# Conditional Convergence and Long-Run Economic Growth

# Conditional Convergence in Practice

- 什么叫做“条件收敛”，或者叫做相对收敛?
  - Growth rate of capital per worker, $\Delta k/k$:
  - $\Delta k/k = \phi[\, k(0)\, ,\, k^*]$ ,

    $(-)\ (+)$
  - 即$\Delta k/k$的大小取决于$k^*/\ k(0)$的比值，或者两者之间的相对距离。

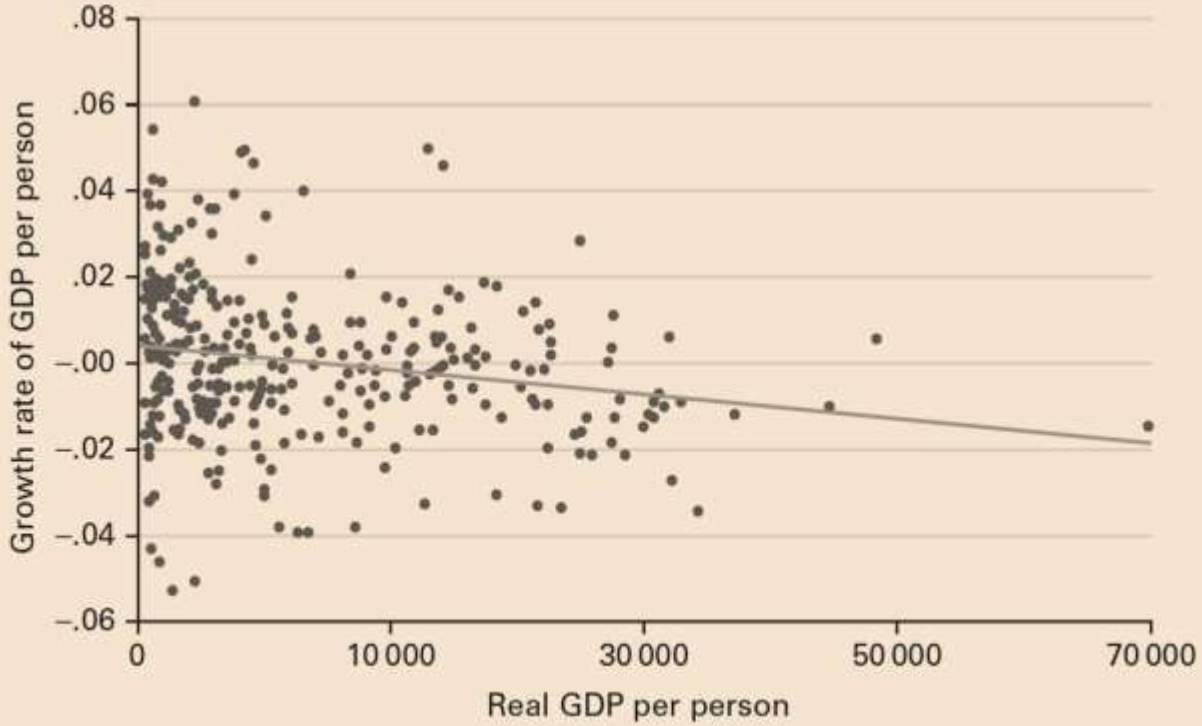
# Conditional Convergence in Practice

- $y = A \cdot f(k)$
- Growth rate of real GDP per worker is a function of initial and steady-state real GDP per worker.
- $\Delta y/y = \phi[\, y(0) \,,\, y^*]$

  $(-) \quad (+)$

**Figure 5.1 Growth rate versus level of real GDP per person: conditional convergence for a broad group of countries**

The horizontal axis shows real GDP per person in 2005 US dollars. The data are for 57 countries in 1965, 62 countries in 1975, 65 countries in 1985, 69 countries in 1995 and 70 countries in 2005. (The sample was based on the availability of data.) The vertical axis shows the corresponding growth rates of GDP per capita for 1965–75, 1975–85, 1985–95, 1995–05 and 2005–10. Each of the growth rates filters out (and, therefore, holds constant) the estimated effects of the variables discussed in the text. The blue line is the straight line that provides a best fit to the relation between the growth rate of real GDP per person (the variable on the vertical axis) and the level of real GDP per person (on the horizontal axis). The line has a clear negative slope. Therefore, once we hold constant the other variables, a lower level of real GDP per person matches up with a higher growth rate of real GDP per person. This relation is called 'conditional convergence'.



