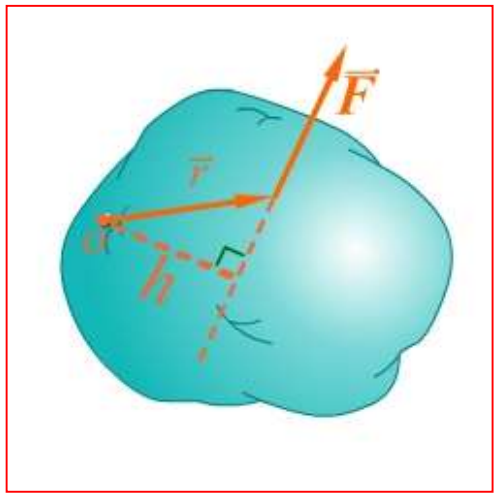
上节课内容回顾

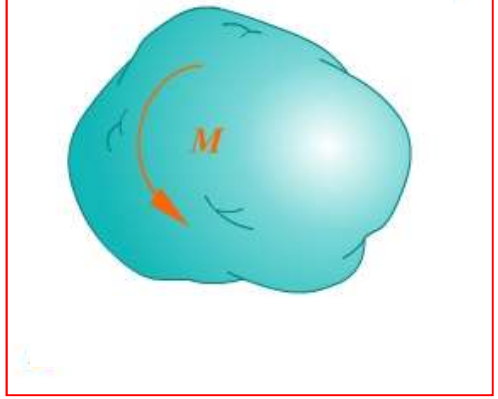
力矩



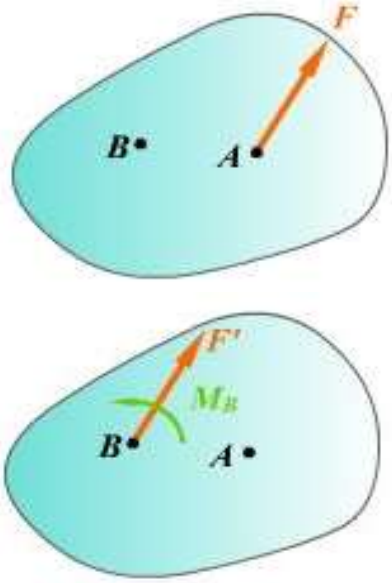
力偶

力偶矩

平面力偶系合成



2. 力的平移定理



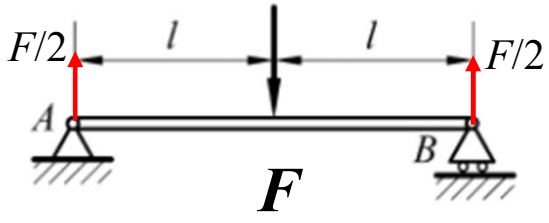
力在刚体内不沿着作用线移动，都会产生力偶

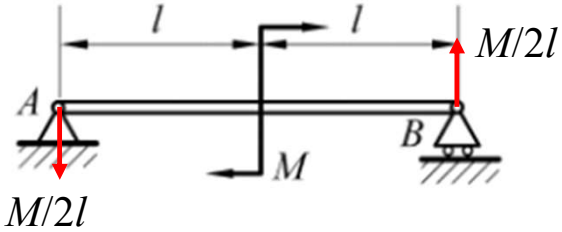
-因为力与力偶独立，所以平移力只能产生力偶

力与力偶是两个独立的静力学要素：

力偶是一对等值方向不共线的平行力

力只能由力平衡，力偶只能由力偶平衡





上节课内容回顾

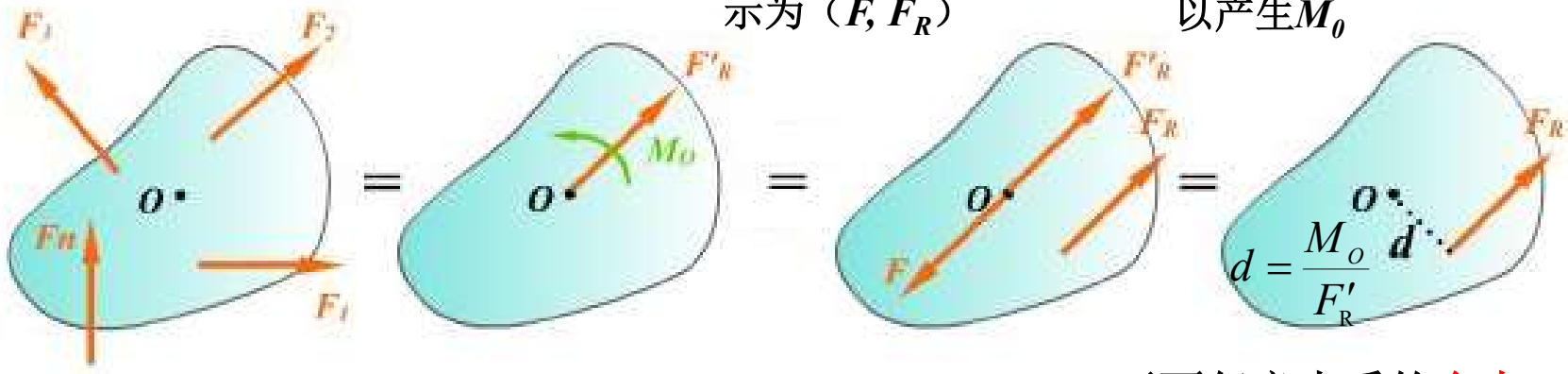
平面任意力系的简化

主矢、主矩→合力

力的平移定理

1. 把力偶M_0表示为（F, F_R）

2. F_R平移到O点可以产生M_0



合力矩定理：平面任意力系的合力对作用面内任一点的矩等于力系中各力对同一点的矩代数和

平面任意力系的合力F_R

$$M_B(\bar{F}_R) = M_B = \sum M_B(\bar{F}_i)$$

平面任意力系的平衡

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_O = 0 \end{cases}$$

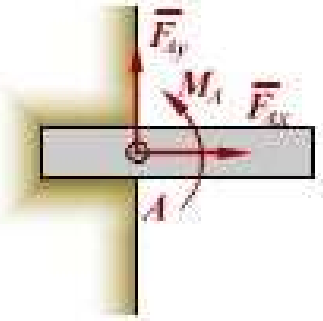
刚体系静力学平衡问题：

多刚体：连接，列多组平衡方程

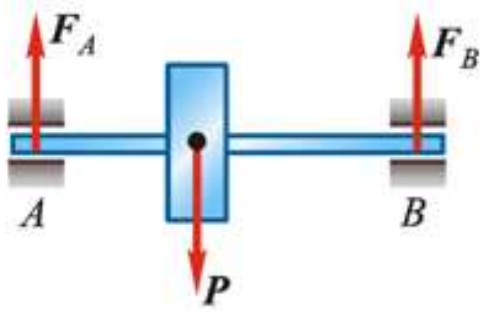
约束：固定/滚动支座，固定端，绳

主动力：集中力、力偶、分布力

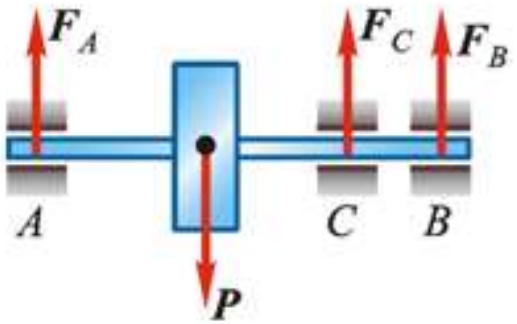
平面固定端约束



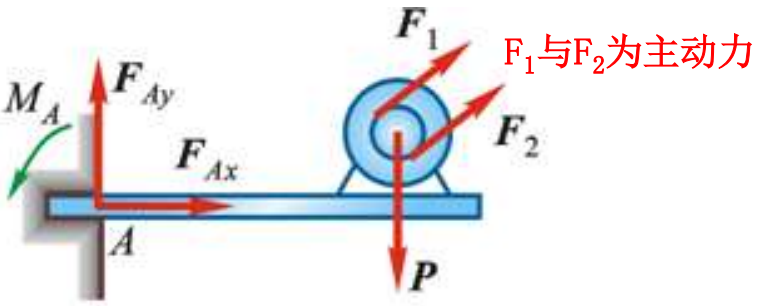
§2-5 物体系的平衡·静定和超静定问题



n=1, 2个未知约束力
因为平面平行力系，水平方向力平衡天然满足，只能列两个平衡方程
静定 (2个约束力）

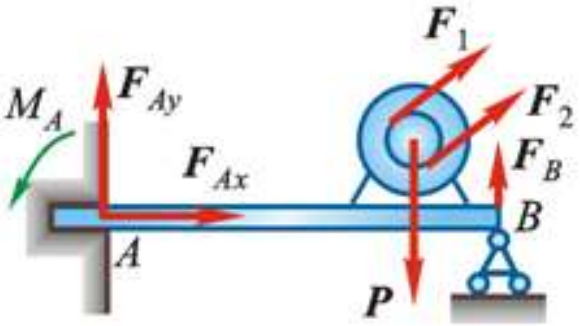


n=1, 3个未知约束力
因为平面平行力系，水平方向力平衡天然满足，只能列两个平衡方程
超静定 (3个约束力）



F_1与F_2为主动力

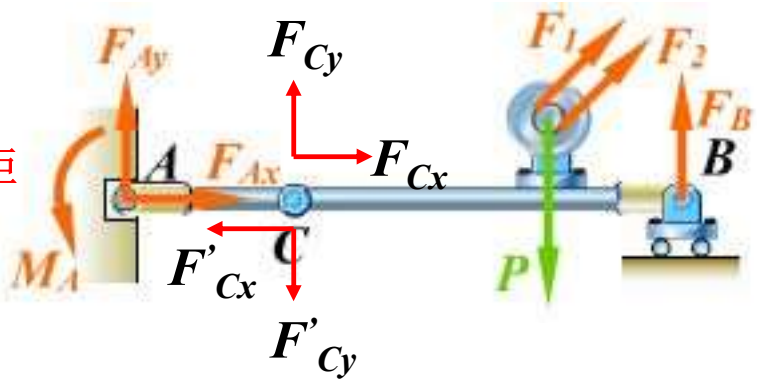
n=1, 3个未知约束力
因为平面任意力系，只能列三个平衡方程
静定 (3个约束力/力偶）



n=1, 4个未知约束力
因为平面任意力系，只能列三个平衡方程
超静定 (4个约束力/力偶）

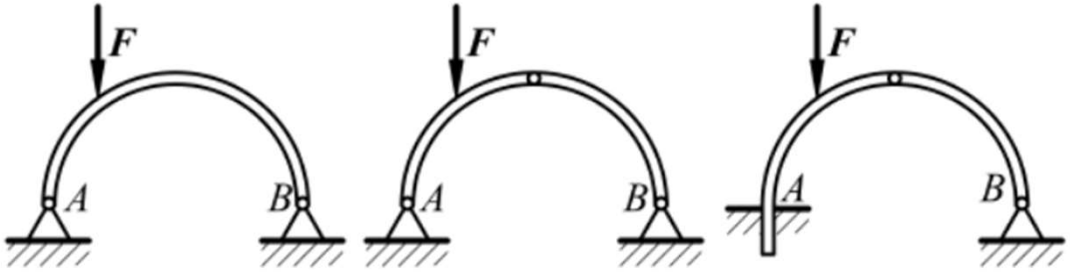
§2-5 物体系的平衡·静定和超静定问题

几个刚体？
2（两点距离不变）



n=2, 6个未知约束力 (铰链C连接）
因为平面任意力系力系，每个刚体能列3个平衡方程，两个刚体
静定 (6个约束力：A3个，C2个，B1个）

练习：判断下列结构是否属于静定结构

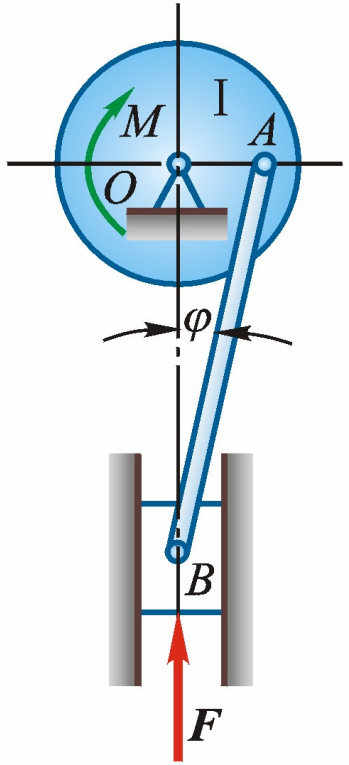


超静定	静定	超静定
n=1, A2个，B2个 3*n*<4	*n*=2, A2个，B2个， 铰链2个 3*n*=6	*n*=2, A3个，B2个， 铰链2个 3*n*<7

§2-5 物体系的平衡·静定和超静定问题

例2-16 已知：$OA=R, AB=l, \vec{F}$, 不计物体自重与摩擦, 系统在图示位置平衡；

求：力偶矩M的大小，轴承O处的约束力，连杆AB受力，冲头给导轨的侧压力.

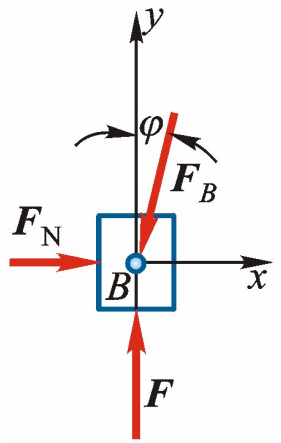


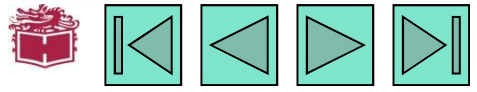
解： 取冲头B, 画受力图.

$$\sum F_y = 0 \qquad F - F_B\cos\varphi = 0$$

$$\sum F_x = 0 \qquad F_N - F_B\sin\varphi = 0$$

$$F_B = \frac{F}{\cos\varphi} = \frac{Fl}{\sqrt{l^2-R^2}} \qquad F_N = F\tan\varphi = \frac{FR}{\sqrt{l^2-R^2}}$$





§2-5 物体系的平衡·静定和超静定问题

取轮, 画受力图.

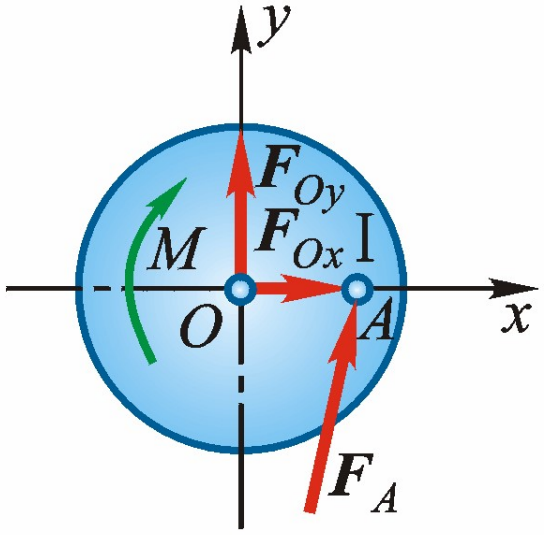
$$\sum F_x = 0 \qquad F_{Ox} + F_A \cos\varphi = 0$$

$$\sum F_y = 0 \qquad F_{Oy} + F_A \sin\varphi = 0$$

$$\sum M_O = 0 \qquad F_A \cos\phi \cdot R - M = 0$$

$$F_{Ox} = -\frac{FR}{\sqrt{l^2 - R^2}} \qquad F_{Oy} = -F$$

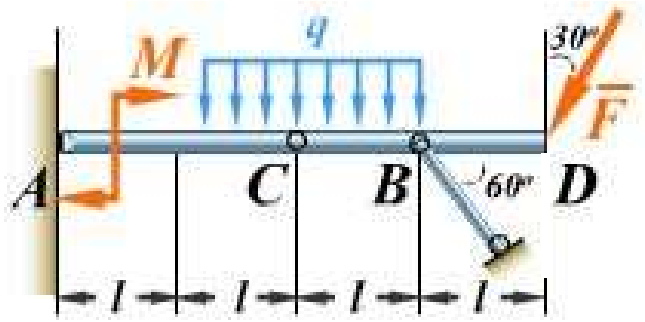
$$M = FR$$



例2-17（分布力跨越连接处）

已知：F=20kN, q=10kN/m, $M=20\text{kN}\cdot\text{m}$, l=1m;

求：A,B 处的约束力.



