This is a description about the content in the tar file.

1 2D Problem Description

For 2D TM wave, the equation for $\vec{E}$ is reduced to a scalar equation

$$\nabla \cdot \left(\frac{1}{\mu_r} \nabla E_z \right) + k^2 \epsilon E_z = 0 \tag{1}$$

with appropriate boundary condition. Similar equation for H_z holds for TE wave. We focus on (1)

1.1 Homogeneous Dirichlet boundary

For homogeneous Dirichlet boundary condition, this equation describes a resonance problem. Finite element discretization of (1) yields an algebraic general eigenvalue problem.

$$Au = k^2 Bu \tag{2}$$

from which we solve for the resonance modes u's and resonance frequencies k^2.

1.2 Inhomogeneous Dirichlet boundary

The discretized equation is

$$Cu = f \tag{3}$$

We solve for the unique solution u. The codes can also solve for Neumann and Robin boundaries, though we do not show examples in the tar file.

2 Folder Circle

A circular geometry is chosen, with homogeneous Dirichlet boundary condition. This folder gives the mesh, resonant modes, and eigenvalues (k^2). For simplicity and verification both ϵ_r and μ_r are chosen to be constant. Notice all the numbers are non-dimensionalized.

3 Folder Complex

A more complex geometry.

3.1 Folder Geometry

As shown in antena_bnd.png, this geometry has interior (red) and exterior (yellow) boundaries. It has 8704 elements. The relative magnetic permeability is a spatial varying field. The relative permettivity is assume a constant field, though a varying permettivity is also straightforward.

3.2 Folder Resonance

When a homogeneous Dirichlet boundary condition is applied to both interior and exterior boundaries, we solve for an eigenvalue problem. The pictures in each folder shows the corresponding eigen-mode: one for magnitude, another for phase angle (normalized by π). It is interesting that the first 6 eigen modes explores the vertical bars one by one. For example, the 1st and 2nd modes are both concentrated in the center vertical bar, while the 1st mode vibrates at the same phase angle up and down, and the 2nd mode vibrates at an opposite phase angle up and down.
We also give an example of higher modes, mode 35.
The eigenvalues, i.e. k^2 for the first 100 modes are plotted in eigmode.png.

3.3 Folder Driven

In this example the interior boundary is still homogeneous Dirichlet, but the exterior boundary is set to be 1, i.e. uniformly driven by an exterior wave. The k^2 just corresponds to the exterior driven frequency. How would the electric field look like as we gradually increase the driven frequency? This is shown in the movie (k^2 from 0 to 2000), only the magnitude is shown here.

When $k^2 = 0$ it is a electrostatic problem, which corresponds to the very start of the movie.