

Definition 1 (Convergence in Probability). *Given a sequence of random variables X_n . We say X_n convergence to a random variable X in probability if and only if $\forall \epsilon > 0$*

$$P(|X_n - X| > \epsilon) \rightarrow 0.$$

Definition 2. *Convergence in Distribution Given a sequence of random variables X_n . We say X_n convergence to a random variable X in distribution if and only if PDF/PMF/CDF of X_n convergence to PDF/PMF/CDF of X .*

Problem 1. *Let U_1, \dots, U_n be random variables. U_n has distribution $\text{Exp}(n)$. Claim U_n convergence to 0 in probability.*

Problem 2. *Let U_1, \dots, U_n be independent $\text{Unif}(0,1)$ random variables, $X_{(n)} := \max\{U_1, \dots, U_n\}$. Prove that $n(1 - X_{(n)})$ convergence in distribution to $\text{Exp}(1)$.*

Problem 3. *Let X_1, \dots, X_{100} be independent $\text{Poisson}(1)$ random variables. Use normal approximation to approximate the probability that $\sum_{i=1}^{100} X_i$ is between 90 and 110.*