

STAT GR5205 – Section 005 HW 4

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#1.

#(a)

#H0 : $Y_i = \beta_0 + \beta_1 x_i + \text{error}_i$ for $i=1, \dots, n$ and $H_a : Y_i = f(x_i) + \text{error}_i$ for $i=1, \dots, n$ error_i are iid $N(0, \sigma^2)$

#(b)

```
filename <- "~/Downloads/copiers_full.txt"
```

```
copier_maintenance<- read.table(file=filename, header=T)
```

```
y <- copier_maintenance$minutes
```

```
x <- copier_maintenance$copiers
```

#Let Y be the total repair time for the i th call on which there were j copiers.

#H0: $Y_{ij} = \beta_0 + \beta_1 x_{ij} + \text{error}_{ij}$

#Ha: $Y_{ij} = \mu + \text{error}_{ij}$, error_{ij} are iid $N(0, \sigma^2)$.

```
reduced <- lm(y ~ x)
```

```
full <- lm(y ~ as.factor(x) - 1)
```

```
anova(reduced, full)
```

```
## Analysis of Variance Table
```

```
##
```

```
## Model 1: y ~ x
```

```
## Model 2: y ~ as.factor(x) - 1
```

```
##   Res.Df    RSS Df Sum of Sq    F Pr(>F)
```

```
## 1      43 3416.4
```

```
## 2      35 2797.7  8    618.72 0.9676 0.4766
```

#The P-value is 0.4766 and thus we accept H0.

#(c)

#The test in part (b) does not detect other departures from the normal SLR model, but the results of the

#2

#(a)

```
filename <- "~/Downloads/copiers_full.txt"
```

```
copier_maintenance<- read.table(file=filename, header=T)
```

```
x <- copier_maintenance$copiers
```

```
y <- copier_maintenance$minutes
```

```
fit <- lm(y ~ x, data=copier_maintenance)
```

```
alpha<-.05
```

```
g <- 2
```

```
confint(fit, level=1-alpha/g)
```

```
##           1.25 %    98.75 %
```

```
## (Intercept) -7.092642  5.932329
```

```
## x           13.913221 16.157275
```

#Hence the 95% confident of beta0 is between -7.092642 and 5.932329 and the 95% confident of beta1 is b

#(b)

*#The intervals in part (a) support this claim,because beta0=0 is in the interval (-7.092642,5.932329)
#and beta1=14 is in the interval (13.913221, 16.157275).*

#(c)

```
alpha <- .05  
g <- 3  
predict(fit,data.frame(x=c(3,5,7)),interval="confidence",level=1-alpha/g)
```

```
##          fit      lwr      upr  
## 1  44.52559  40.35270  48.69848  
## 2  74.59608  71.28313  77.90904  
## 3 104.66658 100.65092 108.68224
```

*#Thus the 95% confidence interval for number of minutes spent when there are 3, 5,
#and 7 copiers to be serviced are (40.35270,48.69848), (71.28313, 77.90904) and (100.65092,108.68224) r*

#(d)

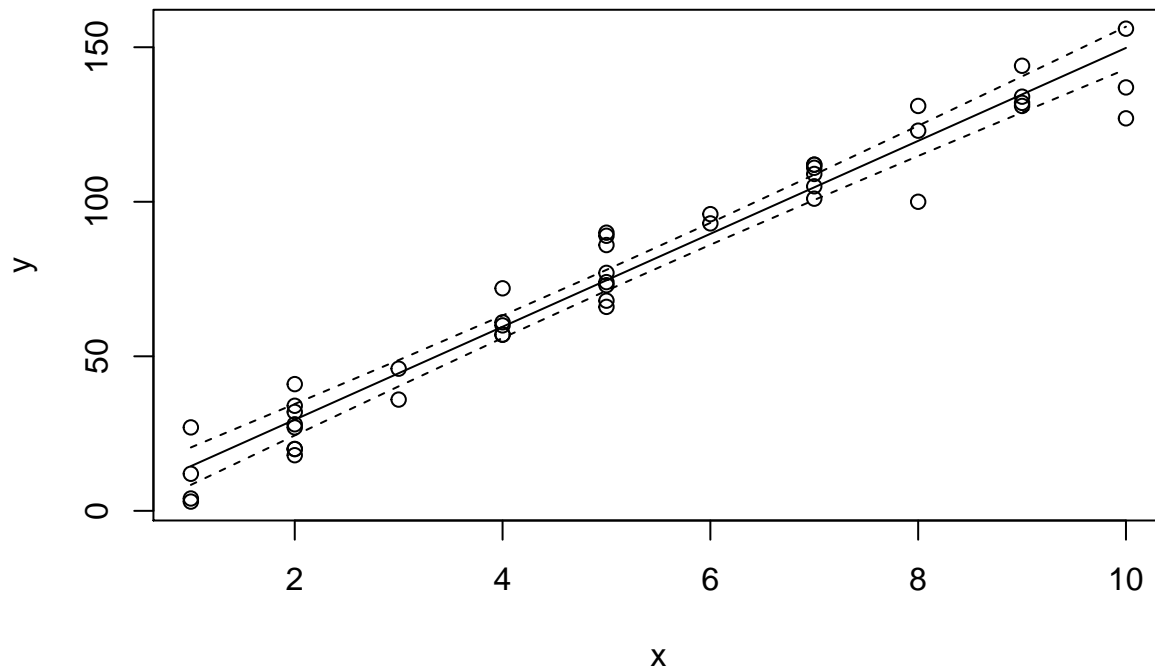
```
alpha <- .05  
g <- 2  
predict(fit, data.frame(x=c(4,6)), interval="prediction", level=1-alpha/g)
```

```
##          fit      lwr      upr  
## 1  59.56084  38.59230  80.52937  
## 2  89.63133  68.67614 110.58652
```

