Regularne operacije in ekvivalentnost avtomatov

$* \cup \cap$

Uroš Čibej

Pregled

- regularne operacije
- tihi prehodi (prehodi ε)
- ekvivalenca NKA in DKA
- reg. izrazi

Literatura

- Sipser razdelka 1.2 in 1.3

Regularni jeziki

Def. Jezik L za katerega obstaja DKA, da velja $L(M) = L$, imenujemo **regularni jezik**.

Množico regularnih jezikov bomo označevali z Reg.

Regularne operacije

Operacije, ki ohranjajo regularnost:

- unija
- stik
- Kleenejevo zaprtje

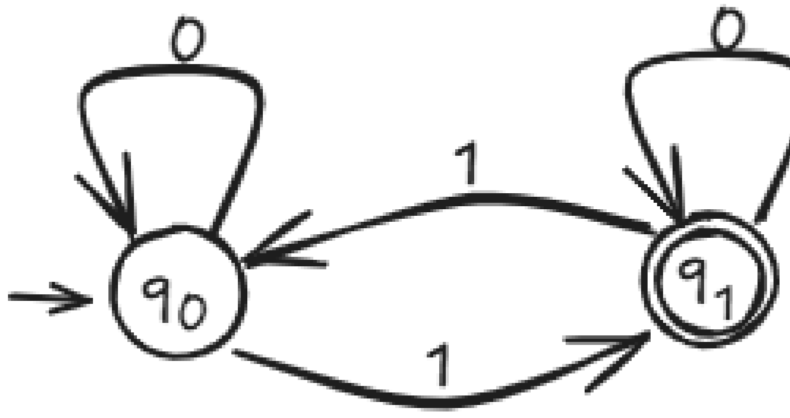
Unija

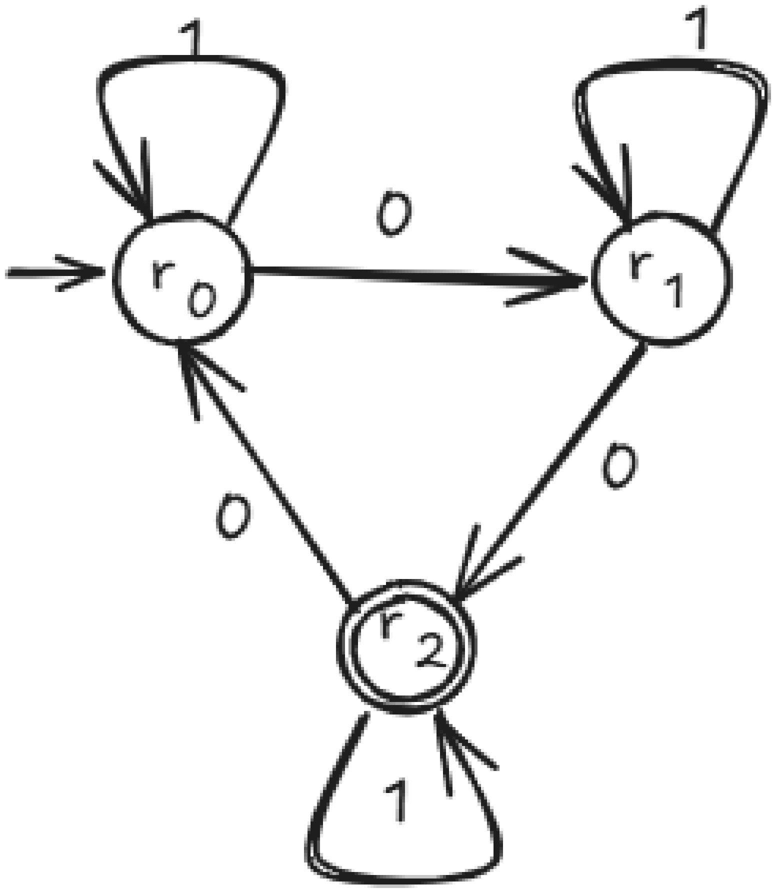
Izrek:

