1 MODEL

μ	::= wk	(Weak)	ς	::= cta	(Thread group)
	| rlx	(Relaxed)		| gpu	(Processor)
	| ra	(Release/Acquire)		| sys	(System)
	| sc	(Sequentially Consistent)			

Orders/Relations in model

- $\trianglelefteq$ is the old $\leq$ (without coherence stuff from f4 and p5b).
  This provides the NO-TAR axiom.
- $\leq$ is a the happens-before suborder, which only includes rf when they are morally strong.
  This serves as a cross-location transitive kernel for the per-location order.
- $\sqsubseteq$ is a per-location order that relates morally strong and poloc accesses
  This includes $\leq$ for morally strong accesses.
  This provides the SC-PER-LOC axiom.

Write $d \vartriangle e$ if they conflict (ie, read/write or write/write, same location).
Write $d \blacktriangle e$ if they conflict and are morally strong

Definition 1.1. A pomset with preconditions is a tuple $(E, \lambda, \leq, \trianglelefteq, \sqsubseteq)$ where

(m1) E is a set of events
(m2) $\lambda : E \to (\Phi \times \mathcal{A})$ is a labeling from which we derive functions
- $\Phi : E \to \Phi$ (formulae)
- $\mathcal{A} : E \to \mathcal{A}$ (actions)

(m3) $\leq \subseteq (E \times E)$, $\trianglelefteq \subseteq (E \times E)$, and $\sqsubseteq \subseteq (E \times E)$ are partial orders
(m4) $\bigwedge_e \Phi(e)$ is satisfiable (consistency)
(m5) if $d \trianglelefteq e$ then $\Phi(e)$ implies $\Phi(d)$ (causal strengthening)
(m6) if $d \leq e$ then $d \trianglelefteq e$
(m7) if $d \leq e$ and d conflicts with e then $d \sqsubseteq e$

We say $d < e$ when $d \leq e$ and $d \neq e$, and similarly for $\vartriangleleft$ and $\sqsubset$.

Definition 1.2 (Strong fulfillment). We say $\mathcal{A}(d) = (\mathsf{W}xv)$ fulfills $\mathcal{A}(e) = (\mathsf{R}xv)$ if

(f3a) $d \vartriangleleft e$
(f3b) $d < e$ if d is morally strong with e
(f3c) $d \sqsubseteq e$ (if d is not morally strong with e)
(f4) $\forall \mathcal{A}(c) = (\mathsf{W}x..)$ either $c \sqsubseteq d$ or $e \sqsubseteq c$,

Definition 1.3 (Weak fulfillment). We say $\mathcal{A}(d) = (\mathsf{W}xv)$ fulfills $\mathcal{A}(e) = (\mathsf{R}xv)$ if

(f3a) $d \vartriangleleft e$
(f3b) $d < e$ if d is morally strong with e
(f3c) $e \not\sqsubseteq d$ (if d is not morally strong with e)
(f4) $\forall \mathcal{A}(c) = (\mathsf{W}x..)$ either $c \precsim d$ or $e \precsim c$, where

$$d \precsim e \text{ when } \begin{cases} d \sqsubseteq e & \text{if } d \text{ is morally strong with } e \\ e \not\sqsubset d & \text{otherwise} \end{cases}$$

If all accesses are morally strong with each other, weak fulfillment degenerates to

(f3) $d < e$
(f4) $\forall \mathcal{A}(c) = (\mathsf{W}x..)$ either $c \sqsubseteq d$ or $e \sqsubseteq c$

If no accesses are morally strong with each other, weak fulfillment degenerates to

(f3) $e \not\sqsubseteq d$
(f4) $\nexists \mathcal{A}(c) = (\mathsf{W}x..)$ both $d \sqsubset c$ and $c \sqsubset e$

Note that the difference between strong and weak fulfillment is limited to $\sqsubseteq$. We sometimes write $\boxed{\mathsf{S}}$ for strong fulfillment and $\boxed{\mathsf{W}}$ for weak fulfillment.

Prefixing is as in OOPSLA, using $\leq$ for order everywhere except p5b, which has $\sqsubseteq$.

