Comparison between different local tests: Simes, Simes with Storey and Wilcoxon-Mann-Whitney using the Lehmann alternative distribution with k=2

2023-07-24

The aim is to compare on real datasets the performance of three closed testing procedures, which respectively use Simes local test with and without Storey estimator for the proportion of true null hypotheses and Wilcoxon-Mann-Whitney local test. We will simulate outliers distribution so that it will be to the Lehmann's alternative with $k = 2$. Denoting inliers distribution by F, we are going to simulate the outliers distribution corresponding to F^k with $k = 2$ in order to perform a power analysis and to show that closed testing procedure with Wilcoxon-Mann-Whitney local test is more powerful than closed testing with Simes local test with and without Storey estimator.

R functions and libraries

```
library(nout)
library(R.matlab)
library(isotree)
library(farff)
library(tictoc)
library(tidyverse)
library(doSNOW)
library(ggplot2)

compact_results = function(res){
  resT=as.data.frame(t(res))

  results = list()
  for(j in 1:length(n1s)){
    discoveries = as.data.frame(
      cbind("d_BH"=unlist(res[[j]][rownames(res[[j]])=="d_BH",]),
            "d_StoBH"=unlist(res[[j]][rownames(res[[j]])=="d_StoBH",]),
            "d_Sim"=unlist(res[[j]][rownames(res[[j]])=="d_Sim",]),
            "d_StoSimes"=unlist(res[[j]][rownames(res[[j]])=="d_StoSimes",]),
            "d_WMW"=unlist(res[[j]][rownames(res[[j]])=="d_WMW",])
            )
    )
    mean.discoveries = apply(discoveries, MARGIN = 2, FUN = mean)

    power.GlobalNull = as.data.frame(discoveries>0)
    mean.powerGlobalNull = apply(power.GlobalNull, MARGIN = 2, FUN = mean)

    # out_identification = as.data.frame(
    #   cbind("out.identif_WMW"=
    #           unlist(res[[j]][rownames(res[[j]])=="outlier.identified_WMW",]),
    #          "out.identif_StoSimes"=
```

```r
    #           unlist(res[[j]][rownames(res[[j]])=="outlier.identified_StoSimes",]),
    #         "out.identif_Simes"=
    #           unlist(res[[j]][rownames(res[[j]])=="outlier.identified_Simes",])
    #         )
    # )
    # mean.out_identification = apply(out_identification, MARGIN = 2, FUN = mean)
    # mean.out_identification_pos = apply(out_identification>0, MARGIN = 2, FUN = mean)

    results[[j]] = list("discoveries" = discoveries,
                        "mean.discoveries" = mean.discoveries,
                        "power.GlobalNull" = power.GlobalNull,
                        "mean.powerGlobalNull" = mean.powerGlobalNull,
                        # "out_identification" = out_identification,
                        # "mean.out_identification" = mean.out_identification,
                        # "mean.out_identification>0" = mean.out_identification_pos,
                        "pi.not" = res[[j]][rownames(res[[j]])=="pi.not",],
                        "uniques" = res[[j]][rownames(res[[j]])=="uniques",],
                        "n1" = res[[j]][rownames(res[[j]])=="n1",1],
                        "alpha" = res[[j]][rownames(res[[j]])=="alpha",1])
  }
  return(results)
}


TrainingIsoForest = function(l, dataset){

  tr_ind = sample(in_ind, size = l)
  tr = dataset[tr_ind,]
  isofo.model = isotree::isolation.forest(tr, ndim=ncol(dataset), ntrees=10, nthreads=1,