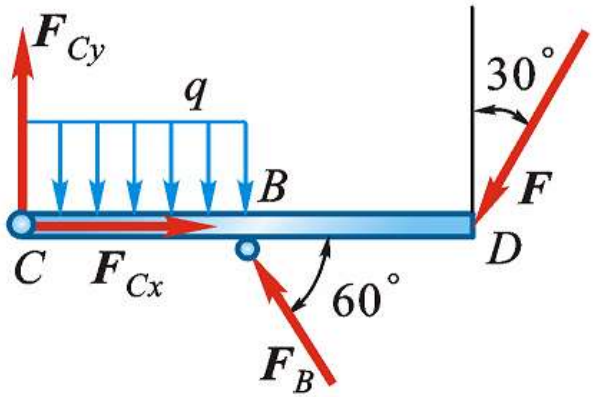
解：取CD梁, 画受力图.

$$\sum M_C = 0$$

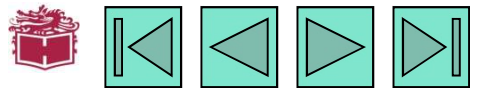
$$F_B \sin 60^\circ \cdot l - ql \cdot \frac{l}{2} - F\cos 30^\circ \cdot 2l = 0$$

$$\Rightarrow \quad F_B = 45.77\text{kN}$$

$$\sum F_x = 0, \sum F_y = 0 \rightarrow F_{Cy}, F_{Cx}$$

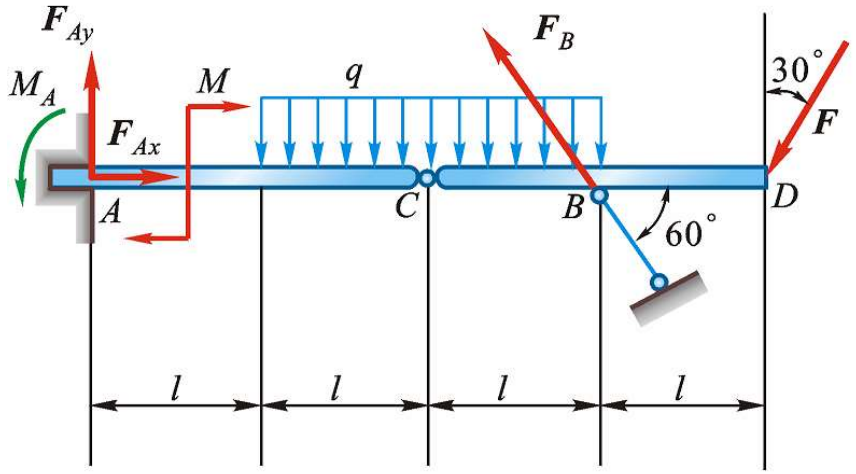


选择未知约束力多的点列力矩平衡方程，分布力只考虑在分离的刚体部分



§2-5 物体系的平衡·静定和超静定问题

取整体, 画受力图.



$$\sum F_x = 0$$

$$F_{Ax} - F_B \cos 60^\circ - F \sin 30^\circ = 0$$

$$\sum F_y = 0$$

$$F_{Ay} - F_B \sin 60^\circ - 2ql - F \cos 30^\circ = 0$$

选择整体时候，不用考虑C处连接（内力）

$$\sum M_A = 0$$

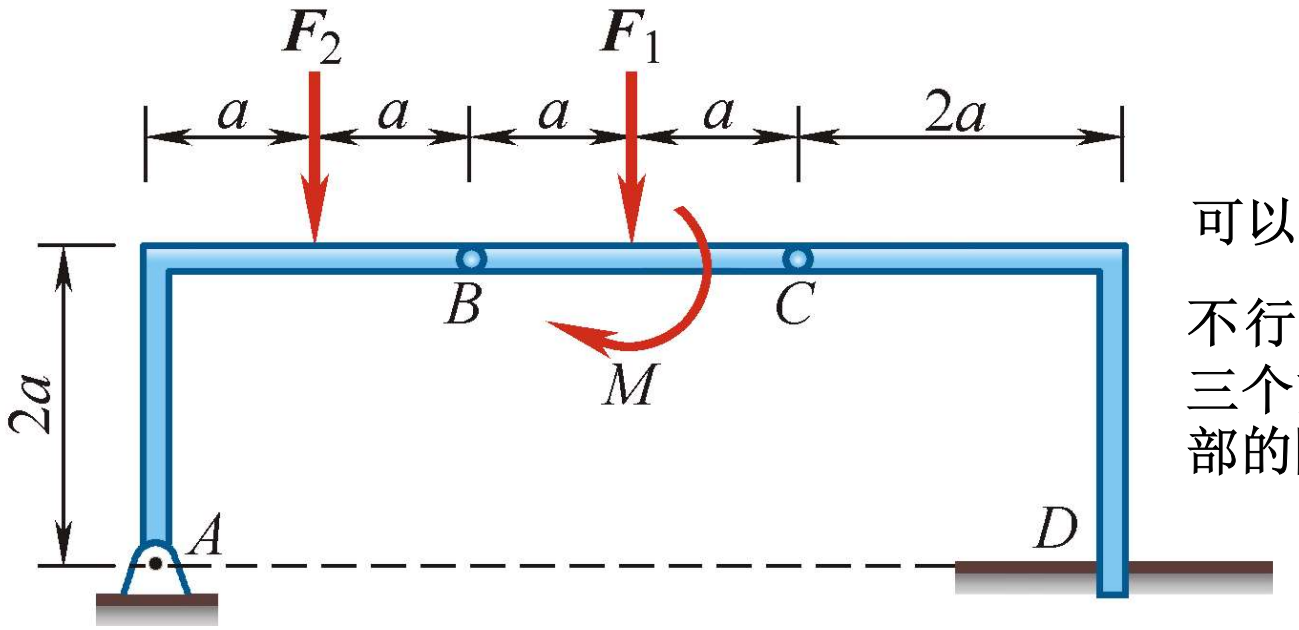
$$M_A - M - 2ql \cdot 2l + F_B \sin 60^\circ \cdot 3l - F \cos 30^\circ \cdot 4l = 0$$

$$\Rightarrow \quad M_A = 10.37\text{kN}\cdot\text{m} \qquad F_{Ax} = 32.89\text{kN} \qquad F_{Ay} = -2.32\text{kN}$$

例2-18（多杆件结构）

已知：如图所示结构，a，$M=Fa$，$F_1=F_2=F$.

求：*A*，*D*处约束力.



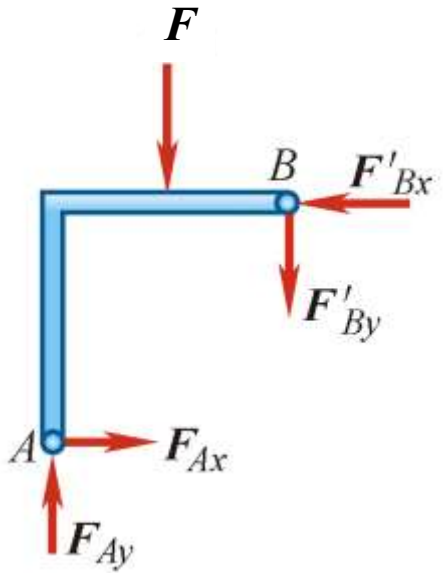
固定支座　　固定端

可以直接整体求解吗？

不行，5个未知约束力，三个方程，必须要靠局部的刚体平衡方程

§2-5 物体系的平衡·静定和超静定问题

解： 以***AB***为研究对象，受力如图所示.

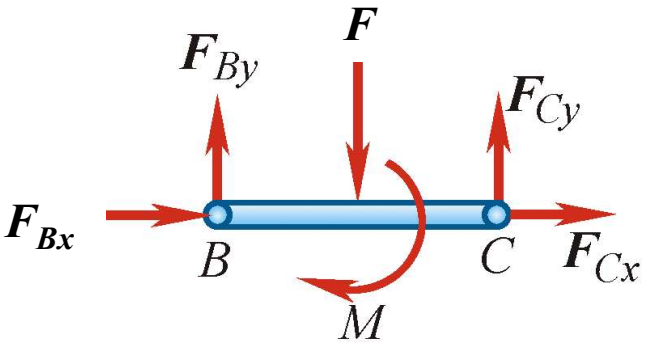


$$\sum M_A = 0 \qquad F'_{Bx} \cdot 2a - F'_{By} \cdot 2a - Fa = 0$$

$$\sum F_x = 0 \qquad F_{Ax} - F'_{Bx} = 0$$

$$\sum F_y = 0 \qquad F_{Ay} - F'_{By} - F = 0$$

以***BC***为研究对象，受力如图所示.



$$\sum M_B = 0 \qquad F_{Cy} \cdot 2a - Fa - M = 0$$

$$\sum F_y = 0 \qquad F_{By} + F_{Cy} - F = 0$$

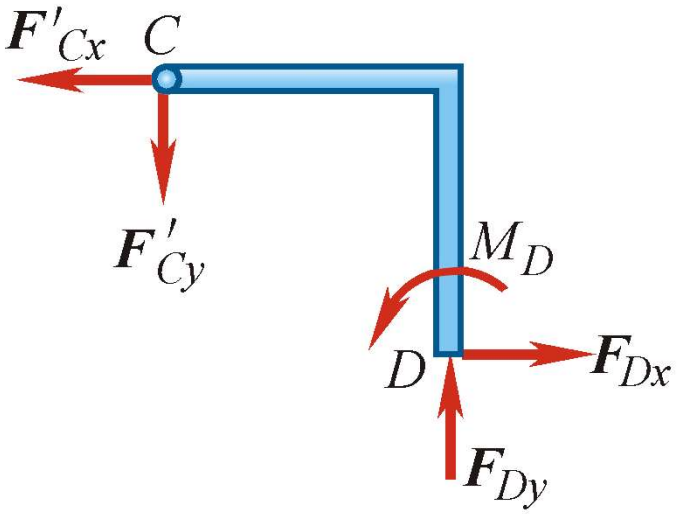
$$\sum F_x = 0 \qquad F_{Cx} + F_{Bx} = 0$$

$$\Rightarrow \quad F'_{Bx} = F_{Ax} = \frac{1}{2}F \qquad F_{Ay} = F$$

$$F_{Cy} = F \qquad F_{By} = 0 \qquad F_{Cx} = -\frac{1}{2}F$$

§2-5 物体系的平衡·静定和超静定问题

以*CD*为研究对象，受力如图所示.



$$\sum F_x = 0 \qquad F_{Dx} - F'_{Cx} = 0$$

$$\sum F_y = 0 \qquad F_{Dy} - F'_{Cy} = 0$$

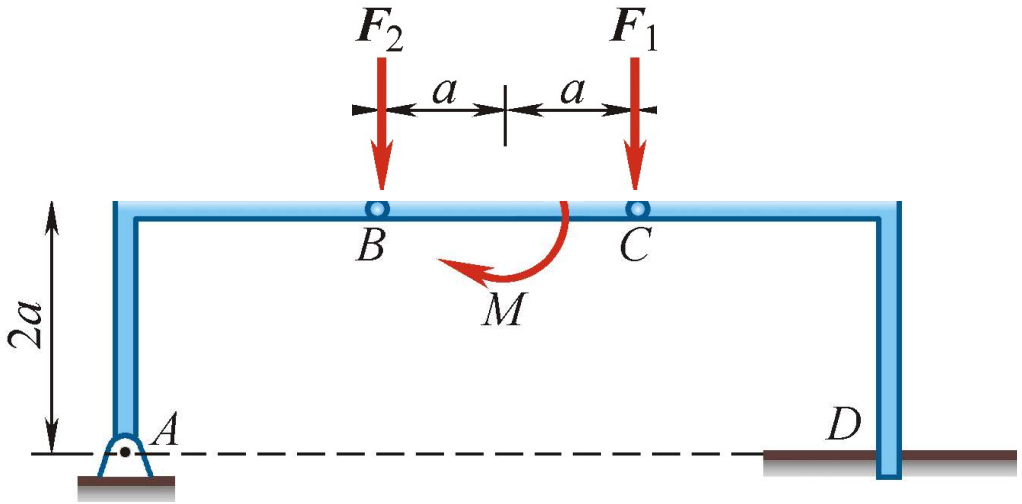
$$\sum M_D = 0 \qquad M_D + F'_{Cy} \cdot 2a + F'_{Cx} \cdot 2a = 0$$

$$\Rightarrow \quad F_{Dx} = -\frac{1}{2}F \qquad F_{Dy} = F \qquad M_D = -Fa$$

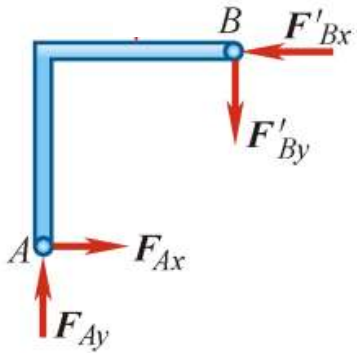
§2-5 物体系的平衡·静定和超静定问题

思考

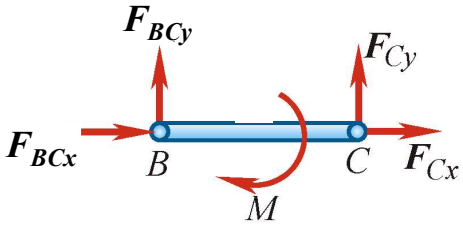
如果集中力F作用在铰链处，该怎么处理



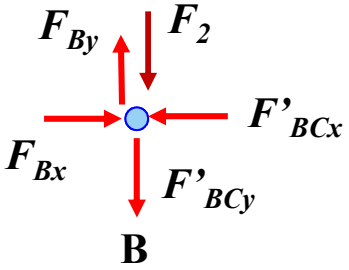
杆AB的受力分析



杆BC的受力分析



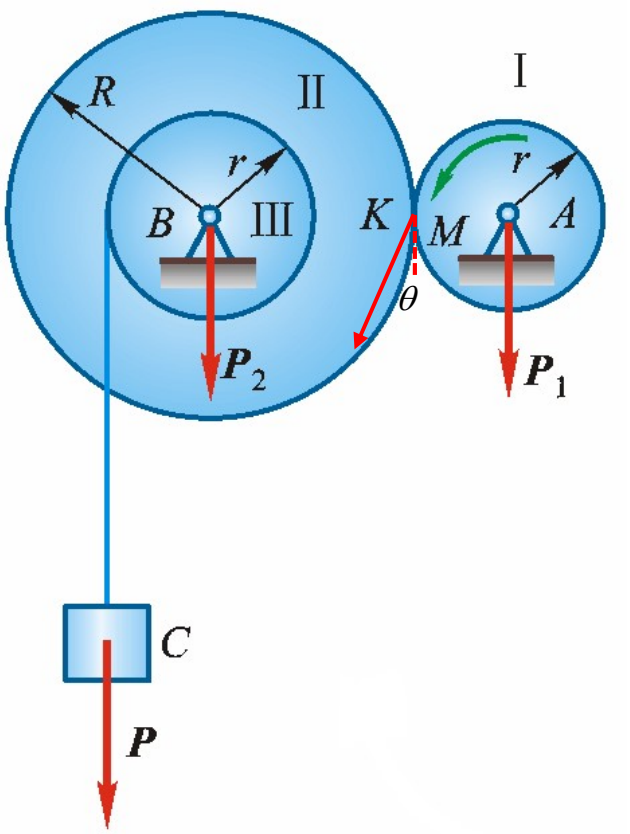
销钉B的受力分析



例2-19（力偶矩）

已知：$P_2=2P_1$，$P=20P_1$，r, $R=2r$，$\theta=20^\circ$;

求：物*C*匀速上升时，作用于小轮上的力偶矩 M ，轴承*A*，*B*处的约束力.



