Pairs Trading

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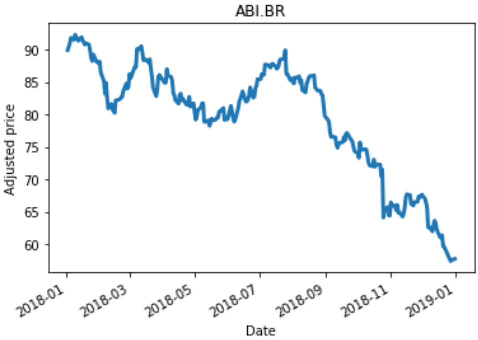
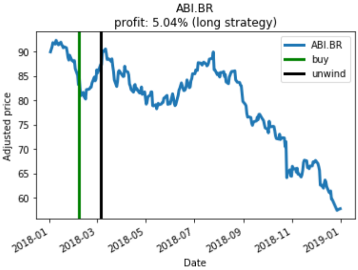
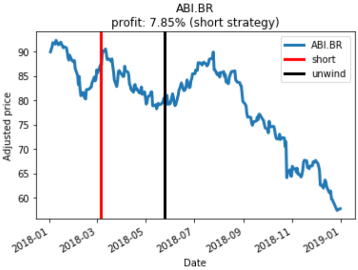
Consider daily (adjusted) closing prices for **ABI.BR** (Inbev).



How can we make money trading **ABI.BR**?. If we forsee trends,

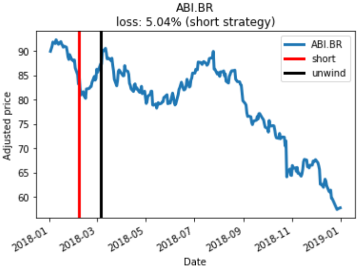
- Buy (go long) before it goes up, and unwind once it has gone up.
- Short (go short) before it goes down, and unwind once it has gone down.



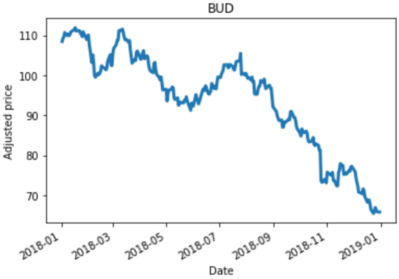


Of course if you guess the trends long, you lose money.

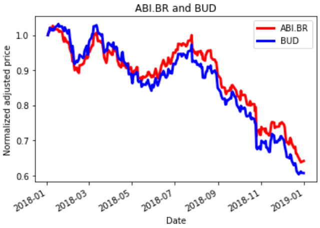
- If you go long (thinking it will go up) and guess wrong, you may have to sell at a loss.
- If you sho short (thinking it will go down) and guess wrong, you may have to cover the short at a higher price.



Consider now a second stock; **BUD** (Anheiser-Busch).



Let's normalize them (start at 1)



and let's do a scatterplot of prices

The stocks seem to be correlated.

These stocks sort of 'track' each other; when one goes up, the other does too (sort of). Let's compare the behavior of **ABI.BR** and **BUD** over a year, defining $\delta \stackrel{\text{def}}{=} \frac{1}{252}$ correspond to one day. Let $\mathbf{ABI.BR}_{n\delta}$ and $\mathbf{BUD}_{n\delta}$ be the prices, respectively of *Inbev* and *Anheiser* $-$ *Busch* on day n.
Let's *regress* Anheiser-Busch on Inbev, writing

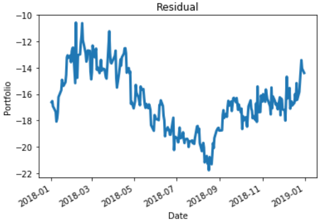
$$\mathbf{BUD}_{n\delta} = m\mathbf{ABI.BR}_{n\delta} + b + z^{\circ}_{n\delta}$$

where m and b are given by regression (i.e., z° is as small as possible in the mean-square sense).

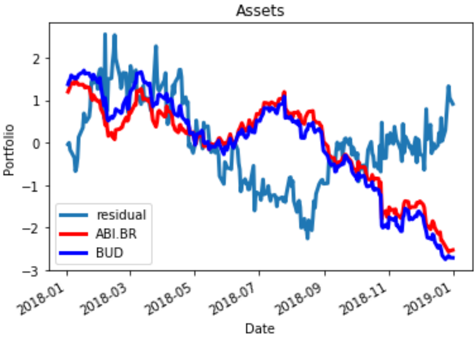
BUD=1.39ABI.BR-16.53

Statistical Arbitrage

The residual $z^\circ_{n\delta}$ should revert to zero, it suggests a *portfolio* which "reverts". We can then hope to "buy low and sell high" or "sell high and buy low". The selection of the portfolio (or more technically the "cointegration") is the "secret sauce" of statistical arbitrage.



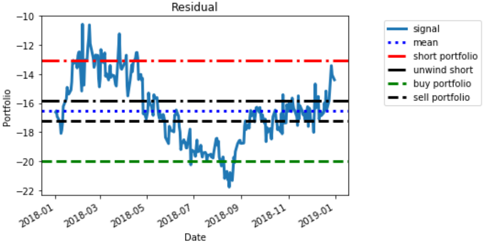
This synthetic "asset" identifies some opportunities which are different than the trends on either **ABI.BR** or **BUD**.