# Conditional Convergence in Practice

- Variables that influence $y^*$ that are held constant.
  - The saving rate（*s*）
  - The fertility rate (*n*)
  - Technology level (A) determines productivity
  - Others also influence economy's productivity
    - maintenance of the rule of law and democracy（民主法治）
    - The government efficiency（政府效率，有为政府）
    - The extent of international openness, measured by the volume of exports and imports (对外开放程度)
    - Changes in the terms of trade(对外开放条件)
    - Measures of investment in education and health （创新基础）
    - The stable rate of inflation (价格稳定，市场作用)

# Long-Run Economic Growth

- Solow model implies that eventually economies will stop growing（in terms of per capita capital and product).
- A case in which capital broadly defined to include human and phsycial capital is the only factor input to production:
- AK model:

  $y = Af(k) = Ak$, that is, $f(k) = k$

  $y/k = A$ :

  $\Delta k/k = sA - s\delta - n$ : 不再是减函数

# Long-Run Economic Growth

- AK model relaxes *f(k) being a diminishing function of k,* which is key for Solow's convergence
- The reason is that, the lower and lower efficiency of physical capital when more and more physical capital is put into production, can be continuously mitigated by more and more human capital bundled with physical captial.

# Long-Run Economic Growth

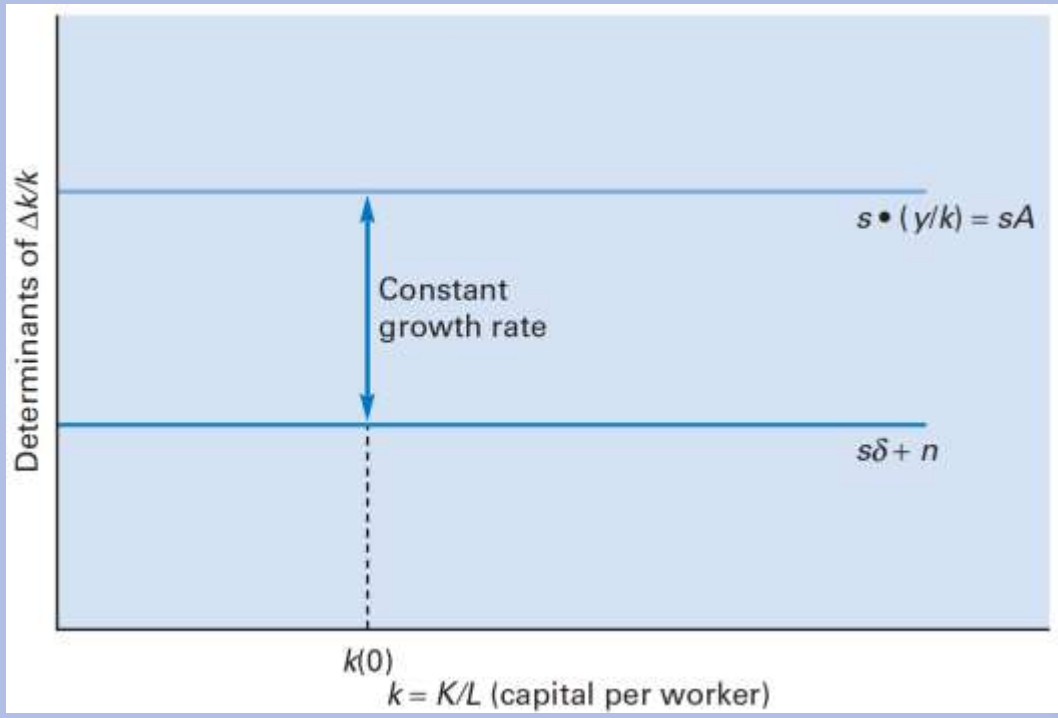



**Figure 5.2** Economic growth with constant average product of capital

This graph modifies Figure 3.9 to allow for an unchanging average product of capital, $y/k$. In this $Ak$ model, $y/k$ equals the technology level, $A$. Therefore, the $s \bullet (y/k)$ curve becomes the horizontal line $sA$. If $sA$ is greater than $s\delta + n$, as shown, the growth rate of capital per worker, $\Delta k/k$, is a positive constant equal to the vertical distance between the two horizontal lines. This distance is shown by the blue arrows.

