A (gentle) introduction to Reinforcement Learning (with some links to causal reasoning)

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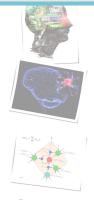
Introduction & Motivation

Markov Decision Process (MDPs)

Planning

Model Free Reinforcement Learning

Causality







WHAT IS REINFORCEMENT LEARNING?

- ► Reinforcement learning is the study of how animals and artificial systems can learn to optimize their behavior in the face of rewards and punishments Peter Dyan, Encyclopedia of Cognitive Science
- ▶ Not supervised learning the animal/agent is not provided with examples of optimal behaviour, it has to be discovered!
- ► Not unsupervised learning either we have more guidance than just observations

LINKS TO OTHER FIELDS

- ► It subsumes most artificial intelligence problems
- ► Forms the basis of most modern intelligent agent frameworks
- ► Ideas drawn from a wide range of contexts, including psychology (e.g., Skinner's "Operant Conditioning"), philosophy, neuroscience, operations research, Cybernetics

- ► Play backgammon/chess/go/poker/any game (at human or superhuman level)
- ► Helicopter control
- ► Learn how to walk/crawl/swim/cycle
- ► Elevator scheduling
- ► Optimising a petroleum refinery
- ► Optimal drug dosage

- ► The primary abstraction we are going to work with is the Markov Decision Process (MDP).
- ► MDPs capture the dynamics of a mini-world/universe/environment
- ▶ An MDP is defined as a tuple $\langle S, A, T, R, \gamma \rangle$ where:
 - \triangleright S, $s \in S$ is a set of states
 - \blacktriangleright A, $a \in A$ is a set of actions
 - ▶ $R: S \times A$, R(s, a) is a function that maps state-actions to rewards
 - ▶ $T: S \times S \times A$, with T(s'|s, a) being the probability of an agent landing from state s to state s' after taking a
 - \triangleright γ is a discount factor the impact of time on rewards

- ► States represent sufficient statistics.
- ► Markov Property ensures that we only care about the present in order to act we can safely ignore past states
- ► Think Tetris all information can be captured by a single screen-shot





- ► An agent is an entity capable of actions
- ► An MDP can capture any environment that is inhabited either by
 - ► Exactly one agent
 - ► Multiple agents, but only one is adaptive
- ► Notice how actions are part of the MDP notice also how the MDP is a "world model"
- ► The agent is just a "brain in a vat"
- ► The agent perceives states/rewards and outputs actions
- ► Transitions specify the effects of actions in the world (e.g., in Tetris, you push a button, the block spins)

- ► Rewards describe state preferences
- ► Agent is happier in some states of the MDP (e.g., in Tetris when the block level is low, a fish in water, pacman with a high score)
- ► Punishment is just low/negative reward (e.g., being eaten in pacman)
- \triangleright γ , the discount factor,
 - ▶ Describes the impact of time on rewards
 - "I want it now", the lower γ is the less important future rewards are
- ► There are no "springs/wells of rewards" in the real world
 - ► What is "human nature"?

- ► Scoring in most video games
- ► The distance a robot walked for a bipedal robot
- ► The amount of food an animal eats
- ► Money in modern societies
- ► Army Medals ("Gamification")
- ► Vehicle routing
 - ► (-Fuel spent on a flight)
 - ► (+ Distance Covered)
- ► Cold/Hot
- ➤ Do you think there is an almost universal reward in modern societies?

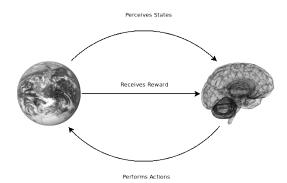
LONG TERM THINKING

- ► It might be better to delay satisfaction
- ► Immediate reward is not always the maximum reward
- ► In some settings there are no immediate rewards at all (e.g., most solitaire games)
- ► MDPs and RL capture this
- ► "Not going out tonight, study"
- ► Long term investment

- ► The MDP (the world) is populated by an agent (an actor)
- ► You can take actions (e.g., move around, move blocks)
- ► The type of actions you take under a state is called the *policy*
- ▶ $\pi: S \times A$, $\pi(s, a) = P(a|s)$, a probabilistic mapping between states and actions
- ► Finding an optimal policy is *mostly* what the RL problem is all about

THE FULL LOOP

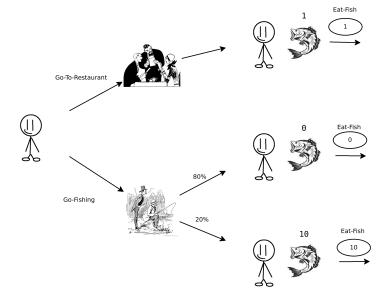
- ► See how the universe described by the MDP defines actions, not just states and transitions
- ► An agent needs to act upon what it perceives
- ▶ Notice the lack of body "brain in a vat". Body is assumed to be part of the world.



