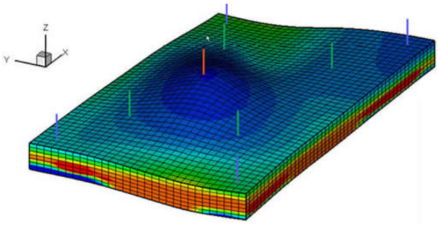
Optimization with Twin Model

September 24, 2014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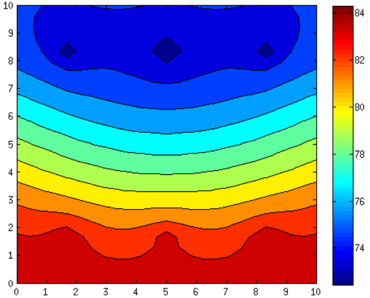
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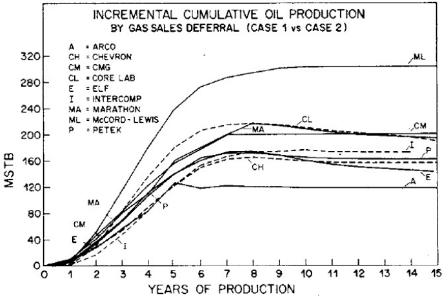
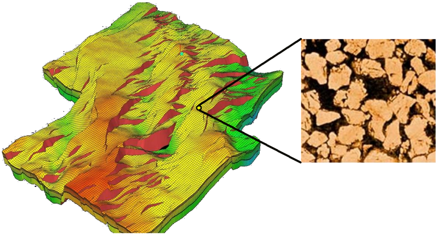
PDEs for oil reservoir simulation is empirical

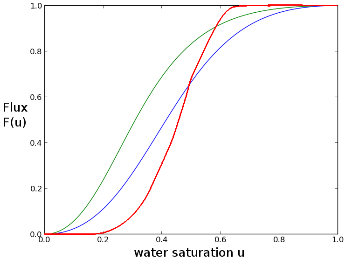


SPE comparative solution project 3
[Kenyon and Behie, 1987]



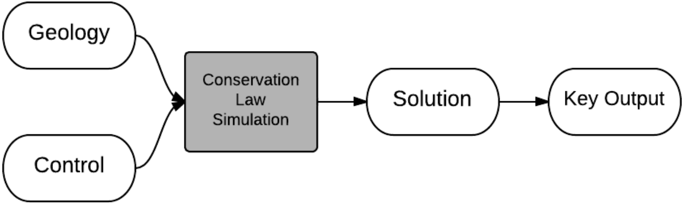
Buckley-Leverett equation is such an example

$$\frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} = 0$$

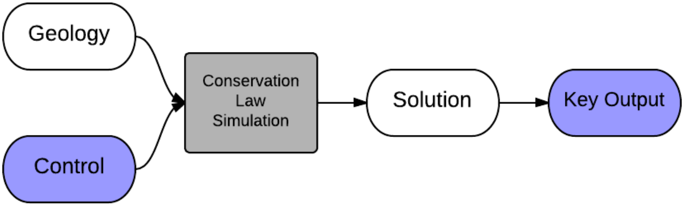


$$\text{Black-box} \begin{cases} \text{PDE} \\ \text{Numerical implementation} \end{cases}$$

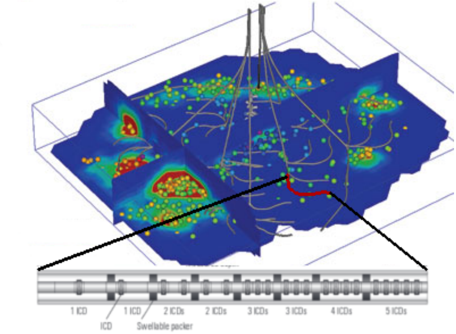
Black-box simulation solves an unknown PDE



Optimization based on black-box simulation requires non-intrusive methods



Conventional non-intrusive optimization suffers from the curse of dimensionality



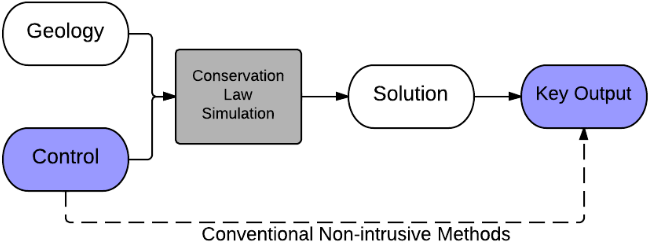
- Control space can be high dimension.
- Gradient-free optimization can be expensive.
- Adjoint-based optimization is more efficient but traditionally intrusive.

