Computational Homework 1

Due: Monday, February 18 (end-of-day 11.59pm)

You should use C++ to write the code.
Submit code and summary of results via Compass2g.

1. **Lattice Methods**

   (a) The *Cox-Ross-Rubinstein* (CRR), *Jarrow-Rudd* and *Tian* parametrizations are all different ways to approximate the Black-Scholes model. For δt sufficiently small, the option prices will be very close to one another. **Write** a program to implement all three parametrizations for the Binomial lattice model and use them to price a *American put option*. **Plot** the put option prices for the three parametrizations (in the same graph) as a function of δt and discuss the results.

   References. CRR/JR – Lecture 1, Slide 24; Tian – Homework 1.

   (b) **Improve** the Binomial approximation of the put option price under the CRR model using the *Binomial Black-Scholes* and the *Binomial Average methods* (both) with *Richardson extrapolation*. Use both methods to price the American put option and **plot** the option price for each method against the number of steps.

   Reference. BBS, BAM, Richardson extrapolation – Lecture 2, Slides 15–19.

   (c) Write a program to implement the *Trinomial model* with risk-neutral probabilities given by

   $$\begin{cases} p_u = \frac{1}{2\lambda^2} + \frac{\nu\sqrt{\delta t}}{2\lambda\sigma} \\ p_m = 1 - \frac{1}{\lambda^2} \\ p_d = 1 - p_u - p_m \end{cases}$$

   where $\nu = r - \frac{1}{2}\sigma^2$. Choose $\lambda = \sqrt{\frac{3}{2}}$. **Compute** the American put option price and plot it as a function of the number of steps. Discuss your results.

   Reference. Trinomial odel – Lecture 3, Slides 26+

   (d) **Compare** the option price you obtained in (b) using the Binomial Black-Scholes method with Richardson extrapolation with the option price you obtained in (c) and plot both in the same graph as a function of the number of steps. Discuss.

   For (a)–(d) use the following parameters to price an American put option: $S = 100$, $\sigma = 20\%$, $r = 3\%$, $K = 105$, $T = 1$. You can use steps from 25 to 25600. If your code takes long to run, feel free to modify the number of steps.

2. **Multidimensional Options**
The payoff of an American *max* option at maturity is

$$\max\left(\max_{1\leq i\leq k} S^i(t) - K, 0\right)$$

where $S^i(t)$, $i = 1, \ldots, k$ is the price of stock i at time t. We assume that each stocks follow a Black-Scholes model with correlation ρ_{ij}.

   (a) **Write** a function that computes the price of an American max option with $k = 3$ stocks. To test your algorithm use the following parameters: $S^i(0) = 100$, $K = 100$, $\sigma_i = 0.2$, $\rho_{ij} = 0.1$, for i, $j = 1, \ldots, k$, $i \neq j$, $r = 0.05$ and $T = 0.5$, and $k = 3$.

   (b) For steps 15, 30, 60, 120, 240, 480, **plot** the option price against the number of steps.

   (c) For the different number of steps, **report** the CPU time that your code needs to run. Summarize your results in a table. Briefly comment on your results.