# Long-Run Economic Growth

- Conclusions
  - The long-run growth rate of capital per worker, $\Delta k/k$, is greater than zero and equal to $sA - s\delta - n$
  - Growth rates of capital and real GDP per worker, $\Delta k/k$ and $\Delta y/y$, do not decrease as capital and real GDP per worker, $k$ and $y$, rise.
  - Consequently, poor economies with low $k$ and $y$ do not necessarily tend to grow faster than rich economies.

# Long-Run Economic Growth

- Technological progress: the regular process of improvement in technology.
  - exogenous technological progress: the improvements in technology were not explained within the model.
  - $\Delta A/A = g$, which is previously assumed =0

# Long-Run Economic Growth

- Endogenous Technological Progress

$\Delta Y/Y = \Delta A/A + \alpha\cdot(\Delta K/K) + (1-\alpha)\cdot(\Delta L/L)$

*Using* $\Delta A/A = g \neq 0$ , $\Delta L/L = n$

$\Delta Y/Y = g + \alpha\cdot(\Delta K/K) + (1-\alpha)\cdot n$

$\Delta y/y = \Delta Y/Y - \Delta L/L = \Delta Y/Y - n$

$\Delta y/y = g + \alpha\cdot(\Delta K/K) + (1-\alpha)\cdot n - n$

$= g + \alpha\cdot(\Delta K/K - n)$

$\Delta k/k = \Delta K/K - \Delta L/L = \Delta K/K - n$

$\Delta y/y = g + \alpha\cdot(\Delta k/k)$

对照：SOLOW MODLE, $\Delta y/y = \alpha\cdot(\Delta k/k)$

# Long-Run Economic Growth

- $\Delta k/k$ previously in the Solow model:

  $\Delta k/k = sA \cdot f(k)/k - s\delta - n$

- Growth rate of real GDP per worker with endogenous technical progress

  $\Delta y/y = g + \alpha \cdot [\, sA \cdot f(k)/k - s\delta - n]$

- 之前$y/k = A \cdot f(k)/k$, 当$k$ 增加的时候，$y/k$ 越来越小（因为$A$是固定不变的常数），可以实现$\Delta k/k = 0$（长期均衡状态下）。

# Long-Run Economic Growth

- 现在$y/k=A\cdot f(k)/k$, 当$k$ 增加的时候，$y/k$越来越小的趋势被抵消，因为$A$是不断增长的。
- 假定$y/k$在长期均衡状态下，可以保持不变（比如AK模型），则:$(\Delta y/y)^* = (\Delta k/k)^* \neq 0$ (因为 $y/k$ 是常数)

又因为：$(\Delta y/y)^* = g + \alpha\cdot(\Delta k/k)^*$

所以：$(\Delta y/y)^* = g + \alpha\cdot(\Delta y/y)^*$

$(\Delta y/y)^* - \alpha\cdot(\Delta y/y)^* = g$

$(1-\alpha)\cdot(\Delta y/y)^* = g$

$(\Delta y/y)^* = g/(1-\alpha) \neq 0$

对照：Solow Model: *g=0, so* $(\Delta y/y)^* = 0$

# Long-Run Economic Growth

- Endogenous Technological Progress
  - Steady-state growth rate with endogenous technological progress
    - $(\Delta y/y)^* = g/(1 - \alpha)$
    - Since $0 < \alpha < 1$, the steady-state growth rate of real GDP per worker, $(\Delta y/y)^*$, is greater than the rate of technological progress, *g*.

# Long-Run Economic Growth

- Endogenous Technological Progress

  $(\Delta k/k)^* = (\Delta y/y)^*$

  $(\Delta k/k)^* = g/(1 - \alpha)$

  Endogenous technological progress at the rate $\Delta A/A = g$ leads to long-term growth in real GDP and capital per worker, $k$ and $y$, at the rate $g/(1 - \alpha)$.