                            scoring_metric = "depth", output_score = TRUE)$model
  in_index2 = setdiff(in_ind, tr_ind)

  return(list("model"=isofo.model, "inlier_remaining" = in_index2))

}



CompareMethodLehmannOutliers = function(B, n1, n, k, out_ind, inlier_remaining, isofo.model, dataset){

  n0 = n-n1
  foreach(b = 1:B, .combine=cbind) %dopar% {

    N = n0 + m + k*n1
    in_index3 = sample(inlier_remaining, size = N)
    cal_ind = in_index3[1:m]
    te_ind.augmented = in_index3[(m+1):N]
    cal = dataset[cal_ind,]
    te = dataset[te_ind.augmented,]
    S_cal = predict.isolation_forest(isofo.model, cal, type = "score")
    augmented.S_te = predict.isolation_forest(isofo.model, te, type = "score")

    if(n1==0)
      S_te = augmented.S_te
    if(n1==n)
      S_te = sapply(1:n1, FUN=function(i) max(augmented.S_te[(1+k*(i-1)):(i*k)]))
    if(0<n1&n1<n)
      S_te = c(augmented.S_te[(1+k*n1):(n0+k*n1)],
                    sapply(1:n1, FUN=function(i) max(augmented.S_te[(1+k*(i-1)):(i*k)])))

      d_WMW = nout::d_MannWhitney(S_Y = S_te, S_X = S_cal, alpha=alpha)
      d_Sim = nout::d_Simes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      StoSimes = nout::d_StoreySimes(S_X = S_cal, S_Y = S_te, alpha = alpha)
      d_StoSimes = StoSimes$d
      pi.not = StoSimes$pi.not
      d_BH = nout::d_benjhoch(S_X = S_cal, S_Y = S_te, alpha = alpha)
      d_StoBH = nout::d_StoreyBH(S_X = S_cal, S_Y = S_te, alpha = alpha)
      uniques = length(unique(c(S_cal, S_te)))

      # # outlier identification with WMW
      # conf.pval = sapply(1:n, function(j) (1+sum(S_cal >= S_te[j]))/(m+1))
      # confvalid.pval = conf.pval<alpha
      # confvalid.index = which(conf.pval<alpha)
      #
      # if(d_WMW>0){
      #   outlierTF = sapply(confvalid.index, function(h)
      #       nout::dselection_MannWhitney(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
      #   outlier.identified_WMW = confvalid.index[as.logical(outlierTF)]
      # }
      # else  outlier.identified_WMW = NULL
      #
      # # outlier identification with Simes
      # if(d_Sim>0){
      #   outlierTF = sapply(confvalid.index, function(h)
      #       nout::dselection_Simes(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
      #   outlier.identified_Simes = confvalid.index[as.logical(outlierTF)]
      # }
      # else  outlier.identified_Simes = NULL
      #
      # # outlier identification with StoreySimes
      # if(d_StoSimes>0){
      #   outlierTF = sapply(confvalid.index, function(h)
      #       nout::dselection_StoreySimes(S_Y = S_te, S_X = S_cal, S = h, alpha=alpha))
      #   outlier.identified_StoSimes = confvalid.index[as.logical(outlierTF)]
      # }
      # else  outlier.identified_StoSimes = NULL

      return(list("d_BH" = d_BH,
                  "d_StoBH" = d_StoBH,
                  "d_Sim" = d_Sim,
                  "d_StoSimes" = d_StoSimes,
                  "d_WMW" = d_WMW,
                  # "outlier.identified_WMW" = length(outlier.identified_WMW),
                  # "outlier.identified_Simes" = length(outlier.identified_Simes),
                  # "outlier.identified_StoSimes" = length(outlier.identified_StoSimes),
                  "uniques" = uniques,
                  "n1" = n1,
                  "pi.not" = pi.not,
                  "alpha" = alpha))
  }
}
```

