

What is union-find?

Union-Find is a data structure with two key operations, as its name suggests: union and find

- `union(x, y)` — Merge two elements into the same group. For example, in the island problem on LeetCode, we can merge adjacent pieces of land into one island.
- `find(x)` — Quickly find which group (or "leader") a particular element belongs to. It tells you the "root" or representative of the group.

Two classic optimizations:

- **Path Compression:** When searching for a representative of an element, all points on the search path are directly linked to the root node, making subsequent searches faster.
- **Union by Rank (or Size):** When merging two trees, always attach the shorter tree under the taller one. This keeps the trees shallow, improving performance.

With both optimizations, each operation becomes nearly constant time $O(\alpha(n))$, where α is the inverse Ackermann function.

Starting from the “simplest idea”

Method 1: use a dictionary to map group IDs to their members

```
group_dict = {
    1: [0, 3, 5],
    2: [1, 4],
    3: [2]
}
```

scan the whole dictionary:

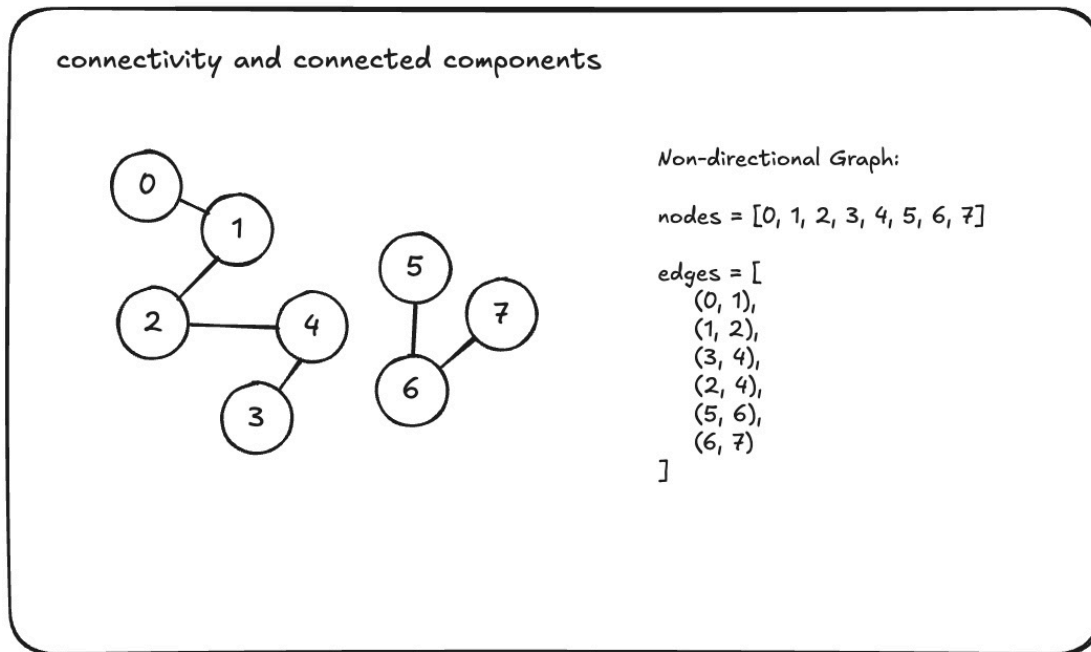
```
for group_id, members in group_dict.items():
    if 3 in members:
        print(group_id)
```

Method 2:

```
belong = [0, 0, 1, 0, 2, 1]
```

Understand Process

Connectivity & Connected Component



We want to know how to use code to:

1. Group these nodes

→ find your biggest boss

2. Check if node a and node b are connected?

How many connected components are in the graph?

→ if biggest boss is the same, these 2 persons are belong to the same gang

What is Connectivity?

In an **undirected graph**, two nodes are **connected** if there's a path between them — even if the path goes through multiple jumps. If nodes can reach each other, we say they belong to the same **connected region**.

What is a Connected Component?

