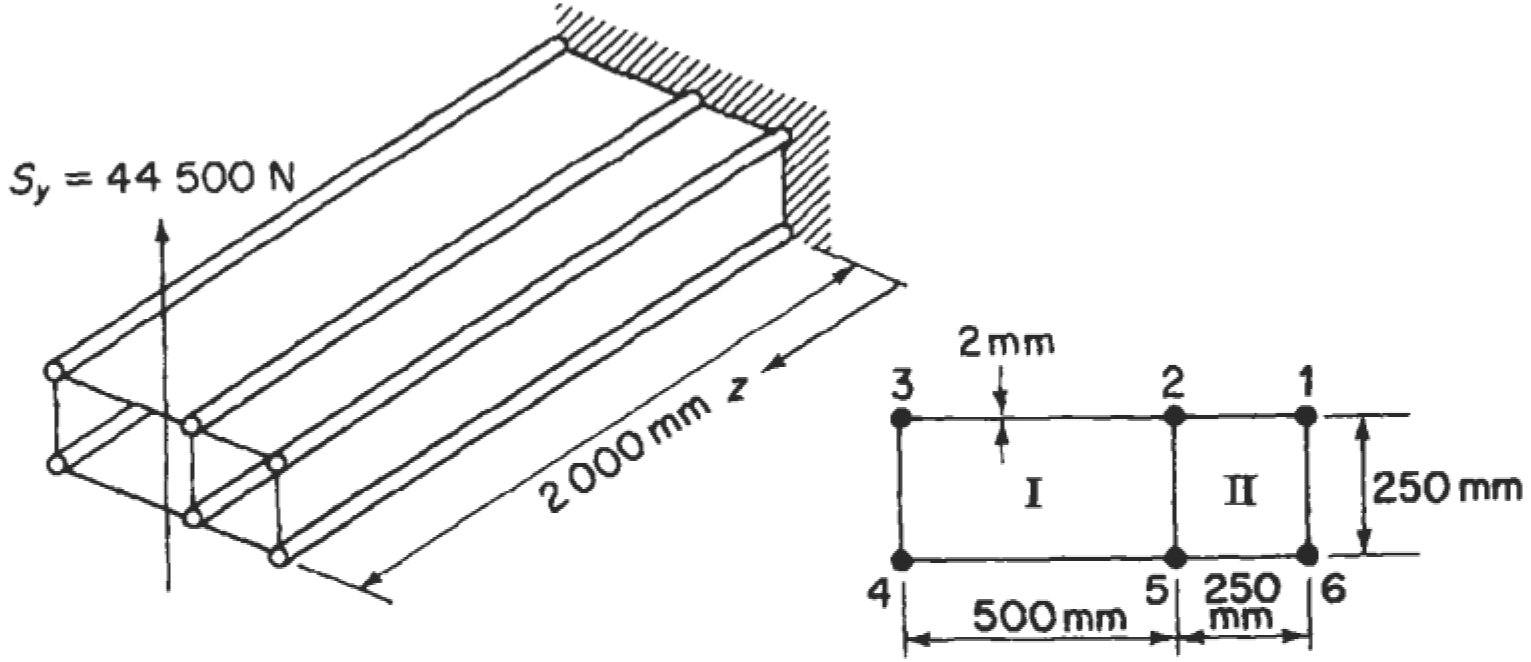
Example 10.12

Calculate the deflection at the free end of the two-cell beam shown in Fig. 10.39 allowing for both bending and shear effects. The booms carry all the direct stresses while the skin panels, of constant thickness throughout, are effective only in shear.

$$\text{Take } E = 69\,000\ \text{N/mm}^2 \text{ and } G = 25\,900\ \text{N/mm}^2$$

$$\text{Boom areas: } -B_1 = B_3 = B_4 = B_6 = 650\ \text{mm}^2,\ B_2 = B_5 = 1300\ \text{mm}^2$$

The beam cross-section is symmetrical about a horizontal axis and carries a vertical load at its free end through the shear centre.



