Pomsets with Preconditions

A Simple Model of Relaxed Memory

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Memory model

What value can be read for x?

$x := 1; x := 2 \quad \| \quad x := 3; x := 4; \quad s := x; x := 5$

- x–z: shared locations, initially 0
- r–s: registers
- po: program order

- Concurrent: 1? 2?
- Sequential: 3? 4? 5?

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- x–z: shared locations, initially 0
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- Concurrent: 1✓ 2✓
- Sequential: 3✗ 4✓ 5✗

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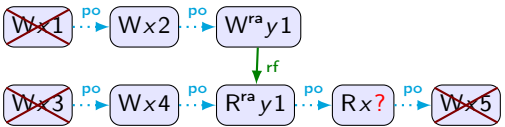
What value can be read for x?

$$x := 1;\ x := 2;\ y^{\mathsf{ra}} := 1 \ \parallel\ x := 3;\ x := 4;\ r := y^{\mathsf{ra}};\ s := x;\ x := 5$$



- x–z: shared locations, initially 0
- r–s: registers
- po: program order
- rf: reads-from
- W^{ra}: release
- R^{ra}: acquire

- Concurrent: 1✗ 2✓
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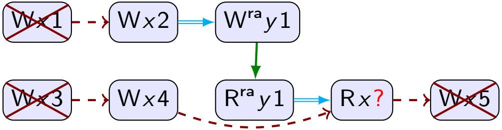
Memory model

What value can be read for x?

$$x := 1\,;\, x := 2\,;\, y^{\mathsf{ra}} := 1 \;\|\; x := 3\,;\, x := 4\,;\, r := y^{\mathsf{ra}}\,;\, s := x\,;\, x := 5$$



- Labelled partial order (Pomset)
  - - -› po ∩ x
  - ⟹ po in to release
  - ⟹ po out of acquire
- Fulfillment
  - ⟶ (W x v) before (R x v)
  - - -› Any other (W x) before (W x v) or after (R x v)

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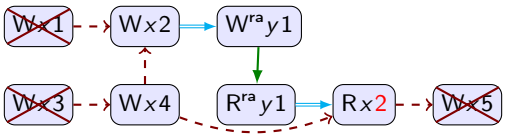

- Labelled partial order (Pomset)
  - --> po $\cap$ x
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- Fulfillment
  - → ($\mathsf{W}x\,v$) before ($\mathsf{R}x\,v$)
  - --> Any other ($\mathsf{W}x$) before ($\mathsf{W}x\,v$) or after ($\mathsf{R}x\,v$)

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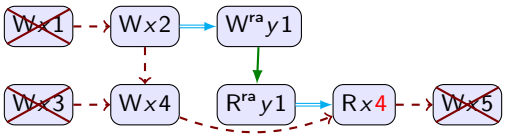

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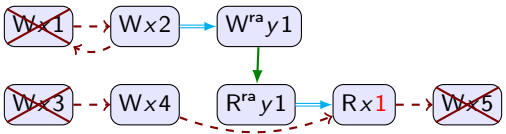

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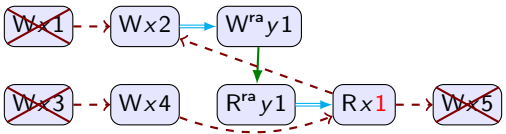

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- Fulfillment
  - → (W$x v$) before (R$x v$)
  - --> Any other (Wx) before (W$x v$) or after (R$x v$)


PRETTY
Image in the Public domain

Preconditions for dependency

Pomset = a single execution

$$s := x\,;\, y := s \;\parallel\; r := y\,;\, x := 1\,;\, z := r$$

- Writes have preconditions

Preconditions for dependency

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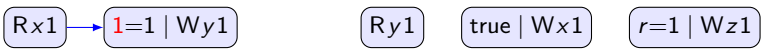

- Writes have preconditions
- Order from $s := (\mathrm{R}x1)$ substitutes $[1/s]$

Preconditions for dependency

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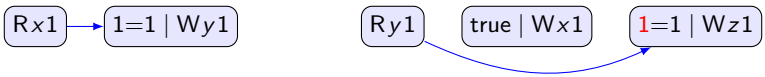

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- Order from $r := (\mathrm{R}y1)$ substitutes $[1/r]$

Preconditions for dependency

Pomset = a single execution

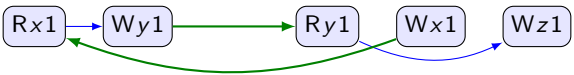
$$y := x \;\|\; r := y\,;\, x := 1\,;\, z := r$$

- Writes have preconditions
- Order from $s := (\mathrm{R}x1)$ substitutes $[1/s]$
- Order from $r := (\mathrm{R}y1)$ substitutes $[1/r]$
- We elide tautologies

Out of order

What value can be written to z?

$$y := x \;\|\; r := y\,;\, x := 1\,;\, z := r \qquad (\star)\ \checkmark$$

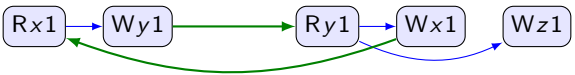


- Partial order
  - - -> per-location
  - ⟹ synchronization
  - → reads-from
  - → dependency
- May reorder $r := y$ and $x := 1$
  - in compiler
  - on ARM

Out of ~~order~~ thin air (OOTA)

What value can be written to z?

$$y := x \;\|\; r := y\,;\, x := r\,;\, z := r$$ (OOTA1) ✗



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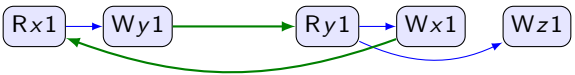
(OOTA1) ✗



- Partial order
  - - -> per-location
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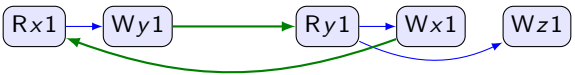
- ~~May reorder $r := y$ and $x := 1$~~
  - ~~in compiler~~
  - ~~on ARM~~
- Partial order prevents!

Out of ~~order~~ thin air (OOTA)

What value can be written to z?

$$\texttt{if}(x)\{y := 1\} \parallel r := y; \texttt{if}(r)\{x := 1; z := r\}$$ (OOTA2) ✗



- Partial order
  - - -> per-location
  - ⟹ synchronization
  - → reads-from
  - → dependency

- ~~May reorder $r := y$ and $x := 1$~~
  - ~~in compiler~~
  - ~~on ARM~~
- Partial order prevents!
- Control flow variant is DRF


VERY
PRETTY
Image in the Public domain

Can this program write 1 to z?

$$y := x \parallel r := y; \qquad x := r; z := r \qquad \text{(OOTA1)}$$

Can this program write 1 to z?

$$y := x \;\|\; r := y;\ \texttt{if}(r)\{x := r;\ z := r\}\ \texttt{else}\ \{x := 1\} \qquad (\star)$$

Can this program write 1 to z?

$$y := x \parallel r := y; \texttt{if}(r)\{x := r; z := r\}\,\texttt{else}\,\{x := 1\} \qquad (\star)$$

x and y can only be 0 or 1

$$y := x \parallel r := y; \texttt{if}(r)\{x := 1; z := 1\}\,\texttt{else}\,\{x := 1\}$$

