

# High-speed Railway Timetabling Model based on Transfer Optimization

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**Abstract**—The development of China's high-speed railway has made remarkable achievements. In this paper, a high-speed railway timetabling model based on transfer optimization is established. First, the current situation of China's high-speed railway development is introduced. To express the model in detail, the event-activity network (EAN) is used. To optimize the transfer, the current situation of passenger transfer in hub stations is studied in detail. The concept 'transferable train pair' is defined and used for modelling, and the ideal connecting time of transfer is calculated. With several assumptions, the constraints and objective function of the model are proposed. This model could guarantee passenger transfer, decrease passenger's transfer waiting time and thereby improve passenger's satisfaction.

**Keywords**—Railway timetabling, Transfer, model, connecting time

## I. INTRODUCTION

In recent years, the development of China's high-speed railway has made remarkable achievements. It is estimated that by 2025, the total length of high-speed railway in China will reach 38,000 kilometers, forming a high-speed railway network with '8 East-West & 8 North-South High-speed Railway Lines' as skeleton, connecting large and medium-sized cities with a population of more than 500,000. Reliability and punctuality are important characteristics of high-speed railway transportation [1].

The train timetable is a plan of all train movements that are supposed to occur in a railway system over a given period of time [2]. A typical train timetable in current railway dispatching system is shown in Fig. 1.

High-speed railway timetabling is the key and core to ensure safety and improve efficiency for high-speed train operation. Different from European countries, China's high-speed railway has been operated as a whole network rather

than by different companies operating specific lines. The research on China's high-speed railway timetabling model should not be limited to a single railway line, but to conduct collaborative and integrated research based on the specific conditions of China's high-speed railway network.

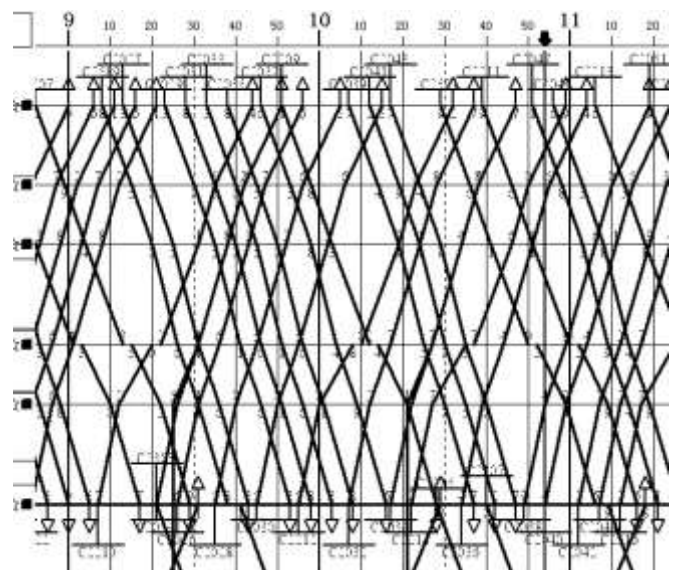


Fig. 1 A typical railway timetable in railway dispatching system .

Since China's high-speed railway network is so large, it's not possible to have trains between every two terminals. Therefore, passengers especially who travel between different high-speed railway lines often need to transfer from one train to another to reach their final destination as no though train between their departure station and terminal station. Thus in this paper we take passenger transfers between different high-speed railway lines into consideration and proposes the concept of 'transferable train pair' to optimize the transfer experience of passengers. A train

timetabling model based on transfer optimization is established.

## II. EVENT-ACTIVITY NETWORK

In the process of train timetabling, event-activity network (EAN) is often used to represent all train operation processes in all stations [3]. An event-activity network consist of nodes and arcs. Nodes represent the operation (event) of the train at the station. Events consist of departure, arrival and through. Arcs directed connecting two adjacent event nodes represent train processes (activities), including a train running from one station to another (running), a train stop at a station (dwelling), operation intervals between adjacent trains on same railway lines at the same station (headway), and transfer between trains on different railway lines (connection). A typical railway timetable and its corresponding event-activity network are shown in Fig. 2 and Fig. 3.

