

# A Unified Memory Model for Heterogenous Systems

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## ACM Reference Format:

Anonymous Author(s). 2022. A Unified Memory Model for Heterogenous Systems. *Proc. ACM Program. Lang.* 0, POPL, Article 0 (January 2022), 9 pages.

## 1 MODEL

### 1.1 Preliminaries

The syntax is built from

- a set of *values*  $\mathcal{V}$ , ranged over by  $v, w, \ell, k$ ,
- a set of *registers*  $\mathcal{R}$ , ranged over by  $r, s$ ,
- a set of *expressions*  $\mathcal{M}$ , ranged over by  $M, N, L$ ,
- a set of *thread ids*  $\mathcal{T}$ , ranged over by  $\alpha, \gamma$ .

*Memory references* are tagged values, written  $[\ell]$ . Let  $\mathcal{X}$  be the set of memory references, ranged over by  $x, y, z$ . We require that:

- values and registers are disjoint,
- values include at least the constants 0 and 1,
- expressions include at least registers and values,
- references do not appear in expressions:  $M[N/x] = M$ ,
- thread ids include the *top-level* id 0.

We model the following language.

$$\begin{aligned} \mu, \nu &::= \text{wk} \mid \text{rlx} \mid \text{rel} \mid \text{acq} \mid \text{ra} \mid \text{sc} & \sigma, \rho &::= \text{cta} \mid \text{gpu} \mid \text{sys} \\ S &::= \text{skip} \mid r := M \mid r := [L]_{\sigma}^{\mu} \mid [L]_{\sigma}^{\mu} := M \mid F_{\sigma}^{\mu} \mid \text{if}(M)\{S_1\} \text{ else } \{S_2\} \mid S_1; S_2 \\ &\mid S_1 \parallel_{\gamma} S_2 \mid r := \text{CAS}_{\sigma}^{\mu, \nu}([L], M, N) \mid r := \text{FADD}_{\sigma}^{\mu, \nu}([L], M) \mid r := \text{EXCHG}_{\sigma}^{\mu, \nu}([L], M) \end{aligned}$$

*Access modes*,  $\mu$ , are weak (wk), relaxed (rlx), release (rel), acquire (acq), release-acquire (ra), and sequentially consistent (sc). Let expressions ( $r := M$ ) only affect thread-local state and thus do not have a mode. Reads ( $r := [L]_{\sigma}^{\mu}$ ) support wk, rlx, acq, sc. Writes ( $[L]_{\sigma}^{\mu} := r$ ) support wk, rlx, rel, sc. Fences ( $F_{\sigma}^{\mu}$ ) support rel, acq, ra, sc. In the atomic update operations,  $\mu$  is a read and  $\nu$  is a write; we require that  $r$  does not occur in  $L$ .

*Scopes*,  $\sigma$ , are thread group (cta), processor (gpu) and system (sys).

*Commands*, aka *statements*,  $S$ , include memory accesses at a given mode, as well as the usual structural constructs. Following [Ferreira et al. 1996],  $\parallel$  denotes parallel composition. If  $(S_1 \parallel_{\gamma} S_2)$  is executed with thread id  $\alpha$ , then  $S_2$  runs with id  $\gamma$  and  $S_1$  continues under id  $\alpha$ . Top level programs run with thread id 0. In examples, we usually drop thread ids. We use the symmetric  $\parallel$  operator when there is no continuation after the parallel composition.

We use common syntax sugar, such as *extended expressions*,  $\mathbb{M}$ , which include memory locations. For example, if  $\mathbb{M}$  includes a single occurrence of  $x$ , then  $y := \mathbb{M}$ ;  $S$  is shorthand for  $r := x$ ;

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2475-1421/2022/1-ART0

<https://doi.org/>

$y := \mathbb{M}[r/x]; S$ . Each occurrence of  $x$  in an extended expression corresponds to an separate read. We also write  $\text{if}(M)\{S\}$  as shorthand for  $\text{if}(M)\{S\} \text{ else } \{\text{skip}\}$ .

The semantics is built from the following.

- a set of *events*  $\mathcal{E}$ , ranged over by  $e, d, c, b$ ,
- a set of *actions*  $\mathcal{A}$ , ranged over by  $a$ ,
- a set of *logical formulae*  $\Phi$ , ranged over by  $\phi, \psi, \theta$ .

Subsets of  $\mathcal{E}$  are ranged over by  $E, D, C, B$ .

- registers include  $\mathcal{S}_{\mathcal{E}} = \{s_e \mid e \in \mathcal{E}\}$  which do not appear in commands:  $S[N/s_e] = S$ ,
- formulae include equalities  $(M=N)$  and  $(x=M)$ ,
- formulae are closed under negation, conjunction, disjunction, and substitutions  $[M/r]$ ,  $[M/x]$ ,
- there is a relation  $\models$  between formulae, capturing entailment,
- $\models$  has the expected semantics for  $=, \neg, \wedge, \vee, \Rightarrow$  and substitution.

We relax the first assumption in examples, assuming that each register is assigned at most once.

Logical formulae include equations over registers, such as  $(r=s+1)$ . For LIR, we also include equations over memory references, such as  $(x=1)$ . Formulae are subject to substitutions; actions are not. We use expressions as formulae, coercing  $M$  to  $M \neq 0$ . Equations have precedence over logical operators; thus  $r=v \Rightarrow s>w$  is read  $(r=v) \Rightarrow (s>w)$ . As usual, implication associates to the right; thus  $\phi \Rightarrow \psi \Rightarrow \theta$  is read  $\phi \Rightarrow (\psi \Rightarrow \theta)$ .

We say  $\phi$  is a *tautology* if  $\text{tt} \models \phi$ . We say  $\phi$  is *unsatisfiable* if  $\phi \models \text{ff}$ .

We also require that there are subsets of actions, distinguishing *read* and *release* actions. We require several binary relations between actions, detailed in the next subsection: *overlaps*, *strongly-overlaps*, *matches*, *strongly-matches*, *strongly-fences*, *blocks*, *sync-delays* and *co-delays*. We require that *strongly-overlaps* implies *overlaps* and that *strongly-matches* implies *matches* implies *blocks* implies *overlaps*.

