Comparison between different local tests: Simes, Simes with Storey and Wilcoxon-Mann-Whitney using the natural outliers distribution and different classes of outliers

2023-08-08

The aim is to compare on real datasets the performance of three closed testing procedures, which respectively use Simes local test with and without Storey estimator for the proportion of true null hypotheses and Wilcoxon-Mann-Whitney local test. We will consider outlier population to be the set of observations tagged as “outlier” in the dataset of interest.

R functions and libraries

```
library(nout)
library(R.matlab)
library(isotree)
library(mlbench)
library(tictoc)
library(tidyverse)
library(doSNOW)
library(ggplot2)
library(hommel)

compact_results = function(res){
  resT=as.data.frame(t(res))

  results = list()
  for(j in 1:length(n1s)){
    lb.d = as.data.frame(
      cbind("d_Sim"=unlist(res[[j]][rownames(res[[j]])=="d_Sim",]),
            "d_StoSimes"=unlist(res[[j]][rownames(res[[j]])=="d_StoSimes",]),
            "d_WMW"=unlist(res[[j]][rownames(res[[j]])=="d_WMW",])
            )
    )
    mean.lb.d = apply(lb.d, MARGIN = 2, FUN = mean)

    power.GlobalNull = as.data.frame(lb.d>0)
    mean.powerGlobalNull = apply(power.GlobalNull, MARGIN = 2, FUN = mean)

    n.disc = as.data.frame(
      cbind("n.disc.Simes" = unlist(res[[j]][rownames(res[[j]])=="n.disc.Simes",]),
             "n.disc.StoSimes" = unlist(res[[j]][rownames(res[[j]])=="n.disc.StoSimes",]),
            "n.disc.WMW" = unlist(res[[j]][rownames(res[[j]])=="n.disc.WMW",]),
            "n.disc.WMW.cpp" = unlist(res[[j]][rownames(res[[j]])=="n.disc.WMW.cpp",])
            )
```

```r
    )
    mean.n.disc = apply(n.disc, MARGIN = 2, FUN = mean)
    #mean.n.disc_pos = apply(n.disc>0, MARGIN = 2, FUN = mean)

    results[[j]] = list("lb.d" = lb.d,
                        "mean.lb.d" = mean.lb.d,
                        "power.GlobalNull" = power.GlobalNull,
                        "mean.powerGlobalNull" = mean.powerGlobalNull,
                        "n.disc" = n.disc,
                        "mean.n.disc" = mean.n.disc,
                        #"mean.n.disc>0" = mean.n.disc_pos,
                        "uniques" = res[[j]][rownames(res[[j]])=="uniques",],
                        "n1" = res[[j]][rownames(res[[j]])=="n1",1],
                        "alpha" = res[[j]][rownames(res[[j]])=="alpha",1])
  }
  return(results)
}


TrainingIsoForest.S = function(l, dataset, in_ind){

  tr_ind = sample(in_ind, size = l)
  tr = dataset[tr_ind,]
  isofo.model = isotree::isolation.forest(tr, ndim=ncol(dataset), ntrees=10, nthreads=1,
                            scoring_metric = "depth", output_score = TRUE)$model
  in_index2 = setdiff(in_ind, tr_ind)

  return(list("model"=isofo.model, "inlier_remaining" = in_index2))

}



CompareMethod.S = function(B, m, n, n1, S, inlier_remaining, isofo.model, dataset, alpha){

  n0 = n-n1
  foreach(b = 1:B, .combine=cbind) %dopar% {
      N = n0 + m
      in_index3 = sample(inlier_remaining, size = N)
      cal_ind = in_index3[1:m]
      if(n0!=0)
        tein_ind = in_index3[(m+1):N]
      else
        tein_ind = NULL
      teout_ind = sample(S, size = n1)
      cal = dataset[cal_ind,]
      te = dataset[c(teout_ind, tein_ind),]
      S_cal = predict.isolation_forest(isofo.model, cal, type = "score")
      S_te = predict.isolation_forest(isofo.model, te, type = "score")