FISHING TOON

- ► Assume a non-player character (let's call her *toon*)
- ► Toon is Hungry!
- ► Eating food is rewarding
- ► Has to choose between going fishing or going to the restaurant (to eat fish)
 - ► Fishing can get you better quality of fish (more reward), but you might also get no fish at all (no reward)!
 - ▶ Going to the restaurant is a low-risk, low-reward alternative

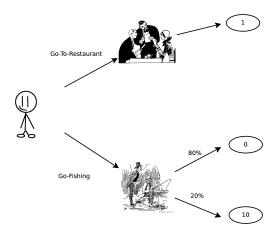
FISHING TOON: PICTORIAL DEPICTION



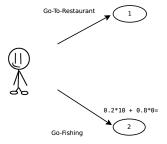
EXPECTED REWARD

- ▶ Our toon has to choose between two different actions
- ► Go-To-Restaurant or Go-Fishing
- ► We assume that toon is interested in maximising the expected sum of happiness/reward
- ▶ We can help the toon reason using the tree backwards

REASONING BACKWARDS (1)



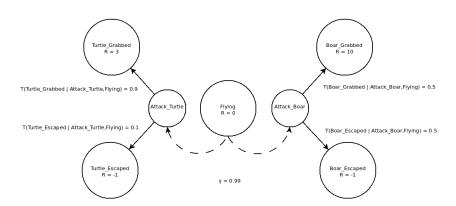
REASONING BACKWARDS (2)



- ► Toon should go Go-Fishing
- ► Would you do the same?
- ► Would a pessimist toon do the same?
- ▶ We just went through the following equation:

$$Q^*(s, a) = R(s, a) + \gamma \sum_{s' \in S} T(s'|s, a) \max_{a' \in A} Q^*(s', a')$$

- ► Looks intimidating but it's really simple
- ▶ Let's have a look at another example
 - ► How about toon goes to the restaurant after failing to fish?
 - ► How would that change the reward structure?



- ► The agent's goal is to maximise its long term reward $\mathbb{E}_{\pi} \left[\sum_{t=0}^{\infty} \gamma^{t} R\left(s^{t}, a^{t}\right) \right]$
- ► Risk Neutral Agent think of the EagleWorld example
- ► Rewards can be anything, but most organisms receive rewards only in a very limited amount of states (e.g., fish in water)
- ▶ What if your reward signal is only money?
 - Sociopathic, egotistic, greed-is-good Gordon Gekko (Wall Street, 1987)
 - ► No concept of "externalities" agents might wreak havoc for marginal reward gains
 - ▶ Same applies to all "compulsive agents" think Chess

- ► One can possibly search through all combinations of policies until she finds the best
- ► Slow, does not work in larger MDPs
- ► Exploration/Exploitation dilemma
 - ► How much time/effort should be spend exploring for solutions?
 - ▶ How much time should be spend exploiting good solutions?

- ► Who was doing the thinking in the previous example (You? The eagle?)
- ► An agent has access to model, i.e., has a copy of the MDP (the outside world) in its mind
- Using that copy, it tries to "think" what is the best route of action
- ▶ It then executes this policy on the real world MDP
- ➤ You can't really copy the world inside your head, but you can copy the dynamics
- ► "This and that will happen if I push the chair"
- ► Thinking, introspection...
- ▶ If the model is learned, sometimes it's called "Model Based RL"

- ► The two most important equations related to MDP
- ► Recursive definitions

►
$$V^{\pi}(s) = \sum_{a \in A} \pi(s, a) \left(R(s, a) + \gamma \sum_{s' \in S} T(s'|s, a) V^{\pi}(s') \right)$$