A **connected component** is a group of nodes that are all reachable from one another. A graph may have several components — like islands, isolated groups, or subnetworks.

In real problems, we often care about two main things:

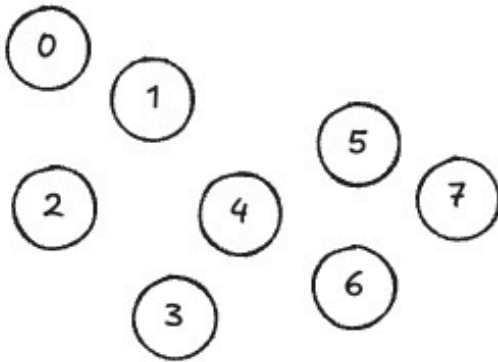
1. **How to group connected nodes together**
2. **How to check if two nodes belong to the same group (component)**

Explain Codes

Step 1: Initialization

Union-Find build

step1: initialize the boss of each person is itself



`boss = []`
one array to store each person's boss

`boss[i]` means person `i`'s boss

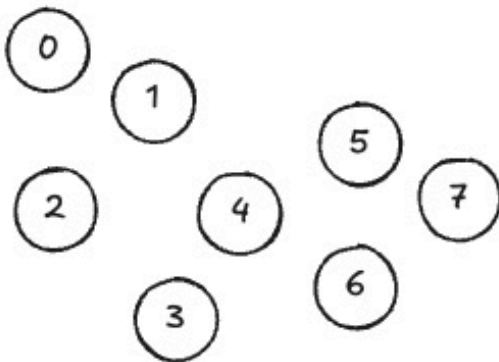
`boss[i] = i`
initialize the boss as person `i` itself

`boss = [0, 1, 2, 3, 4, 5, 6, 7]`

Step 2: The Find Operation

Union-Find build

step2: help person x to find its organization?



```
def find(x):  
    if boss[x] == x:  
        return x  
    return find(boss[x])
```

Optimization

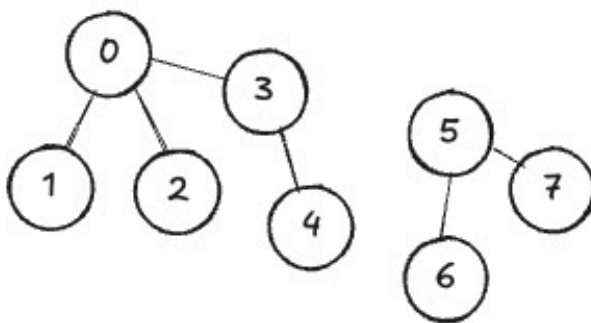
```
def find(x):  
    if boss[x] != x:  
        boss[x] = find(boss[x])  
    return boss[x]
```

`boss = [0, 1, 2, 3, 4, 5, 6, 7]`

Step 3: The Union Operation

Union-Find build

step3: employee merger



```
def union(x, y):  
    rootX = find(x)  
    rootY = find(y)  
    if rootX != rootY:  
        boss[rootX] = rootY
```