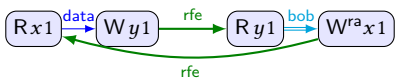
Definition 1.4. Let $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ when $(\exists P \in \mathcal{P})\ (\forall e \in E)$

(p1) $E' = E \cup \{d\}$
(p2) $\leq' \supseteq \leq$, $\trianglelefteq' \supseteq \trianglelefteq$, and $\sqsubseteq' \supseteq \sqsubseteq$
(p3a) $\mathcal{A}'(e) = \mathcal{A}(e)$
(p3b) $\mathcal{A}'(d) = a$
(p4a) $\Phi'(d)$ implies $\phi \wedge (d \notin E \vee \Phi(d))$
(p4b) if $d \neq (\mathsf{R}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$
(p4c) if $d = (\mathsf{R}vx)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)[v/x]$
(p5a) if $d = (\mathsf{R}..)$, $e = (\mathsf{W}..)$ then $e = d$ or $\Phi'(e)$ implies $\Phi(e)$ or $d \leq' e$
(p5b) if d conflicts with e then $d \sqsubseteq' e$
(p5c) if d is an acquire or e is a release then $d \leq' e$
(p5d) if d is an SC write and e is an SC read then $d \leq' e$
(p5e) if d reads, and e is an acquiring fence, then $d \leq' e$
(p5f) if d is a releasing fence, and e writes, then $d \leq' e$

2 ANTON'S RECENT EXAMPLES RELATING IMM AND PTX

It looks like we cannot prove compilation correctness from IMM to PTX. (In this email I assume that all threads are in the same CTA, so any relation is a morally strong one if it is applicable.) The problem is in the LB-data-rel example:

$$r := x\,;\ y := r \ \parallel\ s := y\,;\ x^{\mathsf{ra}} := 1$$



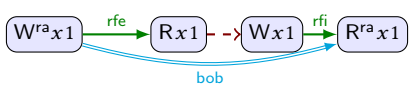
IMM forbids it, but PTX allows it. The point is that IMM mixes dependencies and release/acquire-induced po-order in its NoOOTA axiom, whereas PTX doesn't — release/acquire are only used to have coherence.

The problem is related to the one we have already discussed in the context of the C++ model – if you don't have acquire reads in the program, then you can erase release annotations from writes. In this regard, PTX is closer to PL memory models than to hardware ones.

AFAIU for the same reason we won't be able to show compilation correctness from the Pomset model to PTX even directly, if the Pomset model mixes release/acquire induced order with dependencies in the same causality relation.

Another oddity: PTX includes the bob edge below; IMM does not.

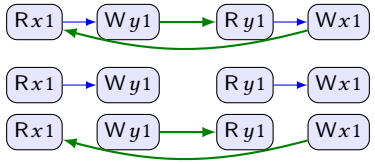
$$x^{\mathsf{ra}} := 1 \ \parallel\ r := x\,;\ x := 1\,;\ 1 := x^{\mathsf{ra}}$$



3 THIN AIR

Need $\trianglelefteq$ to prevent thin air on rlx:

$y := x \parallel x := y$



($\trianglelefteq$) ($\leq$) ($\overline{\text{s}}$)

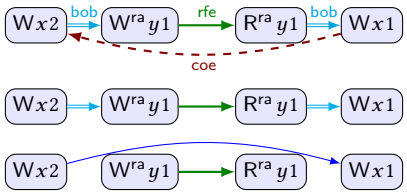
4 IMM EXAMPLES

Interpreting this definition for the imm:

- No wk, default is rlx
- All threads in same cta (only one scope)
- Actions are morally strong when both are ra/sc, mimicking happens-before
- Strong fulfillment may do the right thing

Disallowed by imm:

$x := 2; y^{\mathsf{ra}} := 1 \parallel r := y^{\mathsf{ra}}; x := 1$ (pub-rel-acq-coe)



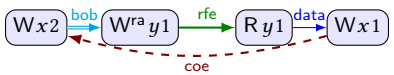
(✗imm)

($\trianglelefteq = \leq$)

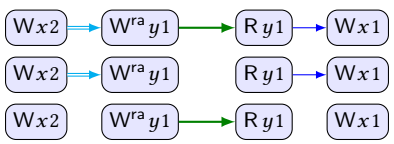
($\overline{\text{s}}$)