# Long-Run Economic Growth

- Endogenous Technological Progress

  $\Delta k/k = s\cdot(y/k) - s\delta - n$

  在长期均衡状态下(In steady-state growth)

  $(\Delta k/k)^* = g/(1-\alpha)$

  所以长期均衡状态下:

  $g/(1-\alpha) = s\cdot (y^*/k^*) - s\delta - n$

  $\Longrightarrow$ $(y^*/k^*) = [s\delta + n + g/(1-\alpha)]$/s: 常数保持不变，即资本生产力保持不变。

  对照：AK模型， $y/k = A$

# Long-Run Economic Growth

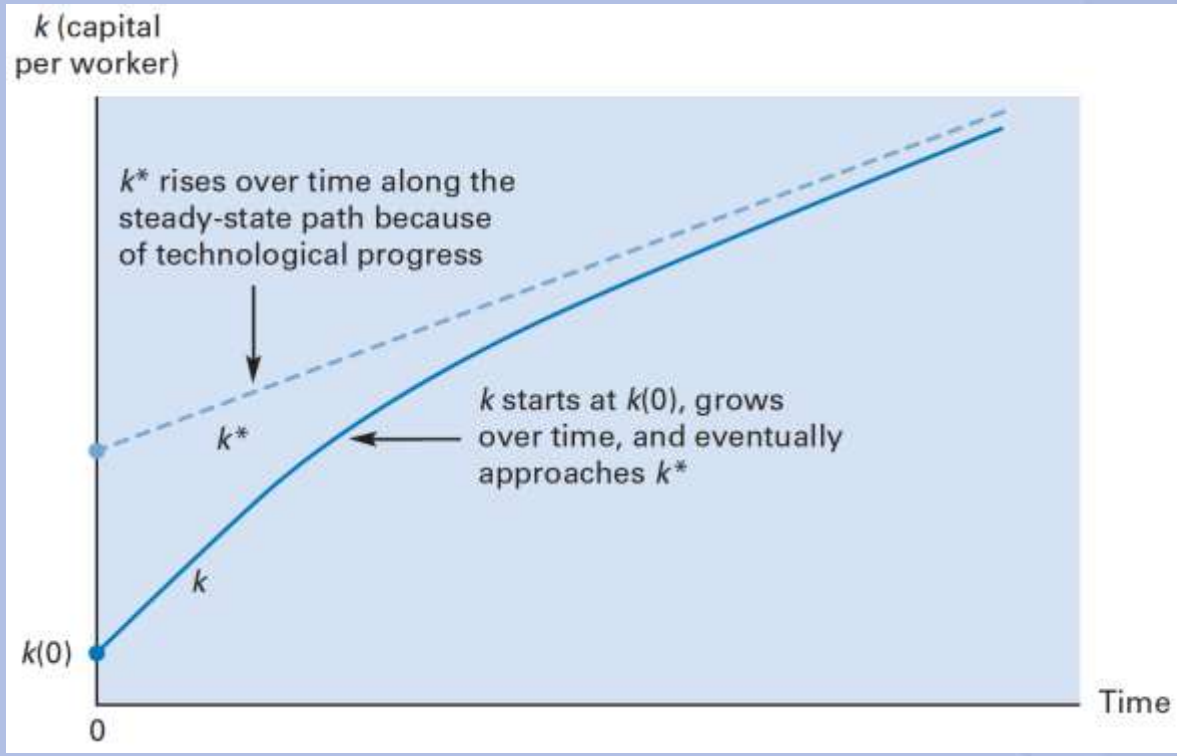
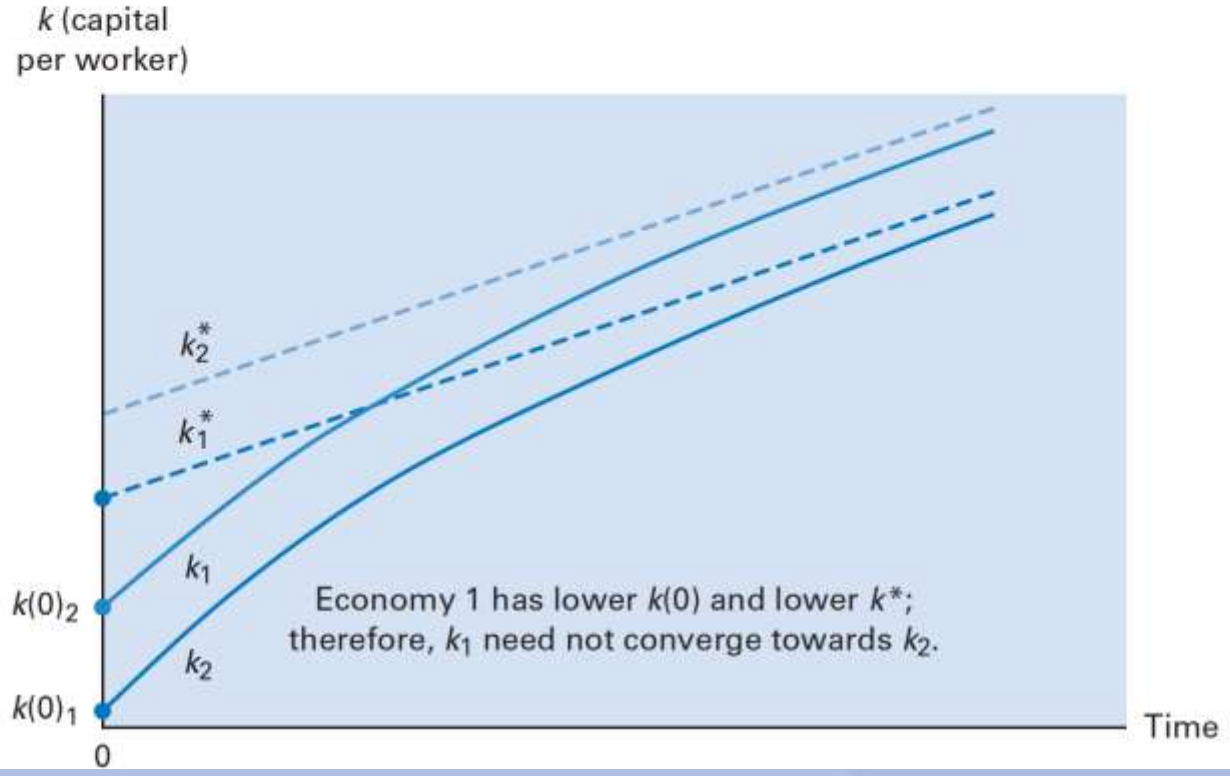