解： 取塔轮及重物 C, 画受力图.

$$\sum M_B = 0 \quad F_t \cdot R - P \cdot r = 0 \qquad F_t = \frac{P \cdot r}{R} = 10P_1$$

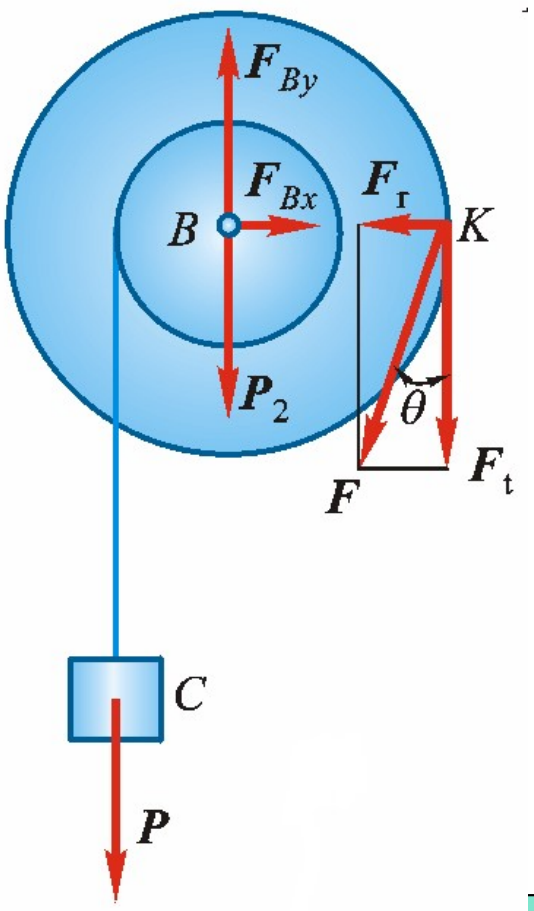
由 $\dfrac{F_r}{F_t} = \tan 20^\circ$

$$F_r = F_t \cdot \tan 20^\circ = 3.64P_1$$

$$\sum F_x = 0 \qquad F_{Bx} - F_r = 0$$

$$\sum F_y = 0 \qquad F_{By} - P - P_2 - F_t = 0$$

$$\Rightarrow \quad F_{Bx} = 3.64P_1 \qquad F_{By} = 32P_1$$



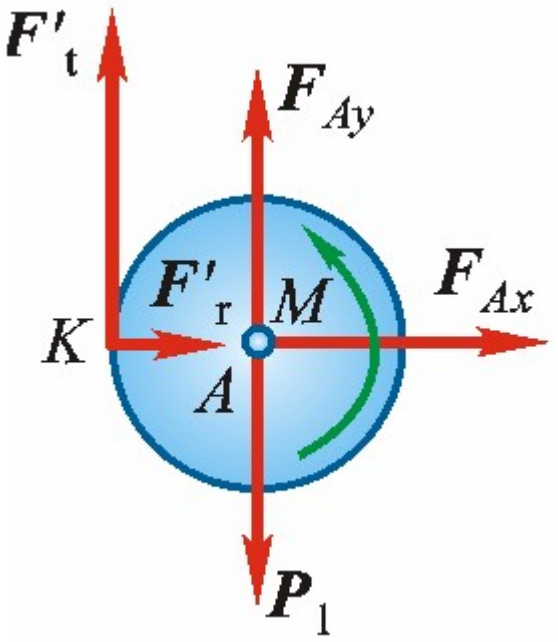
§2-5 物体系的平衡·静定和超静定问题

取小轮，画受力图.

$$\sum F_x = 0 \qquad F_{Ax} + F_r' = 0$$

$$\sum F_y = 0 \qquad F_{Ay} + F_t' - P_1 = 0$$

$$\sum M_A = 0 \qquad M - F_t' \cdot r = 0$$



$$\boldsymbol{F_{Ax} = -3.64P_1}$$

$$\boldsymbol{F_{Ay} = -9P_1}$$

$$M = 10P_1 r$$

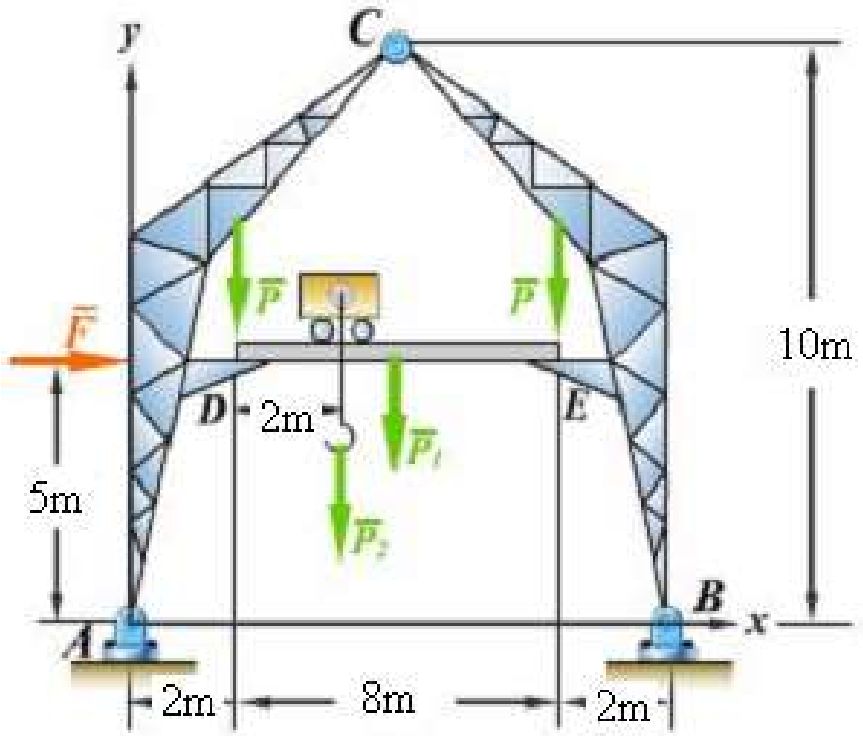
作用力与反作用力！

§2-5 物体系的平衡·静定和超静定问题

例2-20

已知：P=60kN, P_1=20kN, P_2=10kN, 风载F=10kN,
尺寸如图；

求：***A,B***处的约束力.



§2-5 物体系的平衡·静定和超静定问题

解： 取整体, 画受力图.

$$\sum M_A = 0 \qquad 12F_{By} - 10P - 6P_1 - 4P_2 - 2P - 5F = 0$$

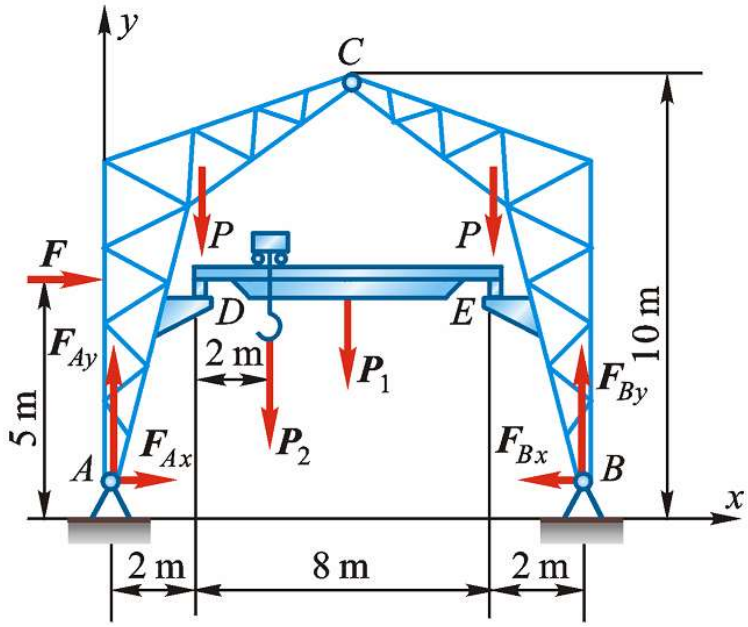
$$\sum F_y = 0 \qquad F_{Ay} + F_{By} - 2P - P_1 - P_2 = 0$$

$$\sum F_x = 0 \qquad F_{Ax} + F - F_{Bx} = 0$$

$$F_{Ay} = 72.5\text{kN}$$

$$F_{By} = 77.5\text{kN}$$

$$F_{Ax} = F_{Bx} - F$$

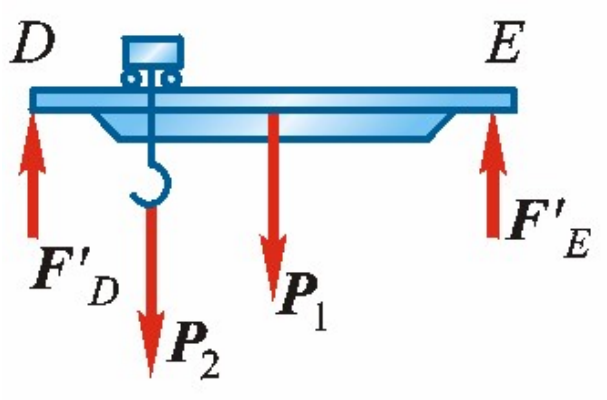


§2-5 物体系的平衡·静定和超静定问题

取吊车梁, 画受力图.

$$\sum M_D = 0 \qquad 8F'_E - 4P_1 - 2P_2 = 0$$

$$\Longrightarrow \qquad F'_E = 12.5\text{kN}$$



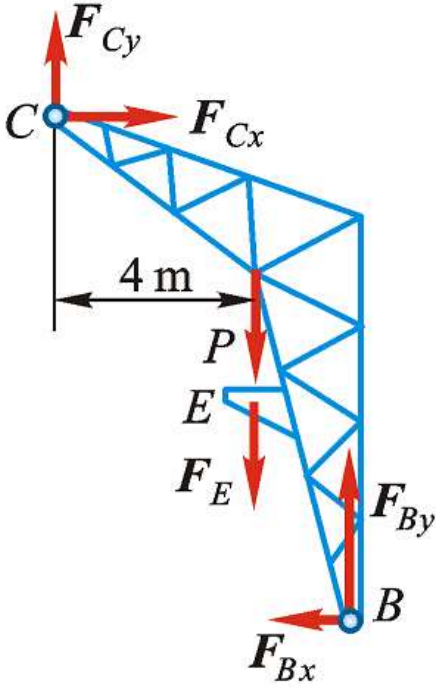
取右边刚架, 画受力图.

$$\sum M_C = 0$$

$$6F_{By} - 10F_{Bx} - 4\left(P + F_E\right) = 0$$

$$\Longrightarrow \qquad F_{Bx} = 17.5\text{kN}$$

$$F_{Ax} = 7.5\text{kN}$$

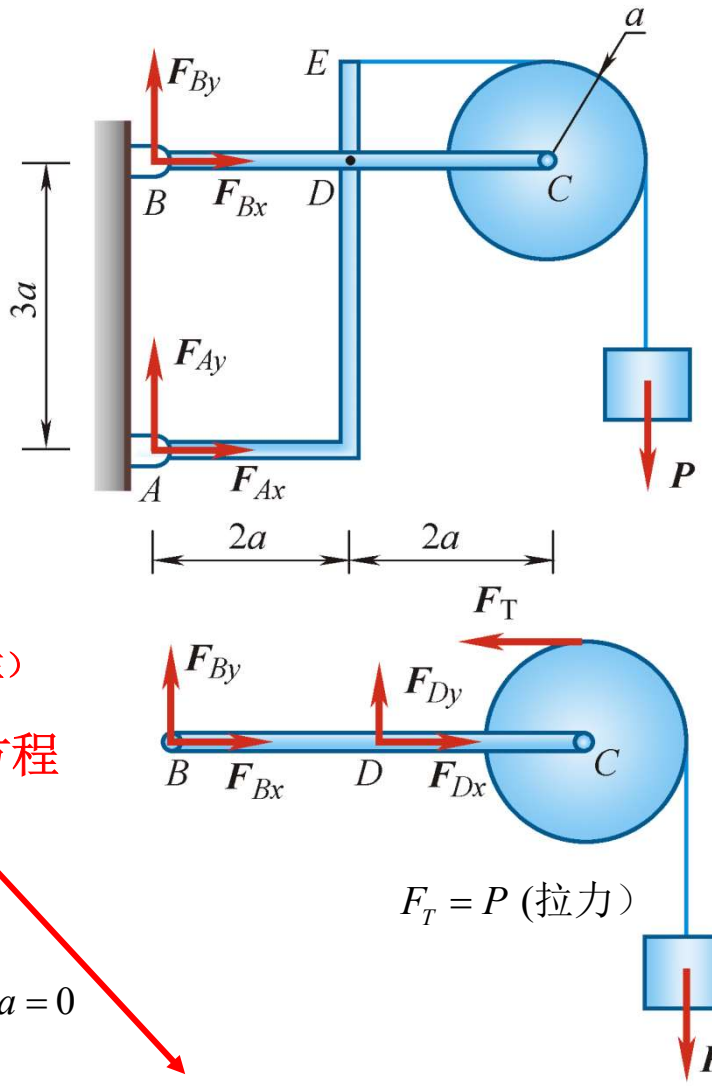


§2-5 物体系的平衡·静定和超静定问题

例2-21（带滑轮结构）

已知：如图所示结构，P和a.

求：支座A，B 处约束力.



解题思路：

先分析整体 ⇒

$$\sum F_x = 0 \quad F_{Bx} + F_{Ax} = 0$$
$$\sum F_y = 0 \quad F_{Ay} + F_{By} = P$$
$$\sum M_A = 0 \quad -P \cdot 5a - F_{Bx} \cdot 3a = 0$$

$F_{Bx} = -\frac{5}{3}P$（向左） ← $F_{Ax} = \frac{5}{3}P$（向右）

F_{Ay}与F_{By}从整体方程无法直接求解—通过局部方程

再分析BC ⇒

$$\sum F_x = 0 \quad F_{Bx} + F_{Dx} - F_T = 0$$
$$\sum F_y = 0 \quad F_{By} + F_{Dy} = P$$
$$\sum M_D = 0 \quad F_T \cdot a - P \cdot 3a - F_{By} \cdot 2a = 0$$

$F_T = P$ (拉力)

$F_{By} = -P$（向下）

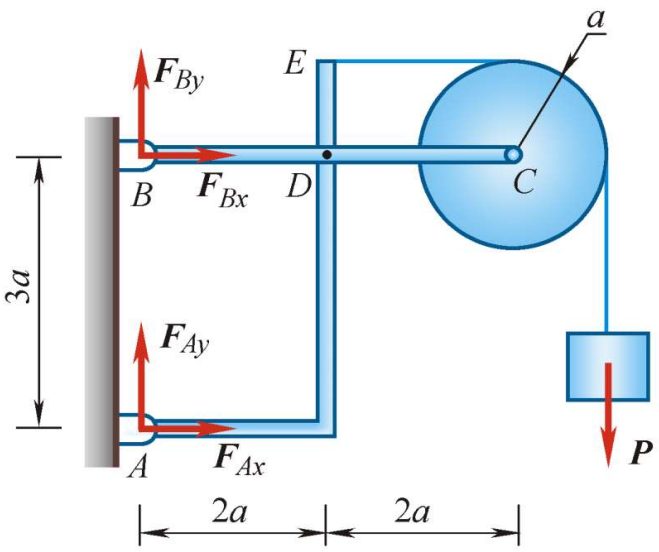
$F_{Ay} = P - F_{By} = 2P$（向上）

$F_{Dx} = F_T - F_{Bx} = P + \frac{5}{3}P = \frac{7}{3}P$（向右）

§2-5 物体系的平衡·静定和超静定问题

总结：

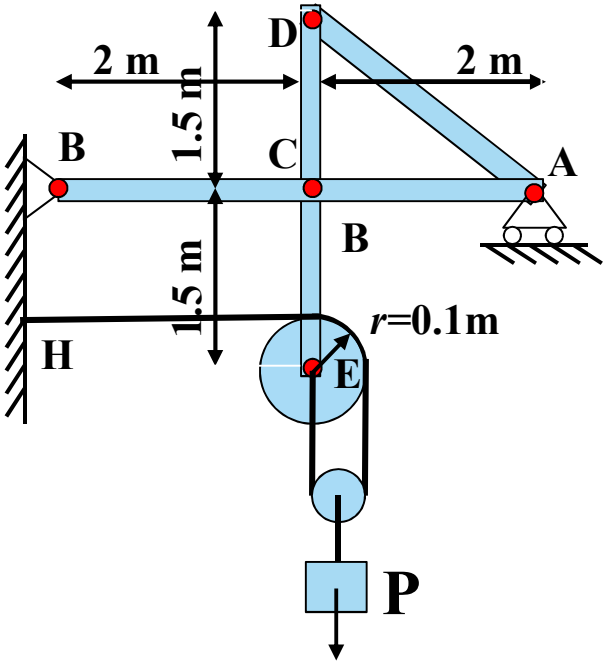
- 一般先分析整体；
- 一般不拆滑轮；
- 一个刚体最多可以列三个方程，求三个未知约束力
- 一个刚体平衡分析，不需要把所有的未知约束力求得，可以通过连接约束传递待求约束力
- 矩心尽量取在较多未知力的交点上；
- 合力投影轴尽量与较多未知力相垂直。

