Formulário MVO-41

Leis de Kepler:

1) trajetórias elípticas
2) Áreas iguais => tempos iguais
3) $T_P^2 \propto r^3$

$$\vec F = -\frac{GMm\vec r}{r^3}$$

Terra elíptica

$b = 6356,1\ Km$
$a = 6378,8\ Km$

$$\vec R = x\cos\theta\,\hat i + x\sin\theta\,\hat j + z\,\hat k$$

$$x = \left|\frac{a}{\sqrt{1-e^2\sin^2 L}} + H\right|\cos L \qquad z = \left|\frac{a(1-e^2)}{\sqrt{1-e^2\sin^2 L}} + H\right|\sin L$$

$$\frac{2\pi(t-T)}{T_P} = E - e\sin E \qquad \cos E = \frac{e+\cos\nu}{1+e\cos\nu}$$

Parábola => $(t-T) = \frac{1}{2\sqrt{\mu}}\left(PD + \frac{1}{3}D^3\right)$ $\quad D = \sqrt{P}\,tg(\nu/2)$

Hipérbole => $(t-T) = \sqrt{\frac{-a^3}{\mu}}\,(e\sinh F - F)$

$$\cosh F = \frac{e+\cos\nu}{1+e\cos\nu}$$

$$\sinh = \frac{e^x-e^{-x}}{2} \qquad \cosh = \frac{e^x+e^{-x}}{2}$$

$$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} \qquad \mathcal{E} = -\frac{\mu}{2a}$$

$\mathcal{E}<0$ elíptica
$\mathcal{E}=0$ parábola
$\mathcal{E}>0$ Hipérbole

$$\vec h = \vec r \times \vec v$$

p/ elipse: $0<e<1$
p/ parábola: $e=1$

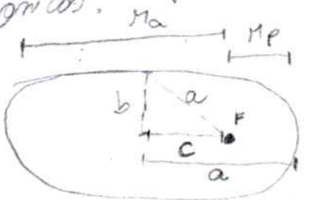
$$e = \frac{B}{\mu}$$

$$r = \frac{P}{1+e\cos\nu} \qquad P = \frac{h^2}{\mu}$$

$$\vec r \times \vec h + \mu\frac{\vec r}{r} = \vec B \qquad e = \sqrt{1+\frac{2\mathcal{E}h^2}{\mu^2}}$$

$$\vec B = \left(v^2 - \frac{\mu}{r}\right)\vec r - (\vec r\cdot\vec v)\vec v$$

Cônicas: Elipse



$\mathcal{E}<0$

$a^2 = b^2 + c^2$

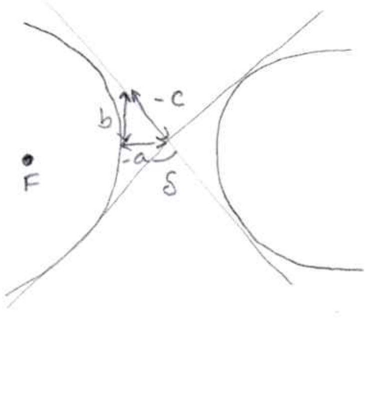
$e = \frac{c}{a}$

$$P = a(1-e^2) = \frac{b^2}{a}$$

$r_p = a(1-e)$ $\quad \frac{r_a+r_p}{2} = a$
$r_a = a(1+e)$ $\quad \frac{r_a-r_p}{2} = c$

$$T_P = \frac{2\pi}{\sqrt{\mu}}a^{3/2}$$

Hiperbólica



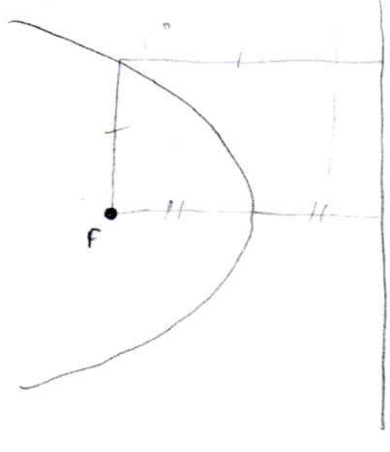
$c^2 = b^2 + a^2$

$\sin\frac{\delta}{2} = \frac{a}{c}$

$V_\infty = \sqrt{2\mathcal{E}}$

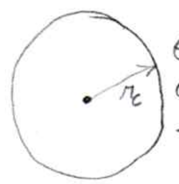
$V_\infty^2 = V^2 - V_{esc}^2$

Parábola



$2r_p = P$
$e = 1$
$\mathcal{E} = 0$
$a = \infty$
$c = \infty$
$V_{esc} = \sqrt{\frac{2\mu}{r}}$