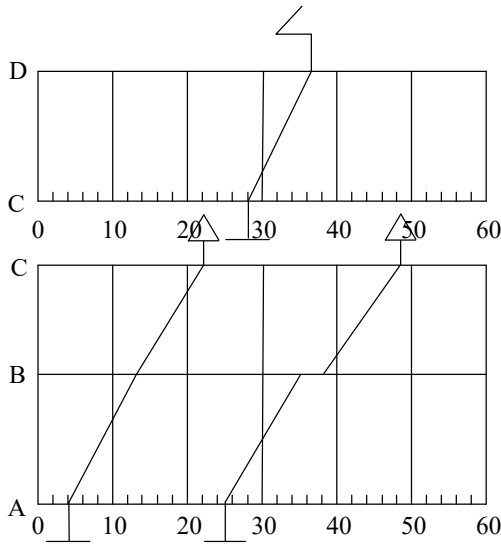


Fig. 2 A typical railway timetable

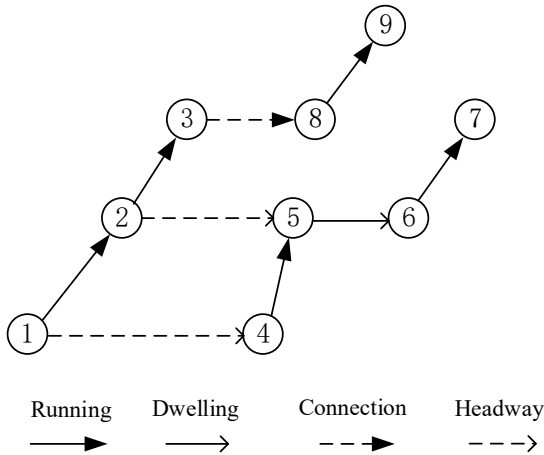


Fig. 3 Corresponding event-activity network

Event-activity network is a directed acyclic graph, often represented as  $N = (\mathcal{E}, A)$ .  $\mathcal{E}$  is the set of all events,  $\mathcal{E}_{dep}$ ,  $\mathcal{E}_{arr}$  and  $\mathcal{E}_{thru}$  are sets of departure, arrival and through. An event in event-activity network is defined by train number, station, event type (departure, arrival and through) and

scheduled time.

$A$  is the set of train activities,  $A_{run}$ ,  $A_{dwell}$ ,  $A_{head}$  and  $A_{con}$  are sets of running, dwelling, headway and connection. An activity  $a_{i,j}$  in event-activity network is defined by starting event  $i$ , end event  $j$ , process type (running, dwelling, connection and headway), scheduled process time interval  $t_{i,j}$  and minimum process time interval  $t_{i,j}^{\min}$ .

In the current practice of timetabling, a new optimized timetable is typically derived from the existing one [4]. The optimized new railway timetable should still retain the original train stop schedule plan and train operation sequence. Thus, we hereby face a problem of optimizing a 'desired' timetable with fixed, pre-determined train orders (sequences) [5]. On the same premise, for the model proposed in this paper, the sequence of all the nodes representing train operations is pre-determined. Therefore, all the arcs representing train operation processes in the event-activity network are pre-determined.

## III. TRANSFERABLE TRAIN PAIR

Nowadays, intersection cities of multiple high-speed railway lines in China, such as Beijing, Zhengzhou, Guangzhou and Chongqing, have become high-speed railway hubs. As mentioned before, passengers sometimes have to transfer at these hubs. In this paper, a pair of trains that can be transferred between in a reasonable travel plan is defined as transferable train pair. A transferable train pair consists two trains, previous train and subsequent train. A typical transferable train pair can be shown in Fig. 4.

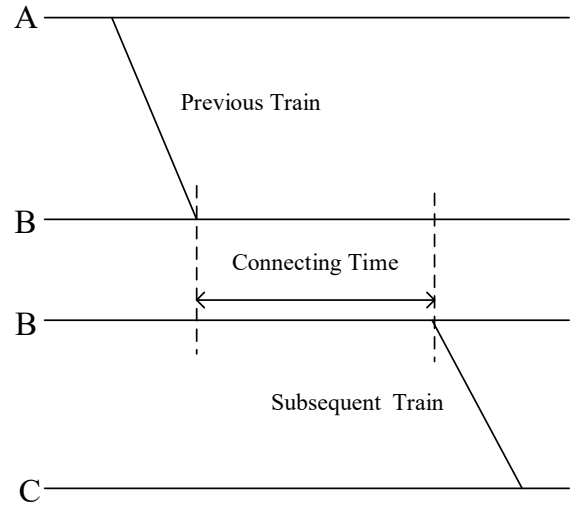


Fig. 4 A typical transferable train pair

In a transfer process, passengers get off the previous train at a hub station, and walk to the subsequent train through the transfer path. The time interval between the scheduled arrive time of the previous train and the scheduled departure time of the subsequent train is defined as the connecting time, as shown in Fig. 4.