**Figure 5.3 The transition path for capital per worker in the Solow model with technological progress**

In the Solow model with technological progress at the rate $g$, the steady-state level of capital per worker, $k^*$, is not fixed; $k^*$ rises over time along the steady-state path shown by the dashed line. (Since we use a proportionate scale on the vertical axis, the straight line means that $k^*$ grows along the steady-state path at a constant rate, given by $g/[1 - \alpha]$.) In the transition, capital per worker, $k$, starts at $k(0)$, rises over time along the solid curve, and gradually approaches the $k^*$ line. (We assume that $k(0)$ lies below the $k^*$ line.)

**Figure 5.5 Failure of convergence and transition paths for two economies in the Solow model with technological progress**

As in Figure 5.4, the first economy starts at $k(0)_1$, and the second economy starts at the higher value $k(0)_2$. However, economy 1 now has a lower steady-state path of capital per worker; that is, the dashed line at $k_1^*$ for lies below the dashed line at $k_2^*$. During the transitions, $k_1$ and $k_2$ gradually approach their respective steady-state paths, $k_1^*$ and $k_2^*$. However, the growth rate of capital per worker, $\Delta k/k$, need not be higher in economy 1 than economy 2. Therefore, $k_1$ (the solid curve at $k_1$) does not necessarily converge towards $k_2$ (the solid curve at $k_2$). Hence, absolute convergence need not hold.



# Long-Run Economic Growth

- Endogenous Growth Theory
  - Extend the model to explain why technological progress occurs.
  - Most endogenous growth models focus on investments in research and development

# Long-Run Economic Growth

- Endogenous Growth Theory
  - 保罗·罗默(Paul M.Romer)，生于1955年，美国经济学家，新增长理论的主要建立者之一。现任纽约大学经济学教授，斯坦福大学经济学教授，胡佛研究所高级研究员.
  - 罗默在1986年建立了内生经济增长模型，把知识完整纳入到经济和技术体系之内，使其做为经济增长的内生变量。罗默提出了四要素增长理论，即新古典经济学中的资本和劳动(非技术劳动)外，又加上了人力资本(以受教育的年限衡量)和新思想(用专利来衡量，强调创新)。

# Long-Run Economic Growth

- Endogenous Growth Theory
  - The private return to R&D investment is higher if the costs of R&D are lower.
  - The reward from successful innovations depends on how much they raise sales revenue or reduce production costs.
  - The private return is higher if intellectual property rights over the use of an invention are more secure and long lasting.

# Long-Run Economic Growth

– The Diffusion of Technology
  - The imitation and adaptation of one country's technology by another country.
  - The rate of technological diffusion to a developing country is high when the country trades a lot with rich countries, has high education levels, and has well functioning legal and political systems.