Allowed by imm, but not by Power/ARMv7/ARMv8/TSO:

$x := 2; y^{\mathsf{ra}} := 1 \parallel r := y; x := 1$ (pub-rel-rlx-coe)



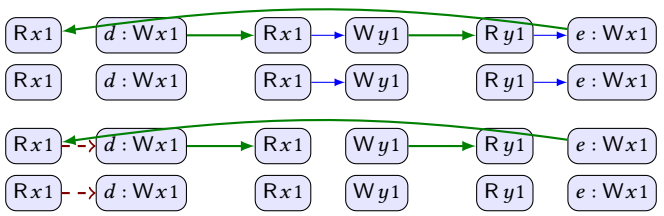
(✓imm)



($\trianglelefteq$) ($\leq$) ($\overline{\text{s}}$)

Example from talk:

$r := x; x := 1 \parallel y := x \parallel x := y$ (arm7-weak)



($\trianglelefteq$) ($\leq$) ($\overline{\text{s}}$) ($\overline{\text{w}}$)

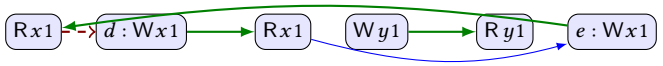
5 TWO ORDER IDEA

The two order idea from OOPSLA talk is:

- Require: $d \sqsubseteq e$ when $d \trianglelefteq e$ and they conflict

This does not work for the imm or ARMv7, but it may work for Power, TSO, ARMv8. That would be nice. Let's write $\sqsubseteq\!\!\!\!\approx$ for this notion, with strong fulfillment.

With this there is a cycle in arm7-weak (weak/strong fulfillment not relevant here):



($\sqsubseteq\!\!\!\!\approx$)

Anton says: arm7-weak is forbidden by Power, TSO, ARMv8, but allowed by ARMv7. Maybe it isn't that important to support it anymore.

There is also a cycle in pub-rel-rlx-coe. Anton says: I checked Power/ARMv7 models in this regard. They disallow the behavior (as well as ARMv8 and TSO), so we can in principle strengthen imm to forbid it as well. For that, we may add axiom to imm forbidding cycles in $\mathsf{co} \cup ([\mathsf{W}]; \mathsf{rfe}^?; ([\mathsf{R}^{\mathsf{acq}}] \cup \mathsf{po}; [\mathsf{FW}^{\mathsf{rel}}]); \mathsf{ar}^*; [\mathsf{W}])$. This works if we have acquire/release accesses on the path since they are compiled with fences to Power.

6 PTX EXAMPLES

Based on [Lustig et al. 2019; NVIDIA 2020].

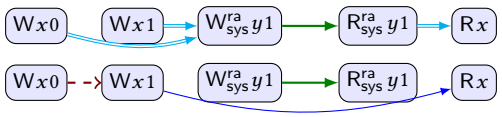
ptx requires weak fulfillment.

Default scope is cta. In examples, all threads in different ctas.

Default mode is wk.

$(\mathsf{R}x0)$ must be forbidden. Before fulfilling the read:

$$x := 0;\ x := 1;\ y^{\mathsf{ra}}_{\mathsf{sys}} := 1 \ \|\ r := y^{\mathsf{ra}}_{\mathsf{sys}};\ s := x \qquad (\mathrm{pub1}_{\mathrm{sys}})$$



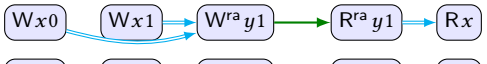
$(\trianglelefteq = \leq)$

$(\sqsubseteq)$

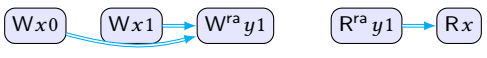
$(\mathsf{W}x1) \sqsubseteq\!\!\!\!\approx (\mathsf{R}x)$ is required by m7, enforcing publication.

$(\mathsf{R}x0)$ must be allowed:

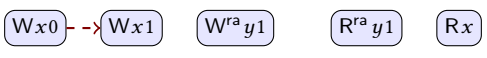
$$x := 0;\ x := 1;\ y^{\mathsf{ra}} := 1 \ \|\ r := y^{\mathsf{ra}};\ s := x \qquad (\mathrm{pub1}_{\mathrm{cta}})$$



$(\trianglelefteq)$



$(\leq)$



$(\sqsubseteq)$

We do not have $(\mathsf{W}^{\mathsf{ra}}y1) \leq (\mathsf{R}^{\mathsf{ra}}y1)$ since f3 only requires order for things that are morally strong.