The connecting time is very crucial for the transfer experience of passengers. Passengers may miss their transfer if the connecting time is not enough, and passengers could have too much waiting time if the connecting time is too long. These two cases apparently impact on the transfer experience

of passengers and reduce passenger satisfaction. When passengers miss their transfer, they have to change their ticket and wait for the next train to their final destination. Therefore, it is necessary to reduce the transfer waiting time of passengers and try not to make them miss their transfer. The connecting time is discussed in detail in the next section.

#### IV. CONNECTING TIME

The connecting time of a transferable train pair is crucial to passengers. When timetabling, the connecting time of all transferable train pair should be assigned properly, the ideal connecting time of a transferable train pair is analyzed in detail in this section.

In our research, the ideal connecting time of a transferable train pair consists of three parts: the delay buffer time, the transfer walking time and the subsequent boarding time. The ideal connecting time  $t_{i,j}^{ideal}$  can be represented as (1). the delay buffer time, the transfer walking time and the subsequent boarding time are represented as  $b_i$ ,  $w_{i,j}$  and  $c_j$ .

$$t_{i,j}^{ideal} = b_i + w_{i,j} + c_j, \forall i, j \in \mathcal{E}, a_{i,j} \in A_{con} \quad (1)$$

The delay buffer time is the time reserved in the ideal connecting time to prevent the delay of the previous train from causing the passengers miss their transfer. For many reason, the previous train may arrive later than scheduled, in this case, the possible delay of the previous train should not affect the transfer at a big chance (such as 90% or 95%). To obtain the delay buffer time  $b_i$ , the delay time of the previous train should be studied. Some scholars have shown that the delay time follows the exponential distribution model [6], and its probability density function is shown in (2).

$$f(t_d^i) = \frac{1}{\lambda} e^{-\frac{t_d^i}{\lambda}}, \forall i, j \in \mathcal{E}, a_{i,j} \in A_{con} \quad (2)$$

Where  $t_d^i$  is the delay time of the previous train.  $\lambda$  is the average delay time of the previous train, this value can be obtained from the data provided by railway bureau. Therefore, based on probability theory, the delay buffer time  $b_i$  can be set according to the required train punctuality (possibility). For example, If the required possibility is 90%, the value of  $b_i$  can be set to cover 90% cases of the previous train delay, guaranteeing passengers not miss their transfer.

The transfer walking time is the time reserved for passengers to get off the previous train and walk to the gate of the subsequent train. This value is related to the walking speed of passengers and the length of the transfer path at the hub station. Many studies show that the distribution of transfer walking time is skewed, which is usually expressed by lognormal distribution, and its probability density function is shown in (3).

$$f(w_{i,j}) = \frac{1}{w_{i,j} \sigma \sqrt{2\pi}} e^{-\frac{(\ln w_{i,j} - u)^2}{2\sigma^2}}, \forall i, j \in \mathcal{E}, a_{i,j} \in A_{con} \quad (3)$$

Where  $w_{i,j}$  is the transfer working time,  $\sigma$ ,  $u$  are two parameters of the lognormal distribution. These two parameters can be calculated by the length of the transfer path and the average walking speed of passengers. Similar to the delay buffer time, in the distribution of transfer walking time, a big possibility should be assigned to cover a large proportion (such as 90% or 95%) of passengers for their transfer. Therefore, based on probability theory, the transfer walking time  $w_{i,j}$  can be set according to the required train punctuality (possibility). For example, If the required possibility is 90%, the value of  $w_{i,j}$  can be set to cover 90% cases of the transfer walking time, guaranteeing passengers not miss their transfer.

The subsequent boarding time is the time passengers are required to arrive at the ticket gate of the subsequent train before the departure of the subsequent train. The value of this time is a unified value of the station, that is, the subsequent boarding time is a fixed value related to the station management. For example, if the station stops checking tickets 3 minutes before the train departure, The subsequent boarding time  $c_j$  is 3 minute.

By calculating the delay buffer time, the transfer walking time and the subsequent boarding time, the ideal connecting time could be obtained. This value can be used in the model. The detail of the high-speed railway timetabling model based on transfer optimization is discussed in the next section.

#### V. MODEL DESIGN

The model consists of assumptions, constraints and objective function.

##### A. Assumptions

To simplify the process of modelling, the assumptions of the model are as follows:

- (1) As mentioned before, the train stop schedule plan and train operation sequence is fixed as the original timetable. all the nodes and arcs in the event-activity network are pre-determined.
- (2) In our research, event are only consists of arrival, through and departure, other possible events(train operations at stations) are not considered.
- (3) Like several scholars, stations are considered with unlimited traffic capacity, the tracks and switches in stations are not considered [7-8].
- (4) Only normal people's transfer are considered, the transfer path and transfer walking time of disabled people are not considered in this paper.

