Magnetometer measurement model for sensor fusion with INS

$\mathbf{B_b} = \mathbf{D_b^{NED}}\mathbf{B_{NED}}$ is the relationship between the representations of the geomagnetic vector **B** in both S_b and S_{NED} coordinate frames with the true DCM.

$\mathbf{B_{m,b}} = \mathbf{D_b^{NED}}\mathbf{B_{NED}} + \mathbf{v}$ where **v** is the additive measurement noise.

The INS computes an estimate of the true DCM, which can be modeled as:

$\mathbf{\hat{D}_b^{NED}} = \mathbf{D_b^{NED}}\mathbf{D_{NEDt}^{NEDp}}$ where S_{NEDt} is the true NED coordinate frame and S_{NEDp} is the platform NED coordinate frame. Then,

$$\mathbf{\hat{D}_b^{NED}}\mathbf{D_{NEDp}^{NEDt}} = \mathbf{D_b^{NED}}$$

The above implies in:

$$\mathbf{B_{m,b}} = \mathbf{\hat{D}_b^{NED}}\mathbf{D_{NEDp}^{NEDt}}\mathbf{B_{NED}} + \mathbf{v}$$

We now relate the DCM $\mathbf{D_{NEDp}^{NEDt}}$ with the misalignment vectors in the Pinson model: $\boldsymbol{\delta\theta}$ that rotates the true NED coordinate frame S_t into alignment with the computed coordinate frame S_c corresponding to the INS-computed position, and the latter coordinate frame into alignment with S_p via the misalignment vector $\boldsymbol{\psi}$ due to rate gyro drift:

$$\mathbf{D_{NEDp}^{NEDt}} = \mathbf{D_{NEDp}^{NEDc}}\mathbf{D_{NEDc}^{NEDt}} = (\mathbf{I} - \boldsymbol{\psi}_{\mathbf{NED}}\times)(\mathbf{I} - \boldsymbol{\delta\theta}_{\mathbf{NED}}\times) \approx \mathbf{I} - \boldsymbol{\psi}_{\mathbf{NED}}\times - \boldsymbol{\delta\theta}_{\mathbf{NED}}\times$$

The approximation discards nonlinear error terms, and therefore:

$$\mathbf{B_{m,b}} = \mathbf{\hat{D}_b^{NED}}(\mathbf{I} - \boldsymbol{\psi}_{\mathbf{NED}}\times - \boldsymbol{\delta\theta}_{\mathbf{NED}}\times)\mathbf{B_{NED}} + \mathbf{v}$$

Rewriting the above, the magnetometer measurement **y** for use in the Kalman filter is the difference between the strapdown reading and the geomagnetic model prediction based on the INS-computed position represented in the S_b coordinate frame with the INS-computed DCM:

$$\mathbf{B_{m,b}} = \mathbf{\hat{D}_b^{NED}}\mathbf{B_{NED}} - \mathbf{\hat{D}_b^{NED}}[\boldsymbol{\delta\theta}_{\mathbf{NED}}\times]\mathbf{B_{NED}} - \mathbf{\hat{D}_b^{NED}}[\boldsymbol{\psi}_{\mathbf{NED}}\times] + \mathbf{v}$$

$$\mathbf{y} = \mathbf{B_{m,b}} - \mathbf{\hat{D}_b^{NED}}\mathbf{B_{NED}} = \mathbf{\hat{D}_b^{NED}}[\mathbf{B_{NED}}\times]\boldsymbol{\delta\theta}_{\mathbf{NED}} + \mathbf{\hat{D}_b^{NED}}[\mathbf{B_{NED}}\times]\boldsymbol{\psi}_{\mathbf{NED}} + \mathbf{v}$$

One is now reminded how the misalignment $\boldsymbol{\delta\theta}_{\mathbf{NED}}$ relates to the position error $\mathbf{\Delta R_{NED}}$ based on the most recent INS-computed position and the corresponding Earth´s model curvature radii:

$$[\delta\theta_N \quad \delta\theta_E \quad \delta\theta_D]^T = [\Delta R_E/(R_E + h) \quad -\Delta R_N/(R_N + h) \quad -\Delta R_E tg\lambda/(R_E + h)]^T$$

Resulting in the following measurement equation:

$$\mathbf{y} = \mathbf{B}_{\mathbf{m,b}} - \hat{\mathbf{D}}_{\mathbf{b}}^{\mathbf{NED}} \mathbf{B}_{\mathbf{NED}} =$$

$$= \hat{\mathbf{D}}_{\mathbf{b}}^{\mathbf{NED}} [\mathbf{B}_{\mathbf{NED}} \times] \left[\begin{array}{ccc|c|c|c} 0 & 1/(R_E + h) & 0 & & & \\ -1/(R_N + h) & 0 & 0 & \mathbf{0}_3 & \mathbf{I}_3 & \mathbf{0}_{3x6} \\ 0 & -\mathrm{tg}\lambda/(R_E + h) & 0 & & & \end{array} \right] \begin{bmatrix} \Delta \mathbf{R}_{\mathbf{NED}} \\ \hline \Delta \mathbf{V}_{\mathbf{e,NED}} \\ \hline \boldsymbol{\Psi}_{\mathbf{NED}} \\ \hline \boldsymbol{\nabla}_{\mathbf{NED}} \\ \hline \boldsymbol{\varepsilon}_{\mathbf{NED}} \end{bmatrix} + \mathbf{v}$$

One notices that the above magnetometer measurement assumes a bias-free sensor. This measurements yields information about the position error. Furthermore, it can be shown from observability analysis that the misalignment along the direction of the geomagnetic vector **B** is not observable. Thus, a linear combination of the misalignment components remains unobservable.

Obs.:

$$[\mathbf{a} \times] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$