If

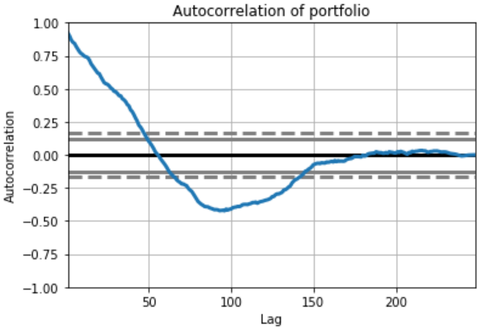
$$z^{\circ}_{n\delta} \stackrel{\text{def}}{=} \mathbf{BUD}_{n\delta} - m\mathbf{ABI.BR}_{n\delta}$$

- Buy z° if it below its mean (i.e., it is underpriced); buy one share of *Anheiser* $-$ *Busch* and short m shares of *Inbev*.
- Short z° if it is above its mean (i.e., it is overpriced); short one share of *Anheiser* $-$ *Busch* and buy m shares of *Inbev*.



The better we model z°, the better we can identify when it is overpriced or underpriced.

Let's try to use autoregression to write the residual in terms of independent increments. Independence is good.

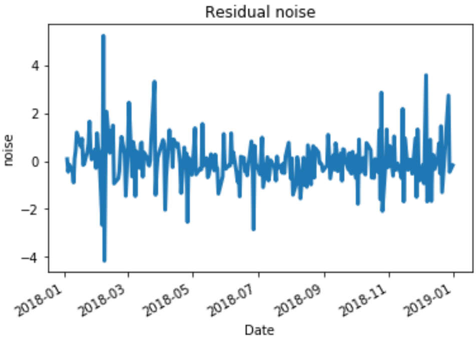


AR(1) means

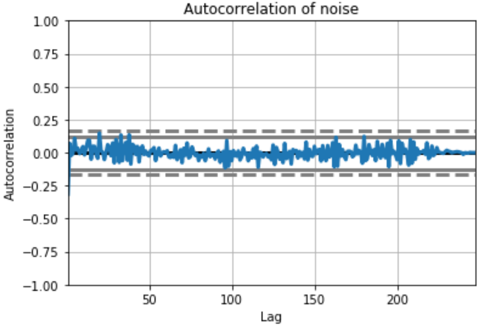
$$z^\circ_{(n+1)\delta} = m_\circ z^\circ_{n\delta} + b_\circ + \sigma_\circ \varepsilon_{(n+1)\delta}$$

where ε is mean zero and variance 1 and $\sigma_\circ$ is as small as possible (in the mean-square sense).

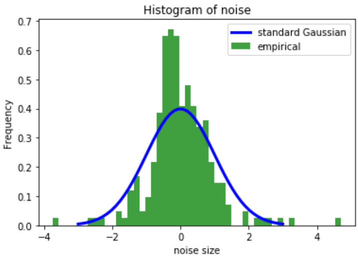
$$\boxed{z^\circ_{(n+1)\delta} = 0.92 z^\circ_{n\delta} + 0.0083 + 0.90 \varepsilon_{(n+1)\delta}}$$

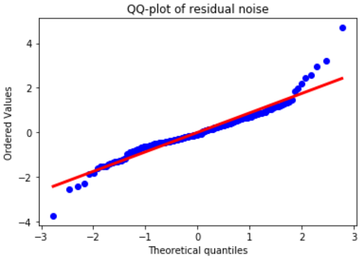


This noise has much lower correlation; the AR(1) decomposition removed a lot of autocorrelation.



The noise process ε looks like a standard Gaussian





In the formula

$$z^{\circ}_{(n+1)\delta} = m_{\circ} z^{\circ}_{n\delta} + b_{\circ} + \sigma_{\circ}\varepsilon_{(n+1)\delta}$$

$$\boxed{z^{\circ}_{(n+1)\delta} = 0.92 z^{\circ}_{n\delta} + 0.0083 + 0.90\varepsilon_{(n+1)\delta}}$$

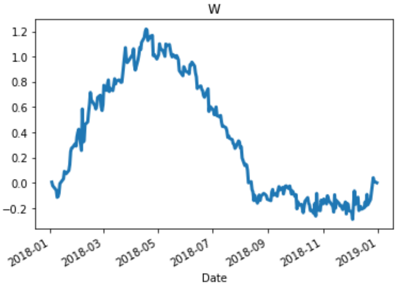
we note that $m_{\circ}$ is close to 1. Let's rewrite the dynamics of z° as

$$\begin{aligned} z^{\circ}_{(n+1)\delta} - z^{\circ}_{n\delta} &= -\underbrace{\frac{1-m_{\circ}}{\delta}}_{-\alpha}\left(z^{\circ}_{n\delta} - \underbrace{\frac{b_{\circ}}{1-m_{\circ}}}_{\bar{z}}\right)\delta + \underbrace{\frac{\sigma_{\circ}}{\sqrt{\delta}}}_{\sigma}\underbrace{\sqrt{\delta}\varepsilon_{(n+1)\delta}}_{W_{(n+1)\delta}-W_{n\delta}} \\ &= -\alpha(z^{\circ}_{n\delta} - \bar{z})\delta + \sigma\left\{W_{(n+1)\delta} - W_{n\delta}\right\} \end{aligned}$$

$$\boxed{z^\circ_{(n+1)\delta} - z^\circ_{n\delta} = -19.16(z^\circ_{n\delta} - 0.11)\delta + 14.27(W_{(n+1)\delta} - W_{n\delta})}$$

Here

$$W_{n\delta} = \sum_{n'=1}^{n-1} \sqrt{\delta}\varepsilon_{n'\delta}$$



Informally, we are modelling z° as an *Ornstein-Uhlenbeck stochastic differential equation*

$$dz_t^\circ = -\alpha(z_t^\circ - \bar{z})dt + \sigma dW_t$$

where W is a *Brownian motion*.
We will see what all of this means a bit later.