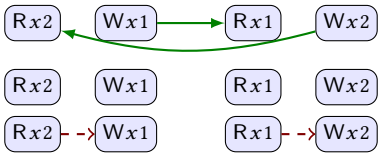
Another example that may be of interest (nothing morally strong). Can this $(\mathsf{R}x0)$?

$$x := 0;\ x := 1 \ \|\ y := x \ \|\ \texttt{if}(y)\{r := x\}$$

ptx allows TC16 for events that are not mutually strong ($\mathrm{tc16}_{\mathrm{wk}}$), but disallows it when events are mutually strong ($\mathrm{tc16}_{\mathrm{sys}}$). Note that $\leq$ imposes no requirements here. Fulfillment

imposes no order. This example shows that f3c cannot be strengthened to require that $d \sqsubseteq e$.

$r := x\,;\ x := 1 \ \parallel\ s := x\,;\ x := 2$ (tc16$_{\text{wk}}$)

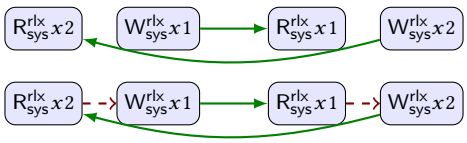


($\trianglelefteq$)

($\leq$)

($\sqsubseteq$)

$r := x^{\text{rlx}}_{\text{sys}}\,;\ x^{\text{rlx}}_{\text{sys}} := 1 \ \parallel\ s := x^{\text{rlx}}_{\text{sys}}\,;\ x^{\text{rlx}}_{\text{sys}} := 2$ (tc16$_{\text{sys}}$)



($\trianglelefteq = \leq$)

($\sqsubseteq$)

About Release-Acquire semantics. Anton confirms that the following example is allowed in C11, but disallowed in the imm. It is apparently allowed in C11 with the intention to allow releasing writes to be downgraded to relaxed in the case that only fulfill relaxed reads.

$r := x^{\text{rlx}}_{\text{sys}}\,;\ y^{\text{ra}}_{\text{sys}} := 1 \ \parallel\ s := y^{\text{rlx}}_{\text{sys}}\,;\ x^{\text{ra}}_{\text{sys}} := 1$ (lb-rel)

($\trianglelefteq = \leq$)

Another example from Anton. This is allowed in PTX because it does not include synchronization in the no-tar axiom, only in coherence and causality.

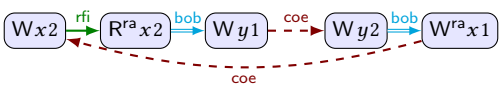
$r := x^{\text{rlx}}_{\text{sys}}\,;\ y^{\text{rlx}}_{\text{sys}} := r \ \parallel\ s := y^{\text{rlx}}_{\text{sys}}\,;\ x^{\text{ra}}_{\text{sys}} := 1$ (lb-data-rel)

($\trianglelefteq = \leq$)

7 RFI EXAMPLES

Anton example 1 (Allowed by ARM) `[rfi-coe-coe]`

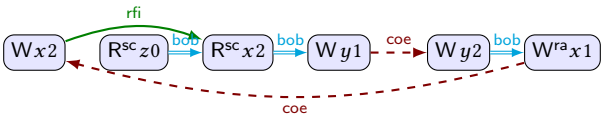
$x := 2\,;\ r := x^{\text{ra}}\,;\ y := 1 \ \parallel\ y := 2\,;\ x^{\text{ra}} := 1$ (rfi-coe-coe)



(✓arm8)

Internal reads survive acquires `[rfi-acq-coe-coe]` (where SC read = `LDAR`)

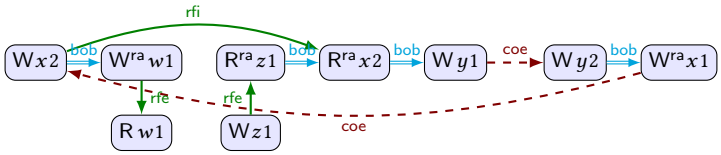
$x := 2\,;\ s := z^{\text{sc}}\,;\ r := x^{\text{sc}}\,;\ y := 1 \ \parallel\ y := 2\,;\ x^{\text{ra}} := 1$ (rfi-acq-coe-coe)