$$L_1, L_2 \in Reg. \implies L_1 \cup L_2 \in Reg$$

Intuicija dokaza

Dva avtomata izvajamo vzporedno





Dokaz

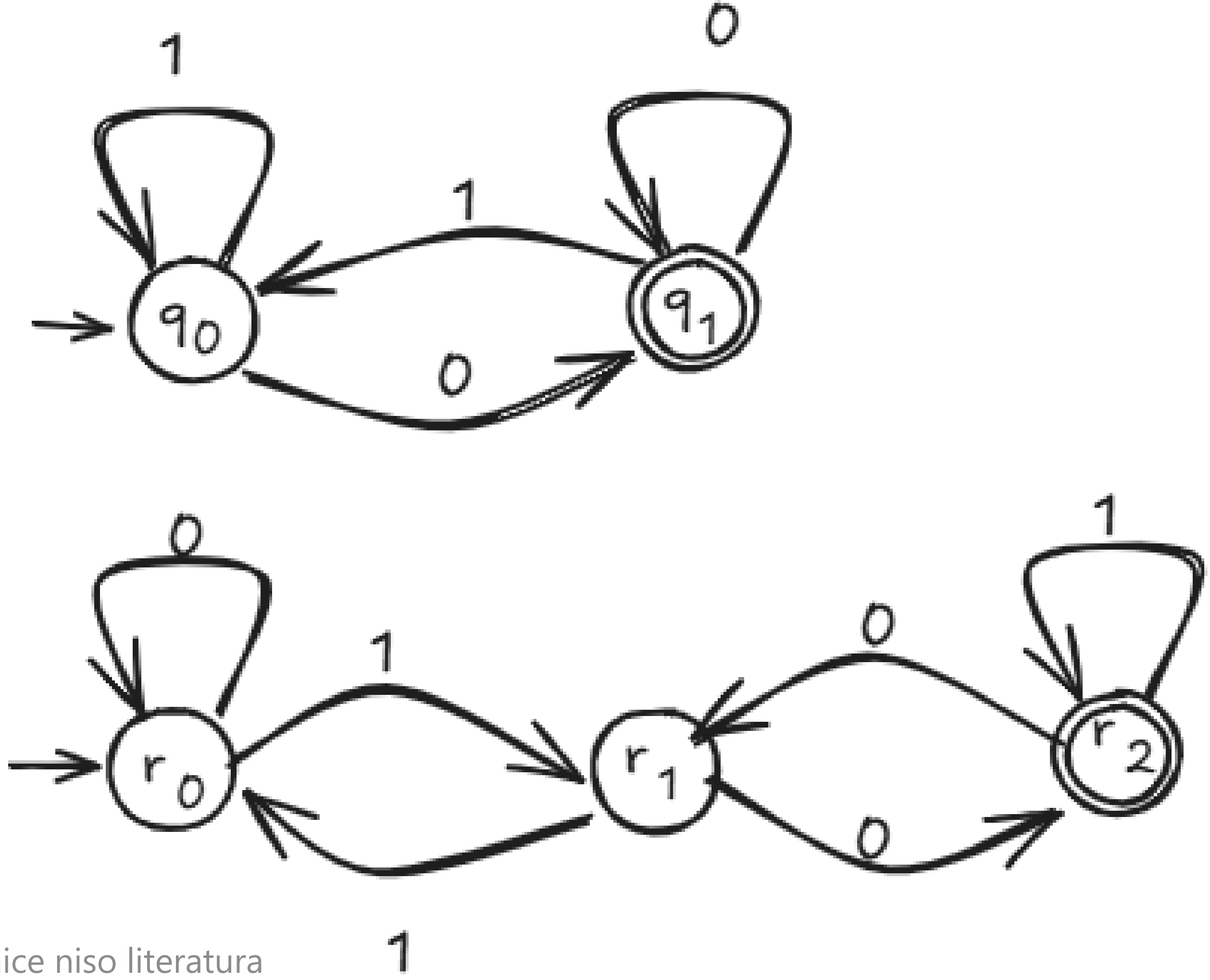
Podana sta avtomata $M_1 = \langle Q_1, \Sigma, \delta_1, q_0, F_1 \rangle$ in $M_1 = \langle Q_2, \Sigma, \delta_2, r_0, F_2 \rangle$

Produktni avtomat:

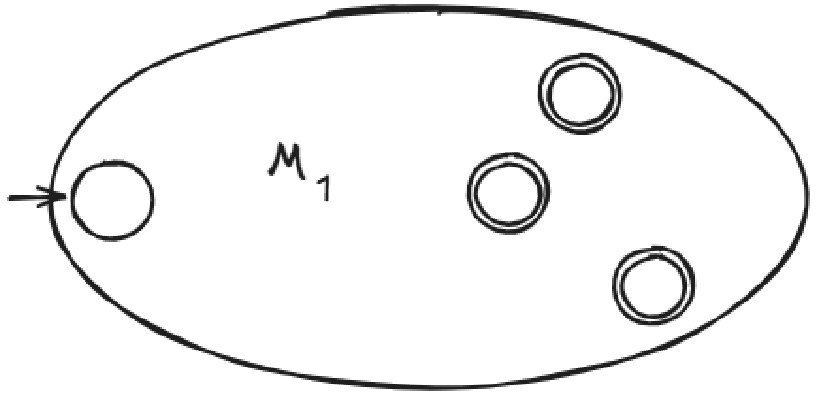
$$M_{1 \cup 2} = \langle Q_{12}, \Sigma, \delta', (q_0, r_0), F' \rangle$$

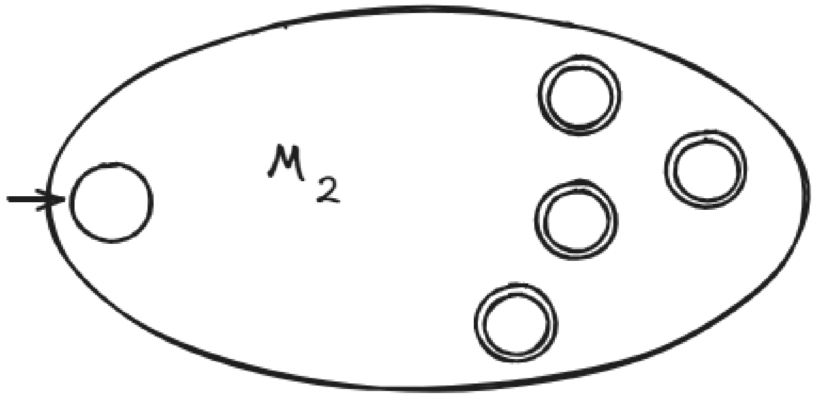
- $Q_{12} = \{(q, r) \mid q \in Q_1, r \in Q_2\}$
- $\delta'((q, r), a) = (\delta_1(q, a), \delta_2(r, a))$
- $F' = \{(q, r) \mid q \in F_1 \vee r \in F_2\}$

Primer



Stik (narobe)