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$$y := x \parallel r := y; \texttt{if}(r)\{x := 1; z := 1\}\,\texttt{else}\,\{x := 1\}$$

lift common code

$$y := x \parallel r := y; x := 1; \texttt{if}(r)\{z := 1\}$$

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commute independent statements

$$y := x \parallel x := 1; r := y; \texttt{if}(r)\{z := 1\}$$

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$$y := x \parallel x := 1; r := y; \texttt{if}(r)\{z := 1\}$$

interleaving

$$x := 1; y := x; r := y; \texttt{if}(r)\{z := 1\}$$

Can this program write 1 to z?

$$y := x \parallel r := y; \texttt{if}(r)\{x := r; z := r\}\,\texttt{else}\,\{x := 2\} \qquad \text{(OOTA3)}$$

x and y can only be 0 or 2

$$y := x \parallel r := y; \texttt{if}(r)\{x := 2; z := 2\}\,\texttt{else}\,\{x := 2\}$$

lift common code

$$y := x \parallel r := y; x := 2; \texttt{if}(r)\{z := 2\}$$

commute independent statements

$$y := x \parallel x := 2; r := y; \texttt{if}(r)\{z := 2\}$$

interleaving

$$x := 2; y := x; r := y; \texttt{if}(r)\{z := 2\}$$

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~~x and y can only be 0 or 1~~

$$\sout{y := x \parallel r := y; \texttt{if}(r)\{x := r; z := 1\}\,\texttt{else}\,\{x := 1\}}$$

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~~interleaving~~

$$\sout{x := 1; y := x; r := y; \texttt{if}(r)\{z := 1\}}$$

Can this program write 1 to z?

$$y := x \parallel r := y;\ \texttt{if}(b)\{x := r;\ z := r\}\ \texttt{else}\ \{x := 1\} \parallel b := 1 \qquad \text{(OOTA4)}$$

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- Existential justification: ARM✓ $(\star)$✓ OOTA1-3✓ OOTA4✗
  - Commitment (JMM - Java Memory Model) [Manson, Pugh, Adve, 2005]
  - Speculation [Jagadeesan, Pitcher, Riely, 2010]
  - Promising [Kang, Hur, Lahav, Vafeiadis, Dreyer 2017]

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- Universal justification: ARM✗ $(\star)$✓ OOTA1-3✓ OOTA4✓
  - Event Structures [Jeffrey, Riely, 2016]

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$$y := x \parallel r := y\,;\ \texttt{if}(b)\{r := \texttt{new D}\}\ \texttt{else}\ \{s := \texttt{new C}\}\,;\ x := r \parallel b := 1 \qquad \text{(OOTA5)}$$

- In the JMM, OOTA5 *"is type correct if it declares x, y and r of type* `D`*. However, it has a legal execution where they reference a* `C` *object."* [Lochbihler 2013]

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    - ~~Realistic memory allocation~~
    - ~~Java Security Architecture (security-sensitive constants)~~

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- *Out Of Thin Air (OOTA)* is a *queasy feeling*

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    - ~~Realistic memory allocation~~
    - ~~Java Security Architecture (security-sensitive constants)~~
- *Out Of Thin Air (OOTA)* is a *queasy feeling*
  Compositionality of proof rules is a *verifiable property*

Can this program write 1 to z?

$y := x \parallel r := y;\ \texttt{if}(r)\{x := r;\ z := r\}\ \texttt{else}\ \{x := 1\}$ $(\star)$

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- In OOTA4, every thread satisfies:
  - $\mathrm{W}y1$ must be preceded by $\mathrm{R}x1$
  - if $\mathrm{W}z1$, then $\mathrm{W}x1$ must be preceded by $\mathrm{R}y1$
  - In PLTL: $[\diamondsuit \mathrm{W}y1 \Rightarrow \diamondsuit \mathrm{R}x1] \wedge [\mathrm{W}z1 \Rightarrow (\diamondsuit \mathrm{R}y1 \wedge \boxminus(\mathrm{W}x1 \Rightarrow \diamondsuit \mathrm{R}y1))]$

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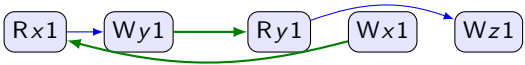
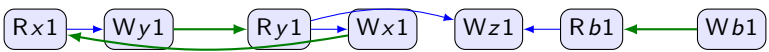
- In OOTA4, every thread satisfies:
  - $\mathrm{W}y1$ must be preceded by $\mathrm{R}x1$
  - if $\mathrm{W}z1$, then $\mathrm{W}x1$ must be preceded by $\mathrm{R}y1$
  - In PLTL: $[\diamondsuit \mathrm{W}y1 \Rightarrow \diamondsuit \mathrm{R}x1] \wedge [\mathrm{W}z1 \Rightarrow (\diamondsuit \mathrm{R}y1 \wedge \boxminus(\mathrm{W}x1 \Rightarrow \diamondsuit \mathrm{R}y1))]$
- Compositionality: every thread satisfies $\Longrightarrow$ program satisfies
  - $\exists$ justification: compositional for predicate logic OOTA1-3
  - $\forall$ justification: compositional for temporal logic OOTA4-5

Comparing attempted executions

$y := x \parallel r := y; \texttt{if}(r)\{x := r; z := r\}\,\texttt{else}\,\{x := 1\}$ (⋆) ✓



$y := x \parallel r := y; \texttt{if}(b)\{x := r; z := r\}\,\texttt{else}\,\{x := 1\} \parallel b := 1$ (OOTA4) ✗




STILL
PRETTY?
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It's hard...

- Programmer desiderata
  - Compositional/local reasoning
  - Sequential Consistency for Data Race Free programs (SC-DRF)

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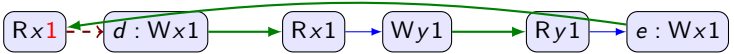
- Implementer desiderata
  - Efficient on ~~hardware~~ MCA hardware
  - Compiler optimizations
- Multi-copy atomicity (MCA)
  - When write published to one processor, published to all
  - ARM✓ x86-64✓ RISC-V✓ POWER✗
  - Prevents anomalies:

$$r := x;\ x := 1 \ \|\ y := x \ \|\ x := y$$



(MCA3) ✗

We've been trying for 20 years...

- Operational with alternate executions: ARM✓ OOTA✗
  - Java 1.1 fails CSE [Pugh 1999]
  - Commitment/Speculation/Promises [2005, 2010, 2017]
- Strong models: ARM✗ OOTA✓
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- Operational with compiler rewrites
  - [Ferreira, Feng, Shao 2010], [Pichon-Pharabod, Sewell 2016]
- Symbolic execution with multiple orders and acyclicly requirements
  - Hardware [Alglave 2010], [Alglave, Maranget, Tautschnig 2014]
  - C++ [Batty, Owens, Sarkar, Sewell, Weber 2011]
- Event Structures
  - WeakestMO [Chakraborty, Vafeiadis 2019]
  - Modularity [Paviotti, Cooksey, Paradis, Wright, Owens, Batty 2020]


PRINCIPLES
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Compositionality

Compositionality

+ Construction


Compositionality
+ Construction
+ Reasoning
+ Local races
+ Safety properties
Image in the Public domain