## 1.2 Actions

We combine access and fence modes into a single order:  $\text{wk} \rightarrow \text{rlx} \xrightarrow{\text{rel}} \text{ra} \rightarrow \text{sc}$ . We write  $\mu \sqsubseteq \nu$  for this order. Let  $\mu \sqcup \nu$  denote the least upper bound of  $\mu$  and  $\nu$ .

Let actions be reads, writes and fences:

$$a, b ::= \alpha W_{\sigma}^{\mu} x v \mid \alpha R_{\sigma}^{\mu} x v \mid \alpha F_{\sigma}^{\mu}$$

In examples, we systematically drop the default mode  $\text{rlx}$  and the default scope  $\text{sys}$ . In definitions, we drop elements of actions that are existentially quantified. We write  $(\alpha A_{\sigma}^{\mu} x)$  to stand for an *access*: either  $(\alpha W_{\sigma}^{\mu} x)$  or  $(\alpha R_{\sigma}^{\mu} x)$ . We write  $(W^{\exists \text{rel}})$  to stand for either  $(W^{\text{rel}})$  or  $(W^{\text{sc}})$ , and similarly for other actions and modes.

We say  $a$  *matches*  $b$  if  $a = (Wxv)$  and  $b = (Rxv)$ .

We say  $a$  *blocks*  $b$  if  $a = (Wx)$  and  $b = (Rx)$ , regardless of value.

We say  $a$  *overlaps*  $b$  if  $a = (Ax)$  and  $b = (Ax)$ , regardless of access type or value.

We say  $a$  *co-delays*  $b$  if  $(a, b) \in \{(Wx, Wx), (Rx, Wx), (Wx, Rx)\} \cup \{(A^{\text{sc}}, A^{\text{sc}})\}$ .

We say  $a$  *sync-delays*  $b$  if  $(a, b) \in \{(a, W^{\exists \text{rel}}), (a, F^{\exists \text{rel}}), (R, F^{\exists \text{acq}}), (R^{\exists \text{acq}}, b), (F^{\exists \text{acq}}, b), (F^{\exists \text{rel}}, W), (W^{\exists \text{rel}}, Wx)\}$ .<sup>1</sup>

Let  $(W^{\exists \text{rel}})$  and  $(F^{\exists \text{rel}})$  be *release* actions. Actions  $(R)$  are *read* actions.

**Definition 1.1.** We assume two equivalences:  $(=_{\text{gpu}}) \subseteq (\mathcal{T} \times \mathcal{T})$  partitions threads by *processor*, and  $(=_{\text{cta}}) \subseteq (=_{\text{gpu}})$  refines the processor partitioning into *thread groups*.

<sup>1</sup>For PTX, one could additionally include  $(Rx, R^{\exists \text{acq}}x)$ , but this is not sound for Arm or IMM.

We say  $(\alpha_1 A_{\sigma_1}^{\mu_1} x)$  *strongly-overlaps*  $(\alpha_2 A_{\sigma_2}^{\mu_2} x)$  when either

- (1)  $\alpha_1 = \alpha_2$ , or (2b) if  $\sigma_1 = \text{cta}$  or  $\sigma_2 = \text{cta}$  then  $\alpha_1 =_{\text{cta}} \alpha_2$ ,
- (2a)  $\mu_1, \mu_2 \neq \text{wk}$ , (2c) if  $\sigma_1 = \text{gpu}$  or  $\sigma_2 = \text{gpu}$  then  $\alpha_1 =_{\text{gpu}} \alpha_2$ .

We say  $(\alpha_1 F_{\sigma_1}^{\mu_1})$  *strongly-fences*  $(\alpha_2 F_{\sigma_2}^{\mu_2})$  when  $\mu_1 = \mu_2 = \text{sc}$  and either (1) or (2) apply, from the definition of *strongly-overlaps*.

We say  $a$  *strongly-matches*  $b$  when  $a$  is a release,  $b$  is an acquire, and either  $a$  *strongly-overlaps*  $b$  or  $a$  *strongly-fences*  $b$ .

Note that for a CPUS, all action have scope sys and mode rx or greater. For this subset of actions, *strongly-overlaps* is the same as *overlaps* and *strongly-fences* applies to any pair of sc fences.

### 1.3 Pomsets with Predicate Transformers

*Definition 1.2.* A predicate transformer is a function  $\tau : \Phi \rightarrow \Phi$  such that

- (x1)  $\tau(\text{ff})$  is ff, (x3)  $\tau(\psi_1 \vee \psi_2)$  is  $\tau(\psi_1) \vee \tau(\psi_2)$ ,
- (x2)  $\tau(\psi_1 \wedge \psi_2)$  is  $\tau(\psi_1) \wedge \tau(\psi_2)$ , (x4) if  $\phi \models \psi$ , then  $\tau(\phi) \models \tau(\psi)$ .

*Definition 1.3.* A family of predicate transformers for  $E$  consists of a predicate transformer  $\tau^D$  for each  $D \subseteq \mathcal{E}$ , such that if  $C \cap E \subseteq D$  then  $\tau^C(\psi) \models \tau^D(\psi)$ .

We write  $\tau$  as an abbreviation of  $\tau^E$ .

*Definition 1.4.* A pomset with predicate transformers is a tuple  $(E, \lambda, \kappa, \tau, \checkmark, \preceq, \sqsubseteq, \text{rmw})$  where

