CSC2/458 Parallel and Distributed Systems Automated Parallelization in Software (Contd.)

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URCS

Outline

Characterizing loop dependences

Identifying Loop Dependences

Current Loop Optimizations

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Identifying Loop Dependences

Current Loop Optimizations

Why characterize dependences?

- The definition of dependence that we have used so far:
  - Two statements have a dependence if:
  - Both access same location (memory or register)
  - And one of the accesses is a write
- This is not sufficient to reason about dependences in loops
- We will extend this definition of dependences to *loop dependence*
  - Study additional characteristics of dependences

Already encountered characteristics of dependences

- True dependence
  - $S_1 \delta S_2$
  - S_1 writes, S_2 reads
- Anti-dependence
  - $S_1 \delta^{-1} S_2$
  - S_1 reads, S_2 writes
- Output dependence
  - $S_1 \delta^o S_2$
  - Both S_1 and S_2 write

Loop-independent dependence

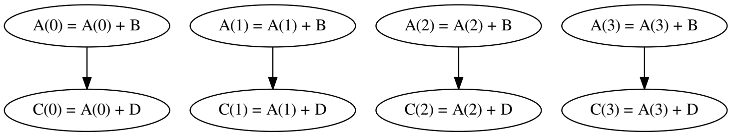
- What are the dependences in the loop body below?
- Can you change the order of the statements in the loop body?

```
DO I = 1, 10
  A(I) = A(I) + B
  C(I) = A(I) + D
ENDDO
```

- Can you change the (execution) order of loop iterations?

Note: FORTRAN uses parentheses in array references: e.g., `A(I)`

Loop-independent dependences visualized



NOTE: Only dependences from first four iterations visualized.

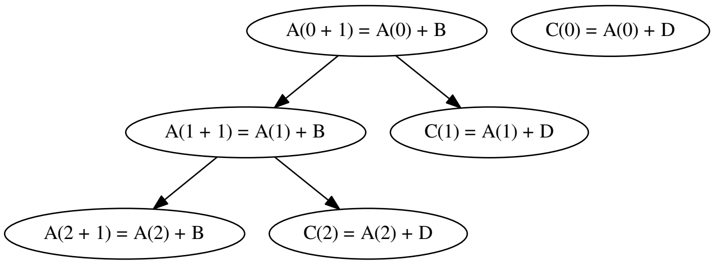
Loop-carried dependences

- What are the dependences in the loop body below?
- Can you change the order of the statements in the loop body?

```
DO I = 1, 10
  A(I + 1) = A(I) + B
  C(I) = A(I) + D
ENDDO
```

- Can you change the (execution) order of loop iterations?

Loop-carried dependences visualized



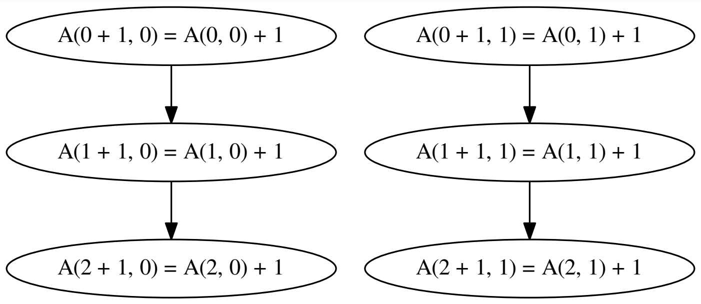
NOTE: Only dependences from first three iterations visualized.

Dependence Level for Loop-Carried Dependences

```
DO I = 1, 10
   DO J = 1, 2
      A(I + 1, J) = A(I, J) + 1
   ENDDO
ENDDO
```

- Can you change the order of inner loop?
- Can you change the order of the outer loop?

Dependences Visualized



NOTE: Only dependences from first three iterations visualized.

Loop Dependences

- Loop-independent dependence
  - In same iteration, independent of loops
- Loop-carried dependence
  - Across different iterations of atleast one loop
- Dependence Level of a Loop-carried Dependence
  - The nesting level k of loop that carries the dependence
  - $S_1 \delta_k S_2$

Iteration Spaces

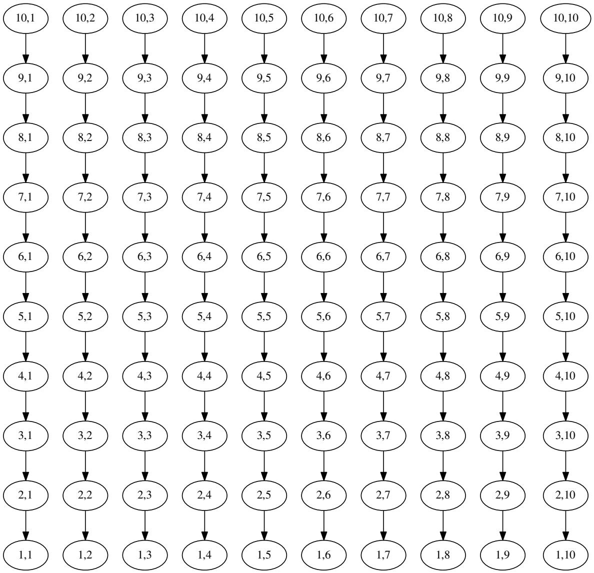
```
DO I = 1, 2
   DO J = 1, 2
      S
   ENDDO
ENDDO
```

- `S` has four instances (I, J): $(1,1), (1,2), (2,1), (2,2)$
- Each of these values represents an *iteration vector*
    - Particular values of loop indices from outermost loop to innermost loop

Iteration Space Example

```
DO J = 1, 10
  DO I = 1, 10
   A(I+1, J) = A(I, J) + X
  ENDDO
ENDDO
```

Iteration Space Figure



Iteration Vector Ordering

For two vectors i and j, each containing n elements, $i < j$ is defined as:

```
def lessthan(i, j, n):
    if n == 1:
        return i[0] < j[0]

    # test prefix for elementwise-equality
    if i[0:n-1] == j[0:n-1]:
        return i[n-1] < j[n-1]
    else:
        return lessthan(i, j, n-1)
```

Can similarly define other order relations.