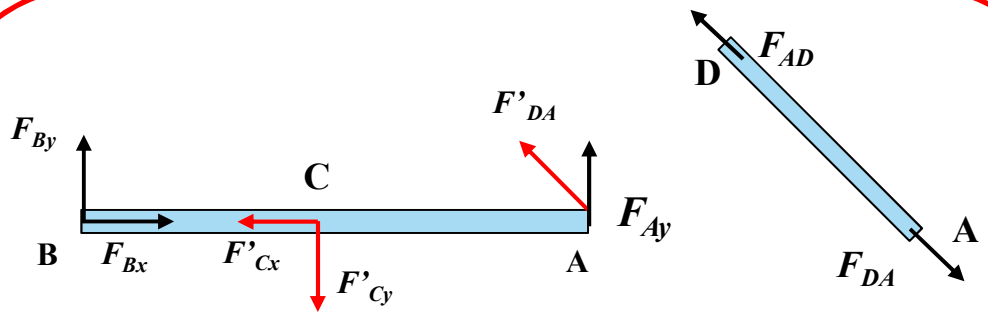


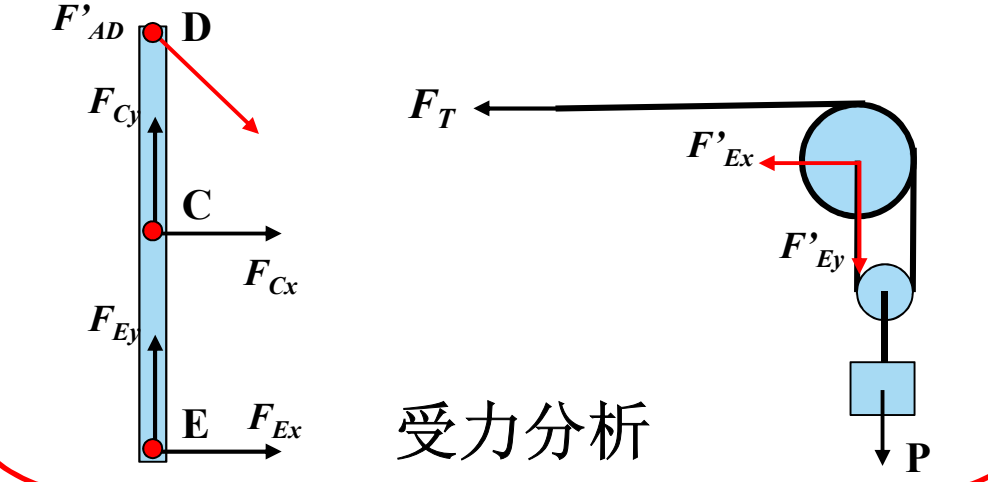
例2-22（支座受力分析）

如图构架，重物P=2400N，由绳索跨过滑轮E后水平系于墙上，不计滑轮与杆重力。

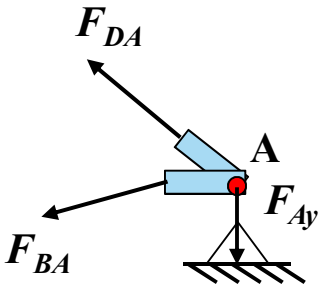
求：支承A与B的约束力，与杆AD内力





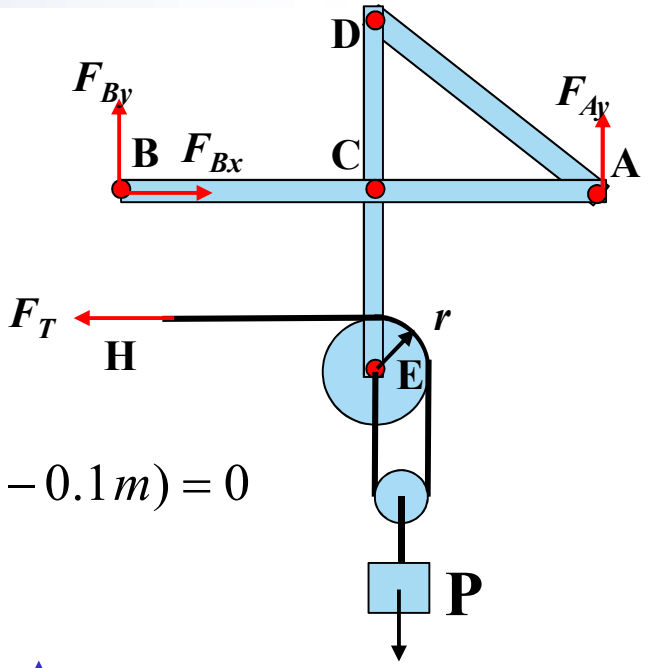


思考：为什么F_{BA}不是水平方向？



§2-5 物体系的平衡·静定和超静定问题

解：取整体，画受力图.



$$\sum F_x = 0 \qquad F_{Bx} - F_T = 0$$

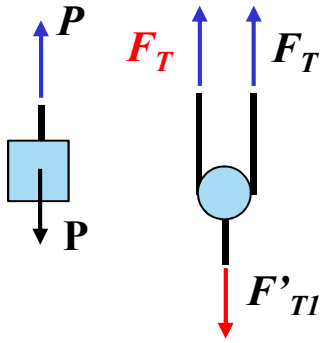
$$\sum F_y = 0 \qquad F_{By} + F_{Ay} - P = 0$$

$$\sum M_B = 0 \qquad -F_{Ay} \times 4\,m - P(2\,m + 0.1\,m) - F_T(1.5\,m - 0.1\,m) = 0$$

$$\Rightarrow \quad F_{Bx} = 600\,N$$

$$F_{Ay} = 840\,N$$

$$F_{By} = 360\,N$$



F_T=0.5P=600N

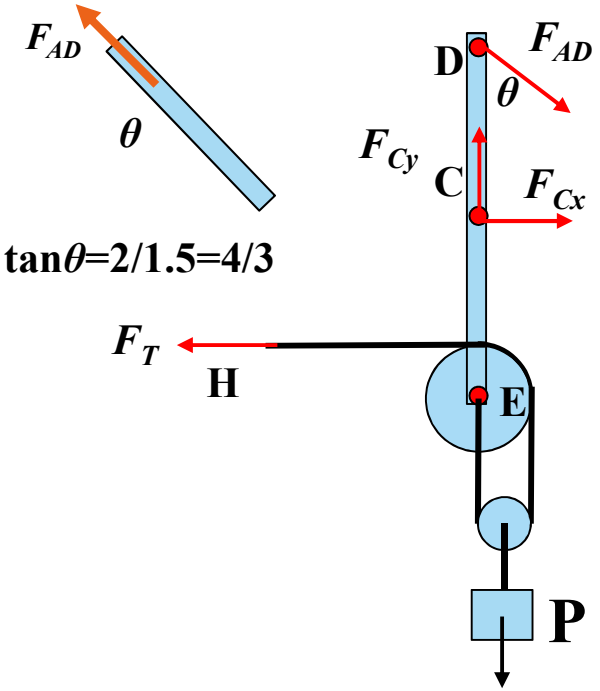
§2-5 物体系的平衡·静定和超静定问题

取***DCE***杆+滑轮，画受力图.

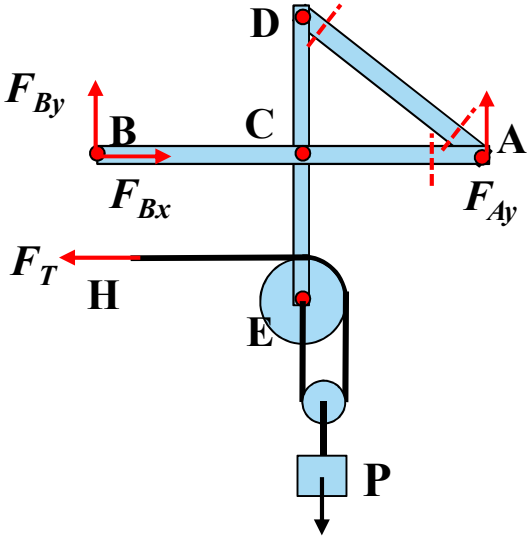
$$\sum M_C = 0$$

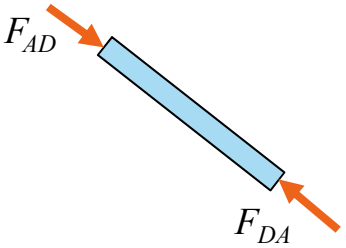
$$-F_{AD}\sin\theta\times 1.5m - P\times 0.05m - F_T\left(1.5m-0.1m\right)=0$$

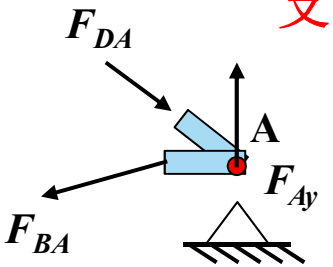
$$F_{AD} = -750N \quad \text{（AD杆受到压力）}$$



对平衡的刚体系，任意一部分都处于平衡状态







支座A的约束力，是指
支座对刚体的作用力

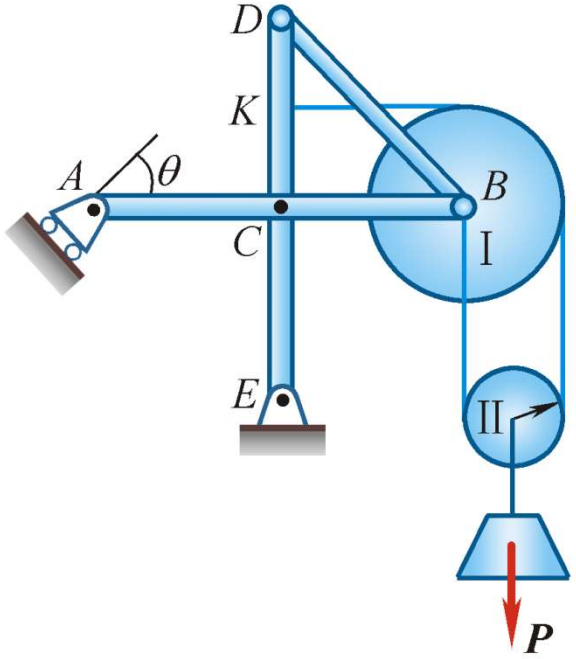
§2-5 物体系的平衡·静定和超静定问题

例2-23

已知：$DC=CE=CA=CB=2l$, $R=2r=l$, $\vec{P}$，各构件自重不计，

$\theta = 45^0$.

求：A, E支座处约束力及BD杆受力.



1. 分析约束力（方向，类型）
2. 找二力杆、三力平衡汇交与柔索张力
3. 不拆滑轮（与杆件连接）
4. 先整体后局部，通过局部构件的平衡方程找出未知约束力。

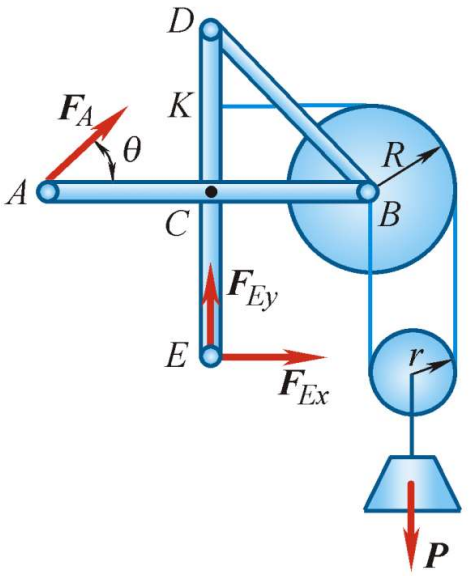
解： 取整体, 画受力图.

$$\sum M_E = 0 \quad -F_A \cdot \sqrt{2} \cdot 2l - P \cdot \frac{5}{2} l = 0$$

$$\sum F_x = 0 \quad F_{Ex} + F_A \cos 45^0 = 0$$

$$\sum F_y = 0 \quad F_{Ey} - P + F_A \sin 45^0 = 0$$

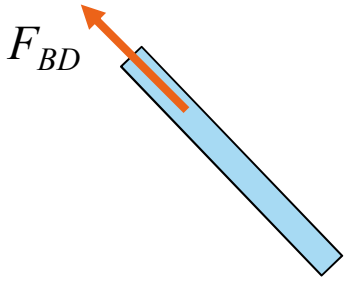
$$\Rightarrow F_A = -\frac{5\sqrt{2}}{8} P \qquad F_{Ex} = \frac{5}{8} P \qquad F_{Ey} = \frac{13}{8} P$$

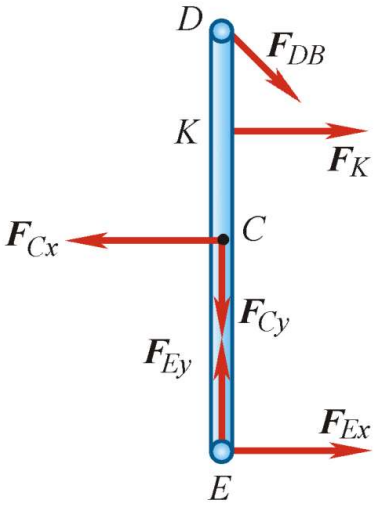


取***DCE***杆, 画受力图.

$$\sum M_C = 0 \quad -F_{DB} \cos 45^0 \cdot 2l - F_K \cdot l + F_{Ex} \cdot 2l = 0$$

$$F_{DB} = \frac{3\sqrt{2}}{8} P \quad \text{(BD杆受到拉力)}$$





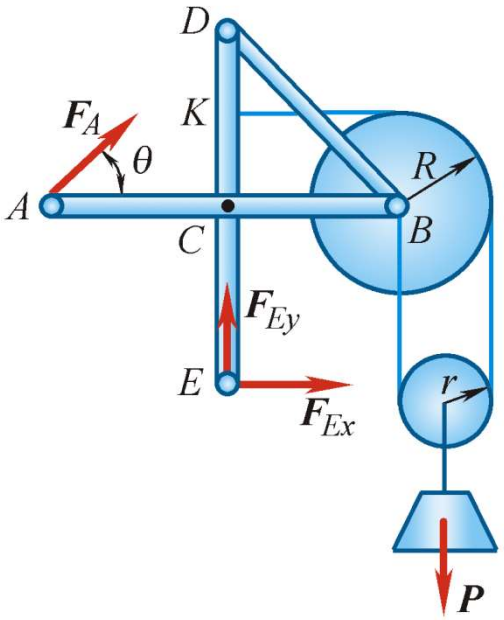
§2-5 物体系的平衡·静定和超静定问题

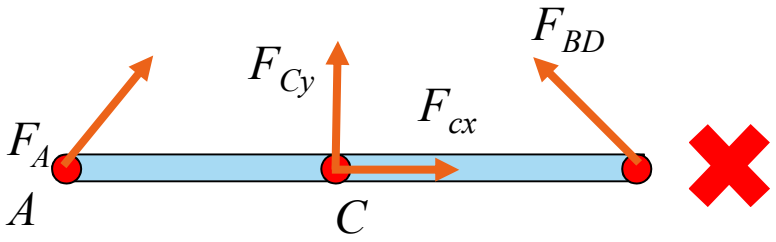
思考

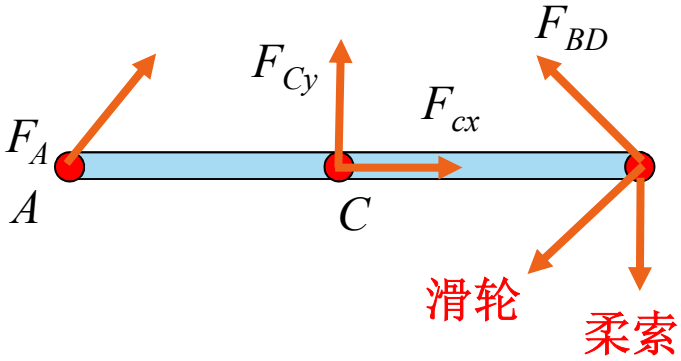
取整体, 画受力图.