- (M1)  $E \subseteq \mathcal{E}$  is a set of events,
- (M2)  $\lambda : E \rightarrow \mathcal{A}$  defines a label for each event,
- (M3)  $\kappa : E \rightarrow \Phi$  defines a precondition for each event, such that
  - (M3a)  $\kappa(e)$  is satisfiable,
- (M4)  $\tau : 2^E \rightarrow \Phi \rightarrow \Phi$  is a family of predicate transformers over  $E$ ,
- (M5)  $\checkmark : \Phi$  is a termination condition, such that
  - (M5a)  $\checkmark \models \tau(\text{tt})$ ,
- (M6)  $\preceq : (E \times E)$  is a partial order capturing dependency,
- (M7)  $\leq : (E \times E)$  is a partial order capturing synchronization,
- (M8)  $\sqsubseteq : (E \times E)$  is a partial order capturing per-location order, such that
  - (M8a) if  $\lambda(d)$  *overlaps*  $\lambda(e)$  then  $d \leq e$  implies  $d \sqsubseteq e$ ,
- (M9)  $\text{rmw} : E \rightarrow E$  is a partial function capturing read-modify-write atomicity, such that
  - (M9a) if  $d \xrightarrow{\text{rmw}} e$  then  $\lambda(e)$  *blocks*  $\lambda(d)$ ,
  - (M9b) if  $d \xrightarrow{\text{rmw}} e$  then  $d \leq e$  and  $d \sqsubseteq e$ ,
  - (M9c) if  $\lambda(c)$  *overlaps*  $\lambda(d)$  then
    - (i) if  $d \xrightarrow{\text{rmw}} e$  then  $c \preceq d$ ,  $c \leq e$  implies  $c \leq d$ ,  $c \sqsubseteq e$  implies  $c \sqsubseteq d$ ,
    - (ii) if  $d \xrightarrow{\text{rmw}} e$  then  $d \preceq c$  implies  $e \preceq c$ ,  $d \leq c$  implies  $e \leq c$ ,  $d \sqsubseteq c$  implies  $e \sqsubseteq c$ .

A pomset is a candidate if there is an injective relation  $\text{rf} : E \times E$ , capturing reads-from, such that

- (c2a) if  $d \xrightarrow{\text{rf}} e$  then  $\lambda(d)$  *matches*  $\lambda(e)$ ,
- (c6) if  $d \xrightarrow{\text{rf}} e$  then  $d \preceq e$ ,
- (c7a) if  $d' \leq d \xrightarrow{\text{rf}} e \leq e'$  and  $\lambda(d')$  *strongly-matches*  $\lambda(e')$  then  $d' \leq e'$ ,
- (c7b) if  $\lambda(d)$  *strongly-fences*  $\lambda(e)$  then either  $d \leq e$  or  $e \leq d$ ,
- (c8a) if  $d \xrightarrow{\text{rf}} e$  then  $d \sqsubseteq e$ ,
- (c8b) if  $d \xrightarrow{\text{rf}} e$  and  $\lambda(c)$  *blocks*  $\lambda(e)$  then either  $c \sqsubseteq d$  or  $e \sqsubseteq c$ ,  
 where  $d' \sqsubseteq e'$  when  $e' \sqsubseteq d'$  implies  $d' = e'$  and  $\lambda(d')$  *strongly-overlaps*  $\lambda(e')$  implies  $d' \sqsubseteq e'$ .

A candidate pomset with  $\text{rf}$  is complete if

- (c2b) if  $\lambda(e)$  is a **read** then there is some  $d \xrightarrow{\text{rf}} e$ ,  
 (c3)  $\kappa(e)$  is a tautology (for every  $e \in E$ ),  
 (c5)  $\checkmark$  is a tautology.

Note that for the IMM model, c8b is equivalent to:<sup>2</sup>

$$\text{if } d \xrightarrow{\text{rf}} e \text{ and } \lambda(c) \text{ blocks } \lambda(e) \text{ then either } c \sqsubseteq d \text{ or } e \sqsubseteq c.$$

Let  $P$  range over pomsets, and  $\mathcal{P}$  over sets of pomsets.

We drop quantifiers when clear from context, such as  $(\forall e \in E)(\forall x \in \mathcal{X})$ . We write  $d < e$  when  $d \leq e$  and  $d \neq e$ , and similarly for  $\triangleleft$  and  $\sqsubset$ . We sometimes use projection functions—for example, if  $\lambda(e) = \alpha W_{\sigma}^{\mu} x v$  then  $\lambda_{\text{thrd}}(e) = \alpha$ ,  $\lambda_{\text{mode}}(e) = \mu$ ,  $\lambda_{\text{scope}}(e) = \sigma$ ,  $\lambda_{\text{loc}}(e) = x$ ,  $\lambda_{\text{val}}(e) = v$ .

## 1.4 Semantics

See Figure 2.

In diagrams, we use different shapes and colors for arrows and events. These are included only to help the reader understand why order is included. We adopt the following conventions:

- $d \xrightarrow{\text{pink}} e$  arises from control/data/address *dependency* (s3, definition of  $\kappa'_2(d)$ ),
- $d \xrightarrow{\text{green}} e$  arises from *sync-delays* (s7a),
- $d \xrightarrow{\text{orange}} e$  arises from *co-delays* (s8a),
- $d \xrightarrow{\text{blue}} e$  arises from *matching* (c6), (c7a) and (c8a),
- $d \xrightarrow{\text{red}} e$  arises from *strong fencing* (c7b),
- $d \xrightarrow{\text{purple}} e$  arises from *blocking* (c8b).

## 1.5 Address Calculation

*Definition 1.5.* If  $P \in \text{WRITE}(L, M, \mu, \sigma)_{\alpha}$  then  $(\exists \ell \in \mathcal{V}) (\exists v \in \mathcal{V})$

- (w1) if  $d, e \in E$  then  $d = e$ , (w4b) if  $E = \emptyset$  then  
 (w2)  $\lambda(e) = \alpha W_{\sigma}^{\mu}[\ell]v$ ,  $(\forall k) \tau^D(\psi) \models (L=k) \Rightarrow \psi[M/[k]]$   
 (w3)  $\kappa(e) \models L=\ell \wedge M=v$ , (w5a) if  $E \neq \emptyset$  then  $\checkmark \models L=\ell \wedge M=v$ ,  
 (w4a) if  $E \neq \emptyset$  then  $\tau^D(\psi) \models (L=\ell) \Rightarrow \psi[M/[\ell]]$ , (w5b) if  $E = \emptyset$  then  $\checkmark \models \text{ff}$ .