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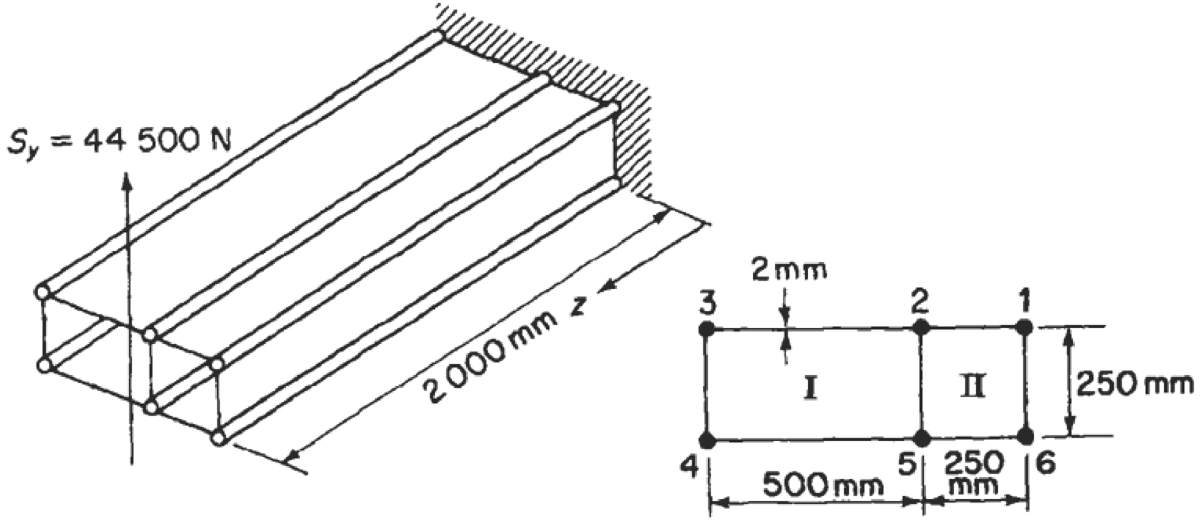


Equação de deslocamento (Princípio das Forças Virtuais)

$$\Delta = \Delta_T + \Delta_M + \Delta_S$$

$$\Delta_T = \int \frac{T\delta T}{GJ} dz \quad \Delta_S = \int \left[\int \frac{q\delta q}{Gt} ds \right] dz$$

$$\Delta_M = \frac{1}{E\left(I_{xx}I_{yy} - I_{xy}^2\right)^2} \left[\Delta_{xx} + \Delta_{yy} + \Delta_{xy_1} + \Delta_{xy_2}\right]$$

$$\Delta_{xx} = \int \left(\delta M_x I_{yy} - \delta M_y I_{xy}\right)\left(M_x I_{yy} - M_y I_{xy}\right) I_{xx} dz$$

$$\Delta_{yy} = \int \left(\delta M_y I_{xx} - \delta M_x I_{xy}\right)\left(M_y I_{xx} - M_x I_{xy}\right) I_{yy} dz$$

$$\Delta_{xy_1} = \int \left(\delta M_y I_{xx} - \delta M_x I_{xy}\right)\left(M_x I_{yy} - M_y I_{xy}\right) I_{xy} dz$$

$$\Delta_{xy_2} = \int \left(\delta M_x I_{yy} - \delta M_y I_{xy}\right)\left(M_y I_{xx} - M_x I_{xy}\right) I_{xy} dz$$