      d_WMW = nout::dselection_MannWhitney(S_Y = S_te, S_X = S_cal, S = 1:n1, alpha=alpha)
      d_Sim = nout::dselection_Simes(S_X = S_cal, S_Y = S_te, S = 1:n1,  alpha = alpha)
      d_StoSimes = nout::dselection_StoreySimes(S_X = S_cal, S_Y = S_te,
                                              S = 1:n1, alpha = alpha)
      uniques = length(unique(c(S_cal, S_te)))

      # outlier identification with WMW
      conf.pval = sapply(1:n, function(j) (1+sum(S_cal >= S_te[j]))/(m+1))
      confvalid.pval = conf.pval<alpha
      confvalid.index = which(conf.pval<alpha)

      n.disc.WMW.cpp=0
      n.disc.WMW=0
      if(d_WMW>0 & length(confvalid.index)!=0){
        outlierTF.WMW.cpp = sapply(confvalid.index, function(h)
          nout::dselection_MannWhitney(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        #outlier.identified_MannWhitney = confvalid.index[as.logical(outlierTF)]
        n.disc.WMW.cpp = sum(outlierTF.WMW.cpp)
        outlierTF.WMW = sapply(confvalid.index, function(h)
          nout::dselection.prova_MannWhitney(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        n.disc.WMW = sum(outlierTF.WMW)
      }

      # outlier identification with Simes
      n.disc.Simes=0
      # n.disc.Simes2=0
      if(d_Sim>0 & length(confvalid.index)!=0){
        outlierTF.Simes = sapply(confvalid.index, function(h)
            nout::dselection_Simes(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        #outlier.identified_Simes = confvalid.index[as.logical(outlierTF)]
        # n.disc.Simes = sum(outlierTF.Simes)
        # p = hommel(conf.pval)
        # n.disc.Simes2 = sum(p@adjusted <= alpha)
      }

      # outlier identification with StoreySimes
      n.disc.StoSimes=0
      if(d_StoSimes>0 & length(confvalid.index)!=0){
        outlierTF.StoSim = sapply(confvalid.index, function(h)
          nout::dselection_StoreySimes(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
        #outlier.identified_StoSimes = confvalid.index[as.logical(outlierTFStoSim)]
        n.disc.StoSimes = sum(outlierTF.StoSim)
      }

      return(list("d_Sim" = d_Sim,
                  "n.disc.Simes" = n.disc.Simes,
                  # "n.disc.Simes2" = n.disc.Simes2,
                  "d_StoSimes" = d_StoSimes,
                  "n.disc.StoSimes" = n.disc.StoSimes,
                  "d_WMW" = d_WMW,
                  "n.disc.WMW" = n.disc.WMW,
                  "n.disc.WMW.cpp" = n.disc.WMW.cpp,
                  "uniques" = uniques,
                  "n1" = n1,
                  "alpha" = alpha))
  }
```

```
}


estimatek.S = function(B, inlier_remaining, S, isofo.model, dataset){
  ress = foreach(b = 1:B, .combine=c) %dopar% {
   inlier_ind = sample(inlier_remaining, size = 1)
   outlier_ind = sample(S, size = 1)
   inlier = dataset[inlier_ind,]
   outlier = dataset[outlier_ind,]
   S_inlier = predict.isolation_forest(isofo.model, inlier, type = "score")
   S_outlier = predict.isolation_forest(isofo.model, outlier, type = "score")

   greater.logi = S_inlier<S_outlier

   return(greater.logi)
  }

  greater.prob = mean(ress)
  k=greater.prob/(1-greater.prob)
  return(k)
}
```

In the following we set the calibration set and the test set size, respectively l and m, so that the nominal level α is proportional to $\frac{m}{l+1}$. The train set size is equal to n and the number of iterations is $B = 10^4$.