*#Thus the 95% confidence interval for number of minutes spent with 4 and 6 copiers to be serviced
#are (38.59230,80.52937) and (68.67614,110.58652) respectively.*

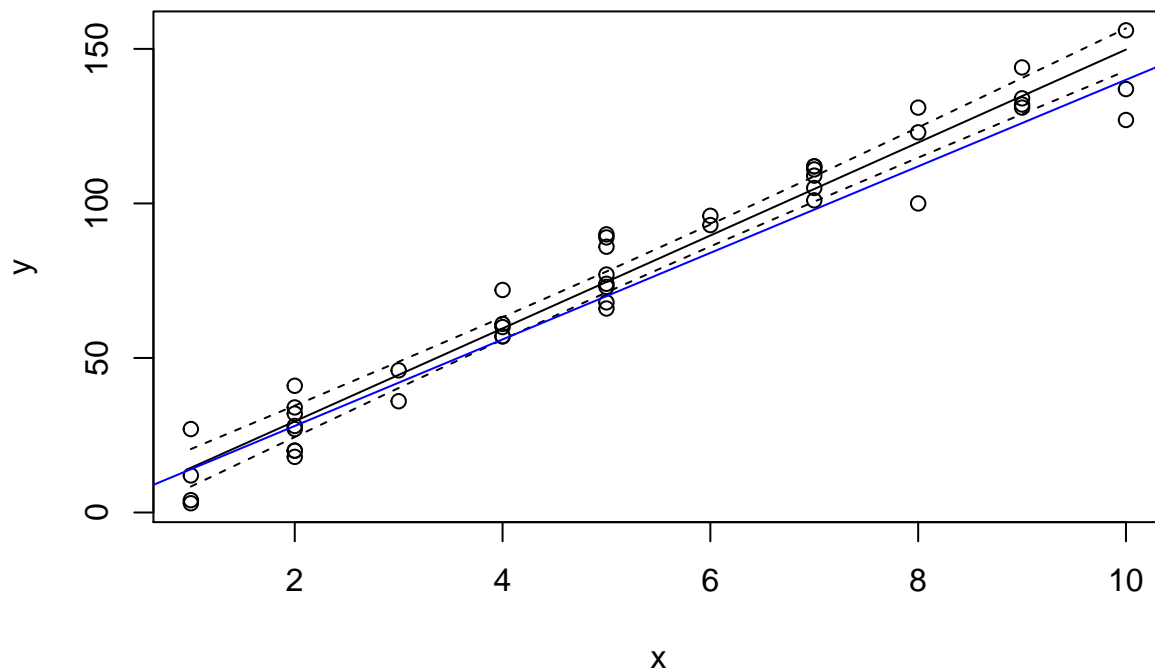
#(e)

```
plot(y ~ x, data=copier_maintenance)  
preds <- predict(fit,data.frame(x=1:10),se.fit=T)  
W.H <- sqrt(2 * qf(.95, df1=2, df2=43))  
lines(1:10, preds$fit)  
lines(1:10, preds$fit + W.H*preds$se.fit, lty=2.5)  
lines(1:10, preds$fit - W.H*preds$se.fit, lty=2.5)
```