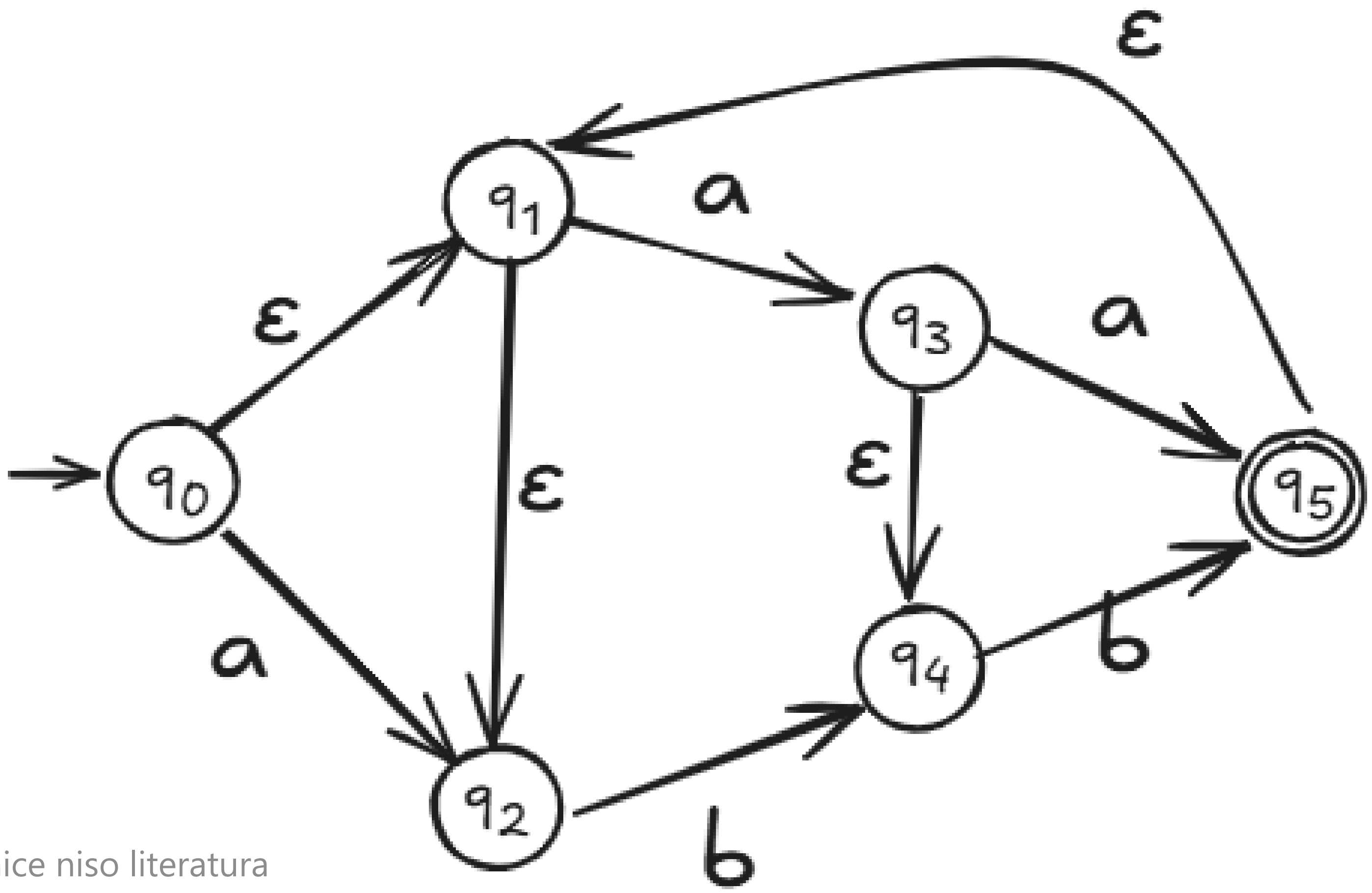
ε-prehodi (tihi prehodi)

Uvedemo prehode preko praznega niza ε

avtomat se nedeterministično "odloči", da skoči v novo stanje, brez porabe znaka na vhodu

Primer

$w = abaa$, napišimo sled izvajanja



ε-pot

Def. Pot med stanjema q in r imenujmo ε pot, če gre samo po tihih prehodih.

ε-ovojnica stanja, množice stanj

$$E(q) = \{r' \in Q \mid \text{obstaja}\ \varepsilon - \text{pot} q, r\}$$

NB: med q in q vedno obstaja ε pot

$$E(A) = \bigcup_{q \in A} E(q)$$

Ekvivalenca računskih modelov

Dva računska modela sta ekvivalentna, če z njima lahko rešujemo isti razred problemov.

Bolj formalno:

1. $\forall I_1 \in Mod_1, \exists I_2 \in Mod_2 : L(I_1) = L(I_2)$
2. $\forall I_2 \in Mod_2, \exists I_1 \in Mod_1 : L(I_1) = L(I_2)$

Ekvivalenca DKA in NKA

Izrek:

$$\forall M \in NKA, \exists M' \in DKA : L(M') = L(M)$$

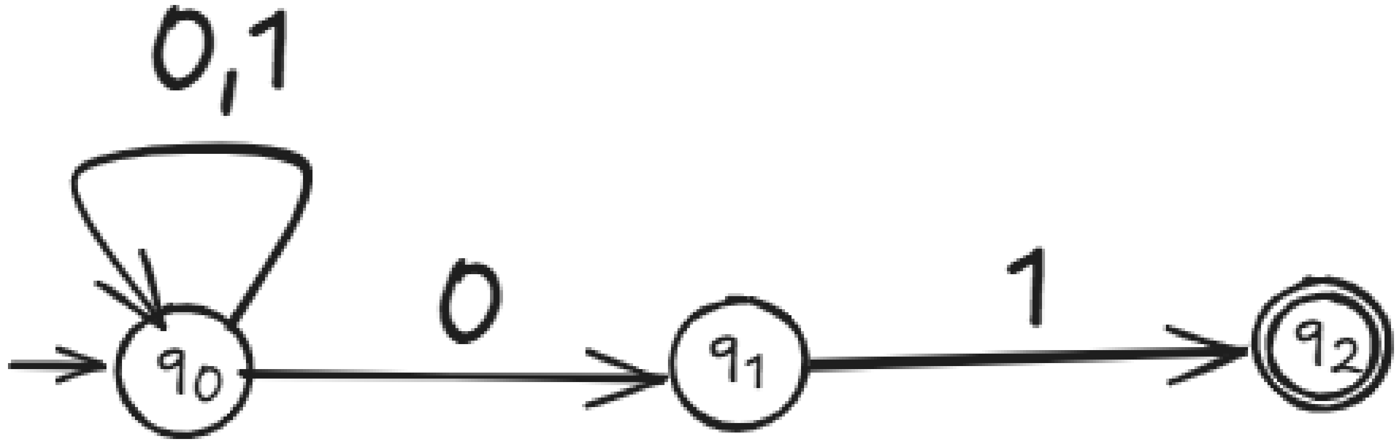
Dokaz za NKA brez ε

Za poljuben NKA $M = \langle Q, \Sigma, \delta, q_0. F\rangle$, zgradimo DKA

$$M' = \langle Q', \Sigma, \delta', \{q_0\}, F'\rangle$$

- $Q' = 2^Q$
- $\delta'(S, a) = \bigcup_{q \in S} \delta(q, a)$
- $F' = \{S \in 2^Q \mid S \cap F \neq \emptyset\}$