Statlog (Shuttle) dataset in library mlbench

The dataset is available in the R library ***mlbench***.

```
# Load the data
data("Shuttle")
levels(Shuttle$Class) = c("1", "2", "3", "4", "5", "6", "7")
table(Shuttle$Class)
```

```
##
##     1     2     3     4     5     6     7
## 45586    50   171  8903  3267    10    13
```

```
# Delete class 4
fours = which(Shuttle[,10]== "4")
Shuttle2 = Shuttle[-fours,]

# DIfferent classes of outliers
out.classes = c("2","3","5","6","7")

out2.ind = which(Shuttle2$Class == levels(Shuttle2[,10])[2])
out3.ind = which(Shuttle2$Class == levels(Shuttle2[,10])[3])
out5.ind = which(Shuttle2$Class == levels(Shuttle2[,10])[5])
out6.ind = which(Shuttle2$Class == levels(Shuttle2[,10])[6])
out7.ind = which(Shuttle2$Class == levels(Shuttle2[,10])[7])

inliers.ind = which(Shuttle2[,10] == "1")
```

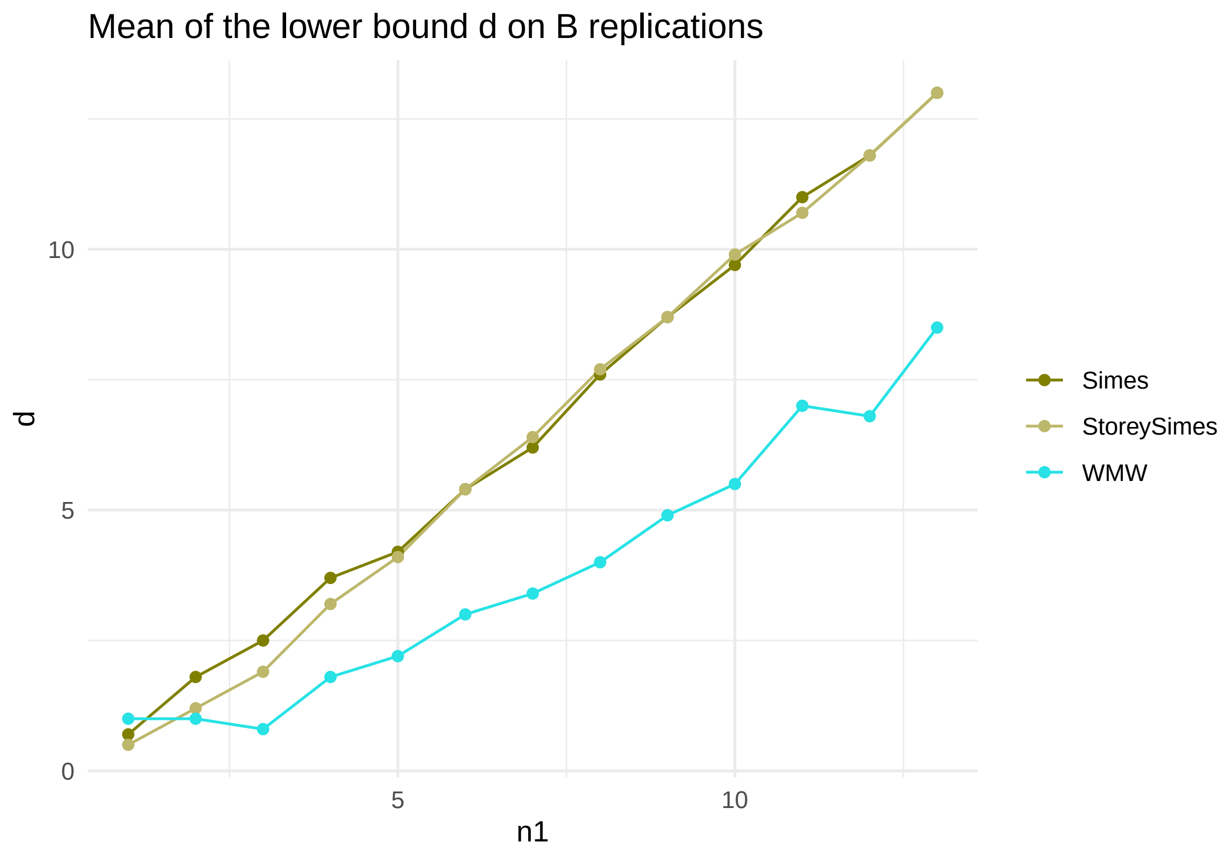
```r
outliers.ind = setdiff(1:nrow(Shuttle2), inliers.ind)
#length(outliers.ind)==length(out2.ind)+length(out3.ind)+length(out5.ind)+length(out6.ind)+length(out7.

# Creating Outlier column
# =1 if observation i is outlier
# =0 if observations i is inlier
outlier = rep(0, times = nrow(Shuttle2))
outlier[outliers.ind] = 1
#sum(outliers)

Shuttle2 = cbind(Shuttle2, "Outlier" = outlier)



set.seed(321)

# Initializing parameters
B = 10
l = 199
m = 199
n = 20
alpha = n/(m+1)

S = out7.ind
s = length(S)
n1s = seq(from = 1, to = s, by = 1)


cluster <- makeCluster(parallel::detectCores())
registerDoSNOW(cluster)
clusterEvalQ(cluster, {list(library(isotree), library(nout), library(hommel))})
```

```
## [[1]]
## [[1]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[1]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[1]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
##
##
## [[2]]
## [[2]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[2]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[2]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
##
##
## [[3]]
## [[3]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[3]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[3]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
##
##
## [[4]]
## [[4]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[4]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[4]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
##
##
## [[5]]
## [[5]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[5]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[5]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics"
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
##
##
## [[6]]
## [[6]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
```