If  $P \in \text{READ}(r, L, \mu, \sigma)_{\alpha}$  then  $(\exists \ell \in \mathcal{V}) (\exists v \in \mathcal{V})$

- (r1) if  $d, e \in E$  then  $d = e$ ,  
 (r2)  $\lambda(e) = \alpha R_{\sigma}^{\mu}[\ell]v$   
 (r3)  $\kappa(e) \wedge L=\ell$ ,  
 (r4a)  $(\forall e \in E \cap D) \tau^D(\psi) \models (L=\ell \Rightarrow v=s_e) \Rightarrow \psi[s_e/r]$ ,  
 (r4b)  $(\forall e \in E \setminus D) \tau^D(\psi) \models ((L=\ell \Rightarrow v=s_e) \vee (L=\ell \Rightarrow [\ell]=s_e)) \Rightarrow \psi[s_e/r]$ ,  
 (r4c)  $(\forall s) \text{ if } E = \emptyset \text{ then } \tau^D(\psi) \models \psi[s/r]$ ,  
 (r5) if  $E = \emptyset$  and  $\mu \neq \text{rlx}$  then  $\checkmark \models \text{ff}$ .

## 1.6 If-closure

*Definition 1.6.* If  $P \in \text{WRITE}(x, M, \mu, \sigma)_{\alpha}$  then  $(\exists v : E \rightarrow \mathcal{V}) (\exists \theta : E \rightarrow \Phi)$

<sup>2</sup>If all accesses are morally strong with each other, weak fulfillment degenerates to

$$\forall \lambda(c) = (Wx) \text{ either } c \sqsubseteq d \text{ or } e \sqsubseteq c$$

If no accesses are morally strong with each other, weak fulfillment degenerates to

$$\exists \lambda(c) = (Wx) \text{ both } d \sqsubseteq c \text{ and } c \sqsubseteq e$$

Note that the difference between strong and weak fulfillment is limited to  $\sqsubseteq$ . We sometimes write  $\boxsubseteq$  for strong fulfillment and  $\boxsubset$  for weak fulfillment.

If  $P \in \text{SKIP}$  then  $E = \emptyset$  and  $\tau^D(\psi) \models \psi$ .

If  $P \in \text{PAR}(\mathcal{P}_1, \mathcal{P}_2)$  then  $(\exists P_1 \in \mathcal{P}_1) (\exists P_2 \in \mathcal{P}_2)$

(p1)  $E = (E_1 \uplus E_2)$ ,

(p2)  $\lambda = (\lambda_1 \cup \lambda_2)$ ,

(p3a) if  $e \in E_1$  then  $\kappa(e) \models \kappa_1(e)$ ,

(p3b) if  $e \in E_2$  then  $\kappa(e) \models \kappa_2(e)$ ,

(p4)  $\tau^D(\psi) \models \tau_1^D(\psi)$ ,

(p5)  $\checkmark \models \checkmark_1 \wedge \checkmark_2$ ,

(p6)  $\trianglelefteq \supseteq (\trianglelefteq_1 \cup \trianglelefteq_2)$ ,

(p7)  $\leq \supseteq (\leq_1 \cup \leq_2)$ ,

(p8)  $\sqsubseteq \supseteq (\sqsubseteq_1 \cup \sqsubseteq_2)$ ,

(p9)  $\text{rmw} = (\text{rmw}_1 \cup \text{rmw}_2)$ .

If  $P \in \text{SEQ}(\mathcal{P}_1, \mathcal{P}_2)$  then  $(\exists P_1 \in \mathcal{P}_1) (\exists P_2 \in \mathcal{P}_2)$

(s1)  $E = (E_1 \cup E_2)$ ,

(s2) (s6) (s7) (s8) (s9) as for  $\text{PAR}$ ,

(s3a) if  $e \in E_1 \setminus E_2$  then  $\kappa(e) \models \kappa_1(e)$ ,

(s3b) if  $e \in E_2 \setminus E_1$  then  $\kappa(e) \models \kappa'_2(e)$ ,

(s3c) if  $e \in E_1 \cap E_2$  then  $\kappa(e) \models \kappa_1(e) \vee \kappa'_2(e)$ ,

(s3d) if  $\lambda_2(e)$  is a **release** then  $\kappa(e) \models \checkmark_1$ ,

(s4)  $\tau^D(\psi) \models \tau_1^D(\tau_2^D(\psi))$ ,

(s5)  $\checkmark \models \checkmark_1 \wedge \tau_1(\checkmark_2)$ ,

(s7a) if  $\lambda_1(d)$  **sync-delays**  $\lambda_2(e)$  then  $d \leq e$ ,

(s8a) if  $\lambda_1(d)$  **co-delays**  $\lambda_2(e)$  then  $d \sqsubseteq e$ ,

where  $\kappa'_2(e) = \tau_1(\kappa_2(e))$  if  $\lambda(e)$  is a read; otherwise  $\kappa'_2(e) = \tau_1^{\downarrow e}(\kappa_2(e))$ , where  $\downarrow e = \{c \mid c \triangleleft e\}$ .

If  $P \in \text{IF}(\phi, \mathcal{P}_1, \mathcal{P}_2)$  then  $(\exists P_1 \in \mathcal{P}_1) (\exists P_2 \in \mathcal{P}_2)$

(i1)  $E = (E_1 \cup E_2)$ ,

(i2) (i6) (i7) (i8) (i9) as for  $\text{PAR}$ ,

(i3a) if  $e \in E_1 \setminus E_2$  then  $\kappa(e) \models \phi \wedge \kappa_1(e)$ ,

(i3b) if  $e \in E_2 \setminus E_1$  then  $\kappa(e) \models \neg\phi \wedge \kappa_2(e)$ ,

(i3c) if  $e \in E_1 \cap E_2$

then  $\kappa(e) \models (\phi \wedge \kappa_1(e)) \vee (\neg\phi \wedge \kappa_2(e))$ ,

(i4)  $\tau^D(\psi) \models (\phi \wedge \tau_1^D(\psi)) \vee (\neg\phi \wedge \tau_2^D(\psi))$ ,

(i5)  $\checkmark \models (\phi \wedge \checkmark_1) \vee (\neg\phi \wedge \checkmark_2)$ .

If  $P \in \text{LET}(r, M)$  then  $E = \emptyset$  and  $\tau^D(\psi) \models \psi[M/r]$ .