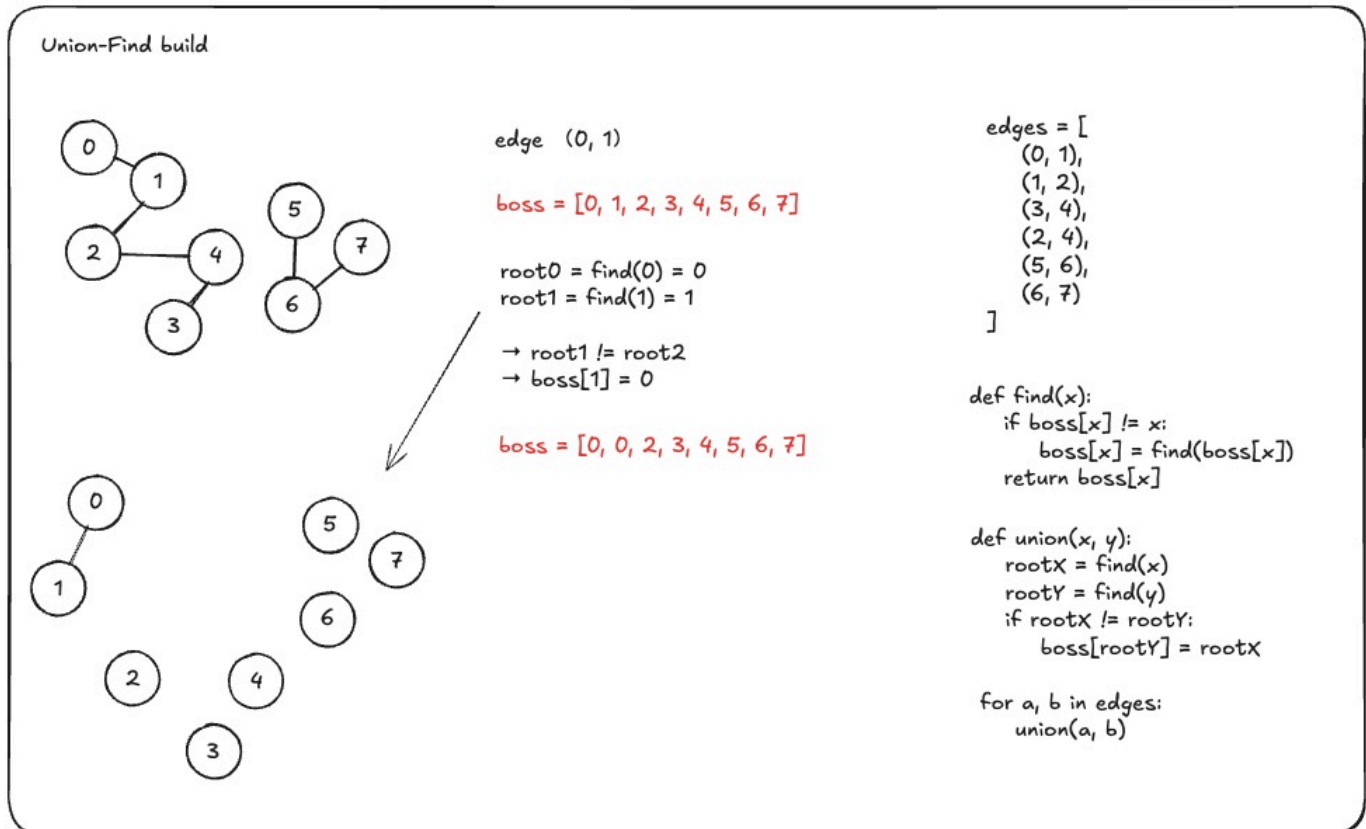
```
for a, b in edges:  
    union(a, b)
```

`boss = [0, 0, 0, 0, 3, 5, 5, 5]`

```
edges = [  
    (0, 1),  
    (1, 2),  
    (3, 4),  
    (2, 4),  
    (5, 6),  
    (6, 7)  
]
```

How to build Union-Find?

Let's walk through a concrete example.



Step 1: No Connections Yet

Initially, all nodes are their own bosses: Each node is its own little company. `boss[i] = i`.

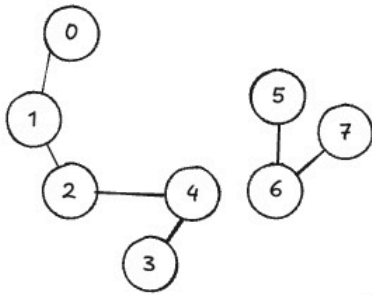
Step 2: Connect (0, 1)

`find(0) = 0`, `find(1) = 1` → different roots

Merge: make 1 report to 0 → `boss[1] = 0`

We got a new array.