Primer

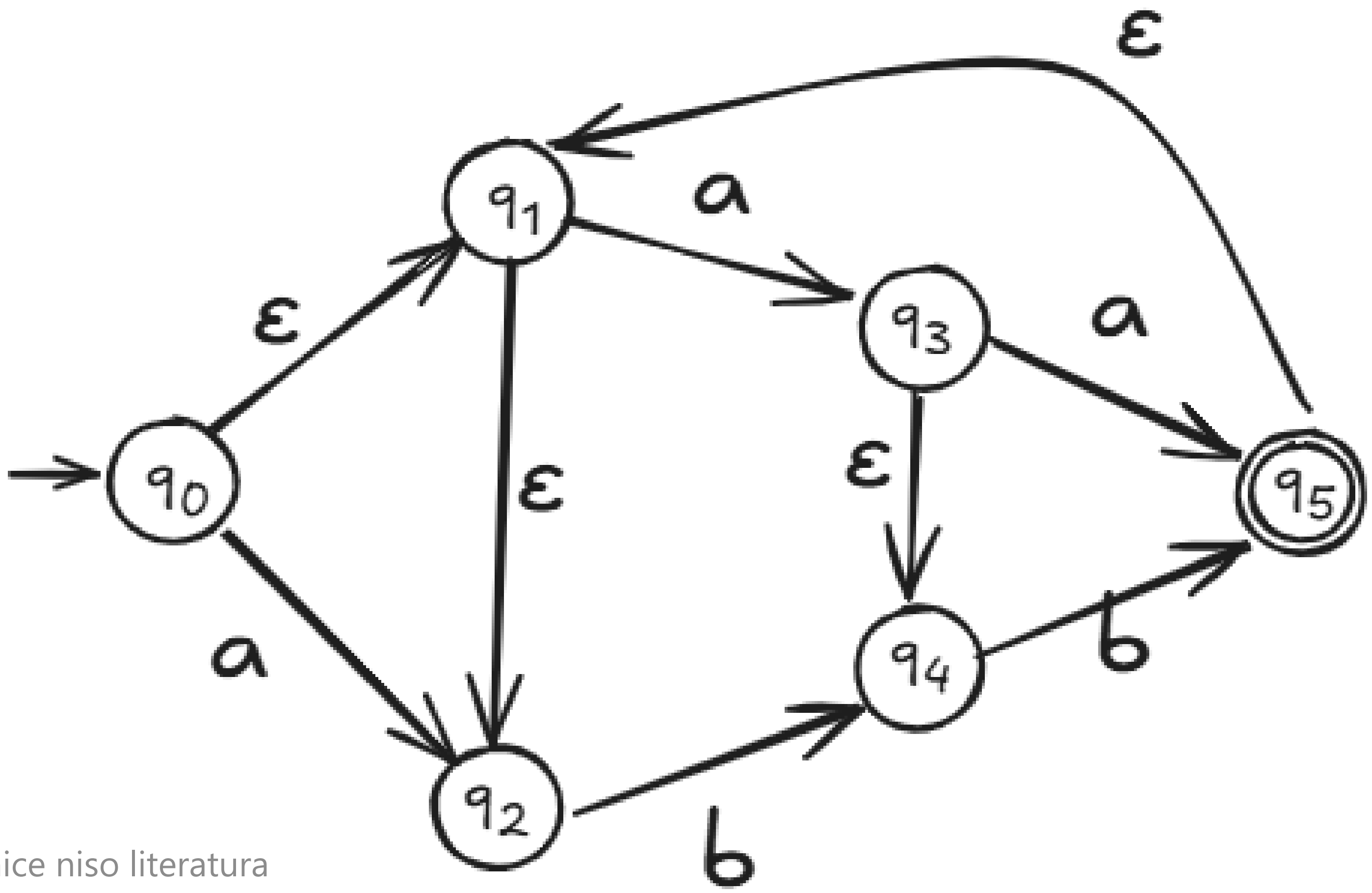


Ekvivalenca z ε

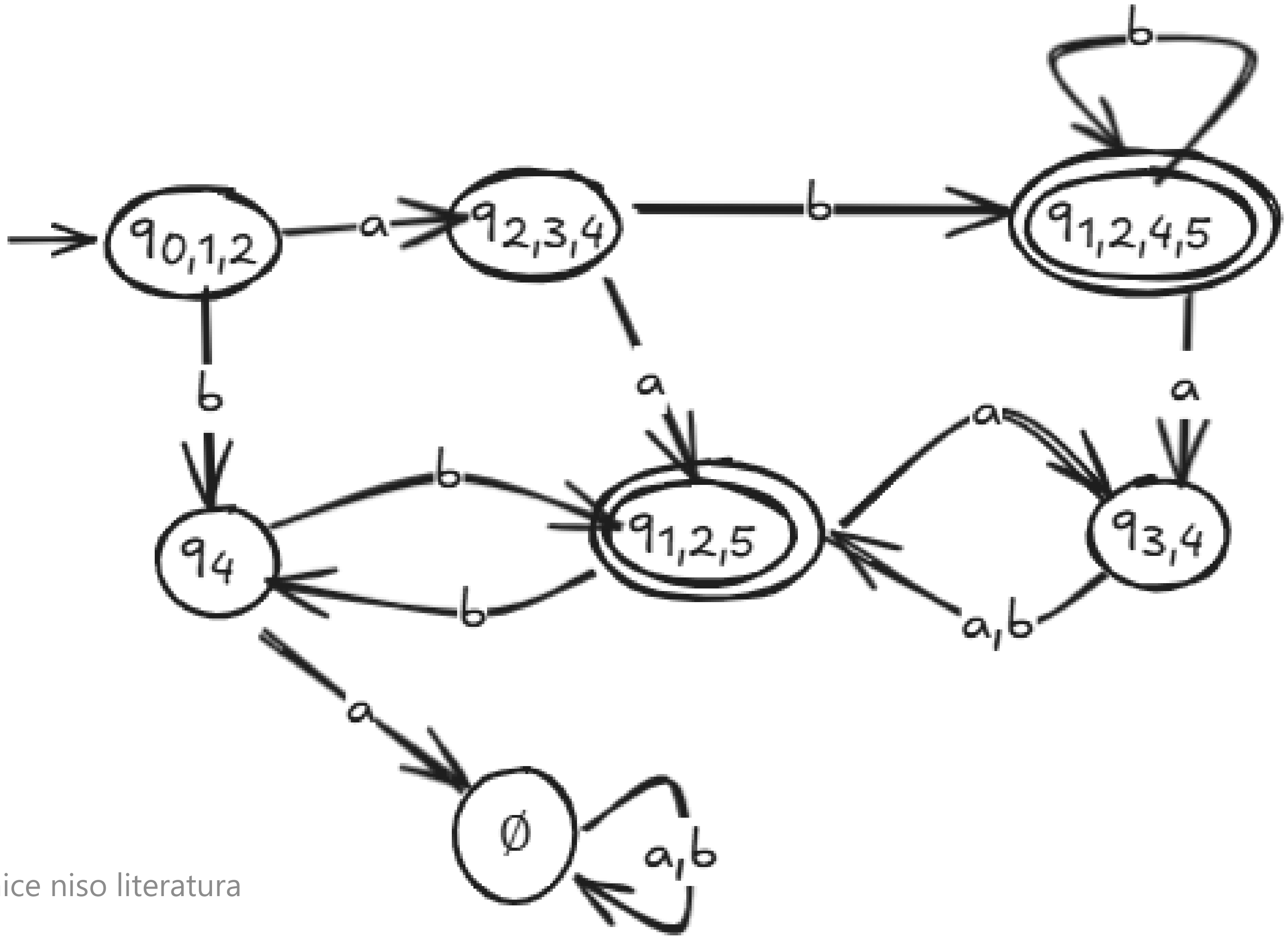
1. Začetno stanje je $q_0' = E(q_0)$
2. $\delta'(S, a) = E(\bigcup_{q \in S} \delta(q, a))$

