機械学習・ディープラーニングのための
基礎数学講座　微分・線形代数

SkillUP AI

3章
線形代数の基礎

例題解答

例題１：ベクトルの基礎

$\boldsymbol{x} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \boldsymbol{y} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$とする

(1) $\boldsymbol{x} \cdot \boldsymbol{y} = 1 \cdot (-2) + 2 \cdot 1 + (-2) \cdot 0 = 0$

(2) 内積が0であるから$\boldsymbol{x}$と$\boldsymbol{y}$は直交している

(3) $\|\boldsymbol{x}\| = \sqrt{1^2 + 2^2 + (-2)^2} = \sqrt{9} = 3$

(4) $\|\boldsymbol{y}\| = \sqrt{(-2)^2 + 1^2 + 0^2} = \sqrt{5}$

例題２：行列の基礎

(1) $AB = \begin{pmatrix} 6 & 3 \\ 6 & 4 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix} = \begin{pmatrix} 6\cdot 3 + 3\cdot 6 & 6\cdot 2 + 3\cdot 4 \\ 6\cdot 3 + 4\cdot 6 & 6\cdot 2 + 4\cdot 4 \end{pmatrix} = \begin{pmatrix} 36 & 24 \\ 42 & 28 \end{pmatrix}$

(2) $|A| = 6\cdot 4 - 3\cdot 6 = 6$より$A$は正則である。

従って逆行列が存在し $A^{-1} = \frac{1}{6}\begin{pmatrix} 4 & -3 \\ -6 & 6 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{2} \\ -1 & \frac{2}{3} \end{pmatrix}$

(3) $|B| = 3\cdot 4 - 2\cdot 6 = 0$より$B$は正則ではない

従って逆行列は存在しない

演習解答

ベクトルと行列

$\boldsymbol{x}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \boldsymbol{x}_2 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \boldsymbol{x}_3 = \begin{pmatrix} -3 \\ -3 \end{pmatrix}, A = (\boldsymbol{x}_1 \quad \boldsymbol{x}_2) = \begin{pmatrix} 1 & 2 \\ 1 & 5 \end{pmatrix}$のとき

(1) $\boldsymbol{x}_1 \cdot \boldsymbol{x}_2 = 1 \cdot 2 + 1 \cdot 5 = 7$

(2) $\|\boldsymbol{x}_3\| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$

(3) $A\boldsymbol{x}_3 = \begin{pmatrix} 1 & 2 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} -3 \\ -3 \end{pmatrix} = \begin{pmatrix} -9 \\ -18 \end{pmatrix}$

機械学習に向けて少し複雑なベクトル・行列の計算

(1) $\overline{x_1} = \frac{1}{3}\{1 + 2 + (-3)\} = 0,\ \overline{x_2} = \frac{1}{3}\{1 + 5 + (-3)\} = 1$より

$$\boldsymbol{\mu} = (\mu_1 \quad \mu_2)^T = (\overline{x_1} \quad \overline{x_2})^T = (0 \quad 1)^T$$

(2)

$$\sigma_{11} = \frac{1}{3}\sum_{n=1}^{3}(x_{n1} - \mu_1)(x_{n1} - \mu_1)$$

$$= \frac{1}{3}\{(1-0)(1-0) + (2-0)(2-0) + (-3-0)(-3-0)\} = \frac{14}{3}$$

機械学習に向けて少し複雑なベクトル・行列の計算

$$\sigma_{22} = \frac{1}{3}\sum_{n=1}^{3}(x_{n2}-\mu_2)(x_{n2}-\mu_2)$$

$$= \frac{1}{3}\{(1-1)(1-1)+(5-1)(5-1)+(-3-1)(-3-1)\} = \frac{32}{3}$$

$$\sigma_{12} = \frac{1}{3}\sum_{n=1}^{3}(x_{n1}-\mu_1)(x_{n2}-\mu_2)$$

$$= \frac{1}{3}\{(1-0)(1-1)+(2-0)(5-1)+(-3-0)(-3-1)\} = \frac{20}{3}$$

機械学習に向けて少し複雑なベクトル・行列の計算

$\sigma_{12} = \sigma_{21}$より

$$\Sigma = \begin{pmatrix} \frac{14}{3} & \frac{20}{3} \\ \frac{20}{3} & \frac{32}{3} \end{pmatrix}$$

