# Slides Outline

**Research background**

1. Motivation:

   - Optimization based on conservation law simulation.
   - Design space is high dimensional.

   Example: optimization of turbulent return bend geometry. (picture)

2. Many conservation law simulators are gray-box. What is gray-box / black-box / open-box. (table)

3. Research scope:

   - conservation law.
   - gray-box simulation.
   - high-dimension design.

4. Review of optimization methods : gradient-free, gradient-based.

5. Challenges of both methods:

   - each simulation can be accurate but costly.
   - if no adjoint, cost of gradient evaluation scales with dimension.
   - grad-free can be costly.

6. When lower-fidelity models are available, can save overall computation time by multi-fidelity optimization.

   - Bayesian calibration / coKriging.
   - Trust region.

   High-fidelity model is the gray-box simulator; low-fidelity model can be constructed by surrogate methods.

7. Review of surrogate methods.

   - physics-based surrogates: use the physics of the underlying system, such as
     - coarser discretization: *require simulator's PDE.*
     - reduced order model (POD, DEIM, balanced truncation): *require simulator's PDE.*
     - simplified physics (RANS, dual porosity, thin airfoil theory).
   - functional surrogate: use the sample value of objective function, such as
     - radial basis function approximation.
     - polynomial approximation.
     - neural network.

   Narrative: functional surrogate not suitable in my research scope because design space is high-dimensional. Focus on physics-based surrogate.

8. Physics-based surrogates can be adaptive.

   - Adaptive discretization: *require simulator's PDE.*
   - Goal oriented reduced order model: match observations, *require simulator's PDE*

**Twin model**

1. Propose a adaptive physics-based surrogate that

   - does not require simulator's PDE, instead infer a PDE.
   - PDE is inferred by matching the gray-box simulation's space-time solution.

2. Feasibility: Infer a general form PDE is not feasible, but we infer a conservation law.

Example: Buckley-Leverett Equation

| no knowledge | $\dfrac{\partial u}{\partial t} + \mathcal{F}(u) = 0$ | Black-box |

| conservation law | $\dfrac{\partial u}{\partial t} + \nabla \cdot G(u) = 0$ | |

| discretized Buckley-Leverett | Discretization of $\dfrac{\partial u}{\partial t} + \dfrac{\partial}{\partial x}\left(\dfrac{u^2}{u^2 + (1-u)^2}\right) = 0$ | Intrusive |