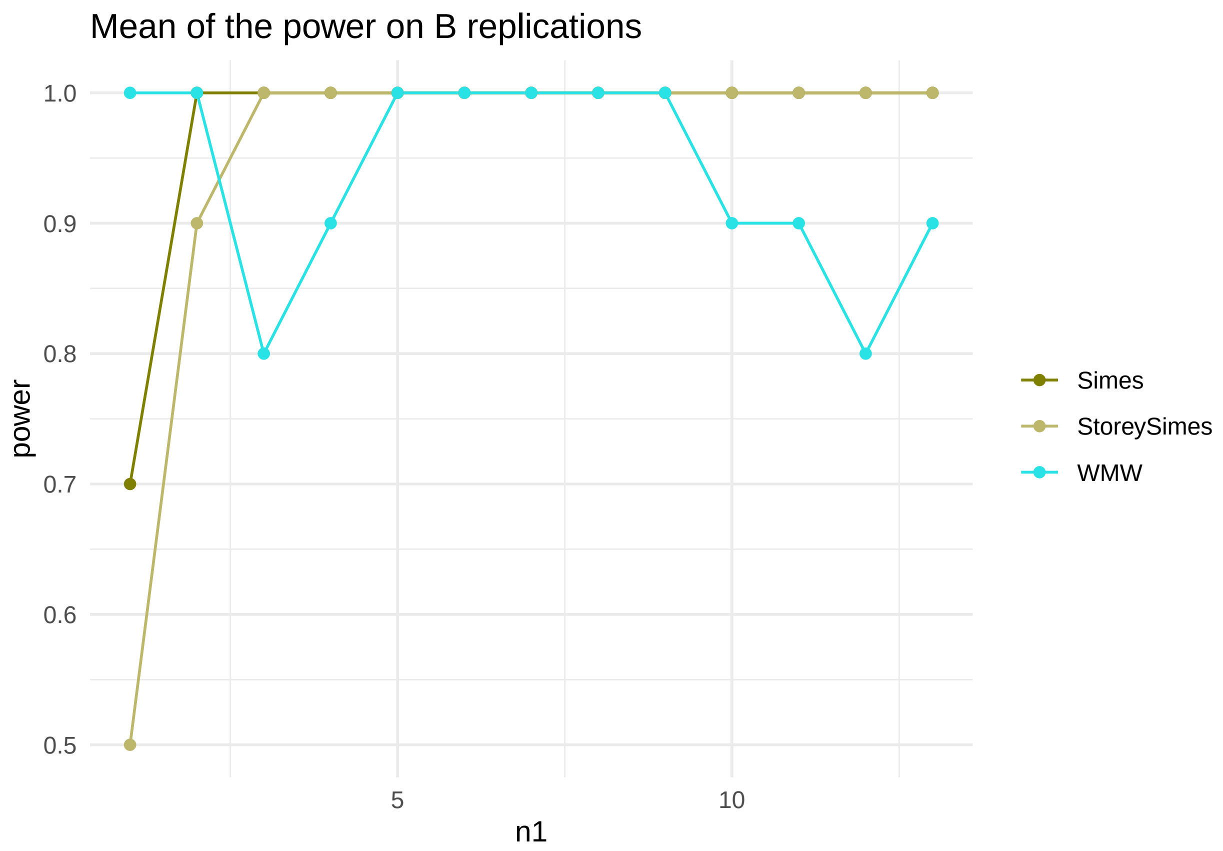
```
## [[6]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"     
## 
## [[6]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics" 
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"     
## 
## 
## [[7]]
## [[7]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"    
## [7] "datasets"  "methods"   "base"     
## 
## [[7]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"     
## 
## [[7]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics" 
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"     
## 
## 
## [[8]]
## [[8]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"    
## [7] "datasets"  "methods"   "base"     
## 
## [[8]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"     
## 
## [[8]][[3]]
##  [1] "hommel"    "nout"      "isotree"   "snow"      "stats"     "graphics" 
##  [7] "grDevices" "utils"     "datasets"  "methods"   "base"
```

```
clusterExport(cluster, list( "l", "Shuttle2", "inliers.ind"))
iso.fo = TrainingIsoForest.S(l=l, in_ind = inliers.ind, dataset=Shuttle2)

# kest = estimatek.S(B=B,
#                    inlier_remaining=iso.fo$inlier_remaining,
#                    S=S,
#                    isofo.model=iso.fo$model,
#                    dataset = Shuttle2)

res = lapply(1:length(n1s),
             function(j) CompareMethod.S(B=B, n1=n1s[j], n=n, m=m, S=S, alpha = alpha,
                                         dataset=Shuttle2,
                                         isofo.model=iso.fo$model,
                                         inlier_remaining=iso.fo$inlier_remaining)
             )

stopCluster(cluster)
```