#The 95% confident interval of mean function falls within the dashed lines.

```
##(f)
plot(y ~ x, data=copier_maintenance)
preds <- predict(fit,data.frame(x=1:10),se.fit=T)
W.H <- sqrt(2 * qf(.95, df1=2, df2=43))
lines(1:10, preds$fit)
lines(1:10, preds$fit + W.H*preds$se.fit, lty=2.5)
lines(1:10, preds$fit - W.H*preds$se.fit, lty=2.5)
abline(0,14,col="blue")
```



#The line doesn't fall within the WH confidence band, thus the data are not consistent with claim.

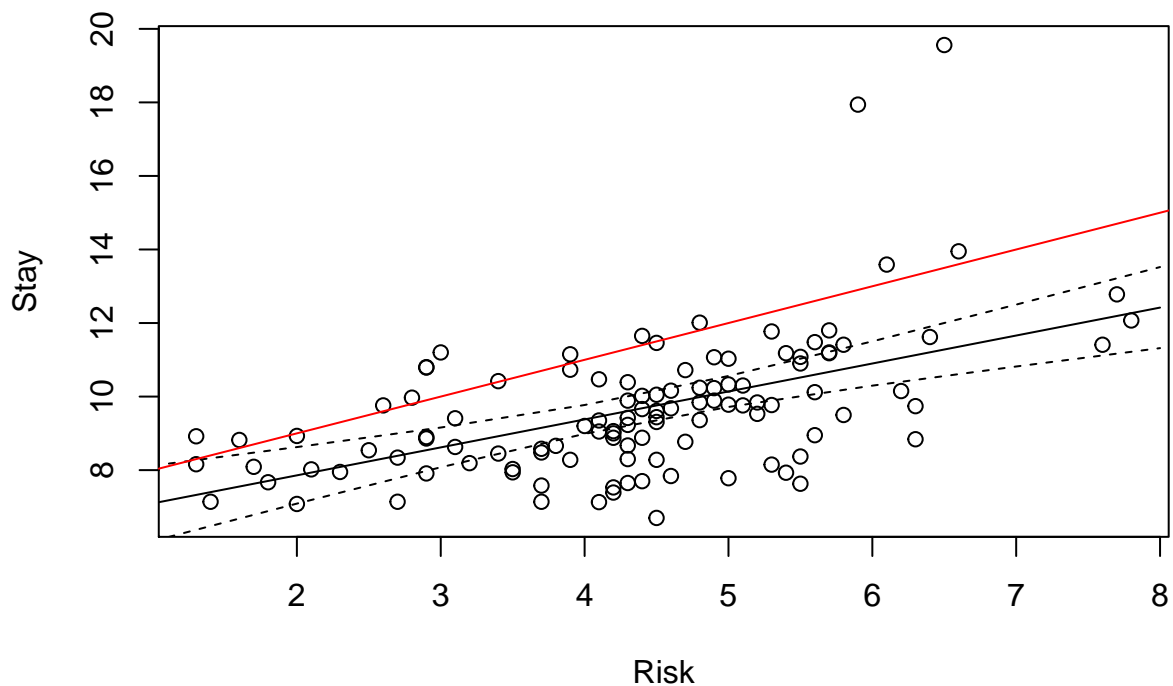
```
#3.  
#(a)  
filename <- "~/Downloads/SENIC.txt"  
SENIC <- read.table(file=filename, header=T)  
fit.Risk <- lm(Stay ~ Risk, data=SENIC)  
alpha<-.05  
g <- 2  
confint(fit.Risk, level=1-alpha/g)
```

```
##              1.25 %  98.75 %  
## (Intercept) 5.1523372 7.521236  
## Risk        0.5003816 1.020460
```

#Hence the 95% confident of beta1 is between 0.5003816 and 1.020460 and the 95% confident of beta0 is b

```
#(b)  
#way 1: Since beta1=1 and beta0=7 are in the interval from part a, the data are consistent with the cla
```

```
preds <- predict(fit.Risk, data.frame(Risk=1:8), se.fit=T)  
W.H <- sqrt(2 * qf(.95, df1=2, df2=111))  
plot(Stay ~ Risk, data=SENIC)  
lines(1:8, preds$fit)  
lines(1:8, preds$fit + W.H*preds$se.fit, lty=2)  
lines(1:8, preds$fit - W.H*preds$se.fit, lty=2)  
abline(7,1, col="red")
```



