Over-fitting (example — polynomial regression)

$n = 2$

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

$p = 2$

$n = 3$

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \varepsilon$$

$p = 3$

$n = 4$

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \varepsilon$$

$p = 4$
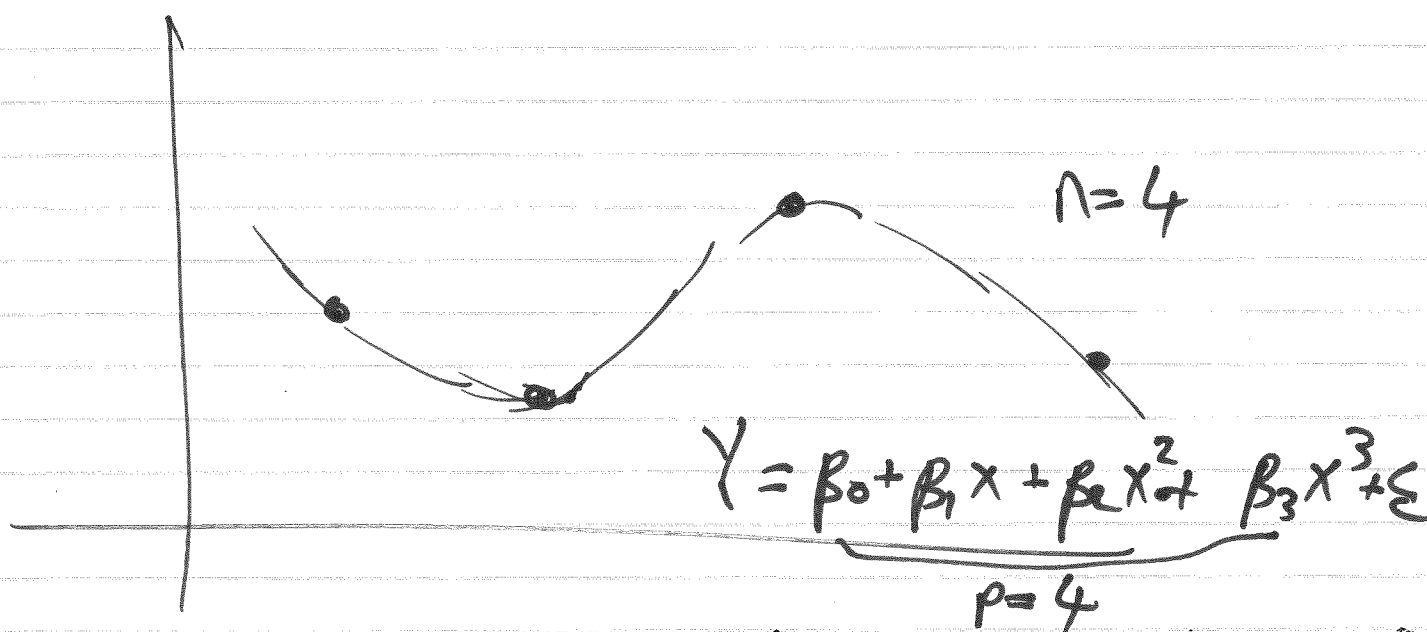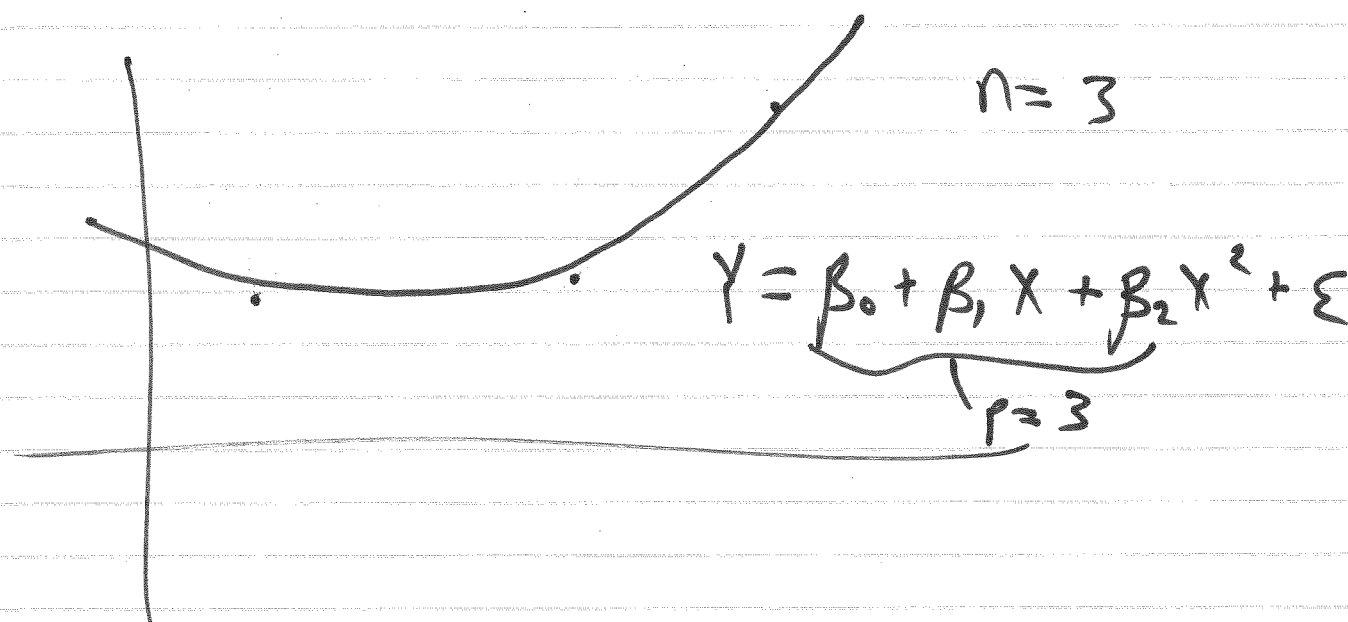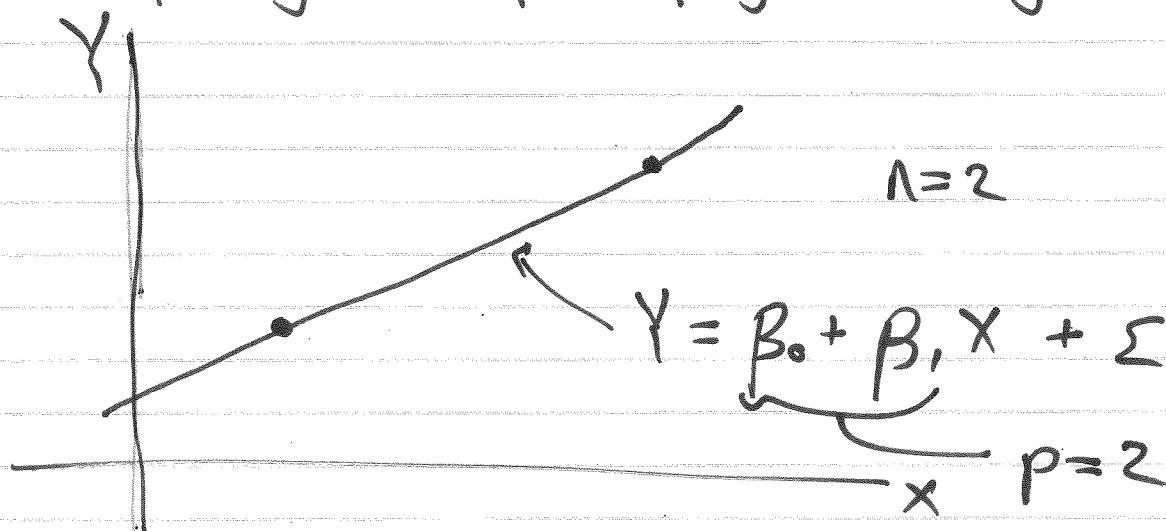
In each case, no degrees of freedom left to estimate σ^2 in $\varepsilon \overset{iid}{\sim} N(0, \sigma^2)$