Primer

Pretvorimo v DKA



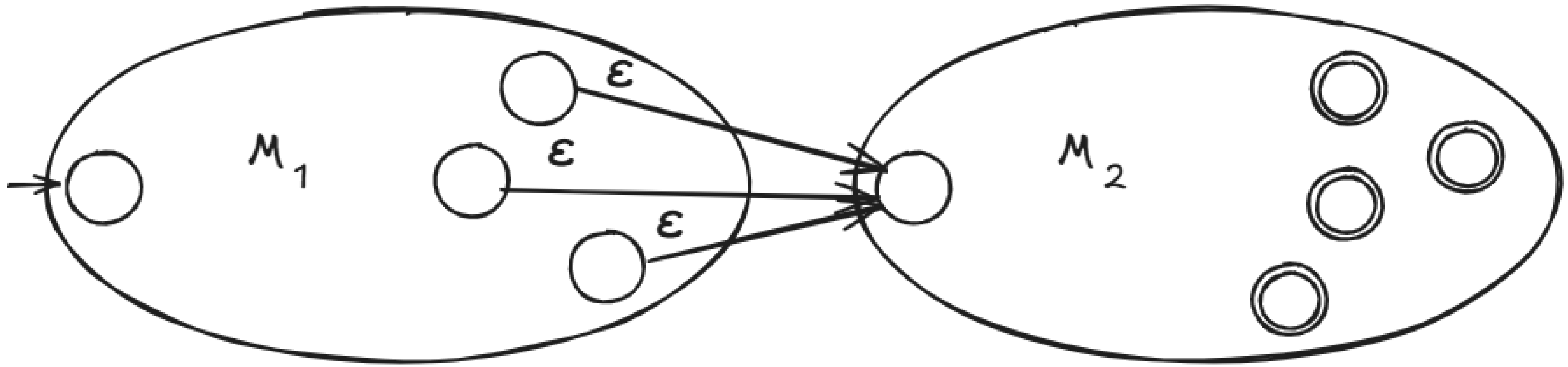
Rešitev



Stik ohranja regularnost

Izrek: $L_1, L_2 \in Reg \implies (L_1 \circ L_2) \in Reg$

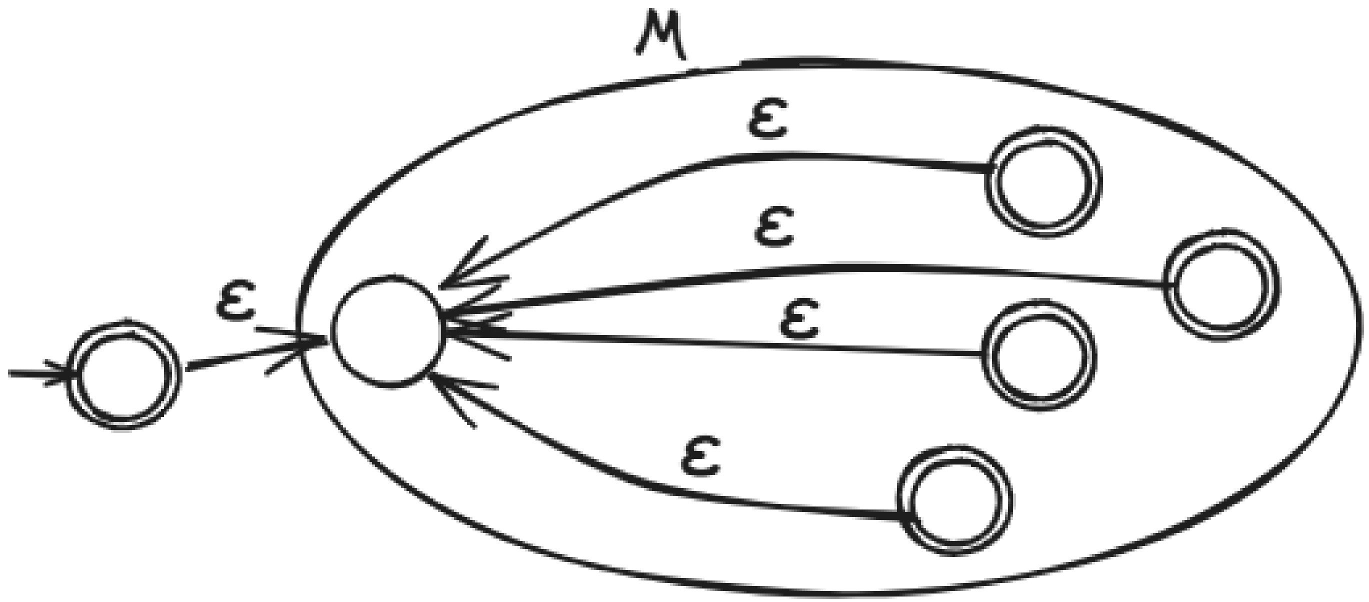
Dokaz



Kleenejevo zaprtje (zvezdica) ohranja regularnost

Izrek: $L \in Reg \implies L^* \in Reg$

Dokaz



Regularni izrazi

- "opisni" način podajanja jezikov
- široko uporabno (in uporabljano) orodje

Formalna definicija - baza

Tri vrste osnovnih regularnih izrazov

sintaksa	**pomen**
$a \in \Sigma$	$\{a\}$
ε	$\{\varepsilon\}$
$\emptyset$	$\emptyset$

Formalna definicija - pravila

Naj bosta r_1, r_2 regularna izraza in njuna jezika L_1 in L_2

sintaksa	pomen
$r_1 + r_2$	$L(r1) \cup L(r_2)$
$r_1 \circ r_2$	$L(r_1) \circ L(r_2)$
r_1^*	$L(r_1)^*$

Primeri

- $(0+1)^*$
- $(0+1)^*0$
- $(001+100)^*(111+\varepsilon)$

Primer iz realnosti (veljavni e-naslov)

- [a-zA-Z0-9._%+-]+@[a-zA-Z0-9.-]+.[a-zA-Z]{2,}

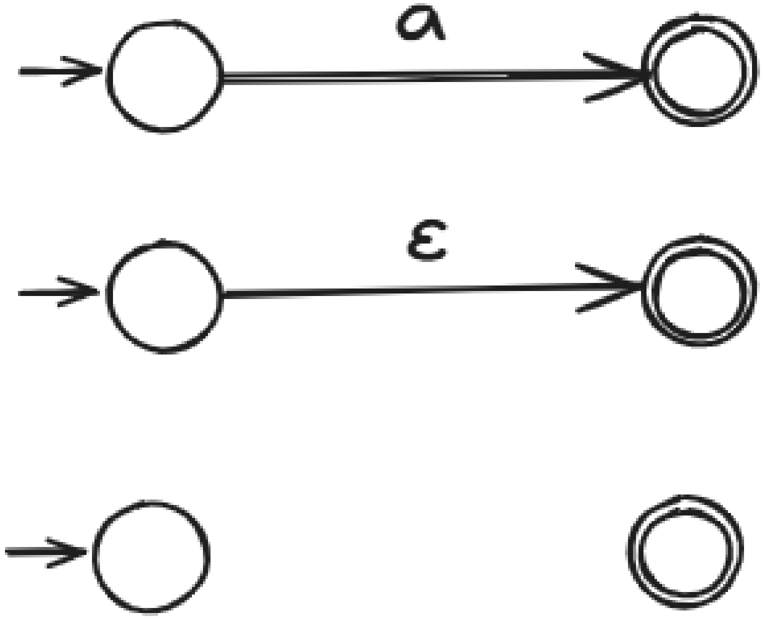
Ekvivalenca RI in KA

Izrek:

1. $\forall r \in RI, \exists M \in KA : L(r) = L(M)$ (danes)
2. $\forall M \in KA, \exists r \in RI : L(r) = L(M)$ (naslednjič)