#way 2: the line doesn't fall within the WH confidence band, thus the data are not consistent with claim

```
##(c)
alpha <- .05
g <- 4
predict(fit.Risk,data.frame(Risk=2:5),interval="confidence",level=1-alpha/g)
```

```
##      fit      lwr      upr
## 1  7.857628 7.071051 8.644206
## 2  8.618049 8.065356 9.170743
## 3  9.378470 8.977088 9.779852
## 4 10.138891 9.708059 10.569723
```

#Thus the 95% confident intervals of the mean stay at hospitals with infection risk $x = 2, 3, 4$ and 5 are $(7.071051, 8.644206)$, $(8.065356, 9.170743)$, $(8.977088, 9.779852)$ and $(9.708059, 10.569723)$ respectively.

```
##4
##(a)
filename <- "~/Downloads/copiers_full.txt"
copier_maintenance<- read.table(file=filename, header=T)
x <- copier_maintenance$copiers
y <- copier_maintenance$minutes
X<-cbind(rep(1,length(x)),x)
beta_value<-solve(t(X)%*%X)%*%t(X)%*%y
beta_value<-as.vector(beta_value)
beta_value
```

```
## [1] -0.5801567 15.0352480
```

#Hence the coefficients are -0.5801567 and 15.0352480.

```
##(b)
y_hat<-X%*%beta_value
y_hat<-as.vector(y_hat)
y_hat
```

```
## [1] 29.49034 59.56084 44.52559 29.49034 14.45509 149.77232 74.59608
## [8] 74.59608 14.45509 29.49034 134.73708 149.77232 89.63133 44.52559
## [15] 59.56084 119.70183 104.66658 119.70183 149.77232 59.56084 74.59608
## [22] 104.66658 104.66658 74.59608 134.73708 104.66658 29.49034 74.59608
## [29] 104.66658 89.63133 119.70183 74.59608 29.49034 29.49034 14.45509
## [36] 59.56084 74.59608 134.73708 104.66658 14.45509 134.73708 29.49034
## [43] 29.49034 59.56084 74.59608
```

```
e<-y-y_hat
e
```

```
## [1] -9.4903394 0.4391645 1.4744125 11.5096606 -2.4550914
## [6] -12.7723238 -6.5960836 14.4039164 -10.4550914 2.5096606
## [11] 9.2629243 6.2276762 3.3686684 -8.5255875 12.4391645
```

```
## [16] -19.7018277  0.3334204  11.2981723 -22.7723238 -2.5608355
## [21] -8.5960836  -3.6665796   4.3334204  -0.5960836  -0.7370757
## [26]  7.3334204 -11.4903394  -1.5960836   6.3334204   6.3686684
## [31]  3.2981723  15.4039164  -9.4903394  -1.4903394 -11.4550914
## [36] -2.5608355  11.4039164  -2.7370757   7.3334204  12.5449086
## [41] -3.7370757   4.5096606  -2.4903394   1.4391645   2.4039164
```

```
#(c)
J<-matrix(rep(1, length(y)^2), length(y), length(y))
SST0<-t(y)%*%y-1/length(y)*t(y)%*%J%*%y
SST0<-as.numeric(SST0)
SST0
```

```
## [1] 80376.8
```

```
SSE<-t(e)%*%e
SSE<-as.numeric(SSE)
SSE
```

```
## [1] 3416.377
```

```
#(d)
MSE<-SSE/(length(y)-2)
MSE<-as.numeric(MSE)
variance_covariance_matrix_of_b<-MSE * solve(t(X) %*% X)
variance_covariance_matrix_of_b
```

```
##                x
##    7.862086 -1.1927966
## x -1.192797  0.2333733
```

```
#(e)
SE.fit<-sqrt(MSE*(1/length(x) + (6-mean(x))^2 / sum((x-mean(x))^2)))
t(beta_value) %*% c(1,6) + c(-1,1)*qt(.975,length(x)-2)*SE.fit
```

```
## [1] 86.81520 92.44746
```

```
#The 95% confidence interval is (86.81520,92.44746).
```

```
#(f)
SE.pred <- sqrt(MSE*(1+1/length(x)+(6-mean(x))^2/sum((x-mean(x))^2)))
t(beta_value)%*%c(1,6)+c(-1,1)*qt(.975,length(x)-2)*SE.pred
```

```
## [1] 71.43628 107.82639
```

```
#The 95% prediction interval is (71.43628,107.82639).
```