LOGIC
+ PRECONDITIONS
Image in the Public domain


LOGIC
+ PRECONDITIONS
+ TEMPORAL SAFETY
Image in the Public domain


4 SLIDES
Image in the Public domain

Semantic Domain

- A *pomset with preconditions* is a tuple $(E, \leq, \lambda)$ where
  - E is a set of *events*
  - $\leq \subseteq (E \times E)$ is a partial order
  - $\lambda : E \to (\Phi \times \mathcal{A})$ is a *labeling* from which we derive functions
    - $\Phi : E \to \Phi$ *(formulae)*
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- We say $\mathcal{A}(d) = (\mathrm{W}x\,v)$ *fulfills* $\mathcal{A}(e) = (\mathrm{R}x\,v)$ if
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- A pomset is *x-closed* if
  - every $\mathcal{A}(e) = (\mathrm{R}x..)$ is fulfilled
  - every $\Phi(e)$ is independent of x: $\big(\forall v.\ \Phi(e) \vDash \Phi(e)[v/x] \vDash \Phi(e)\big)$

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- Let $P' \in (\mathcal{P}^1 \parallel \mathcal{P}^2)$ when $(\exists P^1 \in \mathcal{P}^1)\ (\exists P^2 \in \mathcal{P}^2)$
  $E' = E^1 \cup E^2$, $\leq' \supseteq \leq^1 \cup \leq^2$, and $(\forall e \in E')$ either

$$
\begin{array}{c}
e \notin E^2,\ \mathcal{A}'(e) = \mathcal{A}^1(e) \text{ and } \Phi'(e) \text{ implies } \Phi^1(e), \\
e \notin E^1,\ \mathcal{A}'(e) = \mathcal{A}^2(e) \text{ and } \Phi'(e) \text{ implies } \Phi^2(e), \text{ or} \\
\mathcal{A}'(e) = \mathcal{A}^1(e) = \mathcal{A}^2(e) \text{ and } \Phi'(e) \text{ implies } \Phi^1(e) \vee \Phi^2(e)
\end{array}
$$

Semantic Operations (2/2)

- Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

(p1) $E' = E \cup \{d\}$
(p2) $\leq' \supseteq \leq$
(p3a) $\mathcal{A}'(e) = \mathcal{A}(e)$
(p3b) $\mathcal{A}'(d) = a$
(p4a) $\Phi'(d)$ implies $\phi \wedge (d \notin E \vee \Phi(d))$
(p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$
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(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$
(p5b) if d and e are conflicting actions then $d \leq' e$
(p5c) if d is an acquire or e is a release then $d \leq' e$
(p5d) if d is an SC write and e is an SC read then $d \leq' e$
(p5e) if d reads, and e is an acquiring fence, then $d \leq' e$
(p5f) if d is a releasing fence, and e writes, then $d \leq' e$

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[\![r := M;\ C]\!] &\triangleq [\![C]\!][M/r] \\
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μ	$::=$	rlx	(Relaxed)	κ $::=$	rel (Release)
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4 SLIDES
Image in the Public domain

Sequential Semantics: Preconditions

$$z := r$$

$$\boxed{r{=}0 \mid \mathrm{W}\,z\,0}$$

Precondition records dependence on r

$[\![r := x^{\mu};\, C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu}\,x\,v) \Rightarrow [\![C]\!][x/r]$

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Sequential Semantics: Preconditions

$$z := r$$

$$\boxed{r{=}1 \mid \mathrm{W}\, z\, 1}$$

One pomset for each value

$[\![r := x^{\mu};\, C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu}\, x\, v) \Rightarrow [\![C]\!][x/r]$

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Sequential Semantics: Preconditions

$z := r$

$r{=}2 \mid \mathrm{W}z2$

One pomset for each value

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Sequential Semantics: Preconditions

$$x := r \,;\, z := r$$

$r{=}0 \mid \mathrm{W}x0$ $\quad$ $r{=}0 \mid \mathrm{W}z0$

Two writes = two events per pomset

$[\![r := x^{\mu} \,;\, C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

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Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

(p1) $E' = E \cup \{d\}$

(p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$

(p4c) if $d = (\mathrm{R}\, v\, x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$

(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

Sequential Semantics: Preconditions

$$x := r\,;\ z := r$$

$$\boxed{r{=}1 \mid \mathrm{W}x1} \quad \boxed{r{=}1 \mid \mathrm{W}z1}$$

Two writes = two events per pomset

$[\![r := x^{\mu}\,;\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M\,;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

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(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

Sequential Semantics: Preconditions

$x := r \,;\, z := r$

$r{=}1 \mid \mathrm{W}x1$ $\quad$ $r{=}2 \mid \mathrm{W}z2$

Preconditions must be consistent

Pomset = a single execution

$[\![r := x^{\mu} \,;\, C]\!] \triangleq \bigcup_{v} (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M \,;\, C]\!] \triangleq \bigcup_{v} (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

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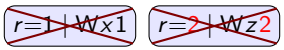
(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

Sequential Semantics: Preconditions

$x := r \,;\, z := r$



Preconditions must be consistent

Pomset = a single execution

$[\![r := x^{\mu} \,;\, C]\!] \triangleq \bigcup_v (\mathsf{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M \,;\, C]\!] \triangleq \bigcup_v (M = v \mid \mathsf{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

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(p5a) if $d = (\mathsf{R}..)$, $e = (\mathsf{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

Sequential Semantics: Preconditions

$x := r\,;\, z := r$

$r{=}1 \mid \mathrm{W}x1$ $\quad$ $r{=}1 \mid \mathrm{W}z1$

$x := 1$

$1{=}1 \mid \mathrm{W}x1$

A separate program

$[\![r := x^{\mu}\,;\, C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M\,;\, C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

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(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

Sequential Semantics: Preconditions

$\texttt{if}(r)\{x := r;\ z := r\}$

$r{\neq}0 \wedge r{=}1 \mid \mathrm{W}x1$ $\quad$ $r{\neq}0 \wedge r{=}1 \mid \mathrm{W}z1$

$\texttt{if}(\neg r)\{x := 1\}$

$r{=}0 \wedge 1{=}1 \mid \mathrm{W}x1$

Adding control

$[\![r := x^{\mu};\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

(p1) $E' = E \cup \{d\}$

(p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$

(p4c) if $d = (\mathrm{R}\, v\, x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$

(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

Sequential Semantics: Preconditions

$\texttt{if}(r)\{x := r;\ z := r\}$

$r{=}1 \mid \mathrm{W}x1$ $\quad$ $r{=}1 \mid \mathrm{W}z1$

$\texttt{if}(\neg r)\{x := 1\}$

$r{=}0 \mid \mathrm{W}x1$

Simplifying

$[\![r := x^{\mu};\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

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Sequential Semantics: Preconditions

$\texttt{if}(r)\{x := r;\ z := r\}$

$r{=}1 \mid \mathrm{W}x1$ $\quad$ $r{=}1 \mid \mathrm{W}z1$

$\texttt{if}(\neg r)\{x := 1\}$

$r{=}0 \mid \mathrm{W}x1$

Combining both sides

$\texttt{if}(r)\{x := r;\ z := r\}\ \texttt{else}\ \{x := 1\}$

$r{=}1 \vee r{=}0 \mid \mathrm{W}x1$ $\quad$ $r{=}1 \mid \mathrm{W}z1$

We can coalesce the writes to x

$[\![r := x^{\mu};\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

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(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