Dokaz - baza

1. $a \in \Sigma$
2. ε
3. $\emptyset$

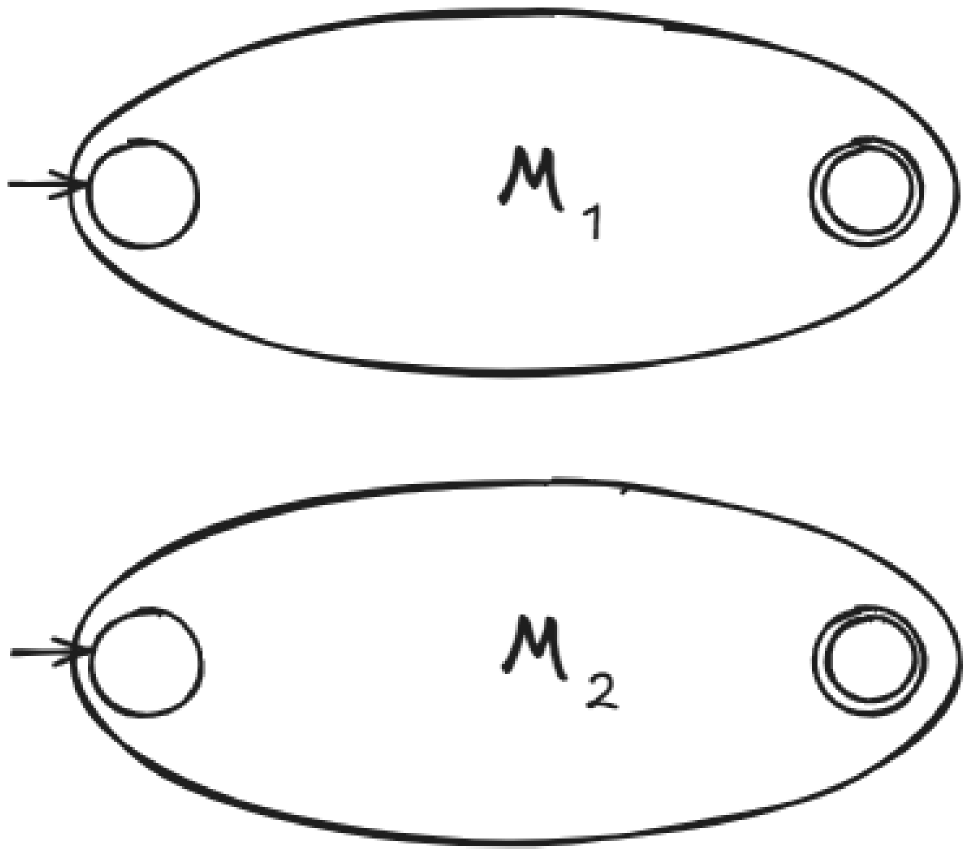


Dokaz - pravila

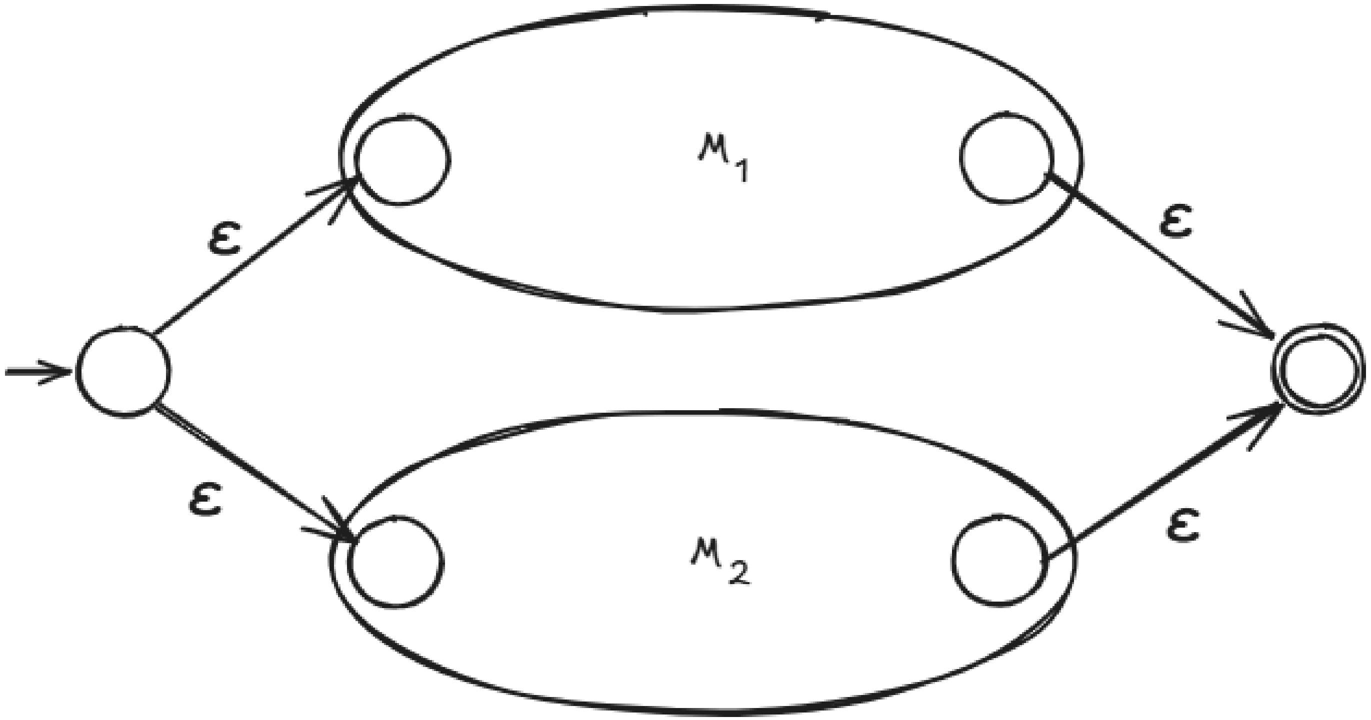
Sicer smo že vse to dokazali

Predpostavka

