Consider a controllable, continuous LTI system with an observation model

$$\begin{cases} \dot{x} = A(\xi)x + B(\xi)u \\ y = Cx \end{cases} \tag{1}$$

with initial condition $x(0) = \mathbf{0}$. $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, $m < n$. For a "true" case, ξ is fixed but unknown. At time 0, it is modeled by a set of samples $\xi^1, \cdots, \xi^N$.
Minimize an objective function:

$$J_T = \int_0^T |x(\tau) - x_0(\tau)|^2 + \lambda |u(\tau)|^2 \mathrm{d}\tau \tag{2}$$

At time 0, a myopic optimal control does not consider the observation model.

$$u^* = \arg\min_u \frac{1}{N} \sum_i J_T \left(x(u, \xi^i; \mathbf{0}), u \right) \tag{3}$$

To improve upon the myopic approach, we use and only use the observation $y(\frac{T}{2})$. Define

$$\begin{aligned} \alpha^* &\equiv u^* \quad \text{for } t \in [0, \frac{T}{2}] \\ \beta^* &\equiv u^* \quad \text{for } t \in [\frac{T}{2}, T] \end{aligned} \tag{4}$$

and

$$J_T \equiv J_\alpha + J_\beta \tag{5}$$

The prediction for the observation at ξ^i and $\frac{T}{2}$ is

$$y^i = C \int_0^{\frac{T}{2}} \exp\left\{ A(\xi^i) \left(\frac{T}{2} - \tau \right) \right\} B(\xi^i) \alpha(\tau) \mathrm{d}\tau \tag{6}$$

Means

$$\begin{aligned} \bar{y} &= \frac{1}{N} \sum_i y^i \\ \bar{\xi} &= \frac{1}{N} \xi^i \end{aligned} \tag{7}$$

Covariances

$$\begin{aligned} Q_{yy} &= \frac{1}{N} \sum_i \left(y^i - \bar{y} \right) \left(y^i - \bar{y} \right)^T \\ Q_{\xi y} &= \frac{1}{N} \sum_i \left(\xi^i - \bar{\xi} \right) \left(y^i - \bar{y} \right)^T \end{aligned} \tag{8}$$

At $\frac{T}{2}$, if ξ^i were the true case, update ξ^j's as

$$\begin{aligned} \Delta \xi_i^j &= Q_{\xi y} Q_{yy}^{-1} \left(y^i - y^j \right) \\ \xi^j &\leftarrow \xi^j + \Delta \xi_i^j \end{aligned} \tag{9}$$

Compute

$$\begin{aligned} \frac{\partial \Delta \xi_i^j}{\partial \alpha} &= \frac{\partial}{\partial \alpha} \left(Q_{\xi y} Q_{yy}^{-1} \right) \left(y^i - y^j \right) \\ &+ Q_{\xi y} Q_{yy}^{-1} C \left(\exp\left\{ A(\xi^i)(\frac{T}{2} - \tau) \right\} B(\xi^i) - \exp\left\{ A(\xi^j)(\frac{T}{2} - \tau) \right\} B(\xi^j) \right) \end{aligned} \tag{10}$$

Also compute

$$\frac{\partial J_\alpha}{\partial \alpha} \tag{11}$$

and

$$x_{\frac{T}{2}}(\xi^i, \alpha^*) \quad \text{and} \quad \frac{\partial}{\partial \alpha} x_{\frac{T}{2}}(\xi^i) \tag{12}$$

Compute a new optimal control for $[\frac{T}{2}, T]$

$$\beta^{**} = \arg\min_\beta \frac{1}{N^2} \sum_i \sum_j J_\beta \left(x(\beta, \xi^j + \Delta\xi_i^j; x_{\frac{T}{2}}(\xi^i, \alpha^*)), \beta \right) \tag{13}$$