Sequential Semantics: Preconditions

$\texttt{if}(r)\{x := r;\ z := r\}$

$r{=}1 \mid \mathrm{W}x1$ $\quad$ $r{=}1 \mid \mathrm{W}z1$

$\texttt{if}(\neg r)\{x := 2\}$

$r{=}0 \mid \mathrm{W}x2$

Combining both sides

$\texttt{if}(r)\{x := r;\ z := r\}\ \texttt{else}\ \{x := 2\}$

$r{=}1 \mid \mathrm{W}x1$ $\quad$ $r{=}1 \mid \mathrm{W}z1$ $\quad$ $r{=}0 \mid \mathrm{W}x2$

We *cannot* coalesce the writes to x

$[\![r := x^{\mu};\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

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Sequential Semantics: Preconditions

$\texttt{if}(r)\{x := r;\ z := r\}$

$r{=}1 \mid \mathrm{W}x1$ $\quad$ $r{=}1 \mid \mathrm{W}z1$

$\texttt{if}(\neg r)\{x := 2\}$

$r{=}0 \mid \mathrm{W}x2$

Combining both sides

$\texttt{if}(r)\{x := r;\ z := r\}\ \texttt{else}\ \{x := 2\}$

~~$r{=}1 \mid \mathrm{W}x1$~~ $\quad$ ~~$r{=}1 \mid \mathrm{W}z1$~~ $\quad$ ~~$r{=}0 \mid \mathrm{W}x2$~~

Preconditions must be consistent

$[\![r := x^{\mu};\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

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Sequential Semantics: Preconditions

$$\texttt{if}\,(r)\,\{x := r;\ z := r\}\,\texttt{else}\,\{x := 1\}$$

$r{=}1 \vee r{=}0 \mid \mathrm{W}x1$ $\qquad$ $r{=}1 \mid \mathrm{W}z1$

Let's start fresh here

$[\![r := x^{\mu};\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$
$[\![x^{\mu} := M;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$
Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$
(p1) $E' = E \cup \{d\}$
(p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$
(p4c) if $d = (\mathrm{R}\, v\, x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$
(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

Sequential Semantics: Preconditions

$$r := y\,;\ \texttt{if}\,(r)\,\{x := r\,;\ z := r\}\ \texttt{else}\ \{x := 1\}$$

$$\boxed{y{=}1 \vee y{=}0 \mid \mathrm{W}x1} \qquad \boxed{y{=}1 \mid \mathrm{W}z1}$$

To prepend, $r := y$, we first substitute $[y/r]$

$[\![r := x^{\mu}\,;\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M\,;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

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Sequential Semantics: Preconditions

$$r := y\,;\ \texttt{if}\,(r)\,\{x := r\,;\ z := r\}\ \texttt{else}\,\{x := 1\}$$

$\boxed{\mathrm{R}\,y\,1}$ $\boxed{(y{=}1 \vee y{=}0) \wedge (1{=}1 \vee 1{=}0) \mid \mathrm{W}\,x\,1}$ $\boxed{(y{=}1) \wedge (1{=}1) \mid \mathrm{W}\,z\,1}$

We then add the action, and its constraint: $\phi' \vDash \phi[v/y]$

$[\![r := x^{\mu}\,;\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu}\,x\,v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M\,;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu}\,x\,v) \Rightarrow [\![C]\!][M/x]$

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Sequential Semantics: Preconditions

$$r := y\,;\ \texttt{if}\,(r)\,\{x := r\,;\ z := r\}\ \texttt{else}\ \{x := 1\}$$

$\mathrm{R}\,y\,2$ $\qquad$ $(y{=}1 \vee y{=}0) \wedge (2{=}1 \vee 2{=}0) \mid \mathrm{W}\,x\,1$ $\qquad$ $(y{=}1) \wedge (2{=}1) \mid \mathrm{W}\,z\,1$

Incompatible reads are disallowed

$[\![r := x^{\mu};\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

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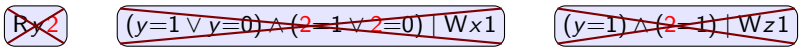
Sequential Semantics: Preconditions

$r := y\,;\ \texttt{if}\,(r)\{x := r\,;\ z := r\}\ \texttt{else}\,\{x := 1\}$



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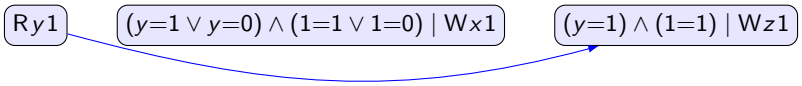
Sequential Semantics: Preconditions

$r := y\,;\ \texttt{if}\,(r)\,\{x := r\,;\ z := r\}\ \texttt{else}\ \{x := 1\}$



Order may be introduced

$[\![r := x^{\mu}\,;\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M\,;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

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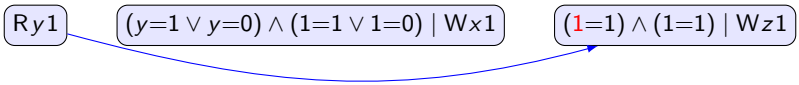
Sequential Semantics: Preconditions

$r := y\,;\ \texttt{if}\,(r)\,\{x := r\,;\ z := r\}\ \texttt{else}\,\{x := 1\}$



Order enables substitution $[v/y]$

$[\![r := x^{\mu}\,;\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M\,;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

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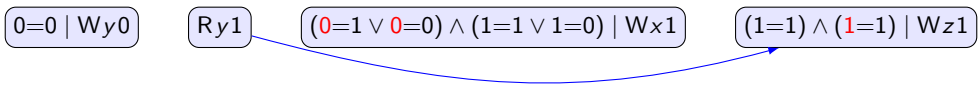
(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

Sequential Semantics: Preconditions

$y := 0;\ r := y;\ \texttt{if}\,(r)\{x := r;\ z := r\}\,\texttt{else}\,\{x := 1\}$



Prepending $y := 0$ substitutes $[0/y]$

Order imposed by sub? Write✗ Read✓

$[\![r := x^{\mu};\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

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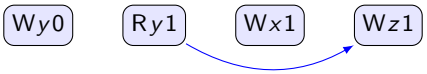
(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

Sequential Semantics: Preconditions

$y := 0;\ r := y;\ \texttt{if}\,(r)\,\{x := r;\ z := r\}\,\texttt{else}\,\{x := 1\}$



Simplifying tautologies

$[\![r := x^{\mu};\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

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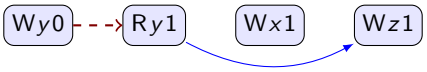
(p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$

(p4c) if $d = (\mathrm{R}\, v\, x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$

(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

Concurrent Semantics: Fulfillment

$y := 0;\ r := y;\ \texttt{if}\,(r)\{x := r;\ z := r\}\ \texttt{else}\,\{x := 1\}$



Must preserve order on *conflicting* accesses (WW, WR)

$[\![r := x^{\mu};\ C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

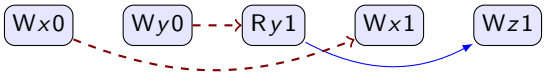
$[\![x^{\mu} := M;\ C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

- (p1) $E' = E \cup \{d\}$
- (p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$
- (p4c) if $d = (\mathrm{R}\, v\, x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$
- (p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$
- (p5b) if d and e are conflicting actions then $d \leq' e$

Concurrent Semantics: Fulfillment

$x := 0; y := 0; r := y;$ `if` $(r)\{x := r; z := r\}$ `else` $\{x := 1\}$



Initializing x

$[\![r := x^{\mu}; C]\!] \triangleq \bigcup_v (\mathsf{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M; C]\!] \triangleq \bigcup_v (M = v \mid \mathsf{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