► $Q^{\pi}(s, a) = R(s, a) + \gamma \sum_{s' \in S} T(s'|s, a) \sum_{a' \in A} \pi(s', a') Q^{\pi}(s', a')$

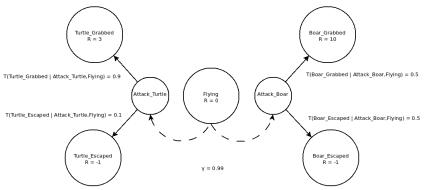
•
$$Q^{\pi}(s, a) = R(s, a) + \gamma \sum_{s' \in S} T(s'|s, a) \sum_{a' \in A} \pi(s', a') Q^{\pi}(s', a')$$

- ► Called V-Value(s) (state-value function) and Q-Value(s) (state-action value function) respectively
- ▶ Both calculate the expected rewards under a certain policy

Link between V^{π} and Q^{π}

- \triangleright V and Q are interrelated
- ► $V^{\pi}(s) = \sum_{a \in A} \pi(s, a) Q^{\pi}(s, a)$ ► $Q^{\pi}(s, a) = R(s, a) + \sum_{s' \in S} T(s'|s, a) V^{\pi}(s')$

Example MDP - EagleWorld - Random Policy



$$\pi(Flying, Attack_Boar) = 0.5, \pi(Flying, Attack_Turtle) = 0.5$$

$$Q(Flying, Attack_Boar) = 0.99 * (10 * 0.5 + 0.5 * -1) = 4.455$$

$$Q(Flying, Attack_Turtle) = 0.99 * (0.9 * 3 + 0.1 * -1) = 2.574$$

$$V^{\pi}(Flying) = 0.5, Q^{\pi}(Flying, Attack_Turtle) + 0.5, Q(Flying, Attack_Boar) = 3.5145$$

- ► An optimal policy can be defined in terms of Q-values
- ▶ It is the policy that maximises Q values

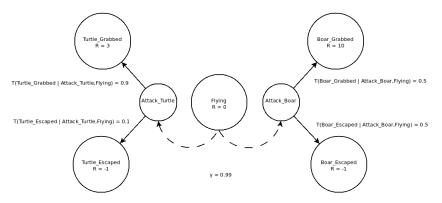
•
$$V^*(s) = \max_{a \in A} R(s, a) + \gamma \sum_{s' \in S} T(s'|s, a) V^*(s')$$

$$\begin{array}{l} \blacktriangleright \ \ V^*(s) = \max_{a \in A} R(s,a) + \gamma \sum_{s' \in S} T(s'|s,a) \, V^*(s') \\ \blacktriangleright \ \ Q^*(s,a) = R(s,a) + \gamma \sum_{s' \in S} T(s'|s,a) \max_{a' \in A} Q^*(s',a') \end{array}$$

Link between V^* and Q^*

- ► Again, they are interrelated
- $\qquad \qquad V(s)^* = \max_{a \in A} \, Q^*(s,a)$
- $Q^*(s, a) = R(s, a) + \gamma \sum_{s' \in S} T(s'|s, a) V^*(s')$

Example MDP - EagleWorld - Optimal Policy



$$\begin{split} &Q(Flying,Attack_Boar) = 0.99*(10*0.5+0.5*-1) = 4.455\\ &Q(Flying,Attack_Turtle) = 0.99*(0.9*3+0.1*-1) = 2.574\\ &\pi^*(Flying,Attack_Boar) = 1,\,\pi^*(Flying,Attack_Turtle) = 0\\ &V^*(Flying) = Q(Flying,Attack_Boar) = 4.455 \end{split}$$

- ► An Agent can be composed of a number of things
- ► A policy
- ► A Q-Value/and or V-Value Function
- ► A Model of the environment (the MDP)
- ► Inference/Learning Mechanisms
- ▶ ...
- ► An agent has to be able to *create a policy* either on the fly or using Q-Values
- ► The Model/Q/V-Values serve as intermediate points towards constructing a policy

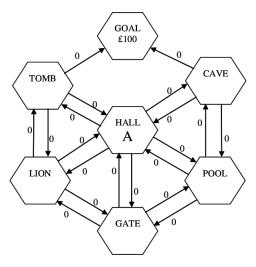
- Assume deterministic transitions
- ► Thus, taking an action on a state will lead only to ONE other possible state for some action a_c
 - $T(s'|s, a_i) = \begin{cases} 1 & \text{if } a_i = a_c \\ 0 & \text{otherwise} \end{cases}$