If  $P \in \text{READ}(r, x, \mu, \sigma)_\alpha$  then  $(\exists v \in \mathcal{V})$

(r1) if  $d, e \in E$  then  $d = e$ ,

(r2)  $\lambda(e) = \alpha R_\sigma^\mu x v$ ,

(r4a) if  $E \neq \emptyset$  and  $(E \cap D) \neq \emptyset$  then

$\tau^D(\psi) \models v = s_e \Rightarrow \psi[s_e/r]$ ,

(r4b) if  $E \neq \emptyset$  and  $(E \cap D) = \emptyset$  then

$\tau^D(\psi) \models (v = s_e \vee x = s_e) \Rightarrow \psi[s_e/r]$ ,

(r4c) if  $E = \emptyset$  then  $(\forall s) \tau^D(\psi) \models \psi[s/r]$ ,

(r5) if  $E = \emptyset$  and  $\mu \supseteq \text{acq}$  then  $\checkmark \models \text{ff}$ .

If  $P \in \text{WRITE}(x, M, \mu, \sigma)_\alpha$  then  $(\exists v \in \mathcal{V})$

(w1) if  $d, e \in E$  then  $d = e$ ,

(w2)  $\lambda(e) = \alpha W_\sigma^\mu x v$ ,

(w3)  $\kappa(e) \models M = v$ ,

(w4)  $\tau^D(\psi) \models \psi[M/x]$ ,

(w5a) if  $E = \emptyset$  then  $\checkmark \models \text{ff}$ ,

(w5b) if  $E \neq \emptyset$  then  $\checkmark \models M = v$ .

If  $P \in \text{FENCE}(\mu, \sigma)_\alpha$  then

(f1) if  $d, e \in E$  then  $d = e$ ,

(f2)  $\lambda(e) = \alpha F_\sigma^\mu$ ,

(f4)  $\tau^D(\psi) \models \psi$ ,

(f5) if  $E = \emptyset$  then  $\checkmark \models \text{ff}$ .

$\llbracket r := M \rrbracket_\alpha = \text{LET}(r, M)$

$\llbracket \text{skip} \rrbracket_\alpha = \text{SKIP}$

$\llbracket r := x^\mu \rrbracket_\alpha = \text{READ}(r, x, \mu, \sigma)_\alpha$

$\llbracket S_1 \parallel_\gamma S_2 \rrbracket_\alpha = \text{PAR}(\llbracket S_1 \rrbracket_\alpha, \llbracket S_2 \rrbracket_\gamma)$

$\llbracket x^\mu := M \rrbracket_\alpha = \text{WRITE}(x, M, \mu, \sigma)_\alpha$

$\llbracket S_1 ; S_2 \rrbracket_\alpha = \text{SEQ}(\llbracket S_1 \rrbracket_\alpha, \llbracket S_2 \rrbracket_\alpha)$

$\llbracket F_\sigma^\mu \rrbracket_\alpha = \text{FENCE}(\mu, \sigma)_\alpha$

$\llbracket \text{if}(M) \{S_1\} \text{else} \{S_2\} \rrbracket_\alpha = \text{IF}(M \neq 0, \llbracket S_1 \rrbracket_\alpha, \llbracket S_2 \rrbracket_\alpha)$

Fig. 1. Semantics of programs

(w1) if  $\theta_d \wedge \theta_e$  is satisfiable then  $d = e$ ,

(w2)  $\lambda(e) = \alpha W_\sigma^\mu x v_e$ ,

(w3)  $\kappa(e) \models \theta_e \wedge M = v_e$ ,

(w4)  $\tau^D(\psi) \models \theta_e \Rightarrow \psi[M/x]$ ,

(w5)  $\checkmark \models \theta_e \Rightarrow M = v_e$ ,

If  $P \in \text{SKIP}$  then  $E = \emptyset$  and  $\tau^D(\psi) \equiv \psi$ .

If  $P \in \text{PAR}(\mathcal{P}_1, \mathcal{P}_2)$  then  $(\exists P_1 \in \mathcal{P}_1) (\exists P_2 \in \mathcal{P}_2)$

(p1)  $E = (E_1 \uplus E_2)$ ,

(p2)  $\lambda = (\lambda_1 \cup \lambda_2)$ ,

(p3a) if  $e \in E_1$  then  $\kappa(e) \equiv \kappa_1(e)$ ,

(p3b) if  $e \in E_2$  then  $\kappa(e) \equiv \kappa_2(e)$ ,

(p4)  $\tau^D(\psi) \equiv \tau_1^D(\psi)$ ,

(p5)  $\checkmark \equiv \checkmark_1 \wedge \checkmark_2$ ,

(p6)  $\trianglelefteq \supseteq (\trianglelefteq_1 \cup \trianglelefteq_2)$ ,

(p7)  $\preceq \supseteq (\preceq_1 \cup \preceq_2)$ ,

(p8)  $\sqsubseteq \supseteq (\sqsubseteq_1 \cup \sqsubseteq_2)$ ,

(p9)  $\text{rmw} = (\text{rmw}_1 \cup \text{rmw}_2)$ .

If  $P \in \text{SEQ}(\mathcal{P}_1, \mathcal{P}_2)$  then  $(\exists P_1 \in \mathcal{P}_1) (\exists P_2 \in \mathcal{P}_2)$

(s1)  $E = (E_1 \cup E_2)$ ,

(s2) (s6) (s7) (s8) (s9) as for  $\text{PAR}$ ,

(s3a) if  $e \in E_1 \setminus E_2$  then  $\kappa(e) \equiv \kappa_1(e)$ ,

(s3b) if  $e \in E_2 \setminus E_1$  then  $\kappa(e) \equiv \kappa'_2(e) \wedge \checkmark_1(e)$ ,

(s3c) if  $e \in E_1 \cap E_2$  then

$\kappa(e) \equiv (\kappa_1(e) \vee \kappa'_2(e)) \wedge \checkmark_1(e)$ ,

(s4)  $\tau^D(\psi) \equiv \tau_1^D(\tau_2^D(\psi))$ ,

(s5)  $\checkmark \equiv \checkmark_1 \wedge \tau_1(\checkmark_2)$ ,

(s7a) if  $\lambda_1(d)$  **sync-delays**  $\lambda_2(e)$  and  $\kappa_1(d) \wedge \kappa_2(e)$  is satisfiable then  $d \leq e$ ,

(s8a) if  $\lambda_1(d)$  **co-delays**  $\lambda_2(e)$  and  $\kappa_1(d) \wedge \kappa_2(e)$  is satisfiable then  $d \sqsubseteq e$ ,

where  $\kappa'_2(e) = \tau_1(\kappa_2(e))$  if  $\lambda(e)$  is a read—otherwise  $\kappa'_2(e) = \tau_1^{\downarrow e}(\kappa_2(e))$ , where  $\downarrow e = \{c \mid c \triangleleft e\}$ ;

where  $\checkmark_1(e) = \checkmark_1$  if  $\lambda(e)$  is a **release**—otherwise  $\checkmark_1(e) = \text{tt}$ .

