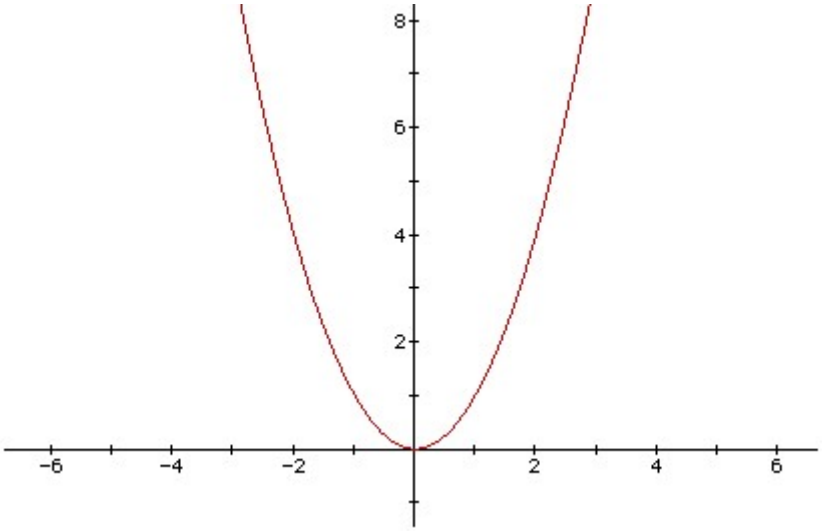
加分題: Problem 1 & Problem 2

Problem 1:
Josephina is a clever girl and addicted to Machine Learning recently. She pays much attention to a method called Linear Discriminant Analysis, which has many interesting properties.

In order to test the algorithm's efficiency, she collects many datasets. What's more, each data is divided into two parts: training data and test data. She gets the parameters of the model on training data and test the model on test data.

To her surprise, she finds each dataset's test error curve is just a parabolic curve. A parabolic curve corresponds to a quadratic function. In mathematics, a quadratic function is a polynomial function of the form $f(x) = ax^2 + bx + c$. The quadratic will degrade to linear function if $a = 0$.




It's very easy to calculate the minimal error if there is only one test error curve. However, there are several datasets, which means Josephina will obtain many parabolic curves. Josephina wants to get the tuned parameters that make the best performance on all datasets. So she should take all error curves into account, i.e., she has to deal with many quadric functions and make a new error definition to represent the total error. Now, she focuses on the following new function's minimal which related to multiple quadric functions.

The new function $F(x)$ is defined as follow:

$$F(x) = max(S_i(x)),\ i = 1...n.$$ The domain of $x$ is [0, 1000]. $S_i(x)$ is a quadric function.

Josephina wonders the minimum of $F(x)$. Unfortunately, it's too hard for her to solve this problem. As a super programmer, can you help her?

Input

The input contains multiple test cases. The first line is the number of cases $T$ ($T < 100$). Each case begins with a number $n$($n \le 10000$). Following $n$ lines, each line contains three integers $a$ ($0 \le a \le 100$), $b$ ($|b| \le 5000$), $c$ ($|c| \le 5000$), which mean the corresponding coefficients of a quadratic function.

## Output

For each test case, output the answer in a line. Round to 4 digits after the decimal point.

## Sample Input

```
2
1
2 0 0
2
2 0 0
2 -4 2
```

## Sample Output

```
0.0000
0.5000
```

n=1, F(x) 是一條拋物線，怎麼求最小值?
注意到 a>=0, 所以可能退化成一直線或者成為開口向上的拋物線. 後者是標準的下凸函數，直線也可視為下凸，所以 n=1 時 可視為下凸函數求極值 .

n=2 or n>2 圖形仍然是下凸. 下凸函數求極值可以使用三分法(ternary search) 求解.
具體方法如下:
取區間[L,R]的兩個三分點 m1 and m2. 比較 F(m1) and F(m2)大小

Ternary Search:
Let a unimodal(單峰函數) function $f(x)$ on some interval $[l; r]$. Take any two points $m1$ and $m2$ in this segment: $l < m1 < m2 < r$. Then there are three possibilities:

- if $f(m1) < f(m2)$, then the required maximum can not be located on the left side - $[l; m1]$. It means that the maximum further makes sense to look only in the interval $[m1;r]$
- if $f(m1) > f(m2)$, that the situation is similar to the previous, up to symmetry. Now, the required maximum can not be in the right side - $[m2; r]$, so go to the segment $[l; m2]$
- if $f(m1) = f(m2)$, then the search should be conducted in $[m1; m2]$, but this case can be attributed to any of the previous two (in order to simplify the code). Sooner or later the length of the segment will be a little less than a predetermined constant, and the process can be stopped.

choice points *m1* and *m2*:

- *m1* = *l* + (*r*-*l*)/3
- *m2* = *r* - (*r*-*l*)/3

Note1: Ternary Search 不僅適用在凸函數，還是用在單峰函數 unimodalfunction.
所謂單峰函數就是先嚴格遞增再嚴格遞減(此時存在唯一的最大值)或者先嚴格遞減再嚴格遞增的函數(此時存在唯一的最大值)

Note2: 注意上面的 100 次反覆運算上限是實驗出來的. 比賽時可以多嘗試幾個值.
如果答案不正確，則加大反覆運算次數;

Problem 2:

有 $n$ 個竹筷，$3 \le n \le 50$ ，其長度分別為 $\ell_1, \ell_2, \ldots, \ell_n$ ，其中 $\ell_i$ ，$1 \le i \le n$ ，為大於 0 的實數。現在請試著以這 $n$ 個竹筷，圍成一“$n$ 邊”多邊形。

請設計一程式
(a) 判斷這 $n$ 個竹筷是否能圍成一 $n$ 邊多邊形？
註：不可將竹筷折斷，並且竹筷不可跨接，只能竹筷的頂端連接另一竹筷的頂端
(b) 若能圍成一 $n$ 邊多邊形，則此 $n$ 邊多邊形的最大面積是多少？
**(誤差的絕對值在 1%之內，即算正確)**

**注意: 本題評分時的執行時間不得超過 10 秒。**

# 提示:

**多邊形版等周問題(Polygonal Isoperimetric Problem):**
**(a) 邊長給定的 $n$ 邊多邊形，當有外接圓時，其面積最大。**
**(b) 也就是頂點共圓時，其面積最大。**

## Input(輸入資訊):

請由檔案 **file1.txt** 讀入每一組竹筷資料。 檔案內可能包含多組竹筷，最後一組的下一列(line)是 0 代表結束。
每一組竹筷的資料包括兩列:
(a) 第一列為 $n$
(b) 第二列為用空白隔開的 $\ell_1$， $\ell_2$， …， $\ell_n$

**Output(輸出資訊) :**

**請從標準輸出裝置(Standard Output;螢幕)輸出.**
假如這組竹筷不能圍成一 *n* 邊多邊形，則輸出 0
否則輸出該組資料所能圍成 *n* 邊多邊形的最大面積

# Sample Input

**4**
**6.0 7.0 8.0 9.0**
**5**
**2.0 3.0 7.0 10.0 4.0**
**7**
**6.0 9.0 3.0 8.0 32.0 2.0 1.0**
**3**
**1.0 1.0 1.732**
**12**
**2.0 6.0 4.0 3.0 8.0 1.0 2.0 30.0 5.0 4.0 7.0 3.0**
**0**

# Sample Output

**54.990916**

**36.783490**

**0.00000**

**0.433038**

**317.646635**