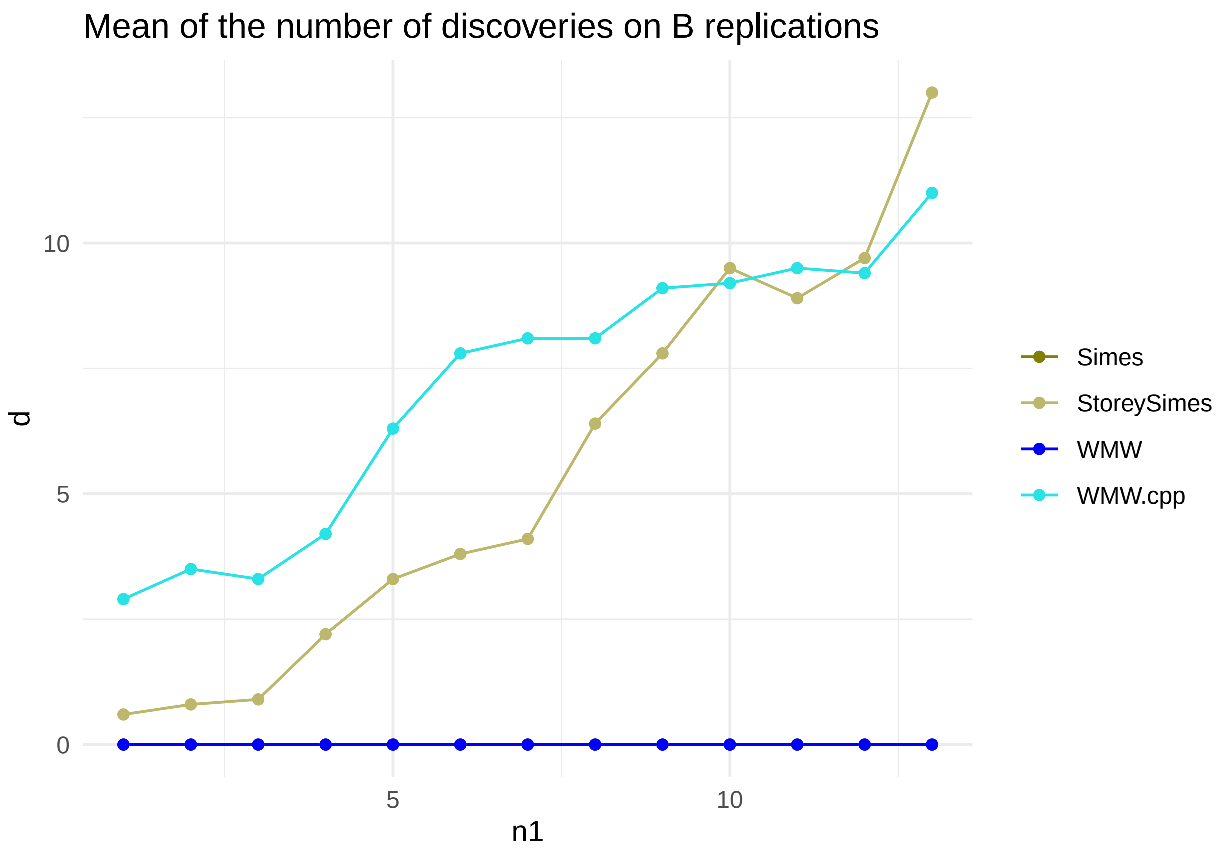
```r
# kest

results = compact_results(res)

d_Sim = vector()
d_StoSimes = vector()
d_WMW = vector()

pow_Sim = vector()
pow_StoSimes = vector()
pow_WMW = vector()

disc_Sim = vector()
disc_StoSimes = vector()
disc_WMW = vector()
disc_WMW.cpp = vector()

for(j in 1:length(n1s)){
  d_Sim[j] = results[[j]]$mean.lb.d[1]
  d_StoSimes[j] = results[[j]]$mean.lb.d[2]
  d_WMW[j] = results[[j]]$mean.lb.d[3]

  pow_Sim[j] = results[[j]]$mean.powerGlobalNull[1]
  pow_StoSimes[j] = results[[j]]$mean.powerGlobalNull[2]
  pow_WMW[j] = results[[j]]$mean.powerGlobalNull[3]

  disc_Sim[j] = results[[j]]$mean.n.disc[1]
  disc_StoSimes[j] = results[[j]]$mean.n.disc[2]
  disc_WMW[j] = results[[j]]$mean.n.disc[3]
  disc_WMW.cpp[j] = results[[j]]$mean.n.disc[4]
}

# Plot lower bound d
df <- data.frame(
  x = n1s,
  Simes = d_Sim,
  StoreySimes = d_StoSimes,
  WMW = d_WMW
)
df_long <- tidyr::pivot_longer(df, cols = -x, names_to = "group", values_to = "y")

ggplot(df_long, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c( "#808000", "#BDB76B", 5)) +
  labs(x = "n1", y = "d", title = "Mean of the lower bound d on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```