 - $V^*(s) = \max_{a \in A} [R(s, a) + \gamma V^*(s')]$ $Q(s, a) = R(s, a) + \gamma \max_{a' \in A} Q(s', a')$
- ▶ It is easier now to solve for problems that have loops in them
- ▶ We can also attempt to learn Q-Values without a model!
- ▶ All we need in order to find the optimal policy is Q(s, a)

- ► The policy is deterministic from start to finish
- ▶ We will use $\pi(s) = \underset{a \in A}{\operatorname{arg max}} Q(s, a)$ to denote the optimal policy
- ► The algorithm now is:
 - ▶ Initialise all Q(s, a) to low values
 - ► Repeat:
 - \blacktriangleright Select an action a using an exploration policy
 - $PQ(s,a) \leftarrow R(s,a) + \gamma \max_{a' \in A} Q(s',a')$
 - $ightharpoonup s \leftarrow s'$
 - ▶ Also known as "Dynamic Programming", "Value Iteration"

AN EXAMPLE (1)

(From Paul Scott's ML lecture notes)



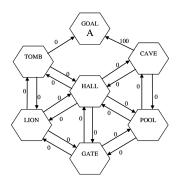
R(HALL, To - CAVE) = 0

AN EXAMPLE (2)

Next suppose the agent, now in state CAVE , selects action To-GOAL

 $R(CAVE, To - GOAL) = 100, \ Q(GOAL, a) = 0$ for all actions (there are no actions)

Hence $Q(CAVE, To - GOAL) = 100 + \gamma * 0 = 100$

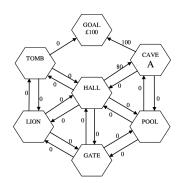


Let's start at hall again and select the same action To-CAVE

$$R(HALL, To - CAVE) = 0, Q(CAVE, GOAL) = 100$$

$$Q(CAVE, a) = 0$$
 for all other actions a

Hence
$$\max_{a \in A} Q(CAVE, a) = 100$$
, if $\gamma = 0.8$, $Q(HALL, To - CAVE) = 0 + \gamma * 100 = 80$



EXPLORATION / EXPLOITATION

- ► How do we best explore?
- ► Choose actions at random but this can be very slow
- $ightharpoonup \epsilon greedy$ is the most common method
- ▶ Act ϵ -greedily

- ϵ -greedy means acting greedily with probability 1ϵ , random otherwise
- ▶ When you are done, act greedily $\pi(s) = \underset{a \in A}{\operatorname{arg max}} Q(s, a)$

- ▶ What can we do if the MDP is not deterministic?
- ► Q-learning

$$\qquad \qquad \bullet \quad Q(s,a) \leftarrow \left. Q(s,a) + \eta \left[R(s,a) + \gamma \max_{a' \in A} \, Q(s',a') - \, Q(s,a) \right] \right.$$

- \triangleright SARSA(0)
- \triangleright SARSA(1)/MC,
 - \triangleright $Q(s,a) \leftarrow Q(s,a) + \eta \left[\mathbf{v}_{\tau} Q(s,a) \right]$
 - $\mathbf{v}_{\tau} \leftarrow R(s, a) + \gamma R(s', a') + ... \gamma^2 R(s'', a'') + \gamma^{\tau 1} R(s^{\tau}, a^{\tau})$
- η is a small learning rate, e.g., $\eta = 0.001$

- ► MC: updated using the whole chain
 - ► Possibly works better when the markov property is violated
- ► SARSA: update based on the next action you actually took
 - ► On Policy learning
- ► Q-Learning: update based on the best possible next action
 - ► Will learn optimal policy even if acting off-policy

- ► Remember Q is just a mean/average
- ► MC (Naive Version)
 - ▶ Start at any state, initialise $Q_0(s, a)$ as you visit states/actions
 - ▶ Act ϵ -greedily
- ▶ Add all reward you have seen so far to $\mathbf{v}_{\tau}^{\mathbf{i}} = R(s', a') + \gamma R(s'', a'') + \gamma^2 R(s''', a''') + \gamma^{\tau-1} R(s^{\tau}, a^{\tau}) \text{ for episode } i$
- $Q_n(s, a) = E_{\pi^{\epsilon}}[v_{\tau}^i] = \frac{1}{n} \sum_{i=1}^n v_{\tau}^i$, where n is the times a state is visited