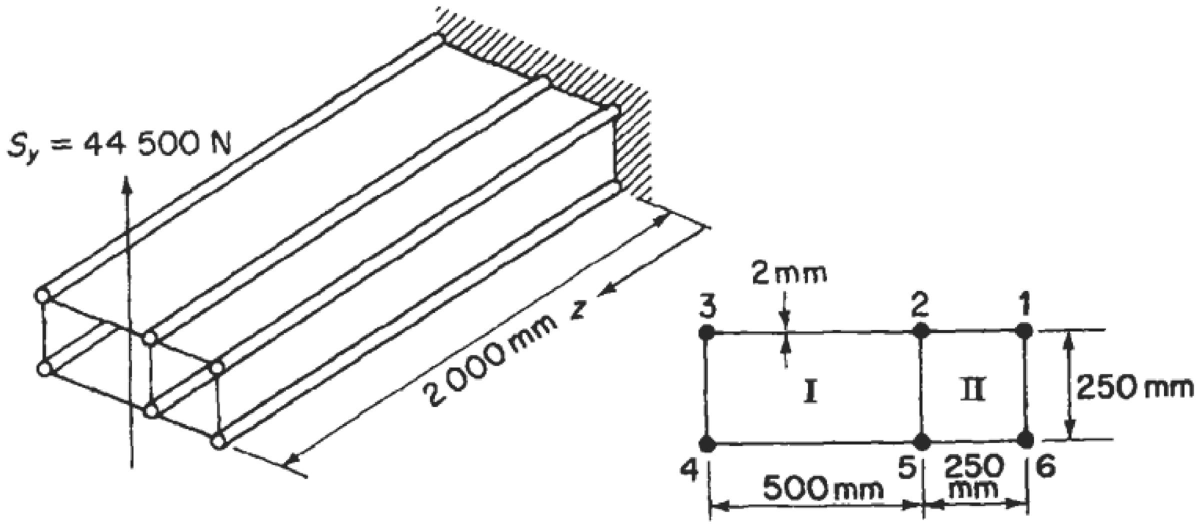
Example 10.12

Calculate the deflection at the free end of the two-cell beam shown in Fig. 10.39 allowing for both bending and shear effects. The booms carry all the direct stresses while the skin panels, of constant thickness throughout, are effective only in shear.

$$\text{Take } E = 69\,000\ \mathrm{N/mm^2} \text{ and } G = 25\,900\ \mathrm{N/mm^2}$$

$$\text{Boom areas: } -B_1 = B_3 = B_4 = B_6 = 650\ \mathrm{mm^2},\ B_2 = B_5 = 1300\ \mathrm{mm^2}$$

The beam cross-section is symmetrical about a horizontal axis and carries a vertical load at its free end through the shear centre.



Equação de deslocamento (Princípio das Forças Virtuais)

$$\Delta = \Delta_M + \Delta_S$$

$$\Delta_S = \int \left[\int \frac{q\delta q}{Gt} ds \right] dz$$

$$\Delta_M = \int \left[\frac{M_x \delta M_x}{EI_{xx}} + \frac{M_y \delta M_y}{EI_{yy}} \right] dz$$

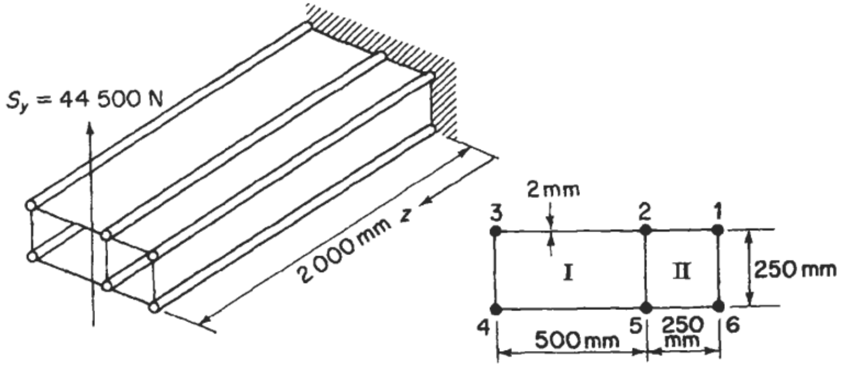
Example 10.12

Calculate the deflection at the free end of the two-cell beam shown in Fig. 10.39 allowing for both bending and shear effects. The booms carry all the direct stresses while the skin panels, of constant thickness throughout, are effective only in shear.

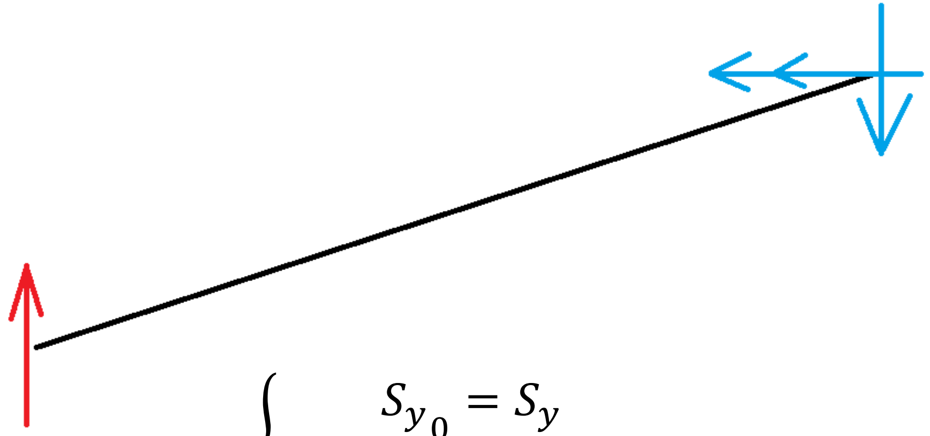
Take $E = 69\,000\ \mathrm{N/mm^2}$ and $G = 25\,900\ \mathrm{N/mm^2}$

Boom areas: $-B_1 = B_3 = B_4 = B_6 = 650\ \mathrm{mm^2}$, $B_2 = B_5 = 1300\ \mathrm{mm^2}$

The beam cross-section is symmetrical about a horizontal axis and carries a vertical load at its free end through the shear centre.

