1 Model

$$
\begin{array}{lllllll}
\mu & ::= & \mathsf{wk} & \text{(Weak)} & \varsigma & ::= \mathsf{cta} & \text{(Thread group)} \\
& | & \mathsf{rlx} & \text{(Relaxed)} & & | \; \mathsf{gpu} & \text{(Processor)} \\
& | & \mathsf{ra} & \text{(Release/Acquire)} & & | \; \mathsf{sys} & \text{(System)} \\
& | & \mathsf{sc} & \text{(Sequentially Consistent)} & & &
\end{array}
$$

Orders/Relations in model

- $\trianglelefteq$ is the old $\leq$ (without coherence stuff from F4 and P5B).
  This provides the NO-TAR axiom.
- $\leq$ is a suborder, which only includes rf when they are morally strong.
  This serves as a cross-location transitive kernel for the per-location order.
- $\sqsubseteq$ is a per-location order that relates morally strong and poloc accesses
  This includes $\leq$ for morally strong accesses.
  This provides the SC-PER-LOC axiom.

▶ **Definition 1.** *A* pomset with preconditions *is a tuple* $(E, \lambda, \leq, \trianglelefteq, \sqsubseteq)$ *where*

(M1) E *is a set of* events

(M2) $\lambda : E \to (\Phi \times \mathcal{A})$ *is a* labeling *from which we derive functions*

- $\Phi : E \to \Phi$ (formulae)
- $\mathcal{A} : E \to \mathcal{A}$ (actions)

(M3) $\leq \subseteq (E \times E)$, $\trianglelefteq \subseteq (E \times E)$, *and* $\sqsubseteq \subseteq (E \times E)$ *are partial orders*

(M4) *if* $d \leq e$ *then* $d \trianglelefteq e$

(M5) *if* $d \leq e$ *and* d *conflicts with* e *then* $d \sqsubseteq e$

(M6) $\bigwedge_e \Phi(e)$ *is satisfiable* (consistency)

(M7) *if* $d \trianglelefteq e$ *then* $\Phi(e)$ *implies* $\Phi(d)$ (causal strengthening)

It is important that M5 covers all conflicting access. See PUB1_{SYS}.

▶ **Definition 2.** *We say* $d < e$ *when* $d \leq e$ *and* $d \neq e$*, and similarly for* $\triangleleft$ *and* $\sqsubset$*.*
Define $\preceq$ *and* $\prec$ *as follows:*

$$d \preceq e \text{ when } \begin{cases} d \sqsubseteq e & \text{if } d \text{ is morally strong with } e \\ e \not\sqsubset d & \text{otherwise} \end{cases}$$

$$d \prec e \text{ when } \begin{cases} d \sqsubset e & \text{if } d \text{ is morally strong with } e \\ e \not\sqsubseteq d & \text{otherwise} \end{cases}$$

▶ **Definition 3.** *We say* $\mathcal{A}(d) = (\mathsf{W}xv)$ fulfills $\mathcal{A}(e) = (\mathsf{R}xv)$ *if*

(F3) $d \prec e$ *and* $d \triangleleft e$

(F4) $\forall \mathcal{A}(c) = (\mathsf{W}x..)$ *either* $c \preceq d$ *or* $e \preceq c$

Note that if all accesses are morally strong with each other, this degenerates to

(F3) $d \sqsubset e$ and $d \triangleleft e$

(F4) $\forall \mathcal{A}(c) = (\mathsf{W}x..)$ either $c \sqsubseteq d$ or $e \sqsubseteq c$

If no accesses are morally strong with each other, this degenerates to

(F3) $e \not\sqsubseteq d$ and $d \triangleleft e$

(F4) $\nexists \mathcal{A}(c) = (\mathsf{W}x..)$ both $d \sqsubset c$ and $c \sqsubset e$

▶ **Definition 4.** *Let* $P' \in (\phi \mid a) \Rightarrow \mathcal{P}$ *when* $(\exists P \in \mathcal{P})$ $(\forall e \in E)$