If  $P \in \text{IF}(\phi, \mathcal{P}_1, \mathcal{P}_2)$  then  $(\exists P_1 \in \mathcal{P}_1) (\exists P_2 \in \mathcal{P}_2)$

(i1)  $E = (E_1 \cup E_2)$ ,

(i2) (i6) (i7) (i8) (i9) as for  $\text{PAR}$ ,

(i3a) if  $e \in E_1 \setminus E_2$  then  $\kappa(e) \equiv \phi \wedge \kappa_1(e)$ ,

(i3b) if  $e \in E_2 \setminus E_1$  then  $\kappa(e) \equiv \neg\phi \wedge \kappa_2(e)$ ,

(i3c) if  $e \in E_1 \cap E_2$

then  $\kappa(e) \equiv (\phi \wedge \kappa_1(e)) \vee (\neg\phi \wedge \kappa_2(e))$ ,

(i4)  $\tau^D(\psi) \equiv (\phi \wedge \tau_1^D(\psi)) \vee (\neg\phi \wedge \tau_2^D(\psi))$ ,

(i5)  $\checkmark \equiv (\phi \wedge \checkmark_1) \vee (\neg\phi \wedge \checkmark_2)$ .

If  $P \in \text{LET}(r, M)$  then  $E = \emptyset$  and  $\tau^D(\psi) \equiv \psi[M/r]$ .

If  $P \in \text{READ}(r, x, \mu, \sigma)_\alpha$  then  $(\exists v \in \mathcal{V})$

(r1) if  $d, e \in E$  then  $d = e$ ,

(r2)  $\lambda(e) = \alpha R_\sigma^\mu x v$ ,

(r3)  $\kappa(e) \equiv \text{tt}$ ,

(r4a) if  $E \neq \emptyset$  and  $(E \cap D) \neq \emptyset$  then

$\tau^D(\psi) \equiv v = s_e \Rightarrow \psi[s_e/r]$ ,

(r4b) if  $E \neq \emptyset$  and  $(E \cap D) = \emptyset$  then

$\tau^D(\psi) \equiv (v = s_e \vee x = s_e) \Rightarrow \psi[s_e/r]$ ,

(r4c) if  $E = \emptyset$  then  $(\forall s) \tau^D(\psi) \equiv \psi[s/r]$ ,

(r5a) if  $E \neq \emptyset$  or  $\mu \sqsubseteq \text{rlx}$  then  $\checkmark \equiv \text{tt}$ .

(r5b) if  $E = \emptyset$  and  $\mu \supseteq \text{acq}$  then  $\checkmark \equiv \text{ff}$ .

If  $P \in \text{WRITE}(x, M, \mu, \sigma)_\alpha$  then  $(\exists v \in \mathcal{V})$

(w1) if  $d, e \in E$  then  $d = e$ ,

(w2)  $\lambda(e) = \alpha W_\sigma^\mu x v$ ,

(w3)  $\kappa(e) \equiv M = v$ ,

(w4)  $\tau^D(\psi) \equiv \psi[M/x]$ ,

(w5a) if  $E = \emptyset$  then  $\checkmark \equiv \text{ff}$ ,

(w5b) if  $E \neq \emptyset$  then  $\checkmark \equiv M = v$ .

If  $P \in \text{FENCE}(\mu, \sigma)_\alpha$  then

(f1) if  $d, e \in E$  then  $d = e$ ,

(f2)  $\lambda(e) = \alpha F_\sigma^\mu$ ,

(f3)  $\kappa(e) \equiv \text{tt}$ ,

(f4)  $\tau^D(\psi) \equiv \psi$ ,

(f5a) if  $E \neq \emptyset$  then  $\checkmark \equiv \text{tt}$ ,

(f5b) if  $E = \emptyset$  then  $\checkmark \equiv \text{ff}$ .

$\llbracket r := M \rrbracket_\alpha = \text{LET}(r, M)$

$\llbracket r := x^\mu \rrbracket_\alpha = \text{READ}(r, x, \mu, \sigma)_\alpha$

$\llbracket x^\mu := M \rrbracket_\alpha = \text{WRITE}(x, M, \mu, \sigma)_\alpha$

$\llbracket F_\sigma^\mu \rrbracket_\alpha = \text{FENCE}(\mu, \sigma)_\alpha$

$\llbracket \text{skip} \rrbracket_\alpha = \text{SKIP}$

$\llbracket S_1 \parallel_Y S_2 \rrbracket_\alpha = \text{PAR}(\llbracket S_1 \rrbracket_\alpha, \llbracket S_2 \rrbracket_Y)$

$\llbracket S_1 ; S_2 \rrbracket_\alpha = \text{SEQ}(\llbracket S_1 \rrbracket_\alpha, \llbracket S_2 \rrbracket_\alpha)$

$\llbracket \text{if}(M) \{S_1\} \text{else} \{S_2\} \rrbracket_\alpha = \text{IF}(M \neq 0, \llbracket S_1 \rrbracket_\alpha, \llbracket S_2 \rrbracket_\alpha)$

Fig. 2. Semantics of programs

If  $P \in \text{READ}(r, x, \mu, \sigma)_\alpha$  then  $(\exists v : E \rightarrow \mathcal{V}) (\exists \theta : E \rightarrow \Phi)$

(R1) if  $\theta_d \wedge \theta_e$  is satisfiable then  $d = e$ ,

(R2)  $\lambda(e) = \alpha R_\sigma^\mu x v_e$

(R3)  $\kappa(e) \models \theta_e$ ,

(R4a)  $(\forall e \in E \cap D) \tau^D(\psi) \models \theta_e \Rightarrow v_e = s_e \Rightarrow \psi[s_e/r]$ ,

(R4b)  $(\forall e \in E \setminus D) \tau^D(\psi) \models \theta_e \Rightarrow (v_e = s_e \vee x = s_e) \Rightarrow \psi[s_e/r]$ ,

(R4c)  $(\forall s) \tau^D(\psi) \models (\bigwedge_{e \in E} \neg \theta_e) \Rightarrow \psi[s/r]$ ,

(R5) if  $E = \emptyset$  and  $\mu \neq \text{rlx}$  then  $\checkmark \models \text{ff}$ .