In the following we set the calibration set and the test set size, respectively l and m, so that the nominal level α is proportional to $\frac{m}{l+1}$. The train set size is equal to n and the number of iterations is $B = 10^4$.

Digits dataset

The dataset is available at http://odds.cs.stonybrook.edu/pendigits-dataset.

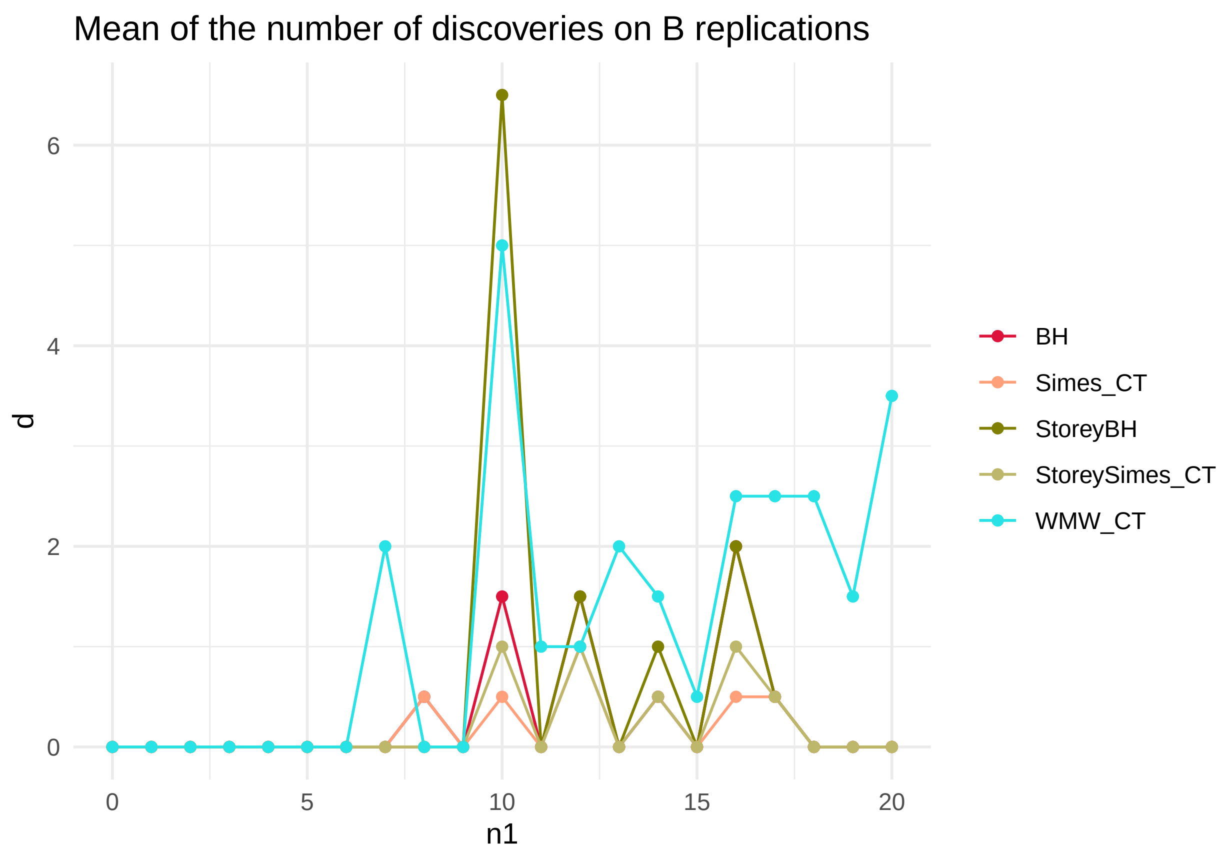
```
set.seed(321)

# Initializing parameters
B = 2
m = 199
l = 199
n = 20
alpha = n/(l+1)
n1s = seq(from=0, to=n, by=1)

data = readMat("~/nout/trials/RealData/Datasets/Dataset digits/pendigits.mat")
dataset = cbind(data$X, data$y); colnames(dataset)[ncol(dataset)] = "y"
in_ind = which(dataset[,ncol(dataset)]==0)
out_ind = which(dataset[,ncol(dataset)]==1)
theta = length(out_ind)/nrow(dataset) # proportion of outliers in the entire dataset

cluster <- makeCluster(parallel::detectCores())
registerDoSNOW(cluster)
clusterEvalQ(cluster, {list(library(isotree), library(nout))})
```

```
## [[1]]
## [[1]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[1]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[2]]
## [[2]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[2]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[3]]
## [[3]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[3]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
##
## [[4]]
## [[4]][[1]]
## [1] "isotree"   "snow"      "stats"     "graphics"  "grDevices" "utils"
## [7] "datasets"  "methods"   "base"
##
## [[4]][[2]]
##  [1] "nout"      "isotree"   "snow"      "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
```