(p1) $E' = E \cup \{d\}$

(p4b) if $d \neq (\mathsf{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$

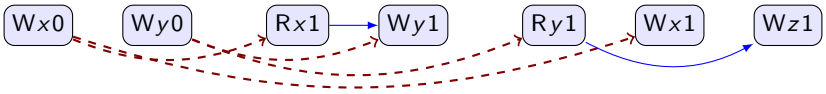
(p4c) if $d = (\mathsf{R}\, v\, x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$

(p5a) if $d = (\mathsf{R}..)$, $e = (\mathsf{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

(p5b) if d and e are conflicting actions then $d \leq' e$

Concurrent Semantics: Fulfillment

$x := 0; y := 0; (y := x \parallel r := y; \texttt{if}(r)\{x := r; z := r\}\ \texttt{else}\ \{x := 1\})$



Introducing $y := x$

$[\![r := x^{\mu}; C]\!] \triangleq \bigcup_v (\mathrm{R}^{\mu} x\, v) \Rightarrow [\![C]\!][x/r]$

$[\![x^{\mu} := M; C]\!] \triangleq \bigcup_v (M = v \mid \mathrm{W}^{\mu} x\, v) \Rightarrow [\![C]\!][M/x]$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

(p1) $E' = E \cup \{d\}$

(p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$

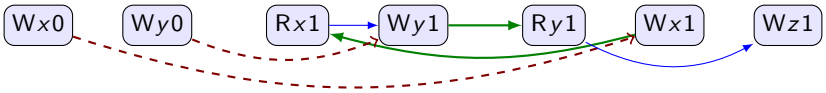
(p4c) if $d = (\mathrm{R}\, v\, x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$

(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

(p5b) if d and e are conflicting actions then $d \leq' e$

Concurrent Semantics: Fulfillment

$x := 0; y := 0; (y := x \parallel r := y; \texttt{if}(r)\{x := r; z := r\}\ \texttt{else}\ \{x := 1\})$ $(\star)$ ✓



Fulfillment requirements

We say $\mathcal{A}(d) = (\mathrm{W}x v)$ *fulfills* $\mathcal{A}(e) = (\mathrm{R}x v)$ if

- $d < e$
- if $\mathcal{A}(c) = (\mathrm{W}x..)$ then either $c \leq d$ or $e \leq c$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

(p1) $E' = E \cup \{d\}$

(p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$

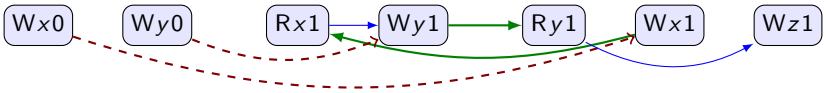
(p4c) if $d = (\mathrm{R}v x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$

(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

(p5b) if d and e are conflicting actions then $d \leq' e$

Concurrent Semantics: Fulfillment

$x := 0; y := 0; (y := x \parallel r := y; \texttt{if}(r)\{x := r; z := r\}\,\texttt{else}\,\{x := 1\})$ $(\star)$ ✓



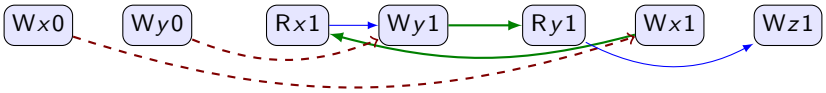
Allowed!

We say $\mathcal{A}(d) = (\mathrm{W}x v)$ *fulfills* $\mathcal{A}(e) = (\mathrm{R}x v)$ if
- $d < e$
- if $\mathcal{A}(c) = (\mathrm{W}x..)$ then either $c \leq d$ or $e \leq c$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$
- (p1) $E' = E \cup \{d\}$
- (p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$
- (p4c) if $d = (\mathrm{R}v x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$
- (p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$
- (p5b) if d and e are conflicting actions then $d \leq' e$

Concurrent Semantics: Fulfillment

$x := 0; y := 0; (y := x \parallel r := y; \texttt{if}(r)\{x := r; z := r\}\,\texttt{else}\,\{x := 1\})$ $(\star)$ ✓



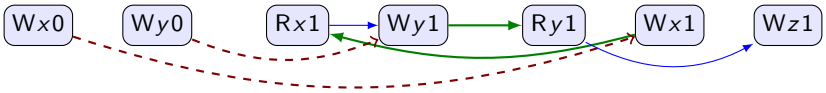
Allowed!
Partial order

We say $\mathcal{A}(d) = (\mathrm{W}x v)$ *fulfills* $\mathcal{A}(e) = (\mathrm{R}x v)$ if
- $d < e$
- if $\mathcal{A}(c) = (\mathrm{W}x..)$ then either $c \leq d$ or $e \leq c$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$
- (p1) $E' = E \cup \{d\}$
- (p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$
- (p4c) if $d = (\mathrm{R}v x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$
- (p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$
- (p5b) if d and e are conflicting actions then $d \leq' e$

Concurrent Semantics: Fulfillment

$x := 0; y := 0; (y := x \parallel r := y; \texttt{if}(r)\{x := r; z := r\}\,\texttt{else}\,\{x := 1\})$ $(\star)$ ✓



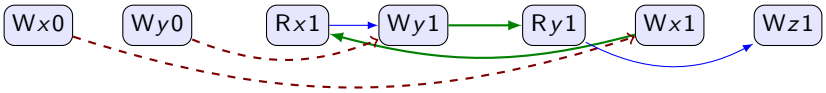
Allowed!
Partial order
All reads fulfilled

We say $\mathcal{A}(d) = (\mathrm{W}x v)$ *fulfills* $\mathcal{A}(e) = (\mathrm{R}x v)$ if
- $d < e$
- if $\mathcal{A}(c) = (\mathrm{W}x..)$ then either $c \leq d$ or $e \leq c$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$
- (p1) $E' = E \cup \{d\}$
- (p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$
- (p4c) if $d = (\mathrm{R}v x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$
- (p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$
- (p5b) if d and e are conflicting actions then $d \leq' e$

Concurrent Semantics: Fulfillment

$x := 0; y := 0; (y := x \parallel r := y; \texttt{if}(r)\{x := r; z := r\}\,\texttt{else}\,\{x := 1\})$ $(\star)$ ✓



Allowed!