- \triangleright ϵ -greedy means acting greedily 1ϵ , random otherwise
- ▶ Better to calculate mean incrementaly

$$Q_n(s, a) = E_{\pi_n}[\mathbf{v}_{\tau}^{\mathbf{i}}]$$

$$Q_n(s, a) = \frac{1}{n} \sum_{i=1}^n \mathbf{v}_{\tau}^{\mathbf{i}}$$

$$Q_n(s, a) = \frac{1}{n} \left(\mathbf{v}_{t}^1 + \mathbf{v}_{\tau}^2 \dots \mathbf{v}_{\tau}^{n-1} + \mathbf{v}_{\tau}^{\mathbf{n}} \right)$$

$$Q_n(s, a) = \frac{1}{n} \left(\sum_{i=1}^{n-1} \mathbf{v}_{\tau}^{\mathbf{i}} + \mathbf{v}_{\tau}^{\mathbf{n}} \right)$$

Monte Carlo Control (3)

by definition

$$Q_{n-1}(s,a) = \frac{1}{n-1} \sum_{i=1}^{n-1} \mathbf{v}_{\tau}^{i} \implies (n-1) Q_{n-1}(s,a) = \sum_{i=1}^{n-1} \mathbf{v}_{\tau}^{i}$$

$$Q_{n}(s, a) = \frac{1}{n} ((n-1)Q_{n-1}(s, a) + \mathbf{v}_{\tau}^{n})$$

$$Q_{n}(s, a) = \frac{1}{n} (Q_{n-1}(s, a)n - Q_{n-1}(s, a) + \mathbf{v}_{\tau}^{n})$$

$$Q_{n}(s, a) = \frac{Q_{n-1}(s, a)n}{n} + \frac{-Q_{n-1}(s, a) + \mathbf{v}_{\tau}^{n}}{n}$$

$$Q_{n}(s, a) = Q_{n-1}(s, a) + \underbrace{\frac{\mathbf{v}_{\tau}^{n} - Q_{n-1}(s, a)}{n}}_{n}$$

▶ But π^n changes continuously, so the distribution of rewards is non-stationary

$$\begin{split} Q_n(s,a) &= Q_{n-1}(s,a) + \frac{1}{n} \left[\mathbf{v}^{\mathbf{n}}_{\tau} - Q_{n-1}(s,a) \right] \to \textbf{Bandit case} \\ Q_n(s,a) &= Q_{n-1}(s,a) + \eta \left[\mathbf{v}^{\mathbf{n}}_{\tau} - Q_{n-1}(s,a) \right] \to \textbf{Full MDP case} \end{split}$$

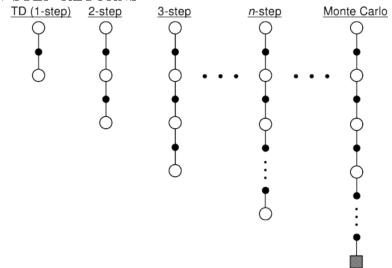
▶ A Bandit can be seen as MDP with a chain of length one (i.e. s) - like the initial EagleWorld, η is a learning rate (e.g., 0.001)

- ▶ Start at any state, initialise $Q_0(s, a)$ as you visit states/actions
- \blacktriangleright Act ϵ -greedily
- ▶ Wait until episode ends, i.e. a terminal state is hit ϵ set to some low value, e.g., 0.1
- ▶ Add all reward you have seen so far to $v_{\tau}^{i} = R(s, a) + \gamma R(s', a') + ... \gamma^{2} R(s'', a'') + \gamma^{\tau 1} R(s^{\tau}, a^{\tau}) \text{ for episode } i$
- $Q_n(s, a) = Q_{n-1}(s, a) + \eta [\mathbf{v}_{\tau}^n Q_{n-1}(s, a)]$

- ▶ With MC we update using the rewards from the whole chain
- ► Can we update incrementally?