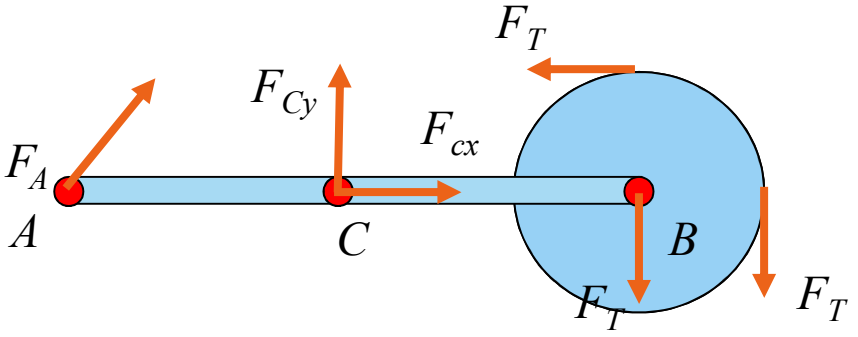
$$F_A = -\frac{5\sqrt{2}}{8}P \quad F_{Ex} = \frac{5}{8}P \quad F_{Ey} = \frac{13}{8}P$$

是否可以直接取*AB*杆求解*BD*杆的力？









§2-5 物体系的平衡·静定和超静定问题

例 2-24(复杂绳索张力)

已知：P=10kN ,a ,杆、轮重不计；

求：A ,C支座处约束力.

解： 取整体，受力图能否这样画？

取整体，画受力图.

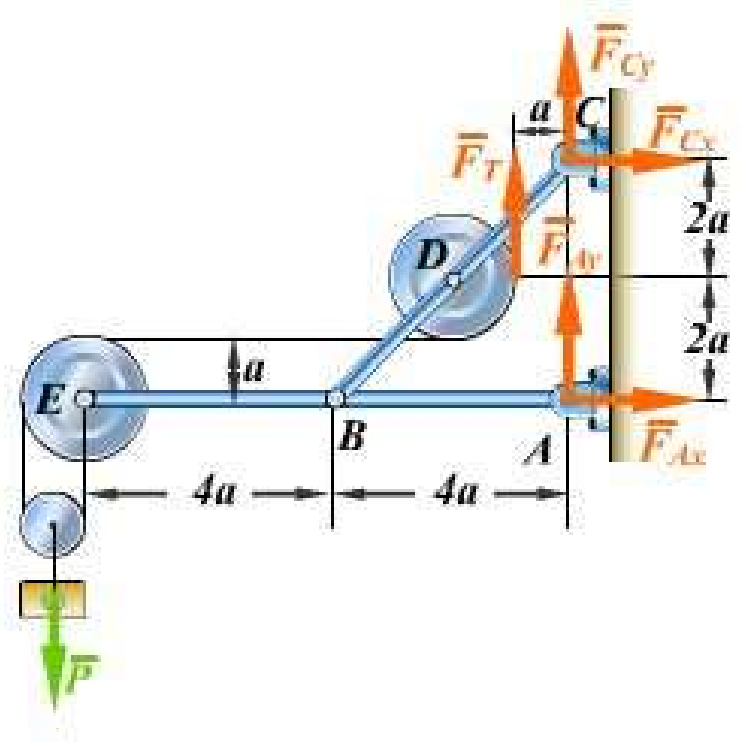
$$\sum M_C = 0 \quad 4aF_{Ax} + 8.5aP - F_T a = 0$$

$$\Rightarrow \quad F_{Ax} = -20\text{kN}$$

$$\sum F_x = 0 \quad F_{Ax} + F_{Cx} = 0$$

$$\Rightarrow \quad F_{Cx} = 20\text{kN}$$

$$\sum F_y = 0 \quad F_{Ay} + F_{Cy} + F_T - P = 0$$



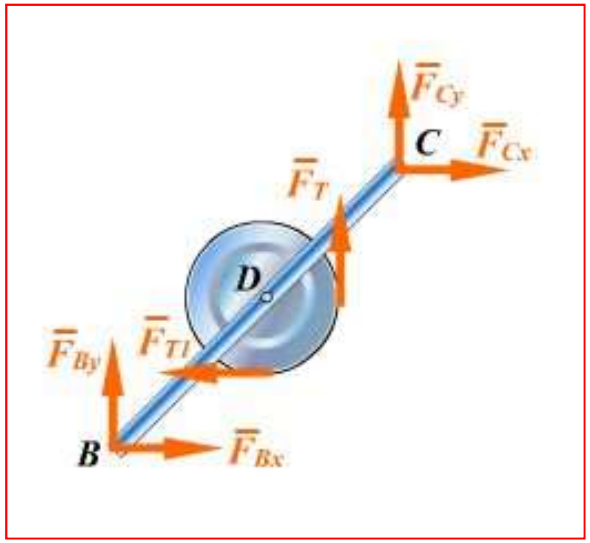
§2-5 物体系的平衡·静定和超静定问题

取***BDC***杆（带着轮）

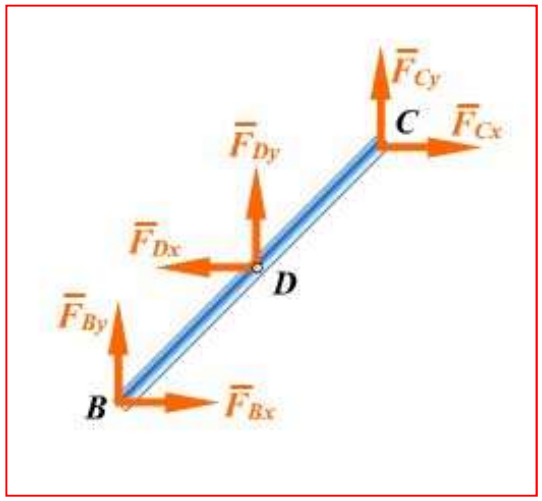
$$\sum M_B = 0$$

$$4aF_{Cy} + F_T \cdot 3a + F_{T1} \cdot a - F_{Cx} \cdot 4a = 0$$

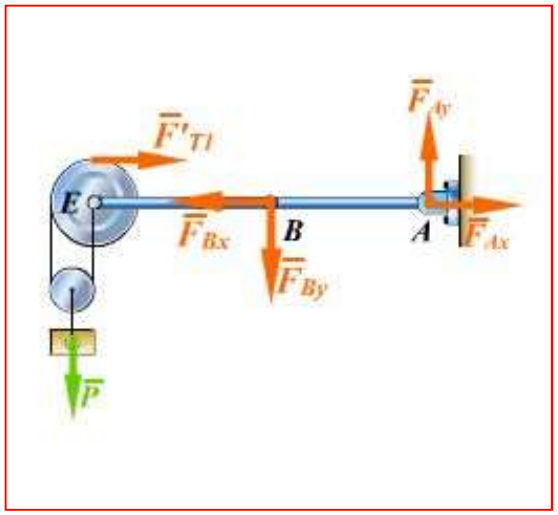
$$\Rightarrow \quad F_{Cy} = 15\text{kN} \qquad F_{Ay} = -10\text{kN}$$



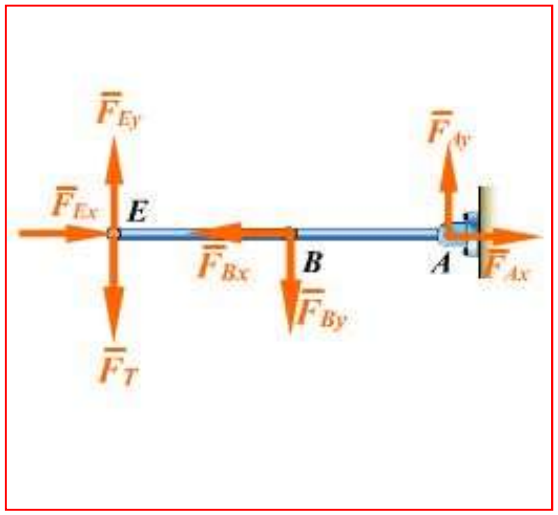
拆滑轮受力分析更复杂



取***BDC*** 杆（不带着轮）



取***ABE***（带着轮）

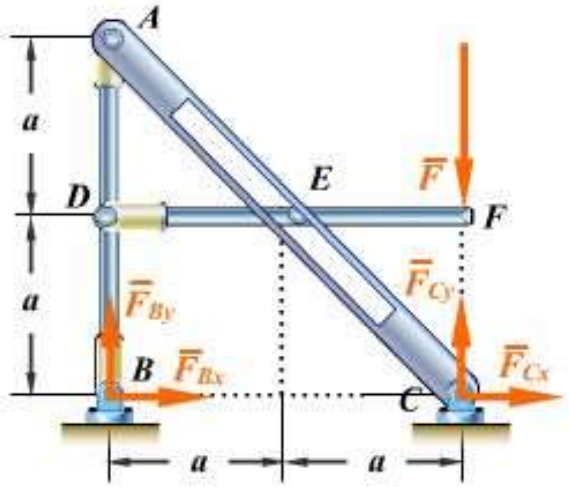


取***ABE***杆（不带着轮）

§2-5 物体系的平衡·静定和超静定问题

例2-25 已知：P, a ,各杆重不计；

求：B 铰处约束力.

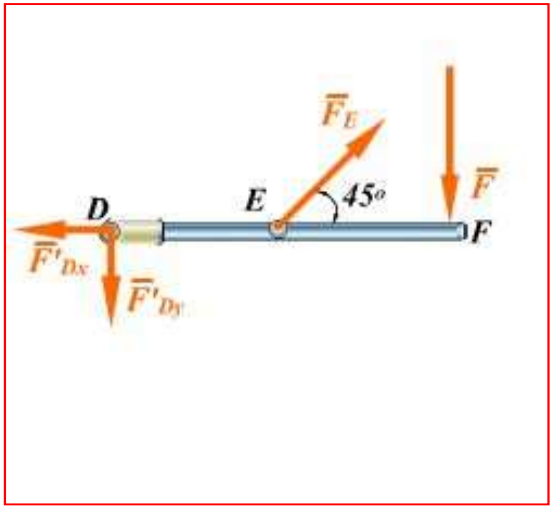


解：取整体，画受力图

$$\sum M_C = 0 \quad -F_{By} \cdot 2a = 0$$

$$\Rightarrow \quad F_{By} = 0$$

取**DEF**杆，画受力图



$$\sum M_D = 0 \quad F_E \sin 45^\circ \cdot a - F \cdot 2a = 0$$

$$\sum F_x = 0 \quad F_E \cos 45^\circ - F'_{Dx} = 0$$

$$\sum M_E = 0 \quad F_{Dy}' \cdot a - F \cdot 2a = 0$$

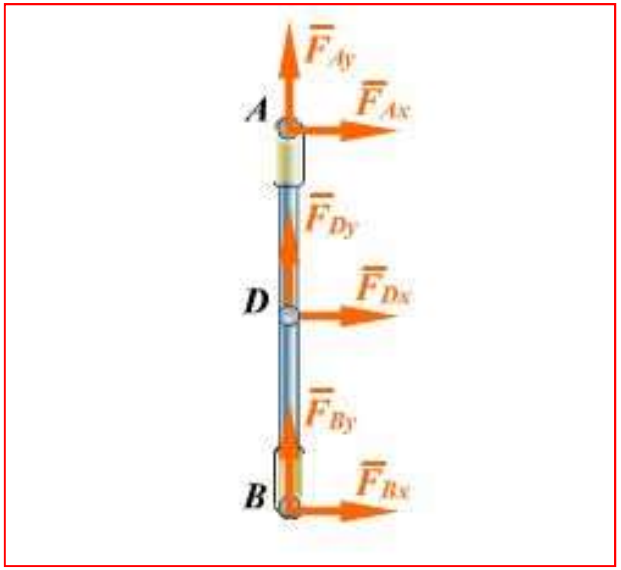
$$\Rightarrow \quad F_E \sin 45^\circ = 2F \quad F'_{Dx} = F_E \cos 45^\circ = 2F \quad F'_{Dx} = 2F$$

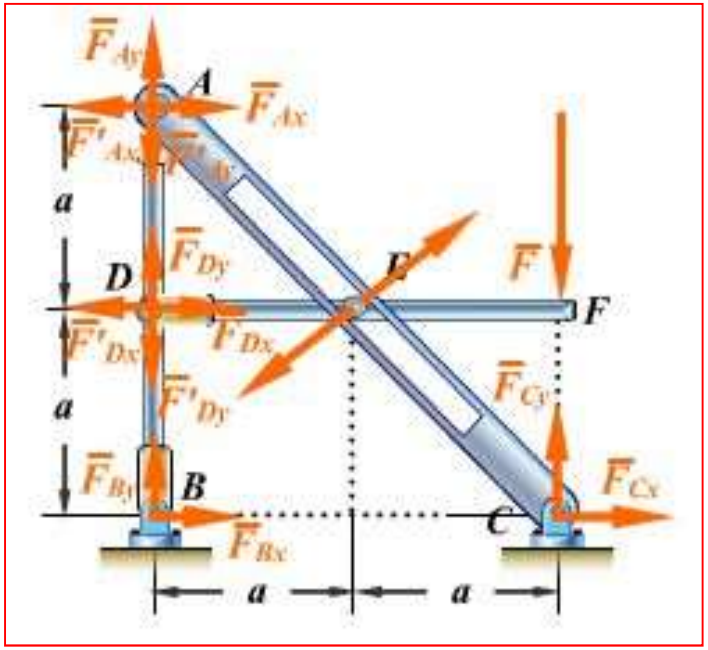
§2-5 物体系的平衡·静定和超静定问题

对***ADB***杆受力图

$$\sum M_A = 0 \qquad F_{Bx} \cdot 2a + F_{Dx} \cdot a = 0$$

$$\Rightarrow \qquad F_{Bx} = -F$$

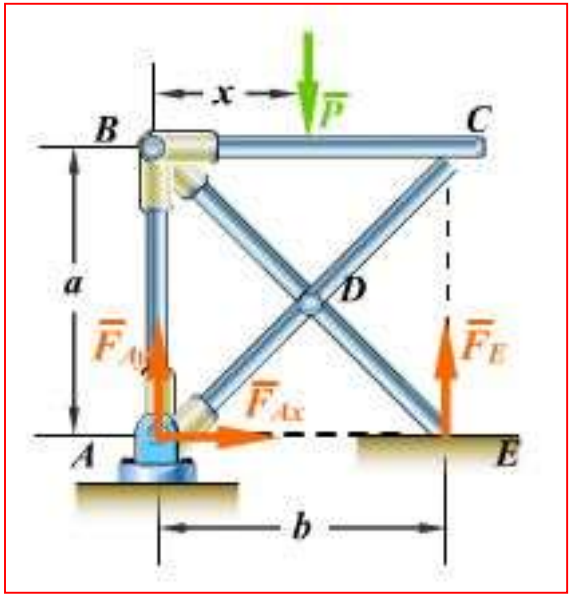




§2-5 物体系的平衡·静定和超静定问题

例2-26 已知：*a* ,*b* ,*P*,各杆重不计， *C*,*E*处光滑；

求证： *AB*杆始终受压，且大小为*P*.