(✓arm8)

And release-acquire pairs `[rfi-ra-coe-coe]` (where acquiring read = `LDAPR`)

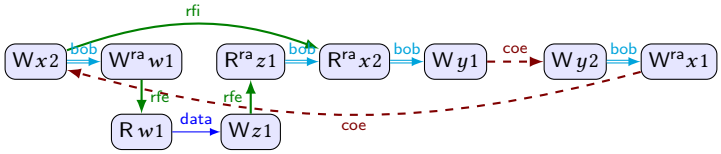
$$x:=2;\ w^{ra}:=1;\ s:=z^{ra};\ r:=x^{ra};\ y:=1 \quad \| \quad y:=2;\ x^{ra}:=1 \quad \| \quad w:=r;\ r:=1; \qquad \text{(rfi-ra-coe-coe2)}$$



(✓arm8)

But not if either acquire is strengthened to SC (where SC read = `LDAR`). The execution is also disallowed if an external thread places order between the `ra` accesses:

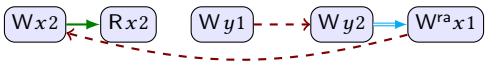
$$x:=2;\ w^{ra}:=1;\ s:=z^{ra};\ r:=x^{ra};\ y:=1 \quad \| \quad y:=2;\ x^{ra}:=1 \quad \| \quad w:=r;\ r:=z; \qquad \text{(rfi-ra-data-coe-coe)}$$



(✗arm8)

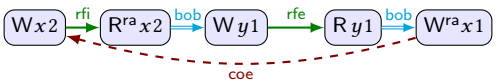
To allow this, weaken `ra` to `rlx` when read fulfilled by relaxed write of same thread (don't need to allow this when the write is part of an rmw).

$$x:=2;\ r:=x^{ra};\ y:=1 \quad \| \quad y:=2;\ x^{ra}:=1$$



RF variant `[rfi-rfe-coe]`:

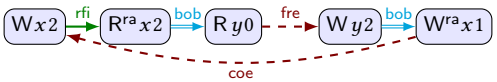
$$x:=2;\ r:=x^{ra};\ y:=1 \quad \| \quad s:=y;\ x^{ra}:=1 \qquad \text{(rfi-rfe-coe)}$$



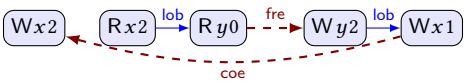
(✓arm8)

tso variant `[rfi-fre-coe]`:

$$x:=2;\ r:=x^{ra};\ s:=y \quad \| \quad y:=2;\ x^{ra}:=1 \qquad \text{(rfi-coe-coe)}$$



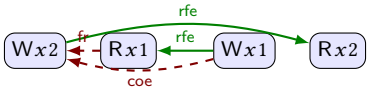
(✓arm8)



(✓tso)

Note that tso does not order W to R in local order, even in poloc. Nonetheless, tso disallows the following because of local visibility in first thread.

$$x:=2;\ r:=x \quad \| \quad x:=1;\ s:=x$$



(✗tso)

[Higham and Kawash 2000] describe tso as a linearization of partial order including:

- poloc
- lws = po; [W]
- $d \overset{\text{po}}{\cdots\blacktriangleright} e$ when $c \xrightarrow{\text{rfe}} d \overset{\text{po}}{\cdots\blacktriangleright} e$

[Alglave et al. 2020] describe tso as linearization of partial order satisfying internal visibility and including

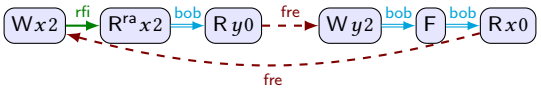
- [W]; po; [W]
- $d \overset{\text{po}}{\cdots\blacktriangleright} e$ when $c \xrightarrow{\text{rfe}} d \overset{\text{po}}{\cdots\blacktriangleright} e$, from `(range(rfe) * _)`
- [R]; po; [W], from `(rfi^-1; lob)`