We propose non-intrusive adjoint-based optimization

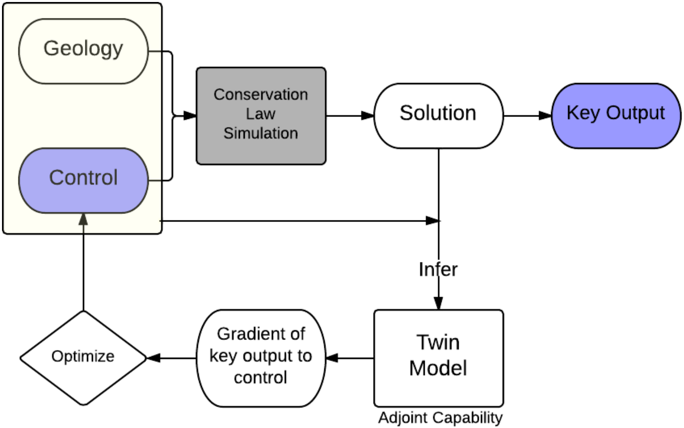
Objective

- Requires no code access.
- Mitigate the curse of dimensionality using gradient information.

How can adjoint be non-intrusive? (1/2)

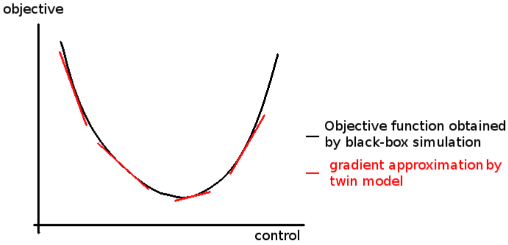


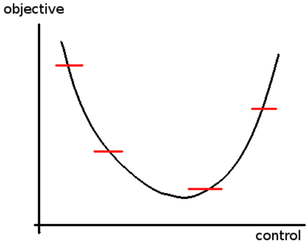
How can adjoint be non-intrusive? (2/2)



Why is good gradient approximation helpful?

The cost of obtaining approximated gradient by twin model is independent of the dimensionality of control.





If the gradient approximation is accurate, it can drive gradient based optimization.

If the gradient approximation is non-informative (e.g. always 0), we can use multi-fidelity optimization [A. March, 2012]. [Need to add reference: how convergence rate depends on gradient accuracy].

Can a non-intrusive method give a “certified” gradient estimation? (1/2)

Black-box model solves (equation unavailable):

$$\frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} = c(t, x) \tag{1}$$

Twin model solves

$$\frac{\partial \hat{u}}{\partial t} + \frac{\partial \hat{F}(\hat{u})}{\partial x} = c(t, x) \tag{2}$$

with the same initial and boundary conditions. $J = J(u, c)$, $\hat{J} = J(\hat{u}, c)$ is the objective function.

Can we find a constant C and proper norms so the gradient computed by the twin model is “certified”?

$$\boxed{\|\nabla_c J - \nabla_c \hat{J}\| < C\|u - \hat{u}\| \qquad \textbf{?}} \tag{3}$$

Can a non-intrusive method give a "certified" gradient estimation? (2/2)

Black-box model solves (equation unavailable):

$$\frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} = c(t, x)$$

Twin model solves

$$\frac{\partial \hat{u}}{\partial t} + \frac{\partial \hat{F}(\hat{u})}{\partial x} = c(t, x)$$

with the same initial and boundary conditions. $J = J(u, c)$, $\hat{J} = J(\hat{u}, c)$ is the objective function. ϕ and $\hat{\phi}$ are corresponding adjoint solutions.

$$\boxed{\begin{aligned} \|\nabla_c J - \nabla_c \hat{J}\| < C_1 \|\phi - \hat{\phi}\| \\ \|\phi - \hat{\phi}\| < C_2 \|F - \hat{F}\| \\ \|F - \hat{F}\| < C_3 \|u - \hat{u}\| \end{aligned}} \tag{4}$$

How to get a small $\|u - \hat{u}\|$?

$$\hat{F}^* = \arg\min_{\hat{F}} \left\{ \|u - \hat{u}\|^2 + \|\hat{F}\|_{\mathcal{R}} \right\} \tag{5}$$

where $\hat{u}$ is the solution of Eqn(13):

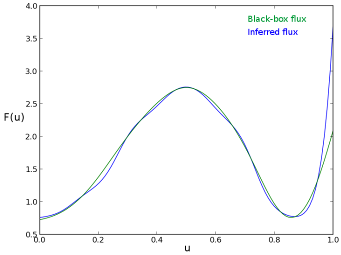
$$\frac{\partial \hat{u}}{\partial t} + \frac{\partial \hat{F}(\hat{u})}{\partial x} = c(t, x)\,,$$

and $\|\hat{F}\|_{\mathcal{R}}$ is a regularization on $\hat{F}$ to avoid ill-posedness.

What distinguishes twin model from conventional non-intrusive methods?

Flux function is independent of control.

$$\frac{\partial u}{\partial t} + \frac{\partial {\color{red}F}(u)}{\partial x} = c \qquad \frac{\partial \hat{u}}{\partial t} + \frac{\partial \hat{F}(\hat{u})}{\partial x} = c$$



Infer twin model's flux using black-box simulation's solution on only one realization of control.