##### B. Constraints

In order to guarantee the operation safety and improve the efficiency of high-speed railway transportation, the constraints of the model are essential. The constraints of the high-speed railway timetabling model based on transfer optimization are as follows:

- (1) Event Sequence constraint.

As mentioned before, all the nodes and arcs in the event-activity network are pre-determined. That is, the order of all

nodes is fixed. Therefore, the event sequence constraint is shown in (4).

$$t_j - t_i > 0, \forall i, j \in \mathcal{E}, a_{i,j} \in A \quad (4)$$

Where  $t_i$  and  $t_j$  are scheduled time of event  $i$  and event  $j$ . And  $a_{i,j}$  is an activity between event  $i$  and event  $j$ . This constraint shows that event  $i$  happens before event  $j$ .

#### (2) Running time constraint.

As mentioned before, in our research, the new optimized timetable is derived from the current one. Therefore, the train running time between each two station is fixed, the running time constraint is shown in (5).

$$t_j - t_i = r_{i,j}, \forall i, j \in \mathcal{E}, a_{i,j} \in A_{run} \quad (5)$$

Where  $t_i$  and  $t_j$  are scheduled time of event  $i$  and event  $j$ ,  $r_{i,j}$  is the running time of activity of  $a_{i,j}$ , the running time is always a fixed value.

#### (3) Dwelling time constraint.

In our research, although the optimized train timetable is derived from the existing one, the dwelling time is flexible in our model. The dwelling time constraint is shown in (6).

$$d_{i,j}^{\min} \leq t_j - t_i \leq d_{i,j}^{\max}, \forall i, j \in \mathcal{E}, a_{i,j} \in A_{dwell} \quad (6)$$

Where  $d_{i,j}^{\min}$  and  $d_{i,j}^{\max}$  are the minimum and maximum dwelling time of the train at the hub station of the activity  $a_{i,j}$ .

#### (4) Headway time constraint.

To guarantee the safety of high-speed railway operation, there should be minimum time intervals between all the events at the same station. the headway time constraint is shown in (7).

$$t_j - t_i \geq h_{i,j}, \forall i, j \in \mathcal{E}, a_{i,j} \in A_{head} \quad (7)$$

Where  $h_{i,j}$  is the minimum time intervals between event  $i$  and event  $j$ .

#### (5) Connecting time constraint.

This constraint is the core of our research. The ideal connecting time of a transferable train pair is discussed in detail. Based on that, the connecting time constraint is shown in (8).

$$t_{i,j}^{ideal} \leq t_j - t_i \leq k \cdot t_{i,j}^{ideal}, \forall i, j \in \mathcal{E}, a_{i,j} \in A_{conn} \quad (8)$$

Where  $t_{i,j}^{ideal}$  is the ideal connecting time between event  $i$

and event  $j$ . In order to guarantee the transfers of passengers, we set the connecting time not less than the ideal connecting time. Therefore, passengers are not likely to miss their transfer by using the model proposed in this paper.

But passengers cannot wait for too long time when transfer. When passengers transfer, as the transfer waiting time increases, passenger satisfaction will decrease. Transfer waiting time is an important factor affecting passenger satisfaction. Therefore, we set an upper limit for the connecting time,  $k$  is a factor larger than one. Suggest values of  $k$  is 1.2 to 1.5.

#### C. Objective Function

In this model, the objective is to minimize the transfer waiting time of passengers. And we also want to balance transfer waiting time between all transferable train pairs. Based on those ideology, the objective function of the model is shown in (9).

$$Z = \min \sum (t_{i,j} - t_{i,j}^{ideal}), \forall i, j \in \mathcal{E}, a_{i,j} \in A_{conn} \quad (9)$$

Where  $t_{i,j}$  is the connecting time between event  $i$  and event  $j$ ,  $t_{i,j}^{ideal}$  is the ideal connecting time between event  $i$  and event  $j$ .

#### VI. CONCLUSION

In this paper, the high-speed railway timetabling model based on transfer optimization is established. The situation of China's high-speed railway development is introduced. To optimize the transfer, the current situation of passenger transfer in hub stations is studied in detail. The concept 'transferable train pair' is defined and used for modelling, and the ideal connecting time of transfer is calculated. With several assumptions and event-activity network, the constraints and objective function of this model are proposed.

This model could guarantee passenger transfer, decrease passenger's transfer waiting time and thereby improve passenger's satisfaction.

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