }

      # outlier identification with StoreySimes
      n.disc.StoSimes=0
      if(d_StoSimes>0 & length(confvalid.index)!=0){
        outlierTF.StoSim = sapply(confvalid.index, function(h)
          nout::dselection_StoreySimes(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        #outlier.identified_StoSimes = confvalid.index[as.logical(outlierTFStoSim)]
        n.disc.StoSimes = sum(outlierTF.StoSim)
      }

      return(list("d_BH" = d_BH,
                  "d_StoBH" = d_StoBH,
                  "d_Sim" = d_Sim,
                  "n.disc.Simes" = n.disc.Simes,
                  "n.disc.Simes2" = n.disc.Simes2,
                  "d_StoSimes" = d_StoSimes,
                  "n.disc.StoSimes" = n.disc.StoSimes,
                  "d_WMW" = d_WMW,
                  "n.disc.WMW" = n.disc.WMW,
                  "n.disc.WMW.cpp" = n.disc.WMW.cpp,
                  "uniques" = uniques,
                  "n1" = n1,
                  "pi.not" = pi.not,
                  "alpha" = alpha))
    }
  }
}


estimatek = function(B, inlier_remaining, out_ind, isofo.model, dataset){
  ress = foreach(b = 1:B, .combine=c) %dopar% {
   inlier_ind = sample(inlier_remaining, size = 1)
   outlier_ind = sample(out_ind, size = 1)
   inlier = dataset[inlier_ind,]
   outlier = dataset[outlier_ind,]
   S_inlier = predict.isolation_forest(isofo.model, inlier, type = "score")
   S_outlier = predict.isolation_forest(isofo.model, outlier, type = "score")
```

```
   greater.logi = S_inlier<S_outlier

   return(greater.logi)
  }

  greater.prob = mean(ress)
  k=greater.prob/(1-greater.prob)
  return(k)
}
```

In the following we set the calibration set and the test set size, respectively l and m, so that the nominal level α is proportional to $\frac{m}{l+1}$. The train set size is equal to n and the number of iterations is $B = 10^4$.

Statlog (Shuttle) dataset

The dataset is available at http://odds.cs.stonybrook.edu/shuttle-dataset.

```
set.seed(321)

# Initializing parameters
B = 10^4
m = 199
l = 199
n = 20
alpha = n/(l+1)
n1s = seq(from=0, to=n, by=1)

data = readMat("~/nout/trials/RealData/Datasets/Dataset shuttle/shuttle.mat")
dataset = cbind(data$X, data$y); colnames(dataset)[ncol(dataset)] = "y"
in_ind = which(dataset[,ncol(dataset)]==0)
out_ind = which(dataset[,ncol(dataset)]==1)

cluster <- makeCluster(parallel::detectCores())
registerDoSNOW(cluster)
clusterEvalQ(cluster, {list(library(isotree), library(nout), library(hommel))})
```

```
## [[1]]
## [[1]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[1]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[1]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
##
##
## [[2]]
## [[2]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
```

```
##
## [[2]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[2]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
##
##
## [[3]]
## [[3]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[3]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[3]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
##
##
## [[4]]
## [[4]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[4]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[4]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
```

```
clusterExport(cluster, list("n", "m", "l", "in_ind", "out_ind", "dataset", "alpha"))

tic()
modeltrain = TrainingIsoForest(l=l, dataset=dataset)
kest = estimatek(B=B, inlier_remaining=modeltrain$inlier_remaining,
          out_ind=out_ind, isofo.model=modeltrain$model, dataset=dataset)
res = lapply(1:length(n1s),
             function(j) CompareMethodNaturalOutliers(B=B, n1=n1s[j], n=n,
                                                      dataset=dataset,
                                                      isofo.model=modeltrain$model,
                                                      out_ind=out_ind,
                                                      inlier_remaining=modeltrain$inlier_remaining))
toc()
```