(P1) $E' = E \cup \{d\}$

(P2) $\leq' \supseteq \leq$, $\trianglelefteq' \supseteq \trianglelefteq$, *and* $\sqsubseteq' \supseteq \sqsubseteq$

(P3A) $\mathcal{A}'(e) = \mathcal{A}(e)$

(P3B) $\mathcal{A}'(d) = a$

(P4A) $\Phi'(d)$ *implies* $\phi \wedge (d \notin E \vee \Phi(d))$

(P4B) *if* $d \neq (\mathsf{R}..)$ *then* $e = d$ *or* $\Phi'(e)$ *implies* $\Phi(e)$

(P4C) *if* $d = (\mathsf{R}vx)$ *then* $e = d$ *or* $\Phi'(e)$ *implies* $\Phi(e)[v/x]$

(P5A) *if* $d = (\mathsf{R}..)$, $e = (\mathsf{W}..)$ *then* $e = d$ *or* $\Phi'(e)$ *implies* $\Phi(e)$ *or* $d \leq' e$

(P5B) *if d conflicts with e then* $d \sqsubseteq' e$

(P5C) *if d is an acquire or e is a release then* $d \leq' e$

(P5D) *if d is an SC write and e is an SC read then* $d \leq' e$

2 Examples

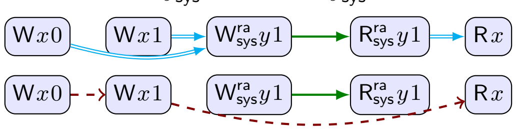
Unless otherwise stated, all threads in different ctas.

Default scope is cta.

Default mode is wk.

$(\mathsf{R}x0)$ must be forbidden. Before fulfilling the read:

$x := 0;\, x := 1;\, y^{\mathsf{ra}}_{\mathsf{sys}} := 1 \parallel r := y^{\mathsf{ra}}_{\mathsf{sys}};\, s := x$ (PUB1$_{\text{SYS}}$)



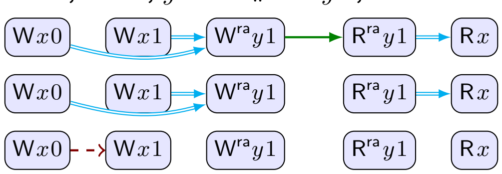
$(\trianglelefteq = \leq)$

$(\sqsubseteq)$

$(\mathsf{W}x1) \preceq (\mathsf{R}x)$ is required by M5, enforcing publication.

$(\mathsf{R}x0)$ must be allowed:

$x := 0;\, x := 1;\, y^{\mathsf{ra}} := 1 \parallel r := y^{\mathsf{ra}};\, s := x$ (PUB1$_{\text{CTA}}$)



$(\trianglelefteq)$

$(\leq)$

$(\sqsubseteq)$

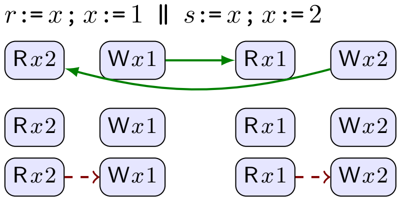
We do not have $(\mathsf{W}^{\mathsf{ra}}y1) \leq (\mathsf{R}^{\mathsf{ra}}y1)$ since F3 only requires order for things that are morally strong.

Another example that may be of interest (nothing morally strong). Can this $(\mathsf{R}x0)$?

$x := 0;\, x := 1 \parallel y := x \parallel \texttt{if}(y)\{r := x\}$

PTX allows TC16 for events that are not mutually strong

$r := x;\, x := 1 \parallel s := x;\, x := 2$ (TC16$_{\text{WK}}$)



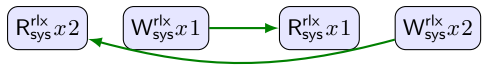
$(\trianglelefteq)$

$(\leq)$

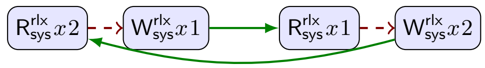
$(\sqsubseteq)$

Note that $\leq$ imposes no requirements here. Fulfillment imposes no order.

$$r := x^{\mathsf{rlx}}_{\mathsf{sys}} \,;\, x^{\mathsf{rlx}}_{\mathsf{sys}} := 1 \;\parallel\; s := x^{\mathsf{rlx}}_{\mathsf{sys}} \,;\, x^{\mathsf{rlx}}_{\mathsf{sys}} := 2 \qquad (\textsc{tc}16_{\textsc{sys}})$$



$(\trianglelefteq = \leq)$



$(\sqsubseteq)$

3 More Model

These definitions need to be updated to include the additional orders.