## 1.7 Address Calculation and If-closure

*Definition 1.7.* If  $P \in \text{WRITE}(L, M, \mu, \sigma)_\alpha$  then  $(\exists \ell : E \rightarrow \mathcal{V}) (\exists v : E \rightarrow \mathcal{V}) (\exists \theta : E \rightarrow \Phi)$

(w1) if  $\theta_d \wedge \theta_e$  is satisfiable then  $d = e$ ,

(w4b)  $(\forall k)$

(w2)  $\lambda(e) = \alpha W_\sigma^\mu[\ell] v_e$ ,

$\tau^D(\psi) \models (\bigwedge_{e \in E} \neg \theta_e) \Rightarrow (L = k) \Rightarrow \psi[M/[k]]$

(w3)  $\kappa(e) \models \theta_e \wedge L = \ell_e \wedge M = v_e$ ,

(w5a)  $\checkmark \models \theta_e \Rightarrow L = \ell_e \wedge M = v_e$ ,

(w4a)  $\tau^D(\psi) \models \theta_e \Rightarrow (L = \ell) \Rightarrow \psi[M/[\ell]]$ ,

(w5b)  $\checkmark \models \bigvee_{e \in E} \theta_e$ .

If  $P \in \text{READ}(r, L, \mu, \sigma)_\alpha$  then  $(\exists \ell : E \rightarrow \mathcal{V}) (\exists v : E \rightarrow \mathcal{V}) (\exists \theta : E \rightarrow \Phi)$

(R1) if  $\theta_d \wedge \theta_e$  is satisfiable then  $d = e$ ,

(R2)  $\lambda(e) = \alpha R_\sigma^\mu[\ell] v_e$

(R3)  $\kappa(e) \models \theta_e \wedge L = \ell_e$ ,

(R4a)  $(\forall e \in E \cap D) \tau^D(\psi) \models \theta_e \Rightarrow (L = \ell_e \Rightarrow v_e = s_e) \Rightarrow \psi[s_e/r]$ ,

(R4b)  $(\forall e \in E \setminus D) \tau^D(\psi) \models \theta_e \Rightarrow ((L = \ell_e \Rightarrow v_e = s_e) \vee (L = \ell_e \Rightarrow [\ell] = s_e)) \Rightarrow \psi[s_e/r]$ ,

(R4c)  $(\forall s) \tau^D(\psi) \models (\bigwedge_{e \in E} \neg \theta_e) \Rightarrow \psi[s/r]$ ,

(R5) if  $E = \emptyset$  and  $\mu \neq \text{rlx}$  then  $\checkmark \models \text{ff}$ .

*Definition 1.8.* Let  $\text{READ}'$  be defined as for  $\text{READ}$ , adding the constraint:

(R4d) if  $(E \cap D) = \emptyset$  then  $\tau^D(\psi) \models \psi$ .

If  $P \in \text{FADD}(r, L, M, \mu, v)$  then  $(\exists P_1 \in \text{SEQ}(\text{READ}'(r, L, \mu), \text{WRITE}(L, r+M, v)))$

(u1) if  $\lambda_1(e)$  is a write then there is a read  $\lambda_1(d)$  such that  $\kappa(e) \models \kappa(d)$  and  $d \xrightarrow{\text{rmw}} e$ .

If  $P \in \text{EXCHG}(r, L, M, \mu, v)$  then  $(\exists P_1 \in \text{SEQ}(\text{READ}'(r, L, \mu), \text{WRITE}(L, M, v)))$

(u1) if  $\lambda_1(e)$  is a write then there is a read  $\lambda_1(d)$  such that  $\kappa(e) \models \kappa(d)$  and  $d \xrightarrow{\text{rmw}} e$ .

If  $P \in \text{CAS}(r, L, M, N, \mu, v)$  then  $(\exists P_1 \in \text{SEQ}(\text{READ}'(r, L, \mu), \text{IF}(r=M, \text{WRITE}(L, N, v), \text{SKIP})))$

(u1) if  $\lambda_1(e)$  is a write then there is a read  $\lambda_1(d)$  such that  $\kappa(e) \models \kappa(d)$  and  $d \xrightarrow{\text{rmw}} e$ .

## 2 PROPERTIES

LEMMA 2.1. (a)  $\mathcal{P} = (\mathcal{P} \parallel \text{SKIP}) = (\mathcal{P}; \text{SKIP}) = (\text{SKIP}; \mathcal{P})$ .

(b)  $(\mathcal{P}_1 \parallel \mathcal{P}_2) \parallel \mathcal{P}_3 = \mathcal{P}_1 \parallel (\mathcal{P}_2 \parallel \mathcal{P}_3)$ .

(c)  $(\mathcal{P}_1; \mathcal{P}_2); \mathcal{P}_3 = \mathcal{P}_1; (\mathcal{P}_2; \mathcal{P}_3)$ .

(d)  $\text{if}(\phi)\{\mathcal{P}_1\}\text{else}\{\mathcal{P}_2\} = \text{if}(\phi)\{\mathcal{P}_1\}; \text{if}(\neg\phi)\{\mathcal{P}_2\} = \text{if}(\neg\phi)\{\mathcal{P}_2\}; \text{if}(\phi)\{\mathcal{P}_1\}$ .

(e)  $\text{if}(\phi)\{\mathcal{P}_1\}\text{else}\{\mathcal{P}_2\} = \mathcal{P}_1$  if  $\phi$  is a tautology.

(f)  $\text{if}(\phi)\{\text{if}(\psi)\{\mathcal{P}\}\} = \text{if}(\phi \wedge \psi)\{\mathcal{P}\}$ .