$$\begin{split} Q_n(s,a) &= Q_{n-1}(s,a) + \eta \left[\mathbf{v}_{\tau}^{\mathbf{n}} - Q_{n-1}(s,a) \right] \\ Q_n(s,a) &= Q_{n-1}(s,a) + \eta \left[R(s,a) + \gamma R(s',a') + \ldots \gamma^2 R(s'',a'') + \gamma^{\tau-1} R(s^{\tau},a^{\tau}) - Q_{n-1}(s,a) \right] \\ Q_n(s,a) &= Q_{n-1}(s,a) + \eta \left[R(s,a) + \gamma (R(s',a') + \ldots \gamma R(s'',a'') + \gamma^{\tau-2} R(s^{\tau},a^{\tau})) - Q_{n-1}(s,a) \right] \\ Q_n(s,a) &= Q_{n-1}(s,a) + \eta \left[R(s,a) + \gamma (\mathbf{v}_{\tau}^{\mathbf{n},(s',a')}) - Q_{n-1}(s,a) \right] \\ Q_n(s,a) &= Q_{n-1}(s,a) + \eta \left[R(s,a) + \gamma Q_{n-1}(s',a') - Q_{n-1}(s,a) \right] \end{split}$$

N-STEP RETURNS



From Temporal Different to Monte Carlo (From Sutton & Burto)

LET'S GO OVER THE TOON EXAMPLE, WITHOUT A MODEL

 $ightharpoonup \epsilon - greedy$, with $\epsilon = 0.1$

- ► There is usually some link between states
- ► We can train function approximators incrementally to model Q(s,a)
- We now have $Q(s, a; \theta)$, where θ are the parameters
- ► Examples include Linear function approximators, Neural Networks, n-tuple networks
- ► Not easy to do, few convergance guarantees
 - ▶ But with some effort, this works pretty well

FAMOUS FUNCTION APPROXIMATION EXAMPLES

- ► Computer GO
- ► Car Driving
- ► Can you name another problem?

PLATFORMS

- ► Let's look at open AI gym
- ► A lot of modern work is a combination of RL with Neural Networks

- ▶ How can one learn a model of the world?
 - ▶ Possibly by breaking it down into smaller, abstract chunks
 - ► Unsupervised Learning
 - ightharpoonup ... and learning what effects ones actions have the environment
 - ► Supervised Learning
- ► RL weaves all fields of Machine Learning (and possibly Artificial Intelligence) into one coherent whole
- ► The purpose of all learning is action!
 - ▶ You need to be able to recognise faces so you can create state
 - ▶ ... and act on it

Causality (bonus)

- ► We often colliqually say "A is caused by B"
- ► Can you discuss the meaning of this?

Counterfactuals

- \blacktriangleright If I take action a I land on state s
- \blacktriangleright What if I don't take action a?'
- ▶ "Experimenter forced you to pick up smoking" vs
- ► "Experimenter observed that you smoked"
- ► Will you get lung disease?
- ► The experimenter takes the actions vs observes

WHAT IS THE LINK?

- ► Off-policy evaluation learning
- ► Let's see an example
 - ► Features are color of hair, height, smoking
 - ► Reward is -1000 (lung disease), 1 (healthy)
- ► This would have been supervised learning if we knew the policy!
- ► Let's see a possible example of data
- ► Can you write down an example policy?

Conclusion

- ► RL is a massive topic
- ▶ We have shown the tip of iceberg
- ightharpoonup Rabbit hole goes deep both on the application level and the theory level

FURTHER STUDY (1)

- ► Tom Mitchell, Chapter 13
- ► David Silver's UCL Course: http://www0.cs.ucl.ac.uk/staff/D.Silver/web/Teaching.html
 - ▶ Some ideas in these lecture notes taken from there
 - ► Probably the best set of notes there is on the subject
 - ► Online at http://www.machinelearningtalks.com/tag/rl-course/
- ► Reinforcement Learning, by Richard S. Sutton and Andrew G. Barto
 - ► Classic book
 - ► Excellent treatment of most subjects

- ► Artificial Intelligence: A Modern Approach by Stuart J. Russell and Peter Norvig
 - ► The Introductory A.I. Textbook
 - ► Chapters 16 and 21
- ► Algorithms for Reinfocement Learning by Csaba Szepesvari
 - Very "Mathematical", but a good resource that provides a very unified view of the field
- ► Reinforcement Learning: State-Of-The-Art by Marco Wiering (Editor), Martijn Van Otterlo (Editor)
 - ► Edited Volume