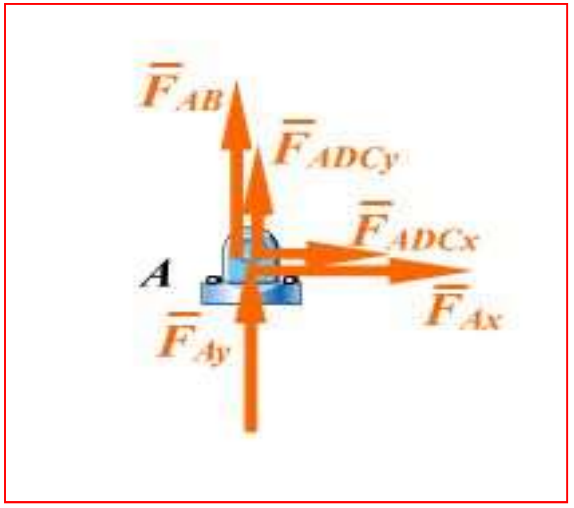
解： 取整体，画受力图.

$$\sum F_x = 0 \qquad F_{Ax} = 0$$

$$\sum M_E = 0 \qquad P\cdot(b-x) - F_{Ay}\cdot b = 0$$

$$\Rightarrow \quad F_{Ay} = \frac{P}{b}(b-x)$$

取销钉*A*，画受力图



$$\sum F_x = 0 \qquad F_{Ax} + F_{ADCx} = 0$$

$$\sum F_y = 0 \qquad F_{AB} + F_{Ay} + F_{ADCy} = 0$$

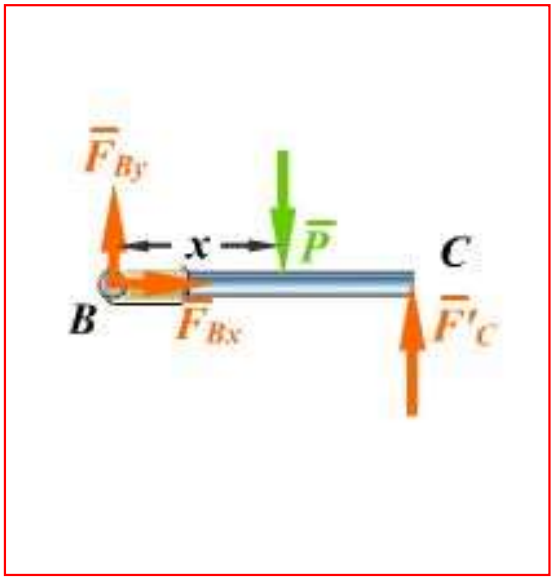
$$\Rightarrow \qquad F_{ADCx} = 0$$

§2-5 物体系的平衡·静定和超静定问题

取***BC***，画受力图.

$$\sum M_B = 0 \qquad F_C' \cdot b - Px = 0$$

$$\rightarrow \qquad F_C' = \frac{x}{b} P$$

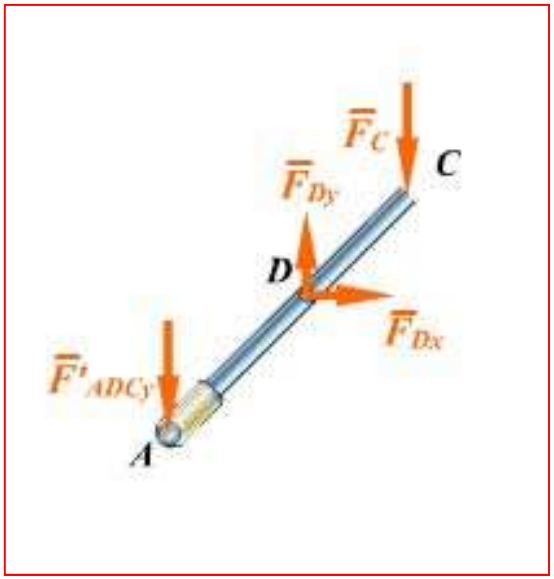


取***ADC***杆，画受力图.

$$\sum M_D = 0 \qquad F'_{ADCy} \cdot \frac{b}{2} - F_C \cdot \frac{b}{2} = 0$$

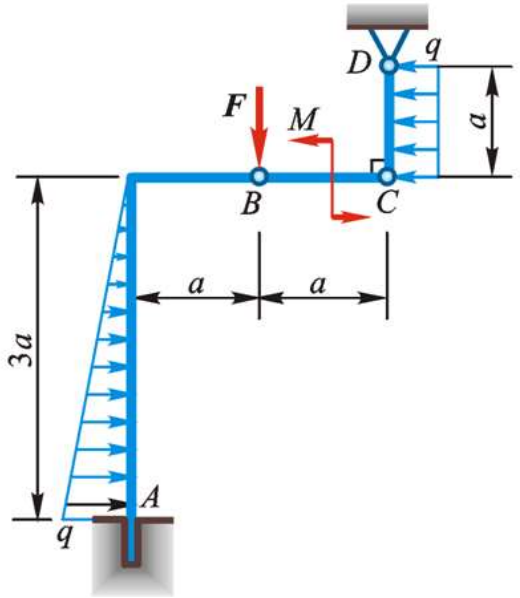
$$\rightarrow \qquad F'_{ADCy} = F_C = \frac{x}{b} P$$

$$\rightarrow \qquad F_{AB} = -P\,(\text{压})$$



§2-5 物体系的平衡·静定和超静定问题

例2-27

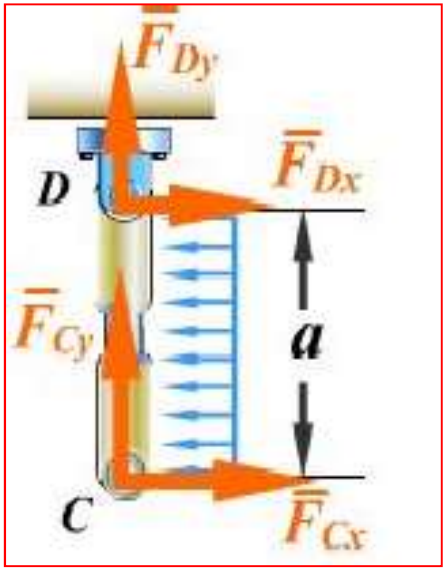


已知：q ,a ,M ， 且$M = qa^2$,

P作用于销钉B上；

求： 固定端A处的约束力和销钉B对BC杆、AB杆的作用力.

解： 取CD杆，画受力图.



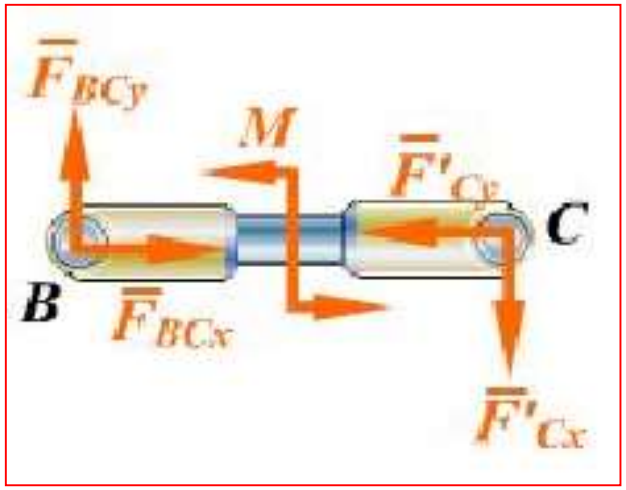
$$\sum M_D = 0$$

$$F_{Cx} \cdot a - qa \cdot \frac{a}{2} = 0$$

$$\rightarrow \quad F_{Cx} = \frac{1}{2} qa$$

§2-5 物体系的平衡·静定和超静定问题

取***BC***杆（不含销钉***B***)，画受力图.

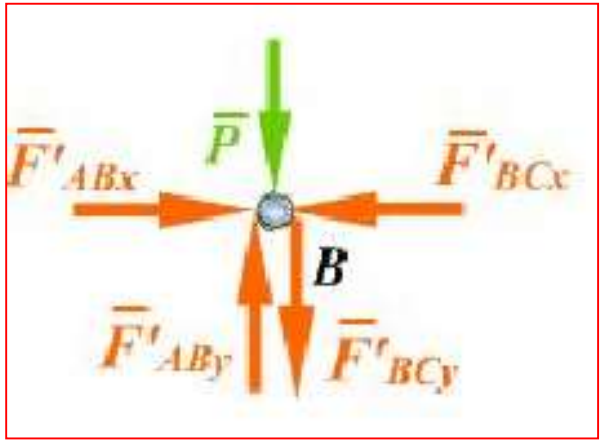


$$\sum F_x = 0 \qquad F_{BCx} - F'_{Cx} = 0$$

$$\sum M_C = 0 \qquad M - F_{BCy}a = 0$$

$$\rightarrow \quad F_{BCx} = \frac{1}{2}qa \qquad F_{BCy} = qa$$

取销钉***B***，画受力图.



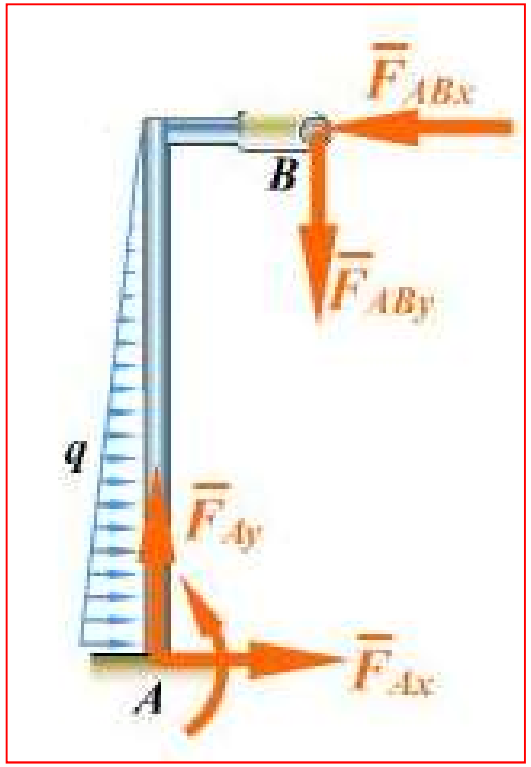
$$\sum F_x = 0 \qquad F'_{ABx} - F'_{BCx} = 0$$

$$\sum F_y = 0 \qquad F'_{ABy} - F'_{BCy} - P = 0$$

$$\rightarrow \quad F'_{ABx} = \frac{1}{2}qa \qquad F'_{ABy} = P + qa$$

$$\rightarrow \quad F_{ABx} = -\frac{1}{2}qa \qquad F_{ABy} = -(P + qa)$$

取*AB*杆（不含销钉*B*），画受力图.



$$\sum F_x = 0 \qquad F_{Ax} + \frac{1}{2}\cdot q \cdot 3a - F_{ABx} = 0$$

$$\rightarrow \qquad F_{Ax} = -qa$$

$$\sum F_y = 0 \qquad F_{Ay} - F_{ABy} = 0$$

$$\rightarrow \qquad F_{Ay} = P + qa$$

$$\sum M_A = 0$$

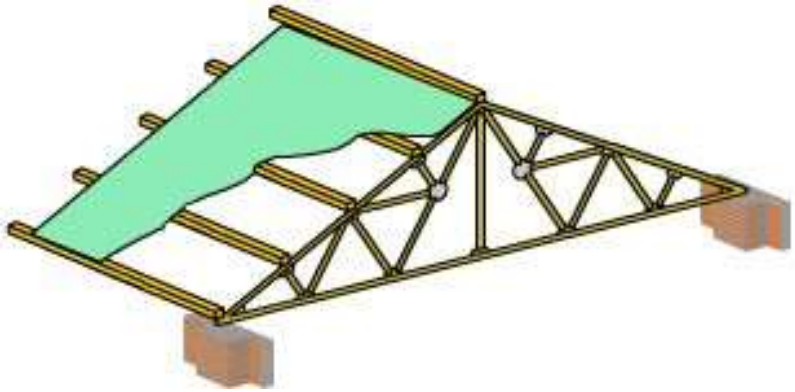
$$M_A - \frac{1}{2}\cdot q \cdot 3a \cdot a + F_{ABx} \cdot 3a - F_{ABy} \cdot a = 0$$