```
## 93351.73 sec elapsed
```

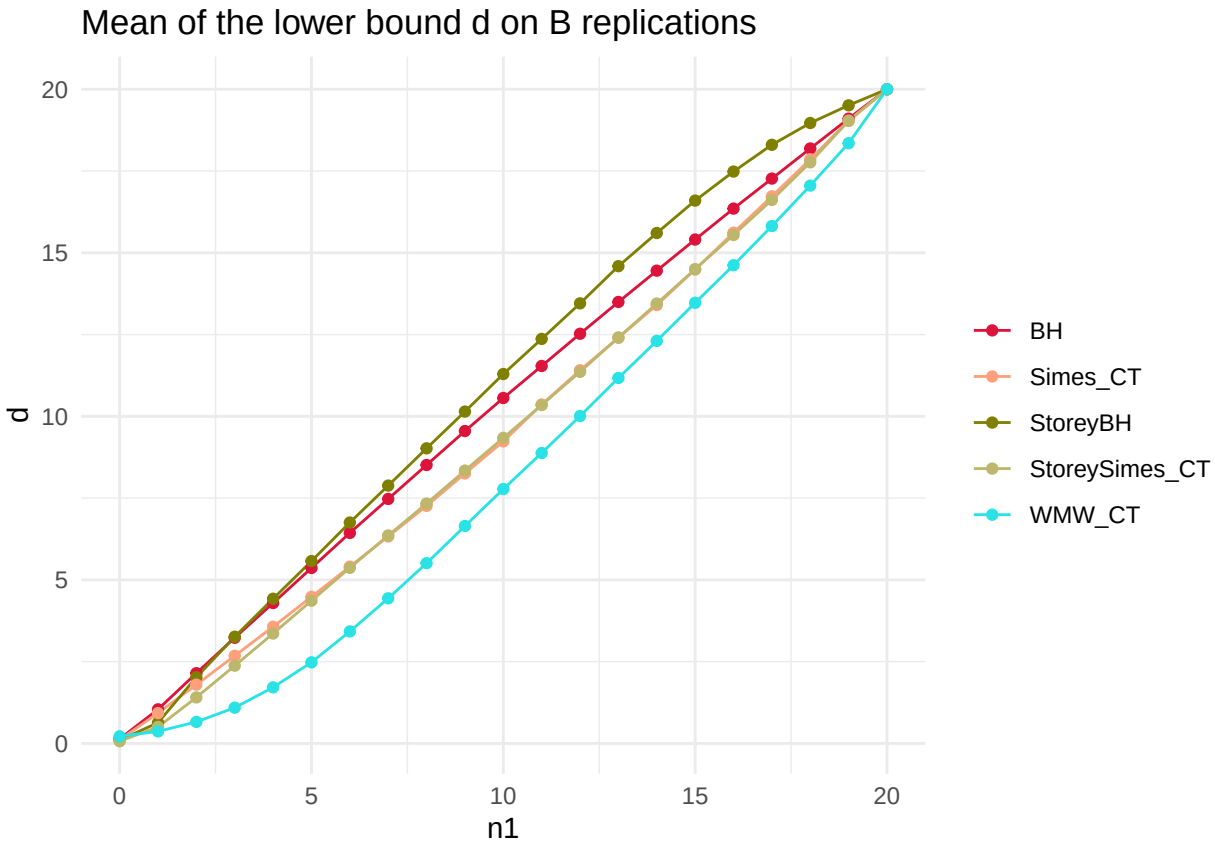
```
stopCluster(cluster)
```

