4. (a) $e_i = y_i - (b_0 + b_1 x_i) = y_i - b_0 - b_1 x_i = y_i - \bar{y} + b_1\bar{x} - b_1 x_i$

$$= y_i - \bar{y} + b_1(\bar{x} - x_i)$$

$$\sum_{i=1}^{n} e_i = \sum_{i=1}^{n}\left[y_i - \bar{y} + b_1(\bar{x} - x_i)\right] = \sum_{i=1}^{n}(y_i - \bar{y}) + \sum_{i=1}^{n} b_1(\bar{x} - x_i)$$

$$= n\bar{y} - n\bar{y} + b_1(n\bar{x} - n\bar{x}) = 0 + b_1 \times (0) = 0$$

(b) $\sum_{i=1}^{n} y_i = n\bar{y}$

$$\sum_{i=1}^{n}\hat{y}_i = \sum_{i=1}^{n}(\bar{y} - b_1\bar{x} + b_1 x_i) = \sum_{i=1}^{n}\bar{y} - \sum_{i=1}^{n} b_1(\bar{x} - x_i) = n\bar{y} - b_1(n\bar{x} - n\bar{x})$$

$$= n\bar{y} - b_1(0) = n\bar{y}$$

Hence $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n}\hat{y}_i$

(c) $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n}\hat{y}_i = \sum_{i=1}^{n}(b_0 + b_1 x_i) = n b_0 + b_1 \sum_{i=1}^{n} x_i$

$$\sum x_i y_i = b_0 \sum_{i=1}^{n} x_i + b_1 \sum_{i=1}^{n} x_i^2$$

$$\sum_{i=1}^{n} x_i e_i = \sum_{i=1}^{n} x_i y_i - b_0 \sum_{i=1}^{n} x_i - b_1 \sum_{i=1}^{n} x_i^2$$

$$= b_0 \sum_{i=1}^{n} x_i + b_1 \sum_{i=1}^{n} x_i^2 - b_0 \sum_{i=1}^{n} x_i - b_1 \sum_{i=1}^{n} x_i^2 = 0$$

(d) $\sum_{i=1}^{n}\hat{y}_i e_i = \sum_{i=1}^{n}(b_0 + b_1 x_i)e_i = b_0 \sum_{i=1}^{n} e_i + b_1 \sum_{i=1}^{n} e_i x_i = b_0 \times 0 + b_1 \times 0 = 0$

(e) $\hat{y} = b_0 + b_1 x = b_0 + b_1\bar{x} = \bar{y} - b_1\bar{x} + b_1\bar{x} = \bar{y}$

Hence the least squares regression line always passes through point $(\bar{x}, \bar{y})$