LIST OF PRESENTATION TOPICS

This is a list of presentation topics for MAT993F: Représentations $\mathrm{SL}(2,\mathbb{C})$ et la géométrie hyperbolique en dimension 3 (MAT993F: $\mathrm{SL}(2,\mathbb{C})$ representations and 3-dimensional hyperbolic geometry). We include references for each topic, as well as a brief description of the central ideas.

- Proof of Mostow rigidity via quasiconformal extension to S^2_∞

  In class, we discussed a proof of Mostow rigidity using Gromov's simplicial volume. Mostow's original proof [Mos68] is different and uses quasiconformal maps. The key steps of this approach are explained in Thurston's notes [Thu22, Ch.5.9]. Students are expected to give a talk outlining the argument and explaining the difference between the two proofs.

- Orbifolds

  Orbifolds are a convenient language for discussing various notions in low-dimensional topology. In the context of this class, they arise when one allows hyperbolic metrics to develop singularities. The basics of the topology and geometry of orbifolds can be found in [Mar22, Ch.3.6]. Students are expected to give an introduction to this material, and explain how orbifolds arise in the hyperbolic Dehn surgery theorem when the generalized Dehn filling coefficient is a non-relatively prime pair of integers.

- Canonical triangulations for once-punctured torus bundles

  In [Gué06], Guéritaud constructed geometric triangulations for all hyperbolic once-punctured torus bundles. Students are expected to read through the paper, then give a talk explaining the construction of the triangulation and outlining the proof of these triangulations being geometric. In the appendix of the paper, Futer extended the methods of the main paper to two bridge link complements; students do not need to present this part of the paper.

- Angled block decompositions

  In [FG09], Futer and Guéritaud develop a generalization of the theory of angle structures on ideal triangulations, which they call angled block decompositions. Students are expected to read through the paper, then give a talk expositing this theory, focusing on how aspects of angle structures and geometric triangulations generalize to angled blocks, including the volume functional and its critical points, and mentioning how Futer and Guéritaud use angled blocks to study arborescent link complements.

- Geodesic ideal triangulations exist virtually

  In [LST08], Luo, Schleimer, and Tillmann showed that every finite volume hyperbolic 3-manifold with a nonempty set of cusps has a finite cover that admits a geometric triangulation. The proof uses a group theoretic property called subgroup separability.

Note that [LST08] shows a more general statement allowing for totally geodesic boundary. For a streamlined proof that only shows the cited theorem, see [Mar22, Ch.15.4.6]. Students are expected to explain the subgroup separability property and outline the proof of the theorem.

- Computational aspects of hyperbolic 3-manifolds

  `SnapPy` [CDGW] is a program for studying hyperbolic 3-manifolds. Each hyperbolic 3-manifold is stored as a geometric triangulation, i.e. a combinatorial triangulation and a modulus for each tetrahedron. One can take covers, carry out hyperbolic Dehn fillings, and locate the shortest geodesics of a hyperbolic 3-manifold using `SnapPy`. In [HIK$^+$16], it is explained how one can certify that these computations are rigorous. Students are expected to demonstrate the functions of `SnapPy`, particularly computations that are relevant to the course material, and give a brief account of the contents of [HIK$^+$16].

- The Teichmüller component of the character variety of Seifert fibred manifold groups

  Seifert fibred manifolds form a distinguished class of geometric manifolds characterised by the fact that they support foliations by circles. Of the eight Thurston geometries, six of them correspond to Seifert manifolds: each closed 3-manifold with one of these six geometries is Seifert fibred and each closed Seifert fibred manifold is geometric with respect to (exactly) one of these geometries. Students are expected to describe this family, their base orbifolds, and the relation between the fundamental group of a Seifert manifold and that of its base orbifold [Sco83, Section 3]. Using this they will deduce that when the base orbifold is hyperbolic, its Teichmüller space [Thu22, Ch.13.3] can be used to build high dimensional components in the Seifert manifold's character variety [Boy, Example 4.2(2)].

- Seifert surgery on knots in the 3-sphere

  Cameron Gordon conjectured that the only rational surgeries on a hyperbolic knot which yield a Seifert fibred manifold are integer surgeries. Students are expected to outline how Culler-Shalen seminorms can be used to show that Gordon's conjecture holds if the rational surgery yields a Seifert fibred manifold which contains an essential surface [Boy, Section 10.1].

References

[Boy] Steven Boyer. Applications of character variety methods to Dehn surgery. Available at https://www.cirget.uqam.ca/boyer/Publications/Minicourse_Lectures_ICERM.pdf.

[CDGW] Marc Culler, Nathan M. Dunfield, Matthias Goerner, and Jeffrey R. Weeks. SnapPy, a computer program for studying the geometry and topology of 3-manifolds. Available at http://snappy.computop.org (01/09/2024).

[FG09] David Futer and François Guéritaud. Angled decompositions of arborescent link complements. *Proc. Lond. Math. Soc. (3)*, 98(2):325–364, 2009.

[Gué06] François Guéritaud. On canonical triangulations of once-punctured torus bundles and two-bridge link complements. *Geom. Topol.*, 10:1239–1284, 2006. With an appendix by David Futer.

[HIK$^+$16] Neil Hoffman, Kazuhiro Ichihara, Masahide Kashiwagi, Hidetoshi Masai, Shin'ichi Oishi, and Akitoshi Takayasu. Verified computations for hyperbolic 3-manifolds. *Exp. Math.*, 25(1):66–78, 2016.

[LST08] Feng Luo, Saul Schleimer, and Stephan Tillmann. Geodesic ideal triangulations exist virtually. *Proc. Amer. Math. Soc.*, 136(7):2625–2630, 2008.

[Mar22] Bruno Martelli. An introduction to geometric topology, 2022.

[Mos68] G. D. Mostow. Quasi-conformal mappings in n-space and the rigidity of hyperbolic space forms. *Inst. Hautes Études Sci. Publ. Math.*, (34):53–104, 1968.

[MR03] Colin Maclachlan and Alan W. Reid. *The arithmetic of hyperbolic 3-manifolds*, volume 219 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2003.

[Sco83] Peter Scott. The geometries of 3-manifolds. *Bull. London Math. Soc.*, 15(5):401–487, 1983.

[Thu22] William P. Thurston. *The geometry and topology of three-manifolds. Vol. IV*. American Mathematical Society, Providence, RI, [2022] ©2022. Edited and with a preface by Steven P. Kerckhoff and a chapter by J. W. Milnor.