Partial order

All reads fulfilled

All preconditions tautologies

We say $\mathcal{A}(d) = (\mathrm{W}x v)$ *fulfills* $\mathcal{A}(e) = (\mathrm{R}x v)$ if

- $d < e$
- if $\mathcal{A}(c) = (\mathrm{W}x..)$ then either $c \leq d$ or $e \leq c$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

(p1) $E' = E \cup \{d\}$

(p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$

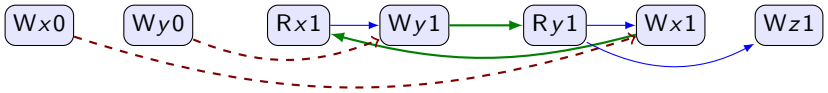
(p4c) if $d = (\mathrm{R}v x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$

(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$

(p5b) if d and e are conflicting actions then $d \leq' e$

Concurrent Semantics: Fulfillment

$x := 0; y := 0; (y := x \parallel r := y; \texttt{if}(r)\{x := r; z := r\}\,\texttt{else}\,\{x := 2\})$ (OOTA3) ✗



Disallowed!
Cycle

We say $\mathcal{A}(d) = (\mathrm{W}x v)$ *fulfills* $\mathcal{A}(e) = (\mathrm{R}x v)$ if

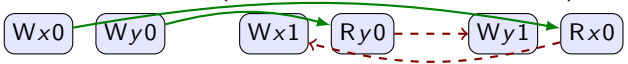
- $d < e$
- if $\mathcal{A}(c) = (\mathrm{W}x..)$ then either $c \leq d$ or $e \leq c$

Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

(p1) $E' = E \cup \{d\}$
(p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$
(p4c) if $d = (\mathrm{R}v x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$
(p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$
(p5b) if d and e are conflicting actions then $d \leq' e$

Buffering

$x := 0; y := 0; (x := 1; r := y \parallel y := 1; r := x)$



(SB) ✓

$r := y; x := 1 \parallel r := x; y := 1$

(LB) ✓

Synchronization and Fences

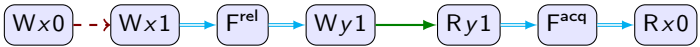
- Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$
  - (p1) $E' = E \cup \{d\}$
  - (p2) $\leq' \supseteq \leq$
  - (p3a) $\mathcal{A}'(e) = \mathcal{A}(e)$
  - (p3b) $\mathcal{A}'(d) = a$
  - (p4a) $\Phi'(d)$ implies $\phi \wedge (d \notin E \vee \Phi(d))$
  - (p4b) if $d \neq (\mathrm{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$
  - (p4c) if $d = (\mathrm{R}\, v\, x)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$
  - (p5a) if $d = (\mathrm{R}..)$, $e = (\mathrm{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$
  - (p5b) if d and e are conflicting actions then $d \leq' e$
  - (p5c) if d is an acquire or e is a release then $d \leq' e$
  - (p5d) if d is an SC write and e is an SC read then $d \leq' e$
  - (p5e) if d reads, and e is an acquiring fence, then $d \leq' e$
  - (p5f) if d is a releasing fence, and e writes, then $d \leq' e$

Publication

$x := 0; x := 1; y^{\mathsf{ra}} := 1 \parallel r := y^{\mathsf{ra}}; s := x$

(PUB1) ✗

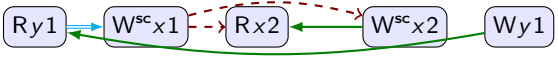
$x := 0; x := 1; \mathsf{F}^{\mathsf{rel}}; y := 1 \parallel r := y; \mathsf{F}^{\mathsf{acq}}; s := x$



(PUB2) ✗

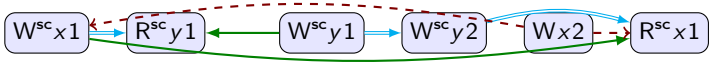
SC Access/Fences

$r := y;\ x^{\mathsf{SC}} := 1;\ s := x \ \parallel\ x^{\mathsf{SC}} := 2;\ y := 1$



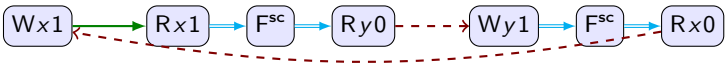
(SC1) ✓

$x^{\mathsf{SC}} := 1;\ r := y^{\mathsf{SC}} \ \parallel\ y^{\mathsf{SC}} := 1;\ y^{\mathsf{SC}} := 2;\ x := 2;\ s := x^{\mathsf{SC}}$



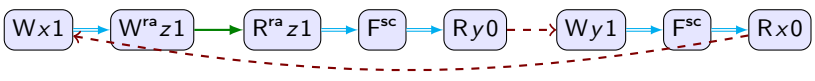
(SC2) ✓

$x := 1 \ \parallel\ r := x;\ \mathsf{F}^{\mathsf{SC}};\ r := y \ \parallel\ y := 1;\ \mathsf{F}^{\mathsf{SC}};\ r := x$



(SC3) ✗

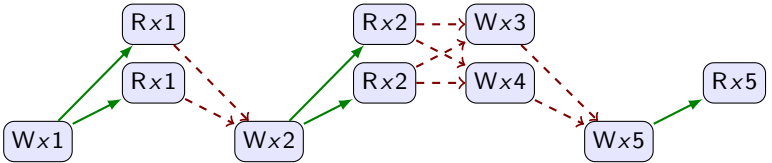
$x := 1;\ z^{\mathsf{ra}} := 1; \ \parallel\ r^{\mathsf{ra}} := z;\ \mathsf{F}^{\mathsf{SC}};\ r := y \ \parallel\ y := 1;\ \mathsf{F}^{\mathsf{SC}};\ r := x$



(SC4) ✗

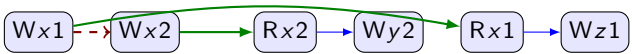
Coherence

$x := 1 \parallel x := 2 \parallel x := 3 \parallel x := 4 \parallel x := 5 \parallel r := x; r := x; r := x; r := x; r := x$



(CO1) ✓

$x := 1; x := 2 \parallel y := x; z := x$



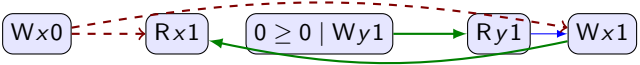
(CO2) ✓

$r := x; x := 1 \parallel s := x; x := 2$

(TC16) ✗

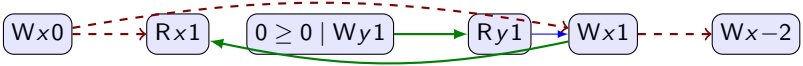
Internal Reads

$x := 0; (r := x; \texttt{if}(r \geq 0)\{y := 1\} \parallel x := y)$



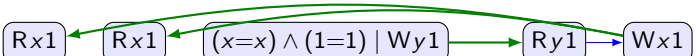
(TC1) ✓

$x := 0; (r := x; \texttt{if}(r \geq 0)\{y := 1\} \parallel x := y \parallel x := -2)$



(TC9) ✓

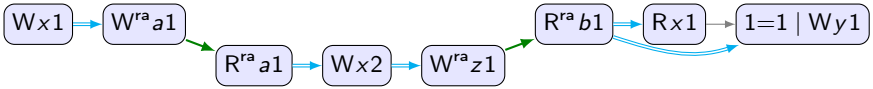
$r := x; s := x; \texttt{if}(r{=}s)\{y := 1\} \parallel x := y$



(TC2) ✓

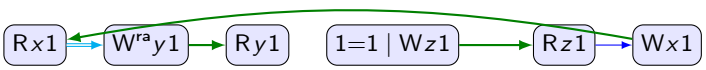
Internal Reads+Synchronization

$x := 1;\ a^{\mathsf{ra}} := 1;\ \mathtt{if}(z^{\mathsf{ra}})\{y := x\} \parallel \mathtt{if}(a^{\mathsf{ra}})\{x := 2;\ z^{\mathsf{ra}} := 1\}$



(IN1) ✗

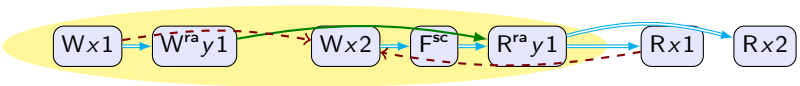
$r := x;\ y^{\mathsf{ra}} := 1;\ s := y;\ z := s \parallel x := z$