```
kest
```

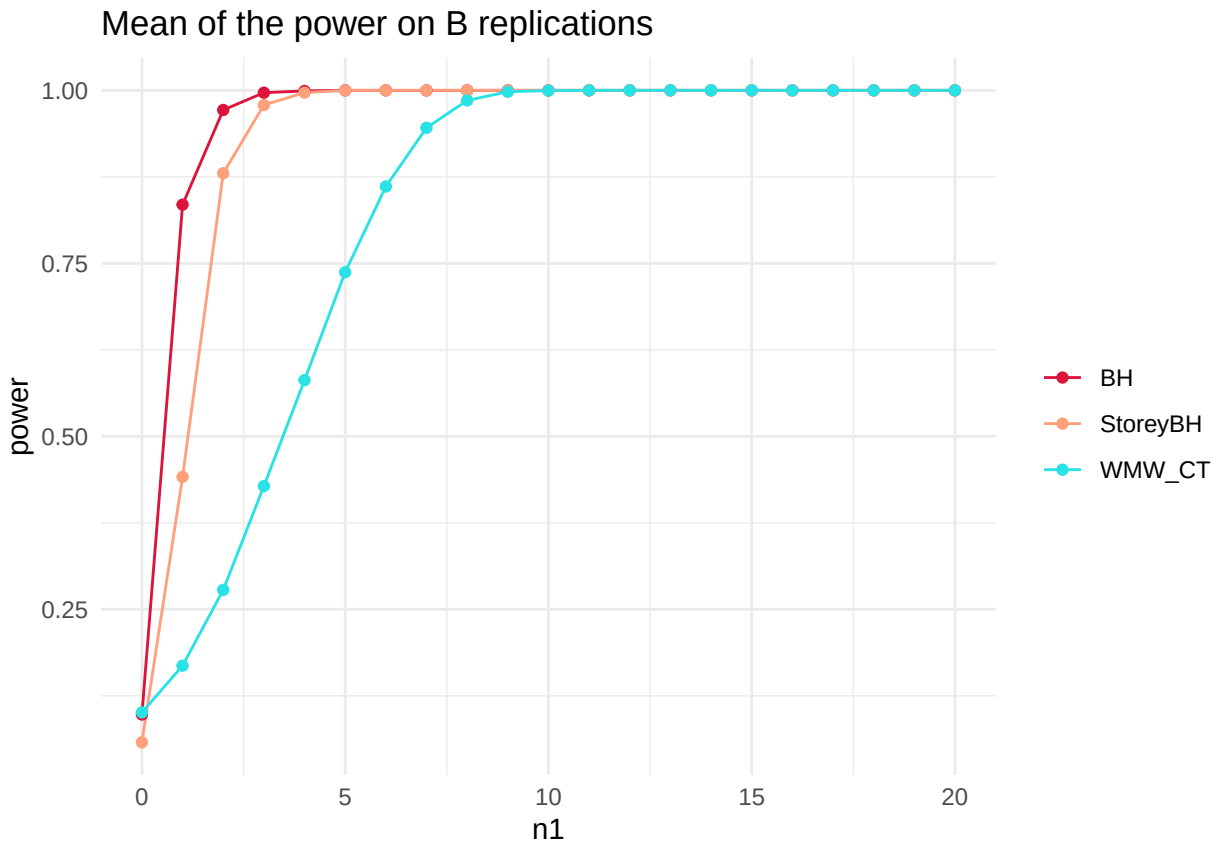
```
## [1] 399
```

```
results = compact_results(res)

d_BH = vector()
d_StoBH = vector()
d_Sim = vector()
d_StoSimes = vector()
d_WMW = vector()

pow_BH = vector()
pow_StoBH = vector()
pow_Sim = vector()
pow_StoSimes = vector()
pow_WMW = vector()

disc_Sim = vector()
disc_StoSimes = vector()
disc_WMW = vector()
disc_WMW.cpp = vector()

for(j in 1:length(n1s)){
  d_BH[j] = results[[j]]$mean.lb.d[1]
  d_StoBH[j] = results[[j]]$mean.lb.d[2]
  d_Sim[j] = results[[j]]$mean.lb.d[3]
  d_StoSimes[j] = results[[j]]$mean.lb.d[4]
  d_WMW[j] = results[[j]]$mean.lb.d[5]

  pow_BH[j] = results[[j]]$mean.powerGlobalNull[1]
  pow_StoBH[j] = results[[j]]$mean.powerGlobalNull[2]
  pow_Sim[j] = results[[j]]$mean.powerGlobalNull[3]
  pow_StoSimes[j] = results[[j]]$mean.powerGlobalNull[4]
  pow_WMW[j] = results[[j]]$mean.powerGlobalNull[5]

  disc_Sim[j] = results[[j]]$mean.n.disc[1]
  disc_StoSimes[j] = results[[j]]$mean.n.disc[3]
  disc_WMW[j] = results[[j]]$mean.n.disc[4]
  disc_WMW.cpp[j] = results[[j]]$mean.n.disc[5]
}

# Plot lower bound d
df <- data.frame(
  x = n1s,
  BH = d_BH,
  StoreyBH = d_StoBH,
  Simes_CT = d_Sim,
  StoreySimes_CT = d_StoSimes,
  WMW_CT = d_WMW
)
df_long <- tidyr::pivot_longer(df, cols = -x, names_to = "group", values_to = "y")