Círculo:



$e = 0$
$a = b = P$
$T_P = \frac{2\pi}{\sqrt{\mu}}r_c^{3/2}$
$V_c = \sqrt{\frac{\mu}{r_c}}$

Elementos da órbita:

$\vec{h} = \vec{r} \times \vec{v}$

$\vec{n} = \hat{k} \times \hat{h}$

$i \to$ inclinação

$\cos i = \dfrac{\hat{k} \cdot \vec{h}}{h}$ $\quad 0 \leq i \leq 180$

$0 \leq i \leq 90 \Rightarrow$ direta

$90 < i \leq 180 \to$ retrógrado

$\Omega \to$ longitude do nó ascendente

$\cos \Omega = \dfrac{\hat{I} \cdot \vec{n}}{n}$ $\quad 0 \leq \Omega \leq 360°$

$\vec{n} \cdot \hat{J} > 0 \Rightarrow \Omega < 180°$

$\omega \to$ Argumento do periapsis

$\cos \omega = \dfrac{\vec{n} \cdot \vec{e}}{n \cdot e}$ $\quad 0 \leq \omega \leq 360°$

$\vec{e} \cdot \hat{k} > 0 \Rightarrow \omega < 180°$

$\nu_0 \to$ Anomalia verdadeira

$\cos \nu_0 = \dfrac{\vec{e} \cdot \vec{r}}{e r}$ $\quad 0° < \nu_0 \leq 360°$

$\vec{r} \cdot \vec{v} > 0 \Rightarrow \nu_0 < 180°$

$u_0 \to$ Argumento de latitude

$\cos u_0 = \dfrac{\vec{n} \cdot \vec{r}}{n \cdot r}$ $\quad$ se $\vec{r} \cdot \vec{k} > 0 \Rightarrow u_0 < 180°$

$l_0 \to$ longitude verdadeira

$$l_0 = \Omega + \underbrace{\omega + \nu_0}_{u_0} = \Omega + u_0$$

OBSERVAÇÃO DE RADAR

$\vec{\rho} = \rho_s \hat{S} + \rho_E \hat{E} + \rho_z \hat{Z}$

$\rho_s = -\rho \cos El \cos Az$

$\rho_e = \rho \cos El \sin Az$

$\rho_z = \rho \sin El$

$\dot{\vec{\rho}} = \dot{\rho}_s \hat{S} + \dot{\rho}_E \hat{E} + \dot{\rho}_z \hat{Z}$

Geocêntrico inercial

$\vec{r} = \vec{R} + \vec{\rho}$

$\vec{v} = \dot{\vec{\rho}} + \vec{\omega}_{\oplus} \times \vec{r}$

Transf.

$$\begin{bmatrix} I \\ J \\ K \end{bmatrix} = T_z(-\theta)\, T_y(\underbrace{L}_{\text{latitude}} - 90) \begin{bmatrix} S \\ E \\ Z \end{bmatrix}$$

$\theta = \underbrace{\theta_g}_{\text{hora de greenwich}} + \underbrace{\lambda_E}_{\text{longitude}}$

Sistema perifocal:

$\vec{r} = r_0 \cos\nu_0 \hat{P} + r_0 \sin\nu_0 \hat{Q}$

$r_0 = \dfrac{p}{1 + e\cos\nu_0}$

$\vec{v} = (\dot{r}_0 \cos\nu_0 - r \sin\nu_0 \dot{\nu}_0)\hat{P} + (\dot{r}_0 \sin\nu_0 + r\cos\nu_0 \dot{\nu}_0)\hat{Q}$

$\vec{v}_0 = \sqrt{\frac{\mu}{p}}\left(-\sin\nu_0 \hat{P} + (e + \cos\nu_0)\hat{Q}\right)$

TRANF. COORDENADAS

$$T_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & s\alpha \\ 0 & -s\alpha & c\alpha \end{bmatrix} \qquad T_z(\alpha) = \begin{bmatrix} c\alpha & s\alpha & 0 \\ -s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_y(\alpha) = \begin{bmatrix} c\alpha & 0 & -s\alpha \\ 0 & 1 & 0 \\ s\alpha & 0 & c\alpha \end{bmatrix}$$

Perifocal $\to$ GEOC. EQ.

$$\begin{bmatrix} \hat{I} \\ \hat{J} \\ \hat{K} \end{bmatrix} = T_z(-\Omega)\, T_x(-i)\, T_z(-\omega) \begin{bmatrix} \hat{P} \\ \hat{Q} \\ \hat{W} \end{bmatrix}$$