Computational Homework 4

Due: Wednesday, April 24 (end-of-day 11.59pm)

You should use C++ to write the code. Submit code and summary of results via Compass2g.

1. *Variance Reduction Techniques: Control variates*

   (a) Price a discrete down-and-out call option using plain vanilla Monte Carlo simulations. Assume that the barrier is monitored once per day (1 day equals 1/252 years; 252 is the number of business days), so in this case there are $m = 25$ barrier monitorings for this option, however your code should be general enough to work with arbitrary m. For this problem you can use: $S_0 = 99$, $r = 0.03$, $\delta = 0$, $K = 105$, $B = 90$, $\sigma = 0.6$, $T = 25/252$. Report the average (i.e. option price estimate), standard error and actual error (you can use 4.647650 as the true option value) for at least 10,000 simulations.

   (b) Repeat part (a) using the plain vanilla European call option as a control variate, with the same number of simulation trials. Report the R^2 of the regression together with the average, standard error and actual error.

2. *Variance Reduction Techniques: Importance Sampling*
   Consider the continuously sampled Asian average call options with payoff at maturity T

$$\left(\frac{1}{L+1} \sum_{i=0}^{L} S_{t_i} - K \right)_+ .$$

   Assume that the underlying stock follows a Geometric Brownian motion described by the following SDE

$$dS_t = rS_t dt + \sigma S_t dW_t$$

   under the risk neutral measure.

   (a) Implement plain vanilla Monte Carlo to price this option. You can use the following parameters: $S_0 = 40$, $r = 0.05$, $\delta = 0$, $\sigma = 0.4$, $T = 90/252$ and 10,000 simulations. Report the Monte Carlo estimate, the standard error and a convergence diagram over 10,000 simulations.

   (b) Repeat part (a) using Importance Sampling. Discuss how you obtained the "new" drift parameter.