Ignoring fences and rmws:

```
let rec lob = po \ ([W]; po; [R])
let IM0 = loc & ((IW * (M\IW)) | ((W\FW) * FW))
let gc-req = (W * _) | ((R * _) & ((range(rfe) * _) | (rfi^-1; lob))
let preorder-gcb = IM0 | lob & gc-req
```

Double FRE variant `[rfi-fre-fre]`:

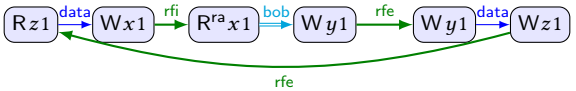
$$x := 2;\ r := x^{\text{ra}};\ s := y \ \|\ y := 2;\ \mathsf{F};\ r := x \qquad \text{(rfi-fre-fre)}$$



(✓arm8)

It does not seem possible to do this only with rfe. ARM disallows this `[data-rfi-rfe-rfe]`:

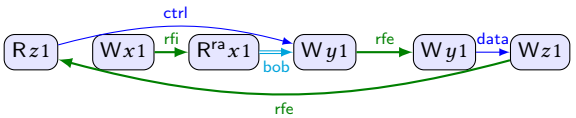
$$x := z;\ r := x^{\text{ra}};\ y := 1 \ \|\ z := y \qquad \text{(data-rfi-rfe-rfe)}$$



(✗arm8)

It also disallows `[ctrl-rfi-rfe-rfe]`:

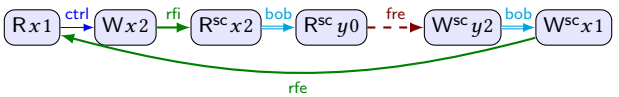
$$\texttt{if}(z)\{\};\ x := 1;\ r := x^{\text{ra}};\ y := 1 \ \|\ z := y \qquad \text{(ctrl-rfi-rfe-rfe)}$$



(✗arm8)

ARM allows some counterintuitive results for SC access `[ctrl-rfi-fre-rfe]`:

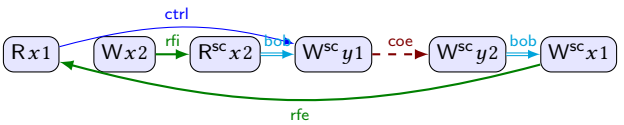
$$\texttt{if}(x)\{\};\ x := 2;\ r := x^{\text{sc}};\ s := y^{\text{sc}} \ \|\ y^{\text{sc}} := 2;\ x^{\text{sc}} := 1 \qquad \text{(ctrl-rfi-fre-rfe)}$$



(✓arm8)

Not possible with coe `[ctrl-rfi-coe-rfe]`:

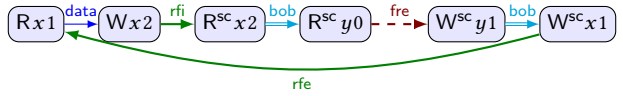
$$\texttt{if}(x)\{\};\ x := 2;\ r := x^{\text{sc}};\ y^{\text{sc}} := 1 \ \|\ y^{\text{sc}} := 2;\ x^{\text{sc}} := 1 \qquad \text{(ctrl-rfi-coe-rfe)}$$



(✗arm8)

This is not allowed with a data dependency instead of a control dependency `[data-rfi-fre-rfe]`:

$$x := x+1;\ r := x^{\mathsf{sc}};\ s := y^{\mathsf{sc}} \ \|\ y^{\mathsf{sc}} := 1;\ x^{\mathsf{sc}} := 1 \qquad \text{(data-rfi-fre-rfe)}$$

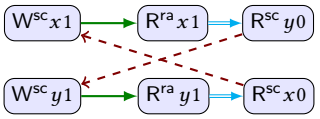


(✗arm8)

8 SC EXAMPLES

iriw-aqc-sc is allowed by trailing-sync compilation to power [Lahav et al. 2017, §1].