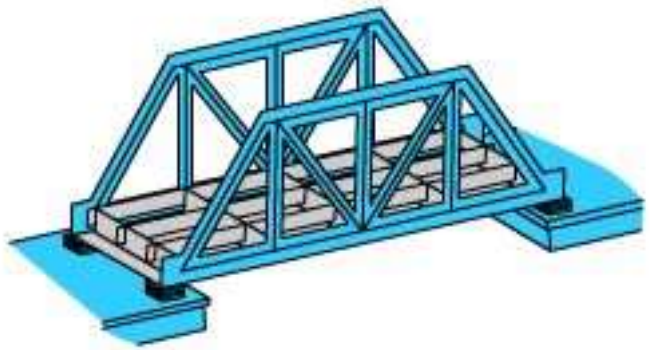
$$\rightarrow \qquad M_A = (P + qa)a$$

平面简单桁架

桁架：一种由杆件彼此在两端用铰链连接而成的结构，它在受力后几何形状不变。

节点：桁架中杆件的铰链接头(几个约束力？）。





§2-6 平面简单桁架的内力计算

关于平面桁架（理想桁架）的几点假设：

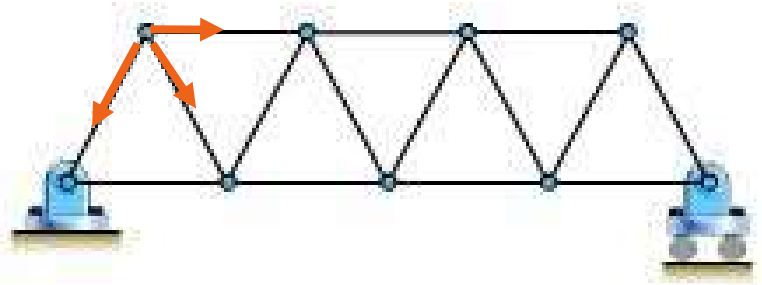
1. 各杆件为直杆，各杆轴线位于同一平面内；

2. 杆件与杆件间均用光滑铰链连接；

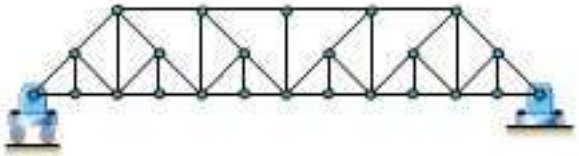
3. 载荷作用在节点上，且位于桁架几何平面内；

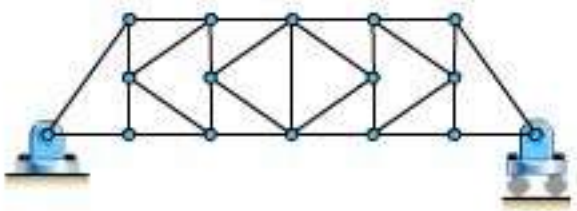
4. 各杆件自重不计或平均分布在节点上。

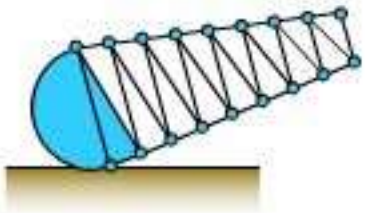
→ 桁架中每根杆件均为二力杆



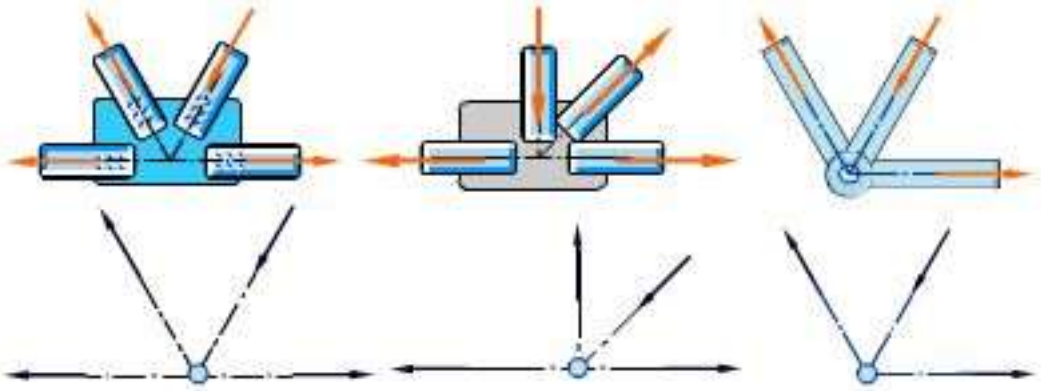
§2-6 平面简单桁架的内力计算



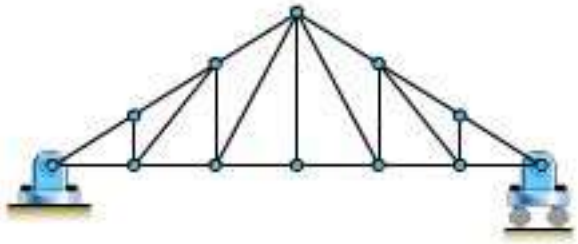


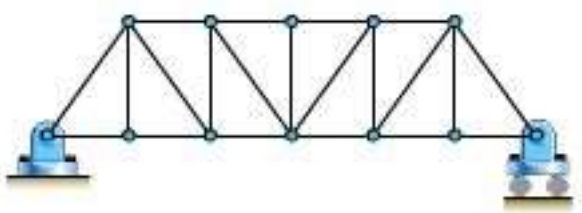


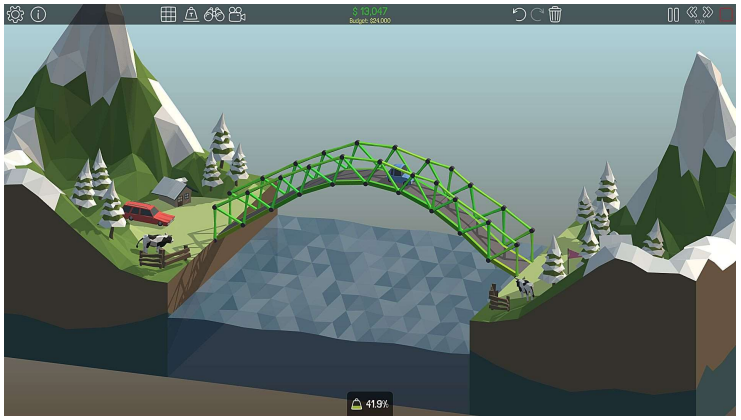
每个铰接点上都
是平面汇交力系



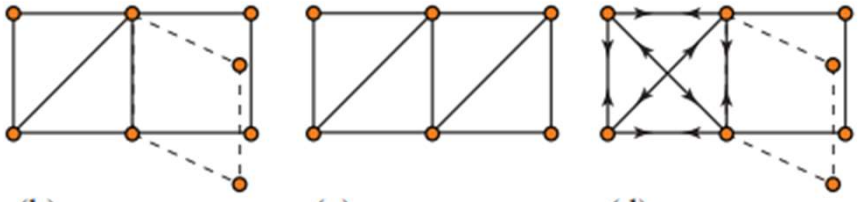
§2-6 平面简单桁架的内力计算







平面桁架结构需要满足什么条件才能处于静定平衡状态？



静定：
方程的未知数个数
=独立平衡方程数

§2-6 平面简单桁架的内力计算

总杆数 m　总节点数 n

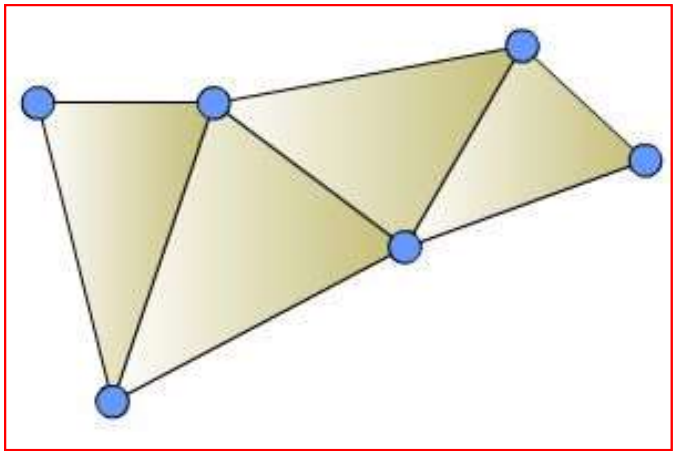
至少需要3个二力杆

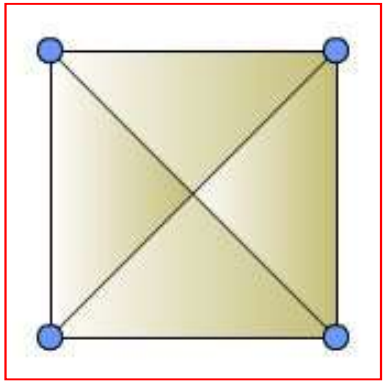
每个节点2个平衡方程

至少3个节点

$$m-3=2(n-3)$$

$$m=2n-3$$

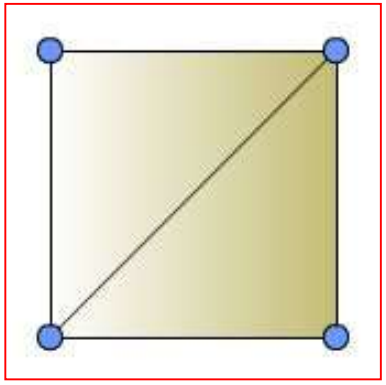




$m>2n-3$

平面复杂

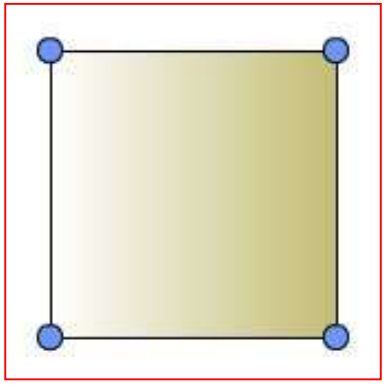
（超静定）桁架



$m=2n-3$

平面简单

（静定）桁架



$m<2n-3$

非桁架（机构）

机构机器人

https://www.strandbeest.com/evolution

Strandbeest by Theo Jansen (Netherland)

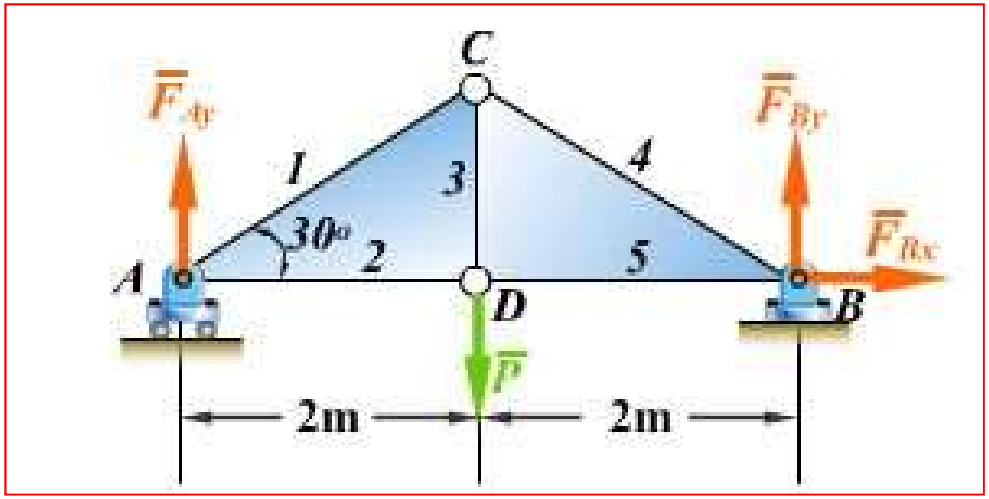
§2-6 平面简单桁架的内力计算

节点法与截面法

例2-28（节点法）

已知: P=10kN,尺寸如图；

求： 桁架各杆件受力.



节点法：对每个节点列平衡方程（2个，平面汇交力系）

解： 取整体，画受力图.

先把约束力求解，再分析节点

$$\sum F_x = 0 \qquad F_{Bx} = 0$$

$$\sum M_B = 0 \qquad 2P - 4F_{Ay} = 0 \qquad F_{Ay} = 5\text{kN}$$

$$\sum F_y = 0 \qquad F_{Ay} + F_{By} - P = 0 \qquad F_{By} = 5\text{kN}$$

§2-6 平面简单桁架的内力计算

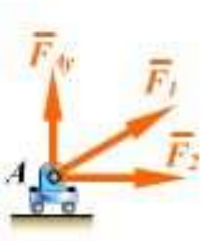
节点法

取节点A，画受力图.

$$\sum F_y = 0 \qquad F_{Ay} + F_1 \sin 30^0 = 0$$

$$\sum F_x = 0 \qquad F_2 + F_1 \cos 30^0 = 0$$

$$F_1 = -10\text{kN}\ (\text{压}) \quad F_2 = 8.66\text{kN}\ (\text{拉})$$

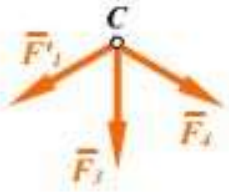


取节点C，画受力图.

$$\sum F_x = 0 \qquad F_4 \cos 30^0 - F_1' \cos 30^0 = 0$$

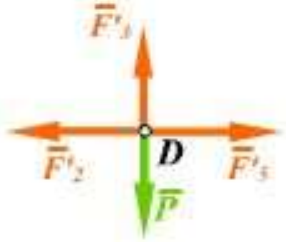
$$\sum F_y = 0 \qquad -F_3 - \left(F_1' + F_4\right)\sin 30^0 = 0$$

$$F_4 = -10\text{kN}(\text{压}) \quad F_3 = 10\text{kN}\ (\text{拉})$$



取节点D,画受力图.

$$\sum F_x = 0 \quad F_5 - F_2' = 0 \quad F_5 = 8.66\text{kN}\ (\text{拉})$$



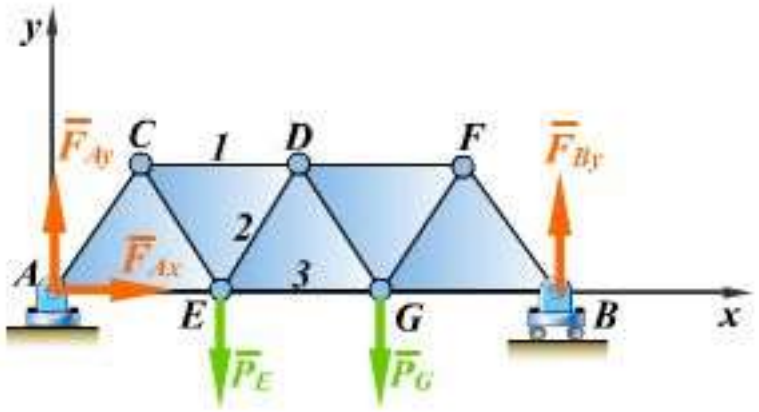
§2-6 平面简单桁架的内力计算

例2-29（截面法）

已知：$P_E = 10\text{kN},\ P_G = 7\text{kN},$ 各杆长度均为**1m;**

求：**1,2,3**杆受力.

解： 取整体,求支座约束力.



$$\sum F_x = 0 \qquad F_{Ax} = 0$$

$$\sum M_B = 0 \quad 2P_E + P_G - 3F_{Ay} = 0$$

$$\sum F_y = 0 \quad F_{Ay} + F_{By} - P_E - P_G = 0$$

$$\Rightarrow \quad F_{Ay} = 9\text{kN} \qquad F_{By} = 8\text{kN}$$

§2-6 平面简单桁架的内力计算

用截面法,取桁架左边部分.

截面经过的每个杆件均为二力杆，截面一侧的刚体系受到平面任意力系作用（3个平衡方程）

截面法

$$\sum M_E = 0 \qquad -F_1 \cdot 1 \cdot \cos 30^0 - F_{Ay} \cdot 1 = 0$$

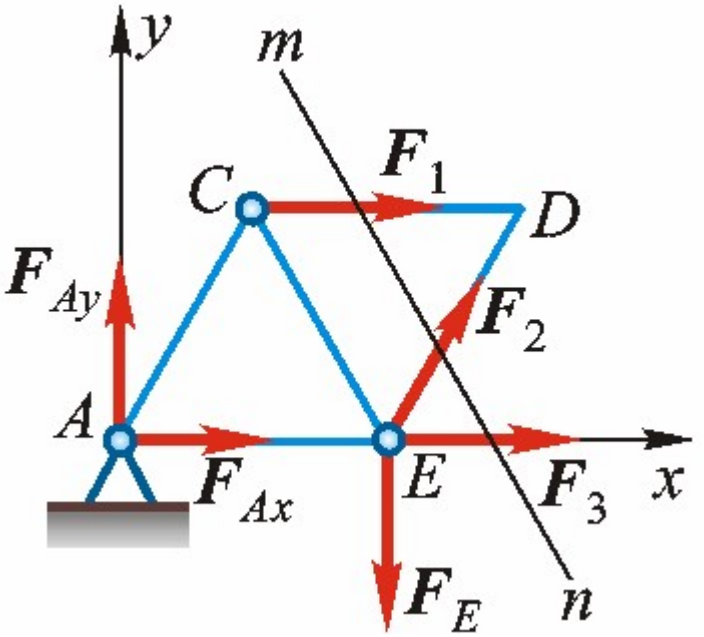
$$\sum F_y = 0 \qquad F_{Ay} + F_2 \cdot \sin 60^0 - P_E = 0$$

$$\sum F_x = 0 \qquad F_1 + F_3 + F_2 \cos 60^0 = 0$$

$$F_1 = 10.4\text{kN(压)}$$

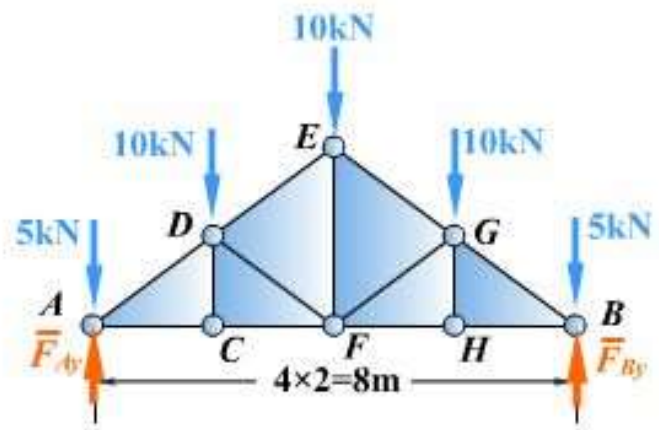
$$F_2 = 1.15\text{kN(拉)}$$

$$F_3 = 9.81\text{kN(拉)}$$



§2-6 平面简单桁架的内力计算

例2-30



已知：　荷载与尺寸如图；

求：　每根杆所受力.

