Computational Homework 2

Due: Wednesday, March 27 (end-of-day 11.59pm)

You should use C++ to write the code. Submit code and summary of results via Compass2g.

1. **Transport Equation**
   Consider the following first order linear PDE
   $$\frac{\partial u}{\partial t} + c\,\frac{\partial u}{\partial x} = 0$$
   where $u = u(x,t)$, $c > 0$ and the initial condition
   $$u(x,0) = f(x) = \begin{cases} 0, & x < -1, \\ x+1, & -1 \le x \le 0, \\ 1, & x > 0. \end{cases}$$
   is given.

   (a) Write a **code** to implement a finite-difference scheme a finite-difference scheme based on a forward approximation with respect to t and backward approximation with respect to x for the transport equation.

   (b) **Plot** the numerical solution you obtain based on your code at different times. You can use the following parameters: $-2 \le x \le 3$, $0 \le t \le 1$ with discretization steps $\delta x = 0.05,\ 0.01,\ 0.005$ and $\delta t = 0.01$ and $c = 1$. Discuss the stability of this scheme.

2. **Down-and-Out Barrier Option**
   Consider a European-type *Down-and-Out Put* option with strike price K and maturity T with payoff function
   $$\begin{cases} \max\{K - S, 0\}, & \text{if } S \ge S_b \\ 0, & \text{if } S < S_b \end{cases}$$
   that is the option becomes void if the asset price falls below the barrier S_b. For this option, an explicit formula is known (Reference: Brandimarte – page 120).

   (a) State the PDE with the appropriate boundary and terminal conditions that correspond to this option.

   (b) Write a code to compute the price of the down-and-out put option using the Crank-Nicolson scheme. Compare your answer with the exact solution for the *down-and-out put* option (You can write a short code to compute this as well). For your calculations, use the following parameters: $S_0 = 50$, $K = 50$, $S_b = 40$ (the barrier), annual interest rate $r = 0.1$, annual volatility $\sigma = 0.4$ and expiry in $T = 5$ months.

3. **American Put Option**
   Implement the Projected SOR method to price an American Put Option. For the pseudocode, refer to Figures 9.2 and 9.3 of the textbook (P. Wilmott et al. book). Test your code with parameters: $K = 10$, $r = 0.1$, $\sigma = 0.4$ and $T = 6$ months.