

(Q1) Suppose you are told:

Probability of someone who has a cold having a high fever is 0.4.

Probability of someone who has flu having a high fever is 0.9.

Proportion of population with a cold at a given time is 4%.

Proportion of population with flu at a given time is 1%.

Proportion of population having a high fever at a given time is 3%.

Assuming that nobody ever has flu and a cold at the same time, on the basis of this data determine the MAP hypothesis over the set {cold, flu} for a patient who has a high fever, and calculate the likelihood that it is correct.

What is the probability that a patient with a high fever is suffering an ailment other than flu or a cold?

Solution.

$$P(H|C) = 0.4$$

$$P(H|F) = 0.9$$

$$P(C) = 0.04$$

$$P(F) = 0.01$$

$$P(H) = 0.03$$

$$P(C \wedge F) = 0$$

$$P(C|H) = P(H|C) P(C) / P(H) = 0.4 * 0.04 / 0.03 = 0.53$$

$$P(F|H) = P(H|F) P(F) / P(H) = 0.9 * 0.01 / 0.03 = 0.3$$

The MAP over the set {C,F} is therefore: C

$$P(\neg C \wedge \neg F | H) = 1 - (P(C|H) + P(F|H)) = 1 - (0.53 + 0.3) = 0.17$$