Identification Design with Non-Binary Treatments

Some caveats I would like to discuss

Alessandro Pizzigolotto

NEK Group Supervision
9th February 2022

Introduction

- I took a 'step back' in my populism paper to analyse all the building blocks
- Focus on the identification design to integrate with the flourishing DIDs and TWFE literature
- Identification design using non-binary treatment and multiple periods
- It would solve some headaches: more convincing identification, soothe some potential endogeneity issues
- People have starting using new estimators in the job market (*e.g* Becker, 2021)

Identification

- Let $t \in \{1, \ldots, T\}$, $g \in \{1, \ldots, G\}$ and $i \in \{1, \ldots, N\}$ placeholders respectively for any time period (year), group (county) and individual.

$$Y_{igt} = \alpha_g + \phi_t + \beta_{post} D_{gt} + \gamma X_{igt} + \varepsilon_{gt}$$

- Y_{igt} denotes the outcome of individual i in group g at period t
- α_g and ϕ_t are respectively group and period fixed effects
- β_{post} denote the coefficient of D_{gt} a continuous treatment at g in period t: in this special case $D_{gt} = 0 \; \forall \; g, t < \tau$ and $D_{gt} > 0 \; \forall \; g, t \geq \tau$, where τ is a constant time period (2009)
- *ignore* X_{igt} covariates for now

Outline

Issue 1: Forbidden Comparisons I

- Until recently, TWFE estimators have been considered equivalent to difference-in-differences under (conditional) parallel trends and no anticipation assumptions
- However, many researchers (Callaway and Sant'Anna, 2021; de Chaisemartin and D'Haultfœuille, 2020; Goodman-Bacon, 2021) have emphasized that the TWFE regressions may not identify a *convex* combination of treatment effects due to 'forbidden extrapolations' (Borusyak et al., 2021)
  - even without variation of treatment timing, forbidden comparisons arise from comparing groups with different treatment intensity at the same period
  - fail to estimate the weighted sum of treatment effects of each group g after treatment by weighting some of the groups negatively

Issue 1: Forbidden Comparisons II

- Following de Chaisemartin and D'Haultfœuille (2018), focusing on non-binary treatments
- two-groups $g \in \{h(igh), l(low)\}$ and two-period setting $t \in \{pre, post\}$ for simplicity

$$\hat{\beta}_{post} = \frac{Y_{h2} - Y_{h1} - (Y_{l2} - Y_{l1})}{D_{h2} - D_{h1} - (D_{l2} - D_{l1})} \qquad \text{(Wald-DID)}$$

Issue 1: Forbidden Comparisons III

- Assume that group h goes from 0 to 2 in second period, whereas l goes from 0 to 1, and potential outcomes are linear in the number of units and constant over time, but differ for groups: $Y_{gt} = T_{gt}(0) + \delta_g d$ (with d placeholder for treatment)
- Denominator becomes 1
- Numerator (under parallel trends) becomes $2\delta_h - \delta_l$, weighted sum of treatment effects BUT group l is weighted negatively subtracting its treatment effect out from $\hat{\beta}_{post}$

Which heterogeneity-robust estimator is correct to use in my case?

Issue 1: Forbidden Comparisons

Potential Solution

- Ruling out dynamic effects (no memory) (de Chaisemartin et al., 2022, with quasi-stayers): IMO not a good strategy
- Special case of estimators allowing for dynamic effects
  - non-binary (continuous) treatment
  - special of staggered design where all groups assume a different treatment at the same period
- de Chaisemartin and D'Haultfoeuille (2021) estimator (`did_multiplegt` in Stata) allows for this estimation
  - equivalent to estimate a weighted sum of DIDs between pairs of groups' treatment intensity
  - interpretation of the parameter as some average of the effect produced by one unit increase of treatment

Issue 1: Forbidden Comparisons

Other Caveats in Estimation

- Callaway et al. (2021) propose a similar estimator but distinguish between
  - *level effect*: treatment effect of dose d, difference between group's potential outcome under treatment d and untreated potential outcome
  - *slope effect*: causal response to an incremental change in the dose d (Angrist and Imbens, 1995) (!)
  - in the binary DIDs they coincide
- **de Chaisemartin and D'Haultfoeuille (2021) does not automatically infer the slope effect!**
- Robustness to repeated cross-sectional data not clear
  - I was following same individuals with age ≥ 16 from pre-treatment wave $(t-3)$ with attrition at the tails of the panel using sampling waves
  - I want to use repeated cross-section of individuals with age ≥ 16 to increase power

Issue 2: Covariates Selection

What are the 'Good' Controls?

- I have been controlling for:
  - Time-varying second-order polynomial of age
  - pre-treatment (time-invariant) individual and household characteristics
  - pre-treatment (time-invariant) group-level (county) characteristics

is the correct way of doing it? Any suggestion?

Issue 3: Pre-Treatment Tests under stronger assumptions

- Both de Chaisemartin and D'Haultfoeuille (2021) and Callaway et al. (2021) use stronger parallel trends assumption (can be conditional):

$$\mathbb{E}\left[Y_{gt}(\mathbf{0}_t) - Y_{gt-1}(\mathbf{0}_{t-1})\right] = \mathbb{E}\left[Y_{g't}(\mathbf{0}_t) - Y_{g't-1}(\mathbf{0}_{t-1})\right], \forall\, g \neq g'$$

- cumbersome to test with standard event-study plot with $R_{gt} = t - \tau + 1$ time relative to treatment : $Y_{igt} = \alpha_g + \phi_t + \sum_{r \neq 0} \mathbb{1}\left[R_{gt} = r\right] \beta_r + \eta_{gt}$
- Rambachan and Roth (2021) propose a sensitivity analysis of pre-trends test
  - a) estimate pre-treatment analogue to the counterfactual first post-treatment period b) impose an (arbitrary) magnitude of the post-treatment violations that needs to be reported in the event-study plot

Other suggestions, comments and references are super welcome!

References I

Angrist, J. D. and Imbens, G. W. (1995), 'Two-Stage Least Squares Estimation of Average Causal Effects in Models with Variable Treatment Intensity', *Journal of the American Statistical Association* **90**(430), 431–442.

Becker, A. (2021), Shamed to Death: Social Image Concerns and War Participation.

Borusyak, K., Jaravel, X. and Spiess, J. (2021), Revisiting Event Study Designs: Robust and Efficient Estimation.

Callaway, B., Goodman-Bacon, A. and Sant'Anna, P. H. C. (2021), Difference-in-Differences with a Continuous Treatment.

Callaway, B. and Sant'Anna, P. H. (2021), 'Difference-in-Differences with multiple time periods', *Journal of Econometrics* **225**(2), 200–230.

de Chaisemartin, C. and D'Haultfœuille, X. (2018), 'Fuzzy Differences-in-Differences', *Review of Economic Studies* **85**(2), 999–1028.

References II

de Chaisemartin, C. and D'Haultfœuille, X. (2020), 'Two-Way Fixed Effects Estimators with Heterogeneous Treatment Effects', *American Economic Review* **110**(9), 2964–2996.

de Chaisemartin, C. and D'Haultfoeuille, X. (2021), Difference-in-Differences Estimators of Intertemporal Treatment Effects.

de Chaisemartin, C., D'Haultfœuille, X., Pasquier, F. and Vazquez-Bare, G. (2022), Difference-in-Differences Estimators for Treatments Continuously Distributed at Every Period.

Goodman-Bacon, A. (2021), 'Difference-in-differences with variation in treatment timing', *Journal of Econometrics* .

Rambachan, A. and Roth, J. (2021), An Honest Approach to Parallel Trends.