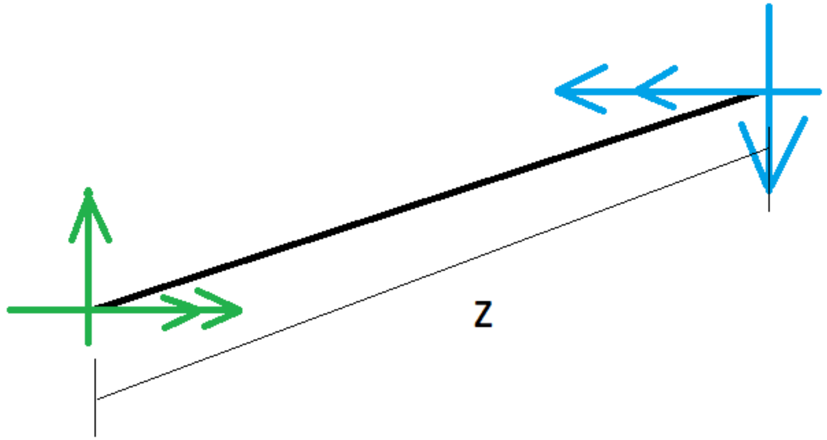


Passo 1: Diagrama de corpo livre



$$\begin{cases} S_{y_0} = S_y \\ M_{x_0} = -2000 \times S_y \end{cases}$$

Passo 2: Análise da seção



$$\begin{cases} S_y(z) = S_{y_0} = S_y \\ M_x(z) = M_{x_0} + S_{y_0} \times z = -2000 \times S_y + S_y \times z = -S_y(2000 - z) \end{cases}$$

$$\begin{cases} S_y(z) = S_y \\ M_x(z) = -S_y(2000 - z) \end{cases}$$

$$\Delta = \int \left\{ \left[\int \frac{q\delta q}{Gt} ds \right] + \frac{M_x \delta M_x}{EI_{xx}} \right\} dz$$

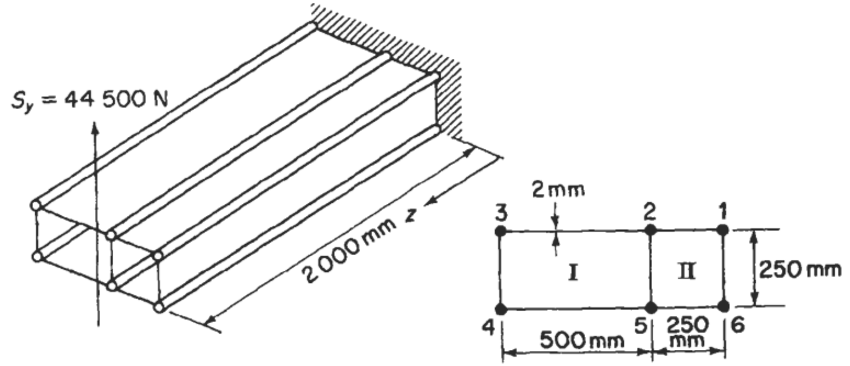
Example 10.12

Calculate the deflection at the free end of the two-cell beam shown in Fig. 10.39 allowing for both bending and shear effects. The booms carry all the direct stresses while the skin panels, of constant thickness throughout, are effective only in shear.

Take $E = 69\,000\ \mathrm{N/mm^2}$ and $G = 25\,900\ \mathrm{N/mm^2}$

Boom areas: $-B_1 = B_3 = B_4 = B_6 = 650\ \mathrm{mm^2}$, $B_2 = B_5 = 1300\ \mathrm{mm^2}$

The beam cross-section is symmetrical about a horizontal axis and carries a vertical load at its free end through the shear centre.