▶ **Definition 5.** *A pomset is x-*closed *if*
- *every $\mathcal{A}(e) = (\mathsf{R}x..)$ is fulfilled*
- *every $\Phi(e)$ is independent of x: $\big(\forall v.\ \Phi(e) \vDash \Phi(e)[v/x] \vDash \Phi(e)\big)$*

▶ **Definition 6.** *Let $P \in (\nu x.\mathcal{P})$ when $P \in \mathcal{P}$ and P is x-closed*

▶ **Definition 7.** *Let $P \in (\phi \triangleright \mathcal{P})$ when $P \in \mathcal{P}$ and $(\forall e \in E)$ $\Phi(e)$ implies ϕ*

▶ **Definition 8.** *Let $P' \in (\mathcal{P}[M/x])$ when $(\exists P \in \mathcal{P})$ $E' = E$, $\leq' = \leq$, $\mathcal{A}' = \mathcal{A}$, and $(\forall e \in E')$ $\Phi'(e) = \Phi(e)[M/x]$*

▶ **Definition 9.** *Let $P' \in (\mathcal{P}^1 \parallel \mathcal{P}^2)$ when $(\exists P^1 \in \mathcal{P}^1)$ $(\exists P^2 \in \mathcal{P}^2)$ $E' = E^1 \cup E^2$, $\leq' \supseteq \leq^1 \cup \leq^2$, and $(\forall e \in E')$ either*

$$\begin{aligned}
&e \notin E^2,\ \mathcal{A}'(e) = \mathcal{A}^1(e) \text{ and } \Phi'(e) \text{ implies } \Phi^1(e),\\
&e \notin E^1,\ \mathcal{A}'(e) = \mathcal{A}^2(e) \text{ and } \Phi'(e) \text{ implies } \Phi^2(e),\ \text{or}\\
&\mathcal{A}'(e) = \mathcal{A}^1(e) = \mathcal{A}^2(e) \text{ and } \Phi'(e) \text{ implies } \Phi^1(e) \vee \Phi^2(e)
\end{aligned}$$

Language

$$\begin{aligned}
[\![\mathtt{skip}]\!] &\triangleq \{\checkmark\}\\
[\![r := M \,;\, C]\!] &\triangleq [\![C]\!][M/r]\\
[\![r := x^{\mu} \,;\, C]\!] &\triangleq \textstyle\bigcup_v\ (\mathsf{R}^{\mu}xv) \Rightarrow [\![C]\!][x/r]\\
[\![x^{\mu} := M \,;\, C]\!] &\triangleq \textstyle\bigcup_v\ (M = v \mid \mathsf{W}^{\mu}xv) \Rightarrow [\![C]\!][M/x]\\
[\![\mathtt{F}^{\nu} \,;\, C]\!] &\triangleq (\mathsf{F}^{\nu}) \Rightarrow [\![C]\!]\\
[\![\mathtt{if}\,(M)\{C\}\,\mathtt{else}\,\{D\}]\!] &\triangleq \big(M \triangleright [\![C]\!]\big) \parallel \big(\neg M \triangleright [\![D]\!]\big)\\
[\![C \parallel D]\!] &\triangleq [\![C]\!] \parallel [\![D]\!]\\
[\![\mathtt{var}\, x \,;\, C]\!] &\triangleq \nu x \,.\, [\![C]\!]
\end{aligned}$$

$$\begin{aligned}
\nu \ ::=\ &\mathsf{rel} && \text{(Release)}\\
\mid\ &\mathsf{acq} && \text{(Acquire)}\\
\mid\ &\mathsf{sc} && \text{(SC)}
\end{aligned}$$