1 Algorithm Analysis

1.1 Common Summation Formulae

Sum of a power of integers — From the big picture perspective, the important thing is that the sum is quadratic, not that the constant is $\frac{1}{2}$. In general,

$$S(n,p) = \sum_{i=1}^{n} i^p = \Theta(n^{p+1})$$

for $p \geq 0$. Thus, the sum of squares is cubic, and the sum of cubes is quartic.

For $p < -1$, this sum $S(n,p)$ always converges to a constant as $n \to \infty$, while for $p \geq 0$ it diverges. The interesting case between these is the harmonic numbers, $H(n) = \sum_{i=1}^{n} \frac{1}{i} = \Theta(logn)$.

Sum of a geometric progression — In geometric progressions, the index of the loop affects the exponent, that is,

$$G(n,a) = \sum_{i=0}^{n} a^i = \frac{(a^{n+1} - 1)}{(a - 1)}$$

How we interpret this sum depends upon the *base* of the progression, in this case a. When $|a| < 1, G(n,a)$ converges to a constant as $n \to \infty$.

This series convergence proves to be the great "free lunch" of algorithm analysis. It means that the sum of a linear number of things can be constant, not linear. For example, $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots \leq 2$ no matter how many terms we add up.

When $a > 1$, the sum grows rapidly with each new term, as in $1 + 2 + 4 + 8 + 16 + 32 = 63$. Indeed, $G(n,a) = \Theta(a^{n+1})$ for $a > 1$.