ggplot(df_long, aes(x = x, y = y, color = group)) +
```

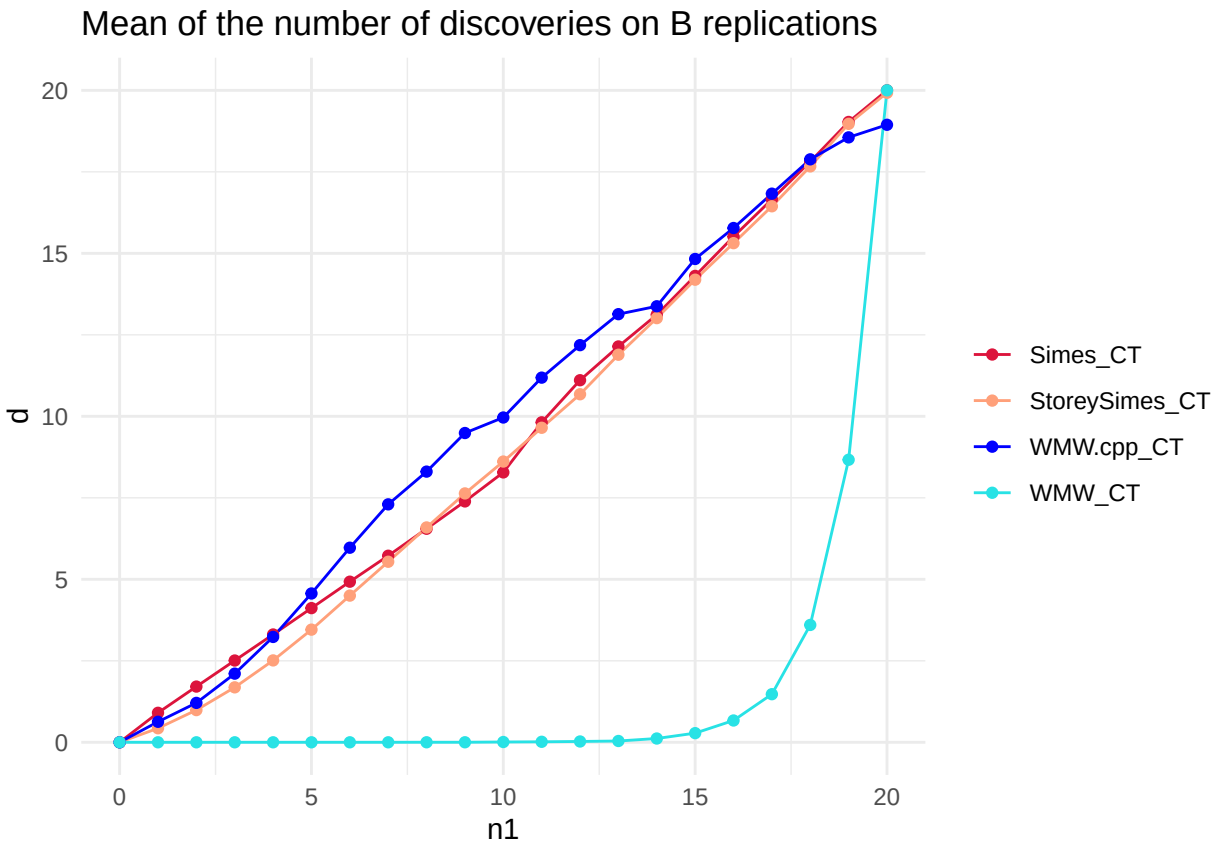
```
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#DC143C", "#FFA07A", "#808000", "#BDB76B", 5)) +
  labs(x = "n1", y = "d", title = "Mean of the lower bound d on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```




```
# Plot power
dfpower <- data.frame(
  x = n1s,
  BH = pow_BH,
  StoreyBH = pow_StoBH,
  WMW_CT = pow_WMW
)
df_long_power <- tidyr::pivot_longer(dfpower, cols = -x, names_to = "group", values_to = "y")

ggplot(df_long_power, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#DC143C","#FFA07A",5)) +
  labs(x = "n1", y = "power", title = "Mean of the power on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```




```
# Plot discoveries
df <- data.frame(
  x = n1s,
  Simes_CT = disc_Sim,
  StoreySimes_CT = disc_StoSimes,
  WMW.cpp_CT = disc_WMW.cpp,
  WMW_CT = disc_WMW
)
df_long <- tidyr::pivot_longer(df, cols = -x, names_to = "group", values_to = "y")