解：　取整体，画受力图.

$$\sum F_x = 0 \qquad F_{Ax} = 0$$

$$\sum M_B = 0 \qquad -8F_{Ay} + 5\times 8 + 10\times 6 + 10\times 4 + 10\times 2 = 0$$

得　$F_{Ay} = 20\mathbf{kN}$

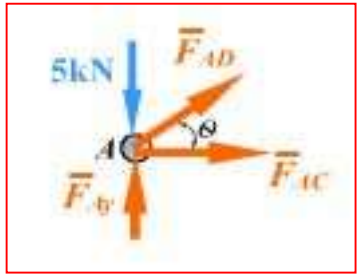
$$\sum F_y = 0 \qquad F_{Ay} + F_{By} - 40 = 0$$

得　$F_{By} = 20\mathrm{kN}$

求各杆内力

取节点A

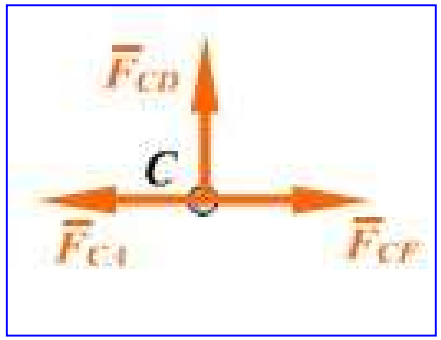
$$\begin{cases} \sum F_y = 0 \rightarrow F_{AD} \\ \sum F_x = 0 \rightarrow F_{AC} \end{cases}$$



§2-6 平面简单桁架的内力计算

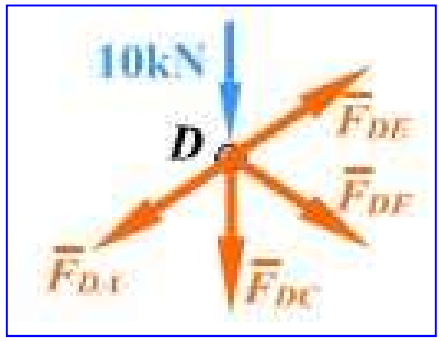
取节点C

$$\begin{cases}\sum F_x = 0 \rightarrow F_{CF} \\ \sum F_y = 0 \rightarrow F_{CD} = 0\end{cases}$$



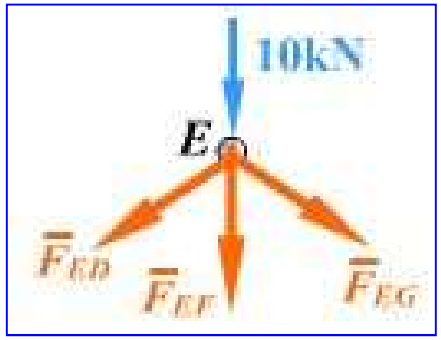
取节点D

$$\begin{cases}\sum F_y = 0 \\ \sum F_x = 0\end{cases} \rightarrow F_{DF}, F_{DE}$$



取节点E

$$\begin{cases}\sum F_y = 0 \rightarrow F_{EG} \\ \sum F_x = 0 \rightarrow F_{EF}\end{cases}$$



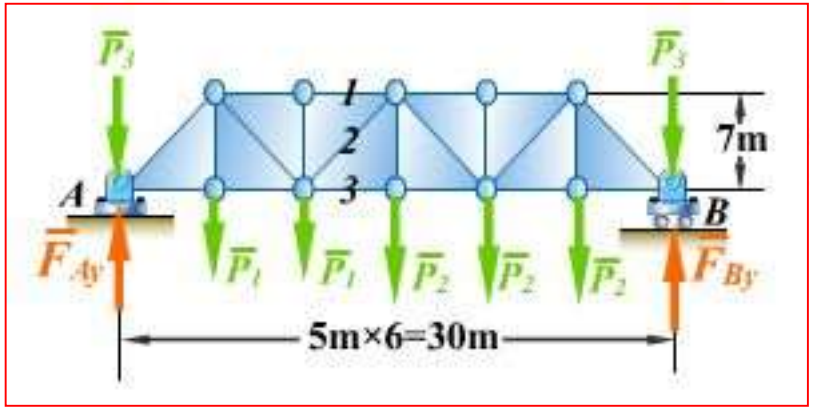
• • • • • •

§2-6 平面简单桁架的内力计算

例2-31

已知：P_1, P_2, P_3，尺寸如图.

求：1，2，3 杆所受力.

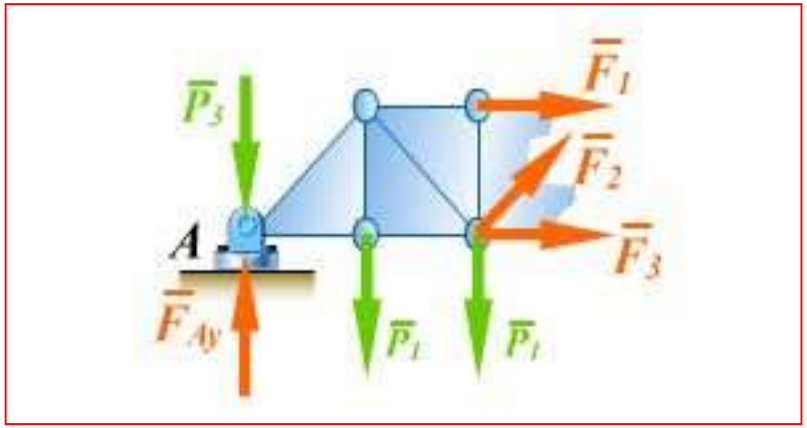


解： 求支座约束力

$$\sum M_A = 0 \quad \rightarrow F_{Ay}$$

$$\sum F_y = 0 \quad \rightarrow F_{By}$$

从1，2，3杆处截取左边部分



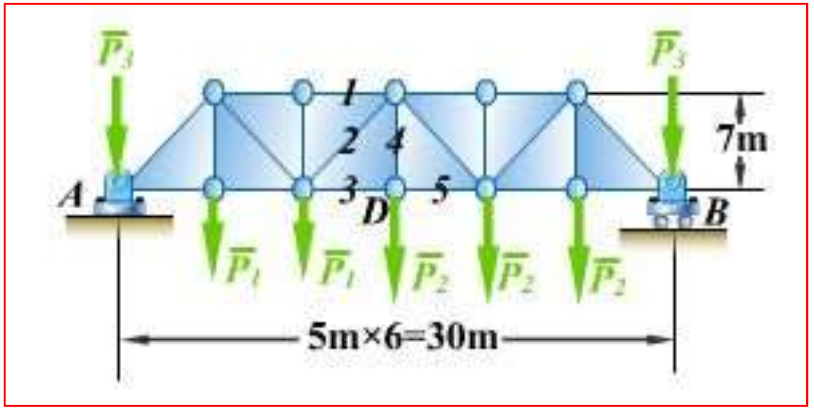
$$\sum F_y = 0 \quad \rightarrow F_2$$

$$\sum M_C = 0 \rightarrow F_1$$

$$\sum F_x = 0 \quad \rightarrow F_3$$

§2-6 平面简单桁架的内力计算

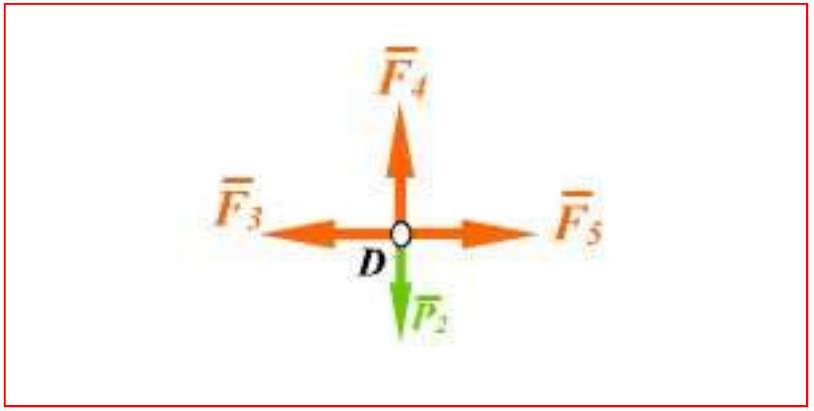
若再求4，5杆受力



取节点*D*

$$\sum F_x = 0 \rightarrow F_5$$

$$\sum F_y = 0 \rightarrow F_4$$



考试例题

判断题：

作用在刚体上的一个力，可以从原来的作用位置平行移动到该刚体内任意指定点，但必须附加一个力偶，附加的力偶的矩等于原力对该指定点的矩。

（正确）

某平面力系的力多边形自行封闭，则该力系必为平衡力系

（错误）

选择题：

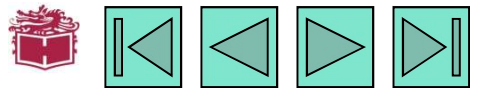
在下述公理、原理与定理中，对所有物体都适用的是

A 二力平衡公理

B 力的平行四边形法则

C 加减平衡力系公理

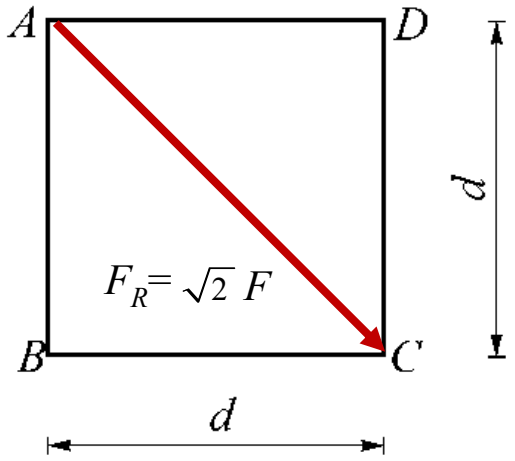
D 力的可传递性



考试例题

简单题：

在边长为***d***的正方形***ABCD***所在平面内，作用一平面任意力系，该力系向***A***点简化有$\sum M_A(\vec{F}_i)=0$，向**B**点简化有$\sum M_B(\vec{F}_i)=-Fd$（顺时针方向），向**D**点简化有$\sum M_D(\vec{F}_i)=Fd$。求此力系简化的最后结果（说明大小与方向）

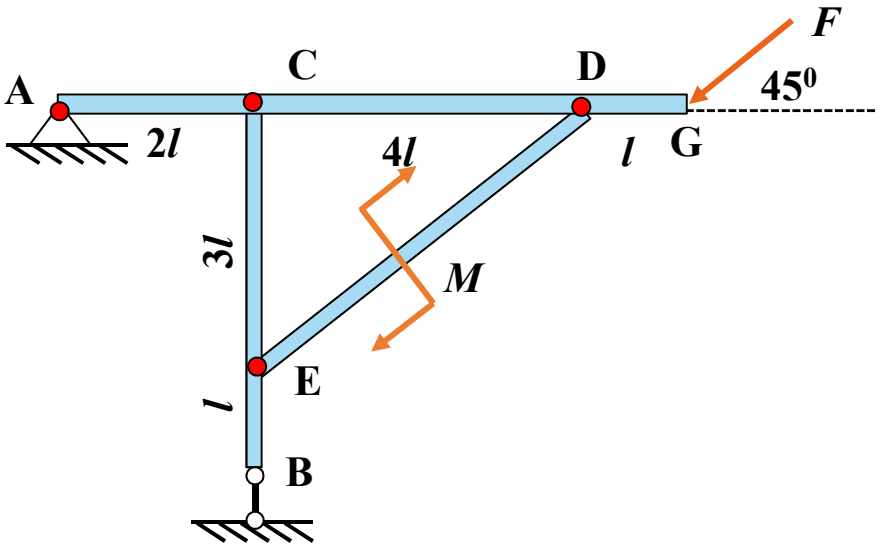


合力矩定理：平面任意力系的合力对作用面内任一点的矩等于力系中各力对同一点的矩代数和

考试例题

计算题：

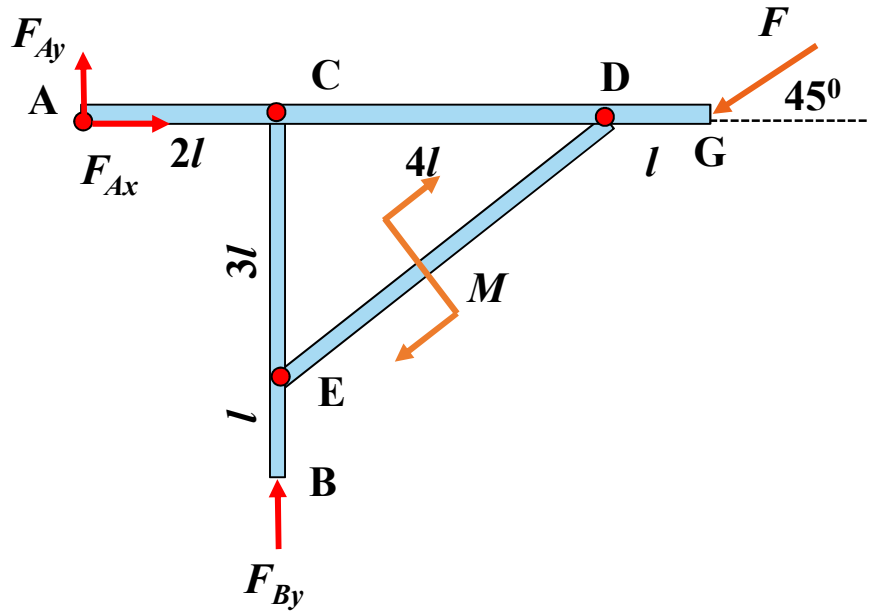
如图所示结构，杆件自重不计，已知F=4kN, M=10kNm, l=1m。求支座A、B以及铰链C、D的约束力。



期末考试例题

计算题：

如图所示结构，杆件自重不计，已知F=4kN, M=10kNm, l=1m。求支座A、B以及铰链C、D的约束力。



解：取整体,求支座约束力.

$$\sum F_x = 0 \quad F_{Ax} - F\cos 45^0 = 0$$

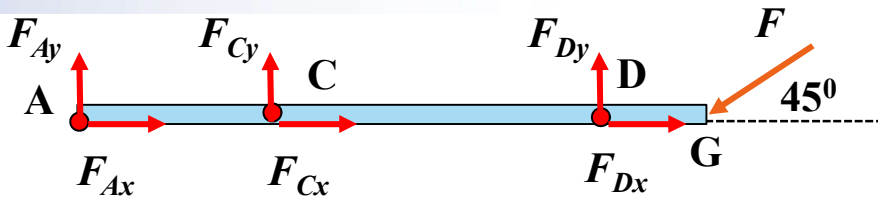
$$\sum F_y = 0 \quad F_{Ay} + F_{By} - F\sin 45^0 = 0$$

$$\sum M_A = 0 \quad F_{By} \cdot 2l - M - F\sin 45^0 \cdot 7l = 0$$

$$\Rightarrow \quad F_{Ax} = 2.83\text{kN} \quad F_{By} = 14.09\text{kN}$$

$$F_{Ay} = -12.07\text{kN}$$

期末考试例题



取**ACDG**杆，受力图如右.

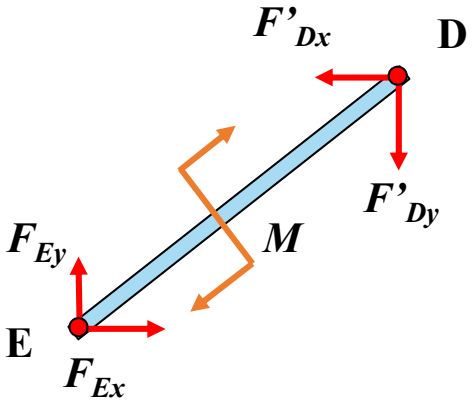
$$\sum F_x = 0 \quad F_{Ax} + F_{Cx} + F_{Dx} - F\cos 45^0 = 0$$

$$\sum F_y = 0 \quad F_{Ay} + F_{Cy} + F_{Dy} - F\sin 45^0 = 0$$

$$\sum M_C = 0 \quad -F_{Ay}\cdot 2l + F_{Dy}\cdot 4l - F\sin 45^0 \cdot 7l = 0$$

取**ED**杆，受力图如右.

$$\sum M_E = 0 \quad F'_{Dx}\cdot 3l - F'_{Dy}\cdot 4l - M = 0$$



$$\Rightarrow \quad F_{Cx} = 0 \qquad F_{Cy} = 17.4\text{kN}$$

$$F_{Dx} = 0 \qquad F_{Dy} = -2.05\text{kN}$$

作业

教材习题：2–39, 2–43,
2–57, 2–60