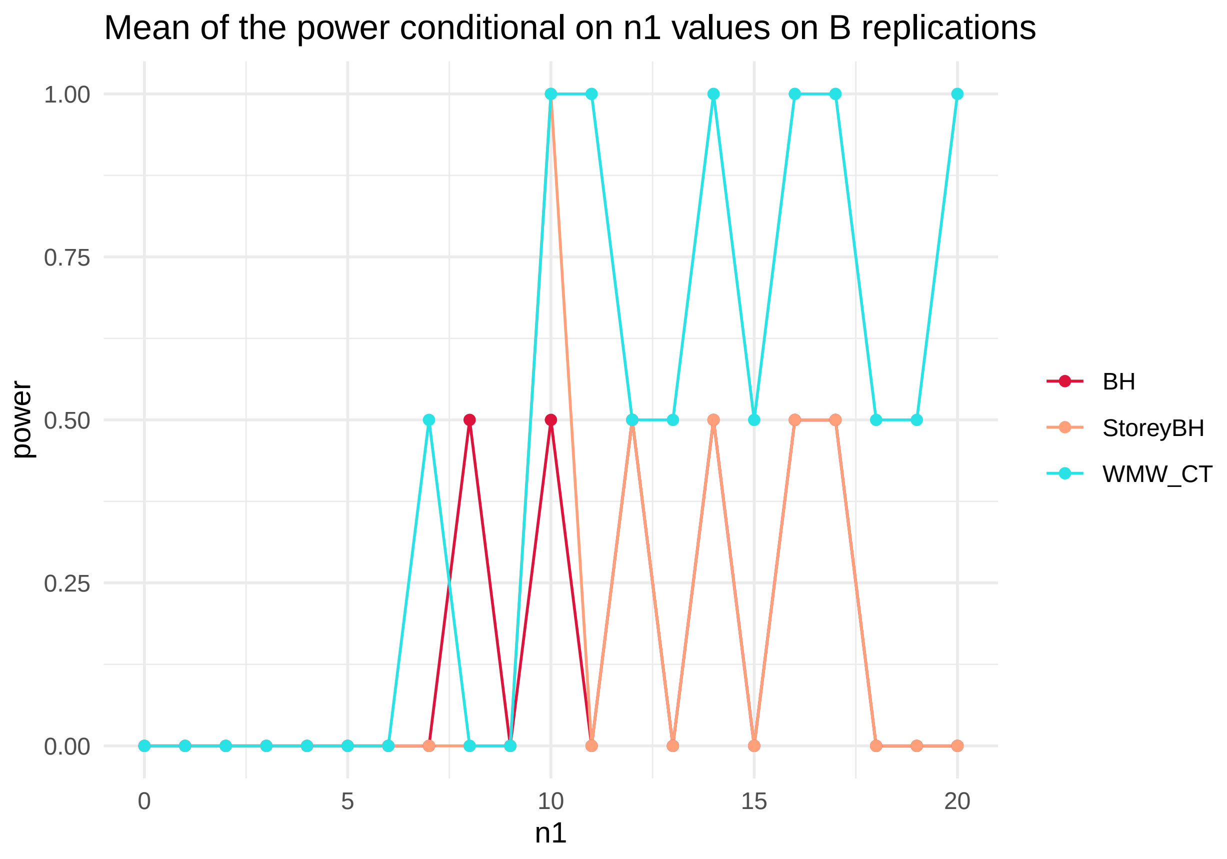
```
clusterExport(cluster, list("n", "m", "l", "in_ind", "out_ind", "dataset", "alpha"))

tic()
modeltrain = TrainingIsoForest(l=l, dataset=dataset)
res = lapply(1:length(n1s),
             function(j) CompareMethodLehmannOutliers(B=B, k=2, n1=n1s[j], n=n,
                               dataset=dataset,
                               isofo.model=modeltrain$model,
                               out_ind=out_ind,
                               inlier_remaining=modeltrain$inlier_remaining))
toc()
```

```
## 6.75 sec elapsed
```

