Why Does Insurance Work?

For an individual, suppose Pr(Claim)=0.1 and the claim amount, if it occurs, is 1000.

1. E(Claim Amount) = 0.1(1000)=100
2. SD(Claim Amount) = $\sqrt{0.9(0-100)^2 + 0.1(1000-100)^2} = 300$
3. To ensure Pr(Claim Amount $>$ P) $<$ 0.01, need P = 1000.

Why Does Insurance Work?

For n independent individuals, suppose Pr(Claim)=0.1 and claim amounts, if they occur, are 1000.

1. Pr(Any Claim) = $1 - (0.9)^n > 0.1$
2. E(Total Claim Amount) = $100n$
3. SD(Total Claim Amount) = $300\sqrt{n}$
4. Pr(Total Claim Amount $> n$P) < 0.01 implies:
   i. 1-$\Phi\{\frac{n\text{P}-100n}{300\sqrt{n}}\} < 0.01$ (provided n is large)
   ii. $\frac{\sqrt{n}(\text{P}-100)}{300} > \Phi^{-1}(0.99)$
   iii. $\text{P} > 100 + (2.33)(300/\sqrt{n})$
   iv.

n	P
10	321.04
25	239.80
50	198.86
100	169.90
1000	122.10
10000	106.99

Issues:

1. Effect magnified when claim distribution is highly variable, and in particular highly skewed. Study skewed distributions.

2. Claim amounts and claim numbers are random. Study random sums.

3. Unused premiums carry over to next period. Study claim processes (Poisson processes) through time.