(IN2) ✓

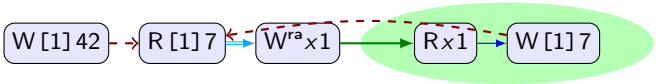
Local data race freedom

$(x := 1;\ y^{\mathsf{ra}} := 1) \parallel (x := 2;\ \mathrm{F}^{\mathsf{sc}};\ \mathtt{if}(y^{\mathsf{ra}})\{r := x;\ s := x\})$



(past) ✗

$(r := 1;\ [r] := 42;\ s := [r];\ x^{\mathsf{ra}} := r) \parallel (r := x;\ [r] := 7)$



(future) ✗

Valid Transformations

$$[\![r := x;\ s := y;\ C]\!] = [\![s := y;\ r := x;\ C]\!] \qquad \text{if } r \neq s$$

$$[\![x := M;\ y := N;\ C]\!] = [\![y := N;\ x := M;\ C]\!] \qquad \text{if } x \neq y$$

$$[\![x := M;\ s := y;\ C]\!] = [\![s := y;\ x := M;\ C]\!] \qquad \text{if } x \neq y \text{ and } s \notin \mathrm{id}(M)$$

$$[\![x^{\mu} := M;\ s := y;\ C]\!] \supseteq [\![s := y;\ x^{\mu} := M;\ C]\!] \qquad \text{if } x \neq y \text{ and } s \notin \mathrm{id}(M)$$

$$[\![x := M;\ s := y^{\mu};\ C]\!] \supseteq [\![s := y^{\mu};\ x := M;\ C]\!] \qquad \text{if } x \neq y \text{ and } s \notin \mathrm{id}(M)$$

$$[\![\mathrm{F}^{\mu};\ \mathrm{F}^{\mu};\ C]\!] \supseteq [\![\mathrm{F}^{\mu};\ s := r;\ C]\!]$$

$$[\![r := x^{\mu};\ s := x^{\mu};\ C]\!] \supseteq [\![r := x^{\mu};\ s := r;\ C]\!]$$

$$[\![r_1 := x;\ s := y;\ r_2 := x;\ C]\!] \supseteq [\![r_1 := x;\ r_2 := r_1;\ s := y;\ C]\!] \qquad \text{if } r_2 \neq s$$

- Reordering: W $\leftrightarrow$ W, R $\leftrightarrow$ R, W $\leftrightarrow$ R
- Roach motel: Relaxed $\leftarrow$ Acquire, Relaxed $\rightarrow$ Release
- Elimination: Redundant fence, Redundant read, Common Subexpression

Valid Transformations

$$[\![C \parallel \texttt{var}\, x; D]\!] = [\![\texttt{var}\, x; (C \parallel D)]\!] \qquad \text{if } x \notin \mathrm{id}(C)$$

$$[\![\texttt{if}(M)\{C\}\,\texttt{else}\,\{C\}]\!] \supseteq [\![C]\!]$$

$$[\![\texttt{if}(M)\{C\}\,\texttt{else}\,\{C\}]\!] \subsetneq^{\star} [\![C]\!]$$

$$[\![\texttt{if}(M)\{C\}\,\texttt{else}\,\{D\}]\!] = [\![C]\!] \qquad \text{if } M \text{ is a tautology}$$

$$[\![x := M; x := N; C]\!] \supseteq^{\star} [\![x := N; C]\!]$$

$$[\![x^{\mu} := M; r := x; C]\!] \supseteq^{\star} [\![x^{\mu} := M; r := M; C]\!]$$

$$[\![r := x; C]\!] \supseteq^{\star} [\![C]\!] \qquad \text{if } r \notin \mathrm{id}(C)$$

- Scope extrusion
- Code lifting, Case analysis* (*See paper)
- Elimination: Dead code, Dead store*, Store forwarding*, Irrelevant read*

Other Transformations

- Valid✓ Proof✗
  - Redundant write after read elimination
- Sound observationally✓ Valid✗
  - Access mode strengthening (rlx $\rightarrow$ ra $\rightarrow$ sc)
  - Commuting/Eliminating some synchronizations
  - Implementing synchronizations using fences
  - Implementing locks using synchronizations
- Sound observationally? Valid✗
  - Access mode weakening, eg (sc $\rightarrow$ ra $\rightarrow$ rlx)
  - Lock elision
- Sound observationally✗
  - Thread inlining
  - Relevant read introduction
  - Write introduction

Other results and limitations

- Results
  - Compositional proof rule for PLTL
  - Efficient ARM implementation
  - Local data race freedom

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  - MCA: ARM✓ x86-64✓ RISC-V✓ POWER✗
    - Add per-location partial order: $\sqsubseteq$
    - Require *observation:* if d/e conflict and $d \leq e$ then $d \sqsubseteq e$
    - Prefixing (p5b): if e conflicts then $d \sqsubseteq' e$
    - Fulfillment (f4): if c conflicts then $c \sqsubseteq d$ or $e \sqsubseteq c$


PRINCIPLES
Image in the Public domain

Compositionality

Compositionality

+ Construction


Compositionality
+ Construction
+ Reasoning
+ Local races
+ Safety properties
Image in the Public domain


LOGIC
+ PRECONDITIONS
Image in the Public domain


LOGIC
+ PRECONDITIONS
+ TEMPORAL SAFETY
Image in the Public domain

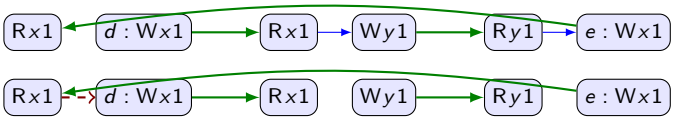

THANKS
Image in the Public domain

Non-MCA Architectures?

- Two partial orders:
  - $\leq$ causal order, as before
  - $\sqsubseteq$ per-location order
- Require: $d \sqsubseteq e$ when $d \leq e$ and they conflict (same location)
- When prefixing d:

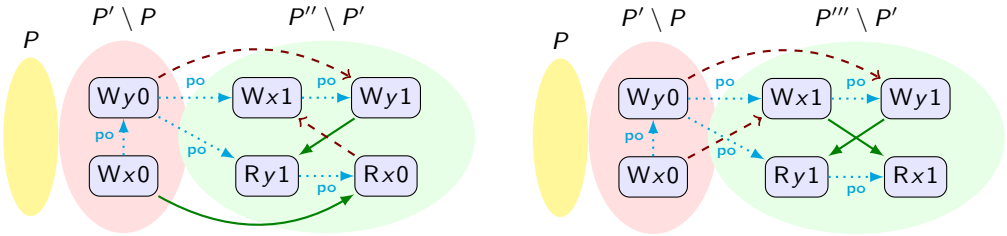
  (p5b) if d and e conflict then $d \sqsubseteq' e$,
- When d *fulfills* e (on x):

  (f4) for every conflicting write c, either $c \sqsubseteq d$ or $e \sqsubseteq c$.
- Example: $r := x;\ x := 1 \ \parallel\ y := x \ \parallel\ x := y$