(3) $\boldsymbol{x}_1 = \binom{x_{11}}{x_{12}} = \binom{1}{1}, \boldsymbol{\mu} = \binom{0}{1}, \sigma_{11} = \frac{14}{3}, \sigma_{22} = \frac{32}{3}$より

$$z_{11} = \frac{x_{11} - \mu_1}{\sqrt{\sigma_{11}^2}} = \frac{1-0}{\sqrt{\frac{14}{3}}} = \frac{\sqrt{42}}{14}, \qquad z_{12} = \frac{x_{12} - \mu_2}{\sqrt{\sigma_{22}^2}} = \frac{1-1}{\sqrt{\frac{32}{3}}} = 0$$

宿題解答

ベクトルと行列

$\boldsymbol{x}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \boldsymbol{x}_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \boldsymbol{x}_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, A = (\boldsymbol{x}_1 \quad \boldsymbol{x}_2) = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$のとき

(1) $\boldsymbol{x}_1 \cdot \boldsymbol{x}_2 = 2 \cdot (-1) + 3 \cdot 2 = 4$

(2) $\|\boldsymbol{x}_3\| = \sqrt{0^2 + 1^2} = 1$

(3) $A\boldsymbol{x}_3 = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

機械学習に向けて少し複雑なベクトル・行列の計算

(1) $\overline{x_1} = \frac{1}{3}\{2 + (-1) + 5\} = 2,\ \overline{x_2} = \frac{1}{3}\{(-4) + 3 + 7\} = 2$より

$$\mu = (\mu_1 \quad \mu_2)^T = (\overline{x_1} \quad \overline{x_2})^T = (2 \quad 2)^T$$

(2)

$$\sigma_{11} = \frac{1}{3}\sum_{n=1}^{3}(x_{n1} - \mu_1)(x_{n1} - \mu_1)$$

$$= \frac{1}{2}\{(2-2)(2-2) + (-1-2)(-1-2) + (5-2)(5-2)\} = 6$$

機械学習に向けて少し複雑なベクトル・行列の計算

$$\sigma_{22} = \frac{1}{3}\sum_{k=1}^{3}(x_{k2}-\mu_2)(x_{k2}-\mu_2)$$

$$= \frac{1}{3}\{(-4-2)(-4-2)+(3-2)(3-2)+(7-2)(7-2)\} = \frac{62}{3}$$

$$\sigma_{12} = \frac{1}{3}\sum_{k=1}^{3}(x_{k1}-\mu_1)(x_{k2}-\mu_2)$$

$$= \frac{1}{3}\{(2-2)(-4-2)+(-1-2)(3-2)+(5-2)(7-2)\} = 4$$

機械学習に向けて少し複雑なベクトル・行列の計算

$\sigma_{12} = \sigma_{21}$より

$$\Sigma = \begin{pmatrix} 6 & 4 \\ 4 & \frac{62}{3} \end{pmatrix}$$

(3) $\boldsymbol{x}_1 = \begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}, \boldsymbol{\mu} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \sigma_{11} = 6, \sigma_{22} = \frac{62}{3}$より

$$z_{11} = \frac{x_{11} - \mu_1}{\sqrt{\sigma_{11}}} = \frac{2-2}{\sqrt{6}} = 0, \qquad z_{12} = \frac{x_{12} - \mu_2}{\sqrt{\sigma_{22}}} = \frac{-4-2}{\sqrt{\frac{62}{3}}} = \frac{-3\sqrt{186}}{31}$$