Union-Find build



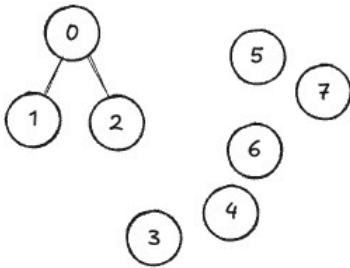
edge (1, 2)

`boss = [0, 0, 2, 3, 4, 5, 6, 7]`

`find(1) → boss[1]=0 → find(0)=0`
`find(2) = 2`

→ `root1 != root2`
 → `boss[2] = 0`

`boss = [0, 0, 0, 3, 4, 5, 6, 7]`



```
edges = [
    (0, 1),
    (1, 2),
    (3, 4),
    (2, 4),
    (5, 6),
    (6, 7)
]
```

```
def find(x):
    if boss[x] != x:
        boss[x] = find(boss[x])
    return boss[x]
```

```
def union(x, y):
    rootX = find(x)
    rootY = find(y)
    if rootX != rootY:
        boss[rootY] = rootX
```

```
for a, b in edges:
    union(a, b)
```

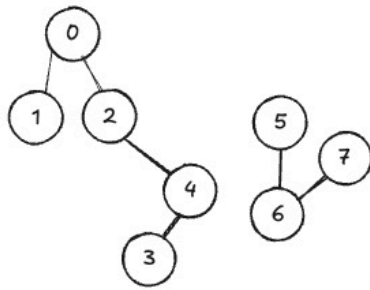
Step 3: Connect (1, 2)

- `find(1) → boss[1] = 0`, and `find(0) = 0` → root is 0
- `find(2) = 2`

Merge: `boss[2] = 0`

Now we've merged 0, 1, and 2 into one group: `boss = [0, 0, 0, 3, 4, 5, 6, 7]`.

Union-Find build



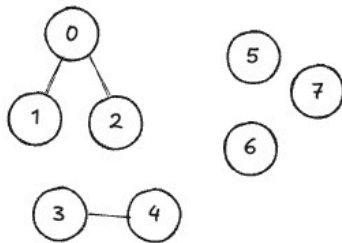
edge (3, 4)

`boss = [0, 0, 0, 3, 4, 5, 6, 7]`

`find(3) = 3`
`find(4) = 4`

→ `root3 != root4`
 → `boss[4] = 3`

`boss = [0, 0, 0, 3, 3, 5, 6, 7]`



```

edges = [
    (0, 1),
    (1, 2),
    (3, 4),
    (2, 4),
    (5, 6),
    (6, 7)
]
  
```

```

def find(x):
    if boss[x] != x:
        boss[x] = find(boss[x])
    return boss[x]
  
```

```

def union(x, y):
    rootX = find(x)
    rootY = find(y)
    if rootX != rootY:
        boss[rootY] = rootX
  
```

```

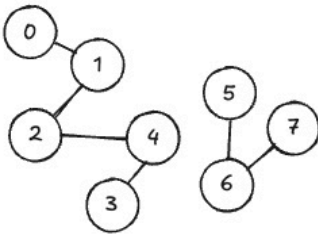
for a, b in edges:
    union(a, b)
  
```

Step 4: Connect (3, 4)

- `find(3) = 3`
- `find(4) = 4`

Merge: `boss[4] = 3`

Union-Find build



edge (2, 4)

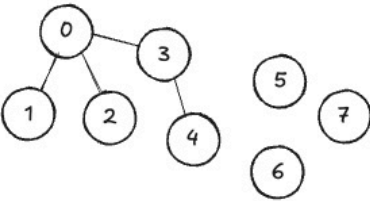
boss = [0, 0, 0, 3, 3, 5, 6, 7]

find(2) → boss[2]=0 → find(0)=0

find(4) → boss[4]=3 → find(3)=3

→ root2 != root4 → boss[3] = 0

boss = [0, 0, 0, 0, 3, 5, 6, 7]



```

edges = [
    (0, 1),
    (1, 2),
    (3, 4),
    (2, 4),
    (5, 6),
    (6, 7)
]

```

```

def find(x):
    if boss[x] != x:
        boss[x] = find(boss[x])
    return boss[x]