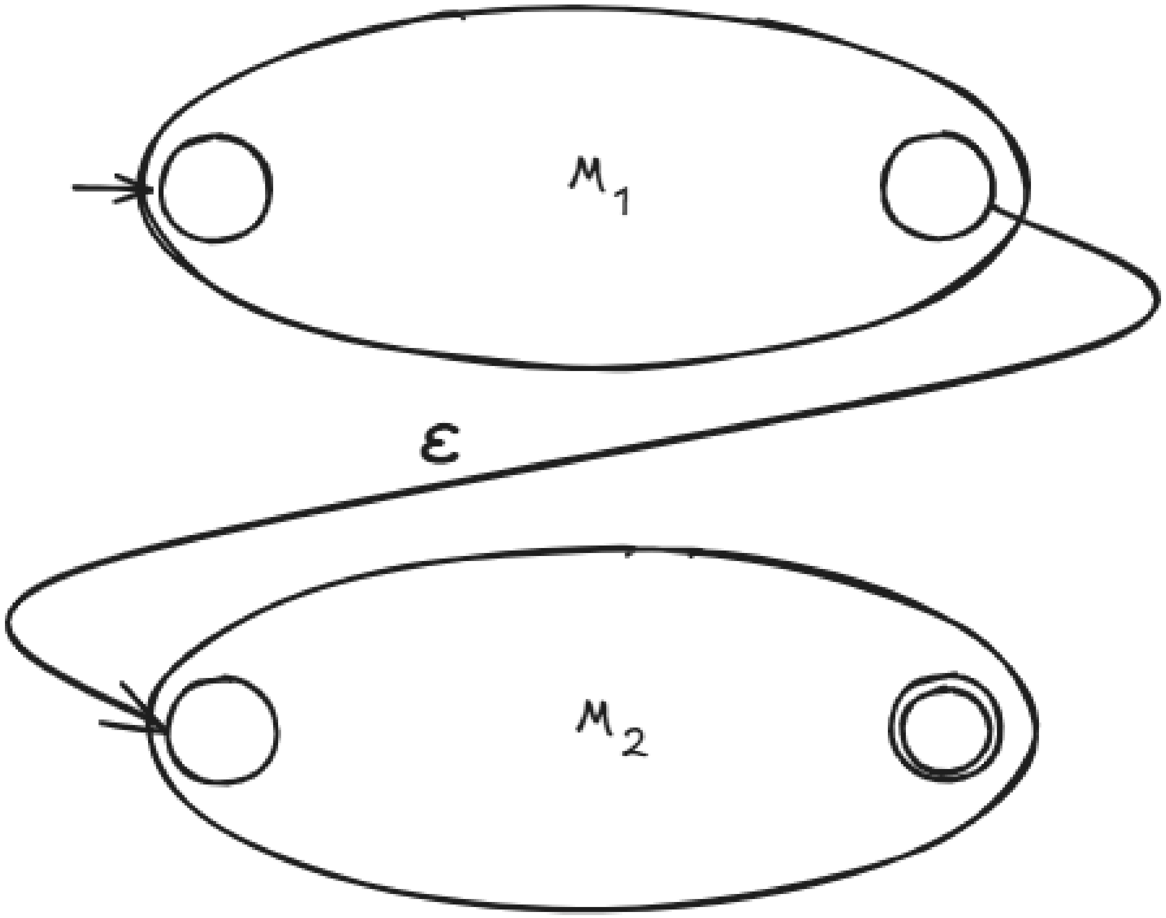
$r_1, r_2 \in RI \rightarrow M_1, M_2 \in KA$



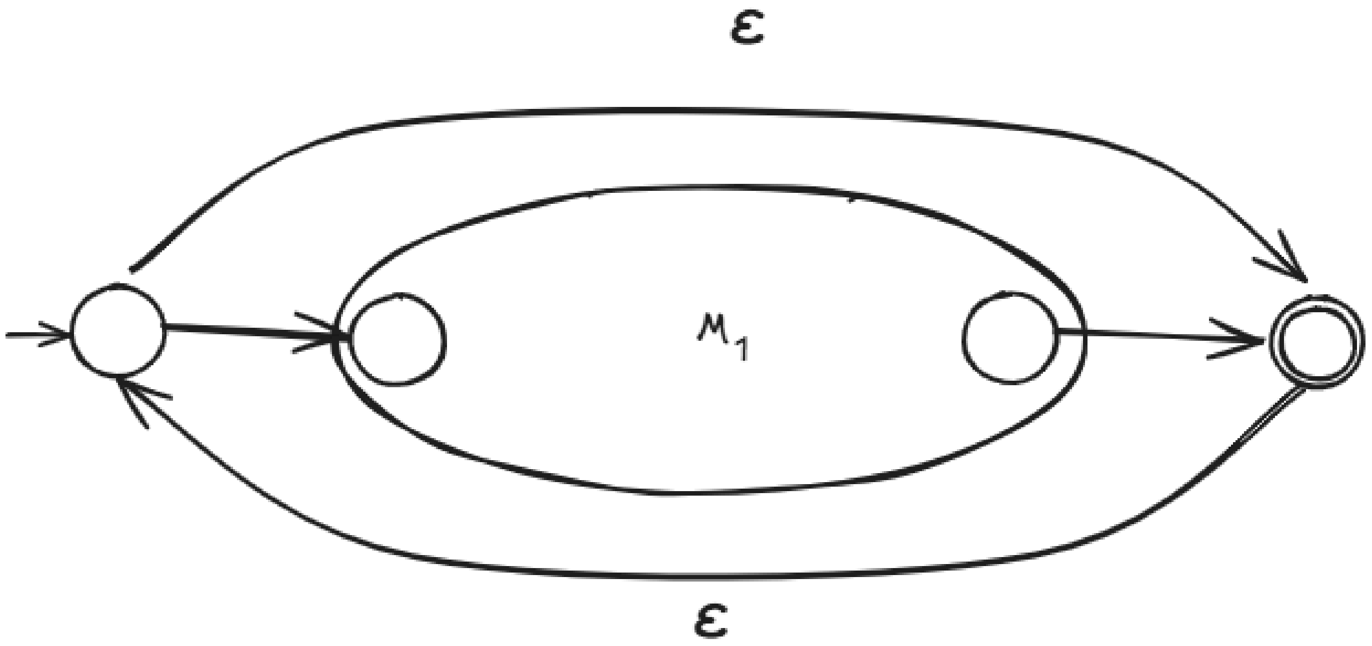
$$r_1 + r_2$$



$$r_1 \circ r_2$$



r_1^*



Primer

$$(a+b)^*b(ab+b)^*$$