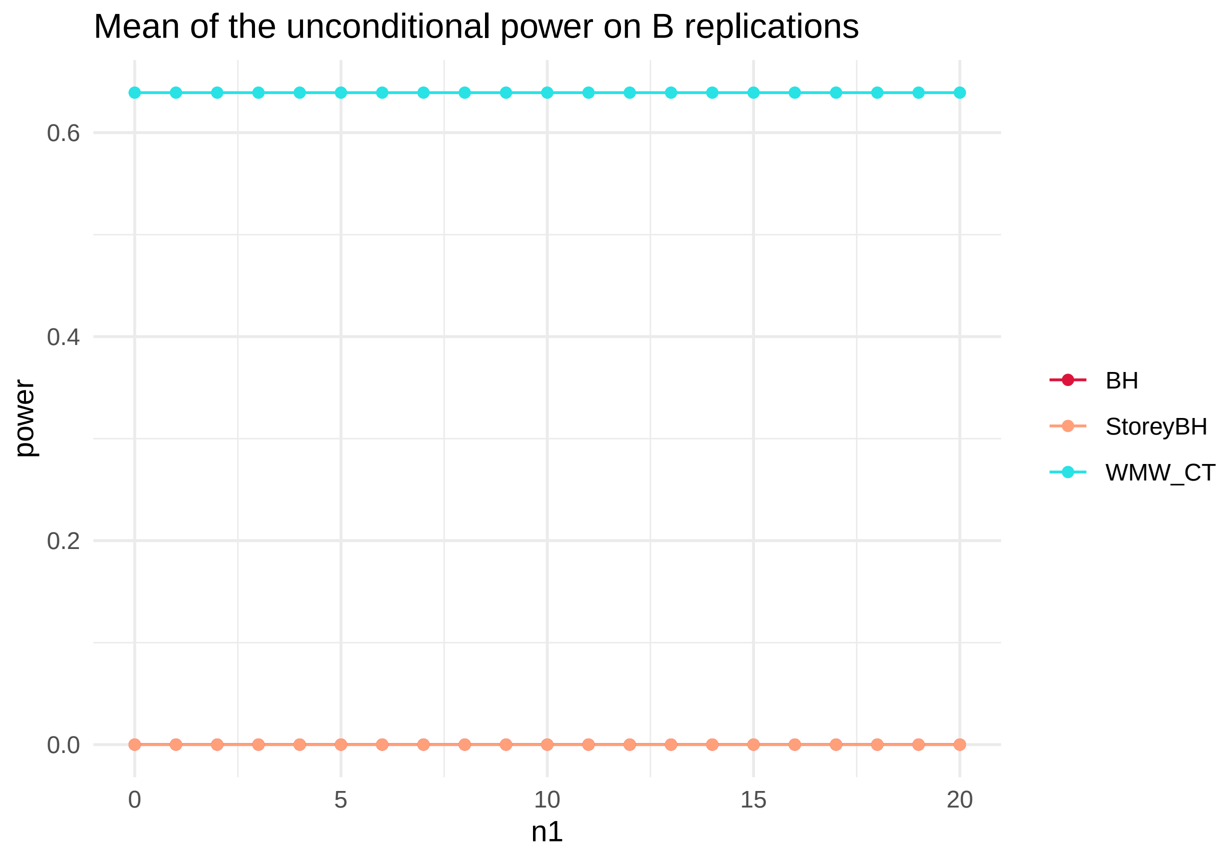
```
stopCluster(cluster)

results = compact_results(res)

d_BH = vector()
d_StoBH = vector()
d_Sim = vector()
d_StoSimes = vector()
d_WMW = vector()

pow_BH = vector()
pow_StoBH = vector()
pow_Sim = vector()
pow_StoSimes = vector()
pow_WMW = vector()

for(j in 1:length(n1s)){
  d_BH[j] = results[[j]]$mean.discoveries[1]
  d_StoBH[j] = results[[j]]$mean.discoveries[2]
  d_Sim[j] = results[[j]]$mean.discoveries[3]
  d_StoSimes[j] = results[[j]]$mean.discoveries[4]
  d_WMW[j] = results[[j]]$mean.discoveries[5]

  pow_BH[j] = results[[j]]$mean.powerGlobalNull[1]
  pow_StoBH[j] = results[[j]]$mean.powerGlobalNull[2]
  pow_Sim[j] = results[[j]]$mean.powerGlobalNull[3]
  pow_StoSimes[j] = results[[j]]$mean.powerGlobalNull[4]
  pow_WMW[j] = results[[j]]$mean.powerGlobalNull[5]

}

# Plot discoveries conditional on n1
df <- data.frame(
  x = n1s,
  BH = d_BH,
  StoreyBH = d_StoBH,
  Simes_CT = d_Sim,
  StoreySimes_CT = d_StoSimes,
  WMW_CT = d_WMW
)
df_long <- tidyr::pivot_longer(df, cols = -x, names_to = "group", values_to = "y")

ggplot(df_long, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#DC143C", "#FFA07A", "#808000", "#BDB76B", 5)) +
  labs(x = "n1", y = "d", title = "Mean of the number of discoveries on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```