```

```

def union(x, y):
    rootX = find(x)
    rootY = find(y)
    if rootX != rootY:
        boss[rootY] = rootX

```

```

for a, b in edges:
    union(a, b)

```

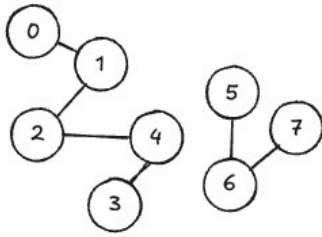
Step 5: Connect (2, 4)

- find(2) → 0
- find(4) → boss[4]=3 → find(3) = 3 → root is 3

Merge: boss[3] = 0 → now 3 and 4 are under 0 too

This subgraph already has 5 members connected together.

Union-Find build



edge (5, 6)

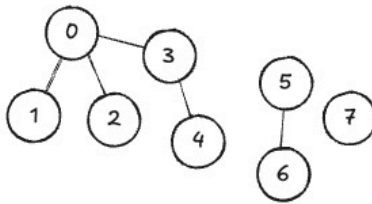
`boss = [0, 0, 0, 0, 3, 5, 6, 7]`

`find(5) = 5`

`find(6) = 6`

$\rightarrow \text{root5} \neq \text{root6} \Rightarrow \text{boss}[6] = 5$

`boss = [0, 0, 0, 0, 3, 5, 5, 7]`



```
edges = [
    (0, 1),
    (1, 2),
    (3, 4),
    (2, 4),
    (5, 6),
    (6, 7)
]
```

```
def find(x):
    if boss[x] != x:
        boss[x] = find(boss[x])
    return boss[x]
```

```
def union(x, y):
    rootX = find(x)
    rootY = find(y)
    if rootX != rootY:
        boss[rootY] = rootX
```

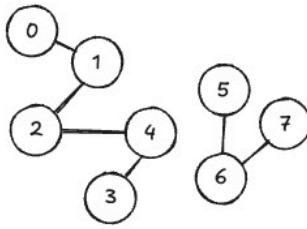
```
for a, b in edges:
    union(a, b)
```

Step 6: Connect (5, 6)

- `find(5) = 5`
- `find(6) = 6`

Merge: `boss[6] = 5` \rightarrow now 5 and 6 are under 5

Union-Find build



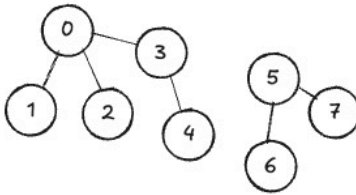
edge (6, 7)

`boss = [0, 0, 0, 0, 3, 5, 5, 7]`

`find(6) → boss[6]=5 → find(5)=5`
`find(7) = 7`

`→ root6 != root7 ⇒ boss[7] = 5`

`boss = [0, 0, 0, 0, 3, 5, 5, 5]`



```
edges = [
    (0, 1),
    (1, 2),
    (3, 4),
    (2, 4),
    (5, 6),
    (6, 7)
]
```

```
def find(x):
    if boss[x] != x:
        boss[x] = find(boss[x])
    return boss[x]
```

```
def union(x, y):
    rootX = find(x)
    rootY = find(y)
    if rootX != rootY:
        boss[rootY] = rootX
```

```
for a, b in edges:
    union(a, b)
```

Step 7: Connect (6, 7)

- `find(6) → boss[6]=5 → the root 5`
- `find(7) = 7`

Merge → `boss[7] = 5` → now 5, 6 and 7 are in the same group

Final structure: `boss = [0, 0, 0, 0, 3, 5, 5, 5]`

We now have two major connected components:

- Group represented by 0: includes nodes 0, 1, 2, 3, 4
- Group represented by 5: includes nodes 5, 6, 7

When to use it?

- Does the problem talk about **groups**, **regions**, or **connectivity**?
- Are we checking whether two things are in the same group?
- Are we merging groups or building connections dynamically?
- Are there a lot of queries like “are these connected?”
- Do the keywords include: **undirected edges**, **components**, **connectivity**?