(g)  $\text{if}(\phi)\{\mathcal{P}_1; \mathcal{P}_3\}\text{else}\{\mathcal{P}_2; \mathcal{P}_3\} \supseteq \text{if}(\phi)\{\mathcal{P}_1\}\text{else}\{\mathcal{P}_2\}; \mathcal{P}_3$ .

(h)  $\text{if}(\phi)\{\mathcal{P}_1; \mathcal{P}_2\}\text{else}\{\mathcal{P}_1; \mathcal{P}_3\} \supseteq \mathcal{P}_1; \text{if}(\phi)\{\mathcal{P}_2\}\text{else}\{\mathcal{P}_3\}$ .

(i)  $\text{if}(\phi)\{\mathcal{P}\}\text{else}\{\mathcal{P}\} \supseteq \mathcal{P}$ .

PROOF. Straightforward calculation. (a) requires m5a for the termination condition in  $(\mathcal{P}; \text{SKIP})$ .

(c) requires both conjunction closure (x2, for the termination condition) and disjunction closure (x3, for the predicate transformers themselves).



(d) requires s7a and s8a not impose order when  $\kappa_1(d) \wedge \kappa_2(e)$  is unsatisfiable, which in turn requires that  $\kappa$  calculates *weakest* preconditions, rather than simple preconditions (see [Jeffrey and Riely 2021]).

(e) requires m3a.

In §1.6, we refine the semantics to validate the reverse inclusions for (g), (h), and (i).  $\square$

**Definition 2.2.**  $P_2$  is an *augment* of  $P_1$  if

- |                                     |  |   |                                 |
|-------------------------------------|--|---|---------------------------------|
| (1) $E_2 = E_1$ ,                   | (3) $\kappa_2(e) \equiv \kappa_1(e)$ ,       | (5) $\checkmark_2 \equiv \checkmark_1$ ,  | (7) $\leq_2 \supseteq \leq_1$ . |
| (2) $\lambda_2(e) = \lambda_1(e)$ , | (4) $\tau_2^D(\psi) \equiv \tau_1^D(\psi)$ , | (6) $\text{rf}_2 \supseteq \text{rf}_1$ , |                                 |

**LEMMA 2.3.** If  $P_1 \in \llbracket S \rrbracket$  and  $P_2$  augments  $P_1$  then  $P_2 \in \llbracket S \rrbracket$ .

**PROOF.** Induction on the definition of  $\llbracket \cdot \rrbracket$ .  $\square$

### 3 ACCESS ELIMINATION

Inspired by [Chakraborty and Vafeiadis 2019, §6.2]

$\text{merge} : \mathcal{A} \times \mathcal{A} \rightarrow 2^{\mathcal{A}}$  be defined as follows, where  $\nu \sqsubseteq \mu$ , using the order on modes from §1.2.

$$\begin{aligned}
 \text{merge}(\alpha W_{\sigma}^{\nu} x w, \alpha W_{\sigma}^{\mu} x v) &= \{\alpha W_{\sigma}^{\mu} x v\} & \text{merge}(\alpha F_{\sigma}^{\mu}, \alpha F_{\sigma}^{\nu}) &= \{\alpha F_{\sigma}^{\mu}\} \\
 \text{merge}(\alpha W_{\sigma}^{\mu} x v, \alpha R_{\sigma}^{\nu} x v) &= \{\alpha W_{\sigma}^{\mu} x v\} & \text{merge}(\alpha F_{\sigma}^{\nu}, \alpha F_{\sigma}^{\mu}) &= \{\alpha F_{\sigma}^{\mu}\} \\
 \text{merge}(\alpha R_{\sigma}^{\mu} x v, \alpha R_{\sigma}^{\nu} x v) &= \{\alpha R_{\sigma}^{\mu} x v\} & \text{merge}(a, b) &= \emptyset, \text{ otherwise}
 \end{aligned}$$

If  $a_0 \in \text{merge}(a_1, a_2)$ , then  $a_1$  and  $a_2$  can coalesce, resulting in  $a_0$ . This allows optimizations such as  $(x := 1; x := 2)$  to  $(x := 2)$  and  $(x := 1; r := x)$  to  $(x := 1; r := 1)$ . For associativity of sequential composition, it is important that merge always take an upper bound on the modes of the two actions. For example, it would invalidate associativity to allow  $(Wxv) \in \text{merge}(Wxv, R^{\text{acq}}xv)$ , although this is considered safe.<sup>3</sup>

Then we can replace s2-s3 in Figure 2 by:

- (s2a) if  $e \in E_1 \setminus E_2$  then  $\lambda(e) = \lambda_1(e)$ ,
- (s2b) if  $e \in E_2 \setminus E_1$  then  $\lambda(e) = \lambda_2(e)$ ,
- (s2c) if  $e \in E_1 \cap E_2$  then  $\lambda(e) \in \text{merge}(\lambda_1(e), \lambda_2(e))$ ,
- (s3a) if  $e \in E_1 \setminus E_2$  then  $\kappa(e) \models \kappa_1(e)$ ,
- (s3b) if  $e \in E_2 \setminus E_1$  then  $\kappa(e) \models \kappa_2'(e)$ ,
- (s3c) if  $e \in E_1 \cap E_2$  then either

- $\kappa(e) \models \kappa_1(e) \wedge \kappa_2'(e)$ , or
- $\kappa(e) \models \kappa_1(e) \vee \kappa_2'(e)$  and  $\lambda(e) = \lambda_1(e) = \lambda_2(e)$ .

Should be allowed:  $\text{if}(M)\{x := 1\}; x := 2$ . Not allowed:  $x := 1; \text{if}(M)\{x := 2\}$ .

Should be allowed:  $x := 1; \text{if}(M)\{r := x\}$ . Not allowed:  $\text{if}(M)\{x := 1\}; r := x$ .

Associativity is a pain. Consider  $x := 1; \text{if}(M)\{x := 2\}; \text{if}(!M)\{x := 2\}$

<sup>3</sup>A list of safe merge operations can be found in [Chakraborty and Vafeiadis 2017, §E] and [Kang 2019, §7.1]. For examples of unsafe merges and reorderings, see [Chakraborty and Vafeiadis 2017, §D].



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