MAT993F UQAM: $\mathrm{SL}(2,\mathbb{C})$ REPRESENTATIONS AND 3-DIMENSIONAL HYPERBOLIC GEOMETRY

Instructors: - Steven Boyer, PK-5212, boyer.steven@uqam.ca
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Meeting time: Thursdays 9am-12nn, room PK-5675

Course webpage: https://sites.google.com/view/chicheuktsang/teaching/uqam-fall-2024

Course description: Studying representations of the fundamental group has had a long tradition in 3-manifold topology. Representations with values in $\mathrm{SL}(2,\mathbb{C})$ have been particularly well-studied because of their computational accessibility and connection to hyperbolic geometry. This course aims to discuss some aspects of this theory. The first part of the course will focus on 3-dimensional hyperbolic geometry, covering topics such as Mostow rigidity and Thurston's hyperbolic Dehn surgery theorem. The second part of the course will focus on Culler and Shalen's character variety machinery, with an eye towards proving the cyclic surgery theorem. The prerequisites for the class are basic algebraic geometry (varieties, valuations) and basic algebraic topology (fundamental group, homology).

Assessment: 50% active participation + 50% presentation

References:

- 'The Geometry and Topology of Three-Manifolds' by William P. Thurston
- 'An Introduction to Geometric Topology' by Bruno Martelli
- 'Foundations of Hyperbolic Manifolds' by John G. Ratcliffe
- 'Applications of Character Variety Methods to Dehn Surgery' by Steven Boyer

Presentation: Each enrolled student is required to give a 50-minute-presentation on one of the presentation topics, listed in the next page, during the last 1-2 weeks of class. See the separate document for more in-depth descriptions and references for the topics.
Students are expected to assimilate the material from the references, and deliver a talk that helps other students understand the main ideas and how they are related to what has been discussed in class.

Tentative schedule:

Week	Date	Topics
1	5 Sep	$\mathrm{SL}(2,\mathbb{C})$, $\mathbb{H}^3$, and character varieties
2	12 Sep	Thick-thin decomposition
3	**16 Sep**	Developing maps
4	26 Sep	Mostow rigidity
5	3 Oct	Triangulations, gluing equations, deformation space
6	10 Oct	Hyperbolic Dehn surgery theorem
7	17 Oct	Hyperbolic volume
Break		
8	31 Oct	Actions on trees and surfaces
9	7 Nov	The SL_2-tree of a discrete valuation
10	14 Nov	Ideal points and discrete valuations
11	21 Nov	Culler-Shalen seminorms I
12	28 Nov	Culler-Shalen seminorms II
13	5 Dec	The cyclic surgery theorem
14	12 Dec	Student presentations

Possible presentation topics:

- Proof of Mostow rigidity via quasiconformal extension to S^2_∞ (Thurston's notes Ch.5)
- Orbifolds (Thurston's notes Ch.13)
- Canonical triangulations for once-punctured torus bundles (Guéritaud)
- Angled block decompositions (Futer-Guéritaud)
- Geodesic ideal triangulations exist virtually (Luo-Schleimer-Tillmann)
- Computational aspects of hyperbolic 3-manifolds (SnapPy, Hoffman-Ichihara-Kashiwagi-Masai-Oishi-Takayasu)
- Group actions on trees and amalgams
- Proof that a finitely generated group acts non-trivially on a tree if and only if some element of the group acts fixed point freely on the tree
- The number field of a hyperbolic 3-manifold
- Teichmüller space and curves of characters of Seifert fibred manifold groups (Boyer-Zhang)
- Culler-Shalen norms of twist knots (Boyer-Mattman-Zhang)
- Seifert surgery on knots in the 3-sphere