機械学習・ディープラーニングのための 基礎数学講座　微分・線形代数 Day5

SkillUP AI

Day 5

ベクトルと行列による微分

解答

問題１：これまでの復習

以下の(1)-(4)の関数をxで微分せよ

(1) $y = -\frac{1}{x}$ $\qquad y' = (-x^{-1})' = x^{-2} = \dfrac{1}{x^2}$

(2) $y = 4^x + \log x$ $\qquad y' = 4^x \log 4 + \dfrac{1}{x}$

(3) $y = e^{-x}$ $\qquad y' = -e^{-x}$

(4) $y = (2x+1)(3x+1)$ $\qquad y' = (2x+1)'(3x+1) + (2x+1)(3x+1)'$

$$= 2(3x+1) + 3(2x+1) = 12x + 5$$

問題２：これまでの復習

以下の関数を x と y それぞれにおいて偏微分せよ

$$f\ (x\ , y) = 3x^2 + 6xy + 8y^4$$

(1) xで偏微分したとき

$$\frac{\partial f}{\partial x} = 6x + 6y$$

(2) yで偏微分したとき

$$\frac{\partial f}{\partial y} = 6x + 32y^3$$

問題３：スカラー・ベクトル・行列

$$A = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}, B = \begin{pmatrix} 2 & 8 \\ 5 & 9 \end{pmatrix}$$ について

(1) $A + B = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix} + \begin{pmatrix} 2 & 8 \\ 5 & 9 \end{pmatrix} = \begin{pmatrix} 7 & 15 \\ 11 & 17 \end{pmatrix}$

(2) $\mathrm{Tr}(A) = 5 + 8 = 13$

(3) $\mathrm{Tr}(2B) = 2\mathrm{Tr}(B) = 2(2 + 9) = 22$

(4) $\mathrm{Tr}(A + 2B) = \mathrm{Tr}(A) + 2\mathrm{Tr}(B) = 35$

問題４：ベクトルによるスカラーの微分

$$(1)\frac{\partial f}{\partial \boldsymbol{x}} = \left(\frac{\partial}{\partial x_1}(x_1^2 + x_2^2 + c)\ \ \frac{\partial}{\partial x_2}(x_1^2 + x_2^2 + c)\right)^{\mathrm{T}} = \begin{pmatrix} 2x_1 \\ 2x_2 \end{pmatrix}$$

$$(2)\ \frac{\partial f}{\partial \boldsymbol{x}} = \left(\frac{\partial}{\partial x_1}(x_1^2 + x_2^2 + x_3)\ \ \frac{\partial}{\partial x_2}(x_1^2 + x_2^2 + x_3)\right)^{\mathrm{T}} = \begin{pmatrix} 2x_1 \\ 2x_2 \end{pmatrix}$$

(3) $f(\boldsymbol{w}, \boldsymbol{x}) = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{x} = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3$

$$\frac{\partial f}{\partial \boldsymbol{w}} = (1\ x_1\ x_2\ x_3)^{\mathrm{T}} = \begin{pmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

問題５：ベクトルによるベクトルの微分

$$\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}} = \begin{pmatrix} \frac{\partial}{\partial x_1}(x_1 + 2x_2 + 3x_3) & \frac{\partial}{\partial x_1}(4x_1 + 5x_2 + 6x_3) & \frac{\partial}{\partial x_1}(7x_1 + 8x_2 + 9x_3) \\ \frac{\partial}{\partial x_2}(x_1 + 2x_2 + 3x_3) & \frac{\partial}{\partial x_2}(4x_1 + 5x_2 + 6x_3) & \frac{\partial}{\partial x_2}(7x_1 + 8x_2 + 9x_3) \\ \frac{\partial}{\partial x_3}(x_1 + 2x_2 + 3x_3) & \frac{\partial}{\partial x_3}(4x_1 + 5x_2 + 6x_3) & \frac{\partial}{\partial x_3}(7x_1 + 8x_2 + 9x_3) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

問題６：行列によるスカラーの微分

$f(a_{11}, a_{12}, a_{21}, a_{222}) = a_{11}a_{12} + a_{21}a_{22},\ A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$のとき$\frac{\partial f}{\partial A}$を求めよ

$$\frac{\partial f}{\partial A} = \begin{pmatrix} \frac{\partial}{\partial a_{11}}(a_{11}a_{12} + a_{21}a_{22}) & \frac{\partial}{\partial a_{12}}(a_{11}a_{12} + a_{21}a_{22}) \\ \frac{\partial}{\partial a_{21}}(a_{11}a_{12} + a_{21}a_{22}) & \frac{\partial}{\partial a_{22}}(a_{11}a_{12} + a_{21}a_{22}) \end{pmatrix} = \begin{pmatrix} a_{12} & a_{11} \\ a_{22} & a_{21} \end{pmatrix}$$

問題７ ：総合問題

(1) $\boldsymbol{x}^{\mathrm{T}} A \boldsymbol{x} = \begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 & a_{12}x_1 + a_{22}x_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} =$

$a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2$

(2) $\frac{\partial}{\partial \boldsymbol{x}} \boldsymbol{x}^{\mathrm{T}} A \boldsymbol{x} = \begin{pmatrix} \frac{\partial}{\partial x_1}(a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2) \\ \frac{\partial}{\partial x_2}(a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2) \end{pmatrix} = \begin{pmatrix} 2a_{11}x_1 + 2a_{12}x_2 \\ 2a_{12}x_1 + 2a_{22}x_2 \end{pmatrix} =$

$2\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 2A\boldsymbol{x}$ （**二次形式の微分の公式**）

問題７：総合問題

(3) $A\boldsymbol{x}\boldsymbol{x}^{\mathrm{T}} = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \begin{pmatrix} x_1 & x_2 \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{12}x_1 + a_{22}x_2 \end{pmatrix} \begin{pmatrix} x_1 & x_2 \end{pmatrix} =$

$$\begin{pmatrix} a_{11}x_1^2 + a_{12}x_1x_2 & a_{11}x_1x_2 + a_{12}x_2^2 \\ a_{12}x_1^2 + a_{22}x_1x_2 & a_{12}x_1x_2 + a_{22}x_2^2 \end{pmatrix}$$

(4) $\mathrm{Tr}\left(A\boldsymbol{x}\boldsymbol{x}^{\mathrm{T}}\right) = a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2 = \boldsymbol{x}^{\mathrm{T}}A\boldsymbol{x}$

（二次形式のトレース表現）