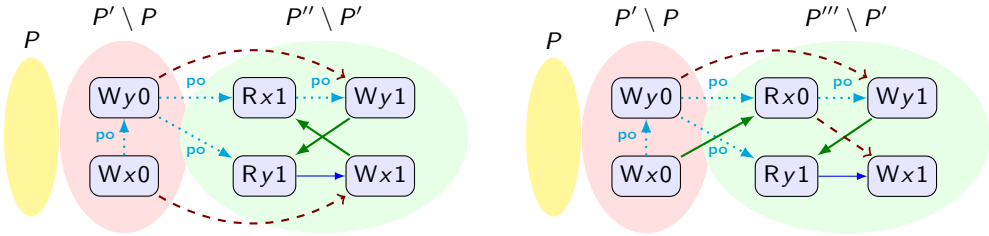
LDRF: Reads from the Past

$x := 0; y := 0; (x := 1; y := 1 \parallel \texttt{if}(y)\{r := x\})$



LDRF: Reads from the Future

$x := 0;\ y := 0;\ (r := x;\ y := 1 \parallel s := y;\ x := s)$

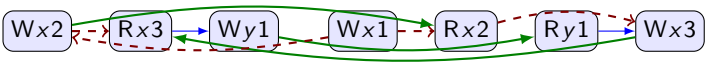


More OOTA

$(y := x + 1 \parallel x := y)$

(OOTA6) ✗

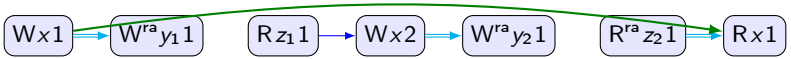
$x := 2; \texttt{if}(x \neq 2)\{y := 1\} \parallel x := 1; r := x; \texttt{if}(y)\{x := 3\}$



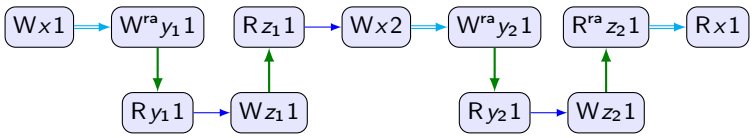
(OOTA7) ✗

Blockers

$$\texttt{var}\, x;\ (x := 1;\ y_1^{\mathsf{ra}} := 1 \ \parallel\ \texttt{if}(z_1)\{x := 2\};\ y_2^{\mathsf{ra}} := 1 \ \parallel\ r := z_2^{\mathsf{ra}};\ s := x)$$



Context $z_1 := y_1 \ \parallel\ z_2 := y_2 \ \parallel\ [-]$

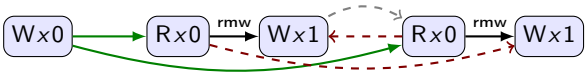


RMWs

$\xrightarrow{\text{rmw}} \subseteq \leq$:

- If $(\mathrm{R}x2) \xrightarrow{\text{rmw}} (\mathrm{W}x3)$ and $(\mathrm{W}x1) \leq (\mathrm{W}x3)$ then $(\mathrm{W}x1) \leq (\mathrm{R}x2)$
- If $(\mathrm{R}x2) \xrightarrow{\text{rmw}} (\mathrm{W}x3)$ and $(\mathrm{R}x2) \leq (\mathrm{W}x3)$ then $(\mathrm{W}x3) \leq (\mathrm{W}x3)$
- $\xrightarrow{\text{rmw}}$ does not coalesce in $\|$ or prefixing

$$x := 0;\ \left(\texttt{FADD}^{\text{rlx,rlx}}(x,1) \ \|\ \texttt{FADD}^{\text{rlx,rlx}}(x,1)\right)$$

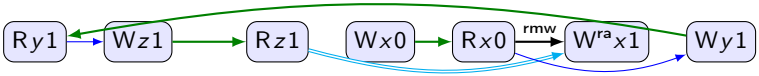


(RMW0) ✗

$$x := 0;\ s := \texttt{FADD}^{\text{rlx,rlx}}(x,1) \ \|\ x := 2;\ s := x$$

(RMW1) ✗

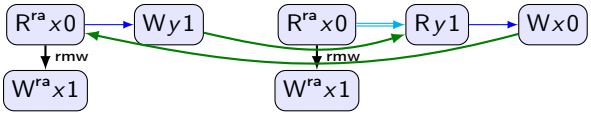
$$r := y;\ z := r \ \|\ r := z;\ x := 0;\ s := \texttt{FADD}^{\text{rlx,ra}}(x,1);\ y := s+1$$



(RMW2) ✓

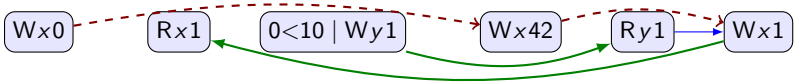
PS2.0 Examples

$r := \texttt{FADD}^{\mathsf{ra},\mathsf{ra}}(x,1);\ \texttt{if}(r{=}0)\{y := 1\} \ \parallel\ r := \texttt{FADD}^{\mathsf{ra},\mathsf{ra}}(x,1);\ \texttt{if}(r{=}0)\{\texttt{if}(y)\{x := 0\}\}$



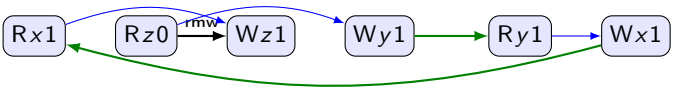
(CDRF) ✗

$x := 0;\ \left(r := \texttt{CAS}^{\mathsf{rlx},\mathsf{rlx}}(x,0,1);\ \texttt{if}(r{<}10)\{y := 1\} \ \parallel\ x := 42;\ x := y\right)$



(GA+E) ✓

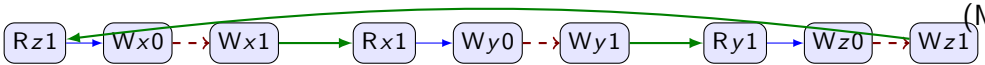
$r := x;\ s := \texttt{FADD}^{\mathsf{rlx},\mathsf{rlx}}(z,r);\ y := s{+}1 \ \parallel\ x := y$



(RP) ✓

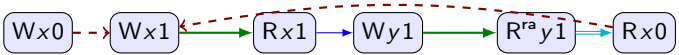
MCA Examples

$\texttt{if}(z)\{x:=0\};\ x:=1 \parallel \texttt{if}(x)\{y:=0\};\ y:=1 \parallel \texttt{if}(y)\{z:=0\};\ z:=1$



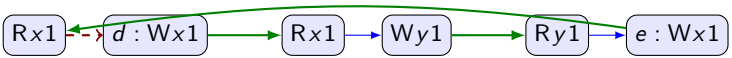
(MCA1) ✗

$x:=0;\ x:=1 \ \parallel\ y:=x \ \parallel\ r:=y^{\mathsf{ra}};\ s:=x$



(MCA2) ✗

$r:=x;\ x:=1 \ \parallel\ y:=x \ \parallel\ x:=y$



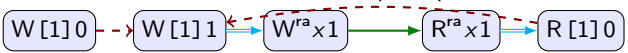
(MCA3) ✗

Address Calculation Examples

$r := y\,;\ s := [r]\,;\ x := s \ \parallel\ r := x\,;\ s := [r]\,;\ y := s$

(ADDR1) ✗

$(r := 1\,;\ [r] := 0\,;\ [r] := 1\,;\ x^{\mathsf{ra}} := r) \ \parallel\ (r := x^{\mathsf{ra}}\,;\ s := [r])$



(ADDR2) ✗