1 Compute $\nabla\times$ of Γ Basis, 2D Zero-th Order

Γ basis vector i (total 3)

$$\frac{1}{\Delta}\hat{N}^T L\Gamma_i$$

The curl is

$$\nabla\times(\frac{1}{\Delta}\hat{N}^T L\Gamma_i) = \hat{z}\begin{pmatrix}0 & -\partial_y & \partial_x\end{pmatrix}(\frac{1}{\Delta}\hat{N}^T L\Gamma_i)$$
$$= -\hat{z}\begin{pmatrix}0 & -\partial_y & \partial_x\end{pmatrix}(\frac{1}{\Delta}\hat{X}^T F^{-T}\Gamma_i)$$

where

$$F^{-T}\Gamma = \begin{pmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{pmatrix}\begin{pmatrix}0 & \zeta_3 & -\zeta_2\\ -\zeta_3 & 0 & \zeta_1\\ \zeta_2 & -\zeta_1 & 0\end{pmatrix} = \begin{pmatrix}-a_2\zeta_3+a_3\zeta_2 & a_1\zeta_3-a_3\zeta_1 & -a_1\zeta_2+a_2\zeta_1\\ -b_2\zeta_3+b_3\zeta_2 & b_1\zeta_3-b_3\zeta_1 & -b_1\zeta_2+b_2\zeta_1\\ -c_2\zeta_3+c_3\zeta_2 & c_1\zeta_3-c_3\zeta_1 & -c_1\zeta_2+c_2\zeta_1\end{pmatrix}$$

Also

$$\nabla\zeta_i = -\frac{\hat{n}_i l_i}{\Delta}$$

where Δ is twice the triangle area. The curl for the 3 Γ basis vectors are

$$\nabla\times_1 = \frac{\hat{z}}{\Delta^2}(b_2 l_3\hat{n}_{3y} - b_3 l_2\hat{n}_{2y} - c_2 l_3 n_{3x} + c_3 l_2 n_{2x})$$

$$\nabla\times_2 = \frac{\hat{z}}{\Delta^2}(-b_1 l_3\hat{n}_{3y} + b_3 l_1\hat{n}_{1y} + c_1 l_3 n_{3x} - c_3 l_1 n_{1x})$$

$$\nabla\times_3 = \frac{\hat{z}}{\Delta^2}(b_1 l_2\hat{n}_{2y} - b_2 l_1\hat{n}_{1y} - c_1 l_2 n_{2x} + c_2 l_1 n_{1x})$$

2 Compute $\nabla\cdot$ of Γ Basis

The div is

$$\nabla\cdot(\frac{1}{\Delta}\hat{N}^T L\Gamma_i) = \begin{pmatrix}0 & \partial_x & \partial_y\end{pmatrix}(\frac{1}{\Delta}\hat{X}^T F^{-T}\Gamma_i)$$

The div for the 3 Γ basis vectors are

$$\nabla\cdot_1 = \frac{1}{\Delta^2}(b_2 l_3\hat{n}_{3x} - b_3 l_2\hat{n}_{2x} + c_2 l_3\hat{n}_{3y} - c_3 l_2\hat{n}_{2y})$$

$$\nabla\cdot_2 = \frac{1}{\Delta^2}(-b_1 l_3\hat{n}_{3x} + b_3 l_1\hat{n}_{1x} - c_1 l_3\hat{n}_{3y} + c_3 l_1\hat{n}_{1y})$$

$$\nabla\cdot_3 = \frac{1}{\Delta^2}(b_1 l_2\hat{n}_{2x} - b_2 l_1\hat{n}_{1x} + c_1 l_2\hat{n}_{2y} - c_2 l_1\hat{n}_{1y})$$