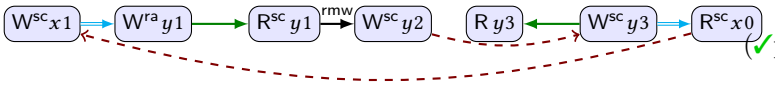
$$x^{\mathsf{sc}} := 1 \ \|\ y^{\mathsf{sc}} := 1 \ \|\ r := x^{\mathsf{ra}};\ s := y^{\mathsf{sc}} \ \|\ r := y^{\mathsf{ra}};\ s := x^{\mathsf{sc}} \qquad \text{(iriw-aqc-sc)}$$



(✓power,rc11)

Leading sync is also unsound in c11 with rmw [Lahav et al. 2017, §2.1].

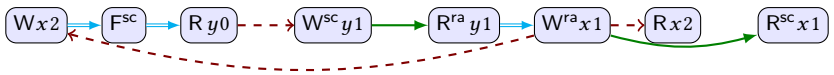
$$x^{\mathsf{sc}} := 1;\ y^{\mathsf{ra}} := 1 \ \|\ \mathsf{FADD}^{\mathsf{sc},\mathsf{sc}}(y, 1);\ s := y \ \|\ y^{\mathsf{sc}} := 3;\ s := x^{\mathsf{sc}} \qquad \text{(z6.u)}$$



(✓power,rc11)

Leading sync is also unsound in c11 with SC fences [Lahav et al. 2017, §A.1].

$$x := 2;\ \mathsf{F}^{\mathsf{sc}};\ r := y \ \|\ y^{\mathsf{sc}} := 1 \ \|\ r := y^{\mathsf{ra}};\ x^{\mathsf{ra}} := 1;\ s := x \ \|\ r := x^{\mathsf{sc}} \qquad \text{(rsync+rsc)}$$

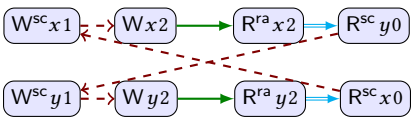


(✓rc11)

Fulfillment of $(\mathsf{R}x2)$ requires that either $(\mathsf{W}^{\mathsf{ra}}x1) \dashrightarrow (\mathsf{W}x2)$ or $(\mathsf{R}x2) \dashrightarrow (\mathsf{W}^{\mathsf{ra}}x1)$. It's interesting that in the pomset, $(\mathsf{R}^{\mathsf{sc}}x1)$ is not needed to get a cycle.

There is a long discussion of this in [Bender and Palsberg 2019, §5.2, Fig. 17], where they also discuss this example:

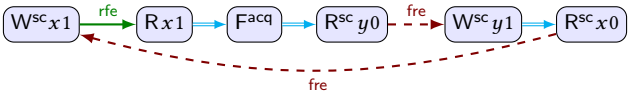
$$x^{\mathsf{sc}} := 1;\ x := 2 \ \|\ y^{\mathsf{sc}} := 1;\ y := 2 \ \|\ r := x^{\mathsf{ra}};\ s := y^{\mathsf{sc}} \ \|\ r := y^{\mathsf{ra}};\ s := x^{\mathsf{sc}} \qquad \text{(iriw-sc-rlx-acq)}$$



(✓rc11)

[Lahav et al. 2017, §A.2] claims that arm8 allows this `[RWC+acq+sc]`, but herd7 rejects it. Reason: they are citing the flowing/pop model [Flur et al. 2016] rather than [Pulte et al. 2018].

$$x^{\mathsf{sc}} := 1 \ \|\ r := x;\ \mathsf{F}^{\mathsf{acq}};\ s := y^{\mathsf{sc}} \ \|\ y^{\mathsf{sc}} := 1;\ r := x^{\mathsf{sc}} \qquad \text{(rwc+acq+sc)}$$



(✗arm8)

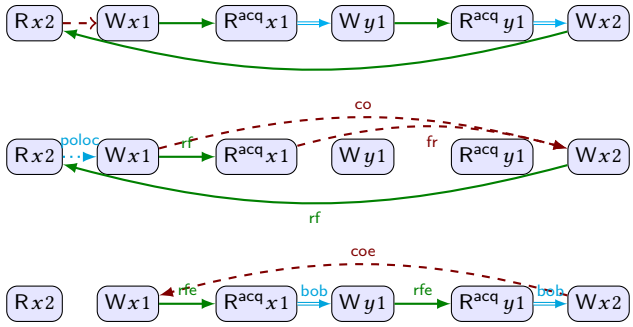
9 RMWS

From [Bender and Palsberg 2019, §3.3]. With partial coherence/weak fulfillment you need to be careful that rmws are totally ordered (if that's a property you want). May not come for free.