```r
# Plot power
dfpower <- data.frame(
  x = n1s,
  Simes = pow_Sim,
  StoreySimes = pow_StoSimes,
  WMW = pow_WMW
)
df_long_power <- tidyr::pivot_longer(dfpower, cols = -x, names_to = "group", values_to = "y")

ggplot(df_long_power, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#808000", "#BDB76B",5)) +
  labs(x = "n1", y = "power", title = "Mean of the power on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```



```
# Plot discoveries
df <- data.frame(
  x = n1s,
  Simes = disc_Sim,
  StoreySimes = disc_StoSimes,
  WMW.cpp = disc_WMW.cpp,
  WMW = disc_WMW
)
df_long <- tidyr::pivot_longer(df, cols = -x, names_to = "group", values_to = "y")

ggplot(df_long, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#808000", "#BDB76B", "blue", 5)) +
  labs(x = "n1", y = "d", title = "Mean of the number of discoveries on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```



```
n.disc.tablelist = list()
for(i in 1:length(n1s)){
  n.disc.tablelist[[i]] = matrix(ncol = 4, nrow = 2)
  colnames(n.disc.tablelist[[i]]) = c("Simes", "StoSimes", "WMW", "WMW.cpp")
  rownames(n.disc.tablelist[[i]]) = c("mean.n.disc", "mean.d")
  n.disc.tablelist[[i]][1,] = apply(results[[i]][["n.disc"]], MARGIN = 2, FUN = mean)
  n.disc.tablelist[[i]][2,] = results[[i]]$mean.lb.d[c(1,2,3,3)]
}

for(i in 1:length(n1s)){
  cat("\n")
  cat(paste("n1=", n1s[i]))
  cat("\n")
  print(n.disc.tablelist[[i]])
}
```

```
##
## n1= 1
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc   0.0      0.6   0     2.9
## mean.d        0.7      0.5   1     1.0
##
## n1= 2
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc   0.0      0.8   0     3.5
## mean.d        1.8      1.2   1     1.0
```

```
##
## n1= 3
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc   0.0      0.9 0.0     3.3
## mean.d        2.5      1.9 0.8     0.8
##
## n1= 4
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc   0.0      2.2 0.0     4.2
## mean.d        3.7      3.2 1.8     1.8
##
## n1= 5
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc   0.0      3.3 0.0     6.3
## mean.d        4.2      4.1 2.2     2.2
##
## n1= 6
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc   0.0      3.8   0     7.8
## mean.d        5.4      5.4   3     3.0
##
## n1= 7
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc   0.0      4.1 0.0     8.1
## mean.d        6.2      6.4 3.4     3.4
##
## n1= 8
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc   0.0      6.4   0     8.1
## mean.d        7.6      7.7   4     4.0
##
## n1= 9
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc   0.0      7.8 0.0     9.1
## mean.d        8.7      8.7 4.9     4.9
##
## n1= 10
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc   0.0      9.5 0.0     9.2
## mean.d        9.7      9.9 5.5     5.5
##
## n1= 11
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc     0      8.9   0     9.5
## mean.d         11     10.7   7     7.0
##
## n1= 12
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc   0.0      9.7 0.0     9.4
## mean.d       11.8     11.8 6.8     6.8
##
## n1= 13
##             Simes StoSimes WMW WMW.cpp
## mean.n.disc     0       13 0.0    11.0
## mean.d         13       13 8.5     8.5
```

```
resShuttle.S7 = list("raw.res"=res,
                        #"k.est" = kest,
                        "compact.results" = results,
                        "n.disc.tablelist" = n.disc.tablelist)
save(resShuttle.S7,
     file="~/nout/trials/RealData/PowerStudy/FinalSimu/Shuttle/resShuttle.S7")
```