```r
# Plot power conditional on n1
dfpower <- data.frame(
  x = n1s,
  BH = pow_BH,
  StoreyBH = pow_StoBH,
  WMW_CT = pow_WMW
)
df_long_power <- tidyr::pivot_longer(dfpower, cols = -x, names_to = "group", values_to = "y")

# Plot the lines with different colors and legends
ggplot(df_long_power, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#DC143C","#FFA07A",5)) +
  labs(x = "n1", y = "power", title = "Mean of the power conditional on n1 values on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```



```r
# Plot unconditional power
probsn1 = sapply(0:n, function(k) (1-theta)^k*theta^(n-k))
uncond.pow_BH = sum(pow_BH*probsn1)
uncond.pow_StoreyBH = sum(pow_StoBH*probsn1)
uncond.pow_WMW = sum(pow_WMW*probsn1)


dfuncondpower <- data.frame(
  x = n1s,
  BH = uncond.pow_BH,
  StoreyBH = uncond.pow_StoreyBH,
  WMW_CT = uncond.pow_WMW
)
df_long_uncondpower <- tidyr::pivot_longer(dfuncondpower, cols = -x, names_to = "group", values_to = "y

# Plot the lines with different colors and legends
ggplot(df_long_uncondpower, aes(x = x, y = y, color = group)) +
  geom_line() +
  geom_point()+
  scale_color_manual(values = c("#DC143C","#FFA07A",5)) +
  labs(x = "n1", y = "power", title = "Mean of the unconditional power on B replications") +
  theme_minimal() +
  theme(legend.title = element_blank())
```



```
# outlier.identification = list()
# for(i in 1:length(n1s)){
#   outlier.identification[[i]] = matrix(nrow = 3, ncol = 4)
#   rownames(outlier.identification[[i]]) = c("WMW", "Simes", "StoSimes")
#   colnames(outlier.identification[[i]]) = c("mean.out.identif", "%successful.identification",
#                                              "mean.d", "mean.d>0(power)")
#   outlier.identification[[i]][,1] = apply(
#     results[[i]][["out_identification"]], MARGIN = 2, FUN = mean)
#   outlier.identification[[i]][,2] = apply(
#     results[[i]][["out_identification"]]>0, MARGIN = 2, FUN = mean)
#   outlier.identification[[i]][,3] = results[[i]]$mean.discoveries[c(3,4,5)]
#   outlier.identification[[i]][,4] = results[[i]]$mean.powerGlobalNull[c(3,4,5)]
# }
#
# for(i in 1:length(n1s)){
#   cat("\n")
#   cat(paste("n1=", n1s[i]))
#   print(outlier.identification[[i]])
# }

resDigits0.1k2 = list("raw.res"=res,
                    "unconditional.power" = cbind("BH" = uncond.pow_BH,
                                                  "StoreyBH" = uncond.pow_StoreyBH,
                                                  "WMW_CT" = uncond.pow_WMW),
                    "compact.results" = results #,
                    #"outlier.identification" = outlier.identification
                    )
save(resDigits0.1k2, file="~/nout/trials/RealData/PowerStudy/FinalSimu/Digits/Lehmannk2/resDigits0.1k2"
```