10 EXAMPLE FROM JAM PAPER

From [Bender and Palsberg 2019, §B]: "Here we demonstrate that it is possible to construct a program that is only forbidden due to the total coherence order"

$r := x\,;\, x := 1 \;\|\; r := x^{\mathsf{ra}}\,;\, x := 1 \;\|\; r := y^{\mathsf{ra}}\,;\, x := 2$ (total-co)



11 MORE MODEL

These definitions need to be updated to include the additional orders.

Definition 11.1. A pomset is x-closed if

- every $\mathcal{A}(e) = (\mathsf{R}x..)$ is fulfilled
- every $\Phi(e)$ is independent of x: $\big(\forall v.\ \Phi(e) \vDash \Phi(e)[v/x] \vDash \Phi(e)\big)$

Definition 11.2. Let $P \in (\nu x.\mathcal{P})$ when $P \in \mathcal{P}$ and P is x-closed

Definition 11.3. Let $P \in (\phi \rhd \mathcal{P})$ when $P \in \mathcal{P}$ and $(\forall e \in E)\ \Phi(e)$ implies ϕ

Definition 11.4. Let $P' \in (\mathcal{P}[M/x])$ when $(\exists P \in \mathcal{P})$
$E' = E$, $\leq' = \leq$, $\mathcal{A}' = \mathcal{A}$, and $(\forall e \in E')\ \Phi'(e) = \Phi(e)[M/x]$

Definition 11.5. Let $P' \in (\mathcal{P}^1 \parallel \mathcal{P}^2)$ when $(\exists P^1 \in \mathcal{P}^1)\ (\exists P^2 \in \mathcal{P}^2)$
$E' = E^1 \cup E^2$, $\leq' \supseteq \leq^1 \cup \leq^2$, and $(\forall e \in E')$ either

$$e \notin E^2,\ \mathcal{A}'(e) = \mathcal{A}^1(e) \text{ and } \Phi'(e) \text{ implies } \Phi^1(e),$$
$$e \notin E^1,\ \mathcal{A}'(e) = \mathcal{A}^2(e) \text{ and } \Phi'(e) \text{ implies } \Phi^2(e), \text{ or}$$
$$\mathcal{A}'(e) = \mathcal{A}^1(e) = \mathcal{A}^2(e) \text{ and } \Phi'(e) \text{ implies } \Phi^1(e) \vee \Phi^2(e)$$

Language

$$\begin{aligned}
[\![\mathtt{skip}]\!] &\triangleq \{\checkmark\} \\
[\![r := M\,;\, C]\!] &\triangleq [\![C]\!][M/r] \\
[\![r := x^{\mu}\,;\, C]\!] &\triangleq \textstyle\bigcup_v (\mathsf{R}^{\mu} x v) \Rightarrow [\![C]\!][x/r] \\
[\![x^{\mu} := M\,;\, C]\!] &\triangleq \textstyle\bigcup_v (M = v \mid \mathsf{W}^{\mu} x v) \Rightarrow [\![C]\!][M/x] \\
[\![\mathsf{F}^{\nu}\,;\, C]\!] &\triangleq (\mathsf{F}^{\nu}) \Rightarrow [\![C]\!] \\
[\![\mathtt{if}(M)\{C\}\mathtt{else}\{D\}]\!] &\triangleq \big(M \rhd [\![C]\!]\big) \parallel \big(\neg M \rhd [\![D]\!]\big) \\
[\![C \;\|\; D]\!] &\triangleq [\![C]\!] \parallel [\![D]\!] \\
[\![\mathtt{var}\, x\,;\, C]\!] &\triangleq \nu x\,.\,[\![C]\!]
\end{aligned}$$

$$
\begin{array}{rll}
\nu & ::= \mathsf{rel} & \text{(Release)} \\
& \mid \mathsf{acq} & \text{(Acquire)} \\
& \mid \mathsf{sc} & \text{(SC)}
\end{array}
$$

A citation: [Jagadeesan et al. 2020]

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