Loop dependence

Statement S1 (source) depends on statement S2 (sink) if:

- There exist iteration vectors i and j such that $i < j$ or $i = j$
- There is a path from S1 to S2 in the loop
- S1 accesses memory M in iteration i
- S2 accesses memory M in iteration j
- and one of the accesses is a write

Distance Vectors

$$d(i,j)_k = j_k - i_k$$

- Where $i, j, d(i,j)$ are n-element vectors
- i_k indicates k-th element of i

Example distance vector: $(0, 1)$

Direction Vectors

$D(i, j)_k =$

- $"<"$, if $d(i, j)_k > 0$
- $"="$, if $d(i, j)_k = 0$
- $">"$, if $d(i, j)_k < 0$

Example direction vector for $(0, 1)$: $(=, <)$

Information we need to track

For every pair of memory references:

- Iteration Vectors i and j which have a dependence, or
- Unique Distance Vectors $d(i, j)$, or
- Unique Direction Vectors $D(i, j)$

Test

- Which of these indicates a loop-independent dependence?
  - $(=,=)$
  - $(=,<)$
- Of the loop-carried dependence in example above, what level is the loop-carried dependence?

Theorems

WARNING: Informal language

- Direction Vector Transform (Theorem 2.3 in AK)
  - If a transformation reorders loop iterations, and preserves the leftmost non-"$=$" component as "$<$", all dependences are preserved.
- Theorem 2.4 in AK
  - If a level-k dependence exists, and a transformation reorders loop iterations while not reordering the level-k loop
  - And does not move loops inside k outside the loop and vice versa
  - It preserves all level-k dependences.
- Iteration Reordering (Theorem 2.6 in AK)
  - Iterations of a level k loop can be reordered if there is no level k dependence.

Outline

Characterizing loop dependences

Identifying Loop Dependences

Current Loop Optimizations

Generalizing Loop Indices

```
DO I_1 = ...
   DO I_2 = ...
      ...
      DO I_N = ...
         A(f1, f2, f3, ..., fM) = ...
         ... = A(g1, g2, g3, ..., gM)
      ENDDO
   ENDDO
ENDDO
```

where A is M-dimensional array, and fX and gX are *index functions* of the form

- fX(I_1, I_2, ..., I_N)
- gX(I_1, I_2, ..., I_N)
- $1 <= X <= M$

Dependence using Iteration Vectors

Let α and β be iteration vectors:

- $\alpha = (i_1, i_2, i_3, ..., i_N)$
- $\beta = (i'_1, i'_2, i'_3, ..., i'_N)$

Then a dependence exists if:

- (vectors) $\alpha < \beta$
- $fX(\alpha) = gX(\beta)$, for $1 <= X <= M$

Example

```
DO J = 1, 10
  DO I = 1, 10
   A(I+1, J) = A(I, J) + X
  ENDDO
ENDDO
```

- $f1(J, I) = I + 1$, $f2(J, I) = J$
- $g1(J, I) = I$, $g2(J, I) = J$
- For $\alpha = (0, 0)$ (i.e. $J = 0, I = 0$) and $\beta = (0, 1)$ (i.e. $J = 0, I = 1$):
  - $f1(\alpha) = g1(\beta)$, i.e. $1 = 1$
  - $f2(\alpha) = g2(\beta)$, i.e. $0 = 0$
  - Many other values of α and β also satisfy these equations.

Dependence Testing

Do iteration vectors α and β exist such that:

- (vectors) $\alpha < \beta$
- $fX(\alpha) = gX(\beta)$, for $1 <= X <= M$

How can we find α and β if they exist?

Restrictions on Index functions

- fX and gX must be decidable
- fX and gX must be "analyzable"
  - to avoid brute force search
- fX and gX must be a linear functions of loop indices:
  - i.e. for $fX(i_1, i_2, i_3, ..., i_N)$
  - $fX = a_1 i_1 + a_2 i_2 + ... + a_n i_n + e$
  - e is optional loop invariant calculation

Dependence Testing on Restricted Index Functions

- Given that *fX* and *gX* are linear functions of loop indices
- Do iteration vectors α and β exist such that:

- (vectors) $\alpha < \beta$
- $fX(\alpha) = gX(\beta)$, for $1 <= X <= M$

How can we find α and β if they exist?

What is this problem better known as?

Dependence Testing

- Integer Linear Programming is NP-complete
- Lots of heuristics invented
  - Profitable to know if no solution exists since it implies no dependence!
  - See Chapter 3 of AK (we will not cover this in this course, take CSC 2/455)

Outline

Current Focus in Compilers

- GCC begin supporting vectorization for C around 4.9
  - `-ftree-vectorize` or `-O3`
  - Can get it to tell you why vectorization failed.
- LLVM also supports vectorization
  - See "Polly" at `http://polly.llvm.org`

More focus on optimization by loop transformation

- More emphasis on *Scheduling*
    - Which iteration of loop executes where
- Classical loop transformations
    - Loop tiling
    - Loop fusion
    - etc.
- Unifying theory and infrastructure
    - polyhedral.info