ggplot(df_long, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#DC143C", "#FFA07A", "blue", 5)) +
  labs(x = "n1", y = "d", title = "Mean of the number of discoveries on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```




```
n.disc.tablelist = list()
for(i in 1:length(n1s)){
  n.disc.tablelist[[i]] = matrix(ncol = 5, nrow = 2)
  colnames(n.disc.tablelist[[i]]) = c("Simes", "Simes2", "StoSimes", "WMW", "WMW.cpp")
  rownames(n.disc.tablelist[[i]]) = c("mean.n.disc", "mean.d")
  n.disc.tablelist[[i]][1,] = apply(results[[i]][["n.disc"]], MARGIN = 2, FUN = mean)
  n.disc.tablelist[[i]][2,] = results[[i]]$mean.lb.d[c(3,3,4,5,5)]
}

for(i in 1:length(n1s)){
  cat("\n")
  cat(paste("n1=", n1s[i]))
  cat("\n")
  print(n.disc.tablelist[[i]])
}
```

```
##
## n1= 0
##              Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 0.0000 0.0000   0.0000 0.0000  0.0000
## mean.d      0.1093 0.1093   0.0669 0.2169  0.2169
##
## n1= 1
##              Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 0.9048 0.9048   0.4307 0.0000  0.6292
## mean.d      0.9271 0.9271   0.5030 0.3679  0.3679
```

```
##
## n1= 2
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 1.7094 1.7094   0.9885 0.0000  1.2089
## mean.d      1.7959 1.7959   1.4065 0.6577  0.6577
##
## n1= 3
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 2.5087 2.5087   1.6859 0.0000  2.1062
## mean.d      2.6793 2.6793   2.3761 1.0923  1.0923
##
## n1= 4
##               Simes Simes2 StoSimes   WMW WMW.cpp
## mean.n.disc 3.3075 3.3075   2.5121 0.000   3.232
## mean.d      3.5670 3.5670   3.3594 1.714   1.714
##
## n1= 5
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 4.1167 4.1167   3.4561 0.0000  4.5658
## mean.d      4.4812 4.4812   4.3633 2.4788  2.4788
##
## n1= 6
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 4.9271 4.9271   4.4998 0.0000  5.9690
## mean.d      5.4048 5.4048   5.3682 3.4237  3.4237
##
## n1= 7
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 5.7233 5.7233   5.5391 0.0000  7.2996
## mean.d      6.3282 6.3282   6.3566 4.4359  4.4359
##
## n1= 8
##               Simes Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc 6.5511 6.5511   6.5910 0.0010  8.3038
## mean.d      7.2621 7.2621   7.3331 5.5106  5.5106
##
## n1= 9
##               Simes Simes2 StoSimes   WMW WMW.cpp
## mean.n.disc 7.3894 7.3894   7.6346 0.001  9.4879
## mean.d      8.2547 8.2547   8.3416 6.647  6.6470
##
## n1= 10
##               Simes Simes2 StoSimes   WMW WMW.cpp
## mean.n.disc 8.2797 8.2797   8.6123 0.009  9.9638
## mean.d      9.2338 9.2338   9.3409 7.777  7.7770
##
## n1= 11
##                Simes  Simes2 StoSimes    WMW WMW.cpp
## mean.n.disc  9.8158  9.8158   9.6486 0.0152 11.1866
## mean.d      10.3598 10.3598  10.3460 8.8790  8.8790
##
## n1= 12
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 11.1074 11.1074  10.6769  0.0254 12.1829
```

```
## mean.d      11.4111 11.4111  11.3603 10.0075 10.0075
##
## n1= 13
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 12.1431 12.1431  11.8891  0.0398 13.1336
## mean.d      12.4024 12.4024  12.4099 11.1732 11.1732
##
## n1= 14
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 13.1162 13.1162  13.0127  0.1163 13.3758
## mean.d      13.4029 13.4029  13.4483 12.3071 12.3071
##
## n1= 15
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 14.3076 14.3076  14.1916  0.2810 14.8272
## mean.d      14.4897 14.4897  14.5002 13.4712 13.4712
##
## n1= 16
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 15.5122 15.5122  15.3118  0.6694 15.7764
## mean.d      15.6145 15.6145  15.5444 14.6203 14.6203
##
## n1= 17
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 16.6555 16.6558  16.4432  1.4759 16.8299
## mean.d      16.7268 16.7268  16.6162 15.8181 15.8181
##
## n1= 18
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 17.8137 17.8144  17.6667  3.6000 17.8829
## mean.d      17.8592 17.8592  17.7656 17.0543 17.0543
##
## n1= 19
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 19.0301 19.0348  18.9715  8.6672 18.5573
## mean.d      19.0400 19.0400  19.0323 18.3505 18.3505
##
## n1= 20
##                Simes  Simes2 StoSimes     WMW WMW.cpp
## mean.n.disc 19.9999 19.9999  19.9245 19.9999 18.9432
## mean.d      19.9999 19.9999  19.9999 20.0000 20.0000
```

```
resShuttle0.1v2 = list("raw.res"=res,
                       "k.est" = kest,
                       "compact.results" = results,
                       "n.disc.tablelist" = n.disc.tablelist)
save(resShuttle0.1v2,
     file="~/nout/trials/RealData/PowerStudy/FinalSimu/Shuttle/resShuttle0.1v2")
```