Passo 3: Análise do Carregamento

$$\begin{cases} S_y(z) = S_y \\ M_x(z) = -S_y(2000 - z) \end{cases} \qquad \Delta = \int \left\{ \left[\int \frac{q\delta q}{Gt} ds \right] + \frac{M_x \delta M_x}{EI_{xx}} \right\} dz$$

Passo 3.1: Contribuição do Momento

a) Propriedades geométricas

$$I_{xx} = 4 \times 650 \times 125^2 + 2 \times 1300 \times 125^2 \rightarrow$$
$$\rightarrow I_{xx} = 81.25 \times 10^6\ \mathrm{mm}^4$$

$$\Delta_\mathrm{M} = \int \frac{M_x \delta M_x}{EI_{xx}} dz = \int_0^{2000} \frac{\left[-S_y(2000 - z)\right]\left[-\delta S_y(2000 - z)\right]}{EI_{xx}} dz = S_y \delta S_y \int_0^{2000} \frac{(2000 - z)^2}{69 \times 10^3 \times 81.25 \times 10^6} dz \rightarrow$$

$$\Delta_\mathrm{M} = \frac{S_y \delta S_y}{69 \times 10^3 \times 81.25 \times 10^6} \int_0^{2000} (2000 - z)^2 dz = \frac{S_y \delta S_y}{69 \times 10^3 \times 81.25 \times 10^6} \left[-\frac{1}{3}(2000 - z)^3 \right] \Bigg|_0^{2000} =$$

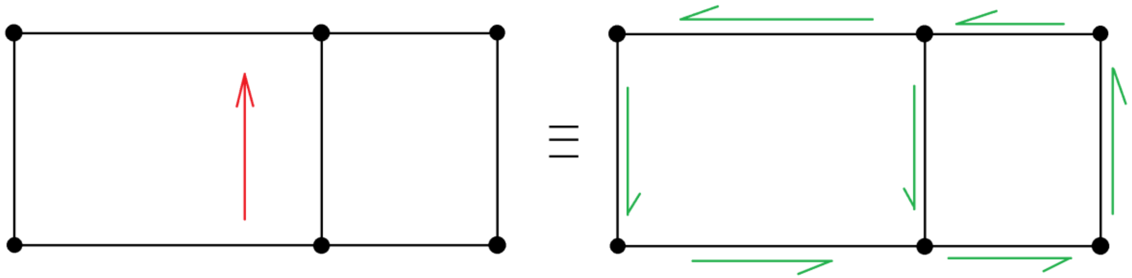
$$\Delta_\mathrm{M} = \frac{S_y \delta S_y \times 8}{3 \times 69 \times 81.25} = 4.76 \times 10^{-4} \times S_y \delta S_y = 4.76 \times 10^{-4} \times 44500 =$$

$$\Delta_\mathrm{M} = 21.17\ \mathrm{mm}$$

Passo 3: Análise do Carregamento

$$\begin{cases} S_y(z) = S_y \\ M_x(z) = -S_y(2000 - z) \end{cases} \qquad \Delta = \int \left\{ \left[\int \frac{q\delta q}{Gt} ds \right] + \frac{M_x \delta M_x}{EI_{xx}} \right\} dz$$

Passo 3.2: Contribuição do Cisalhamento



Dado que os booms não se encontram inclinados, nota-se que toda a carga cisalhante será suportada apenas pela casca

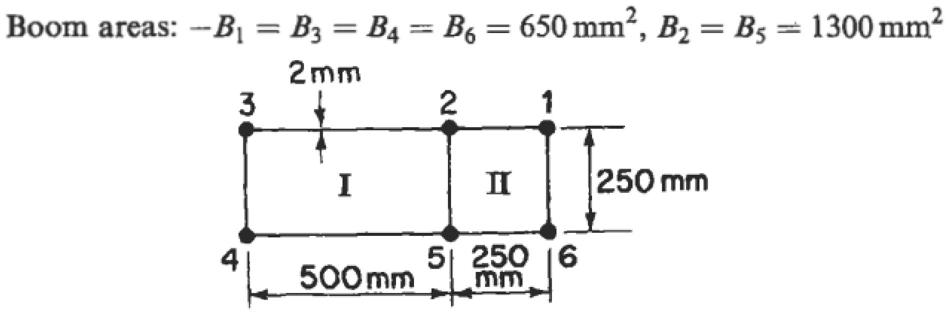
a) Cálculo do Fluxo

$$q_s = -\frac{S_{x_{cis}}.I_{xx} - S_{y_{cis}} I_{xy}}{I_{xx}I_{yy} - I_{xy}^2} . \left(\int_0^s t_d x ds + \sum_{r=1}^{n} B_r x_r \right) - \frac{S_{y_{cis}}.I_{yy} - S_{x_{cis}} I_{xy}}{I_{xx}I_{yy} - I_{xy}^2} . \left(\int_0^s t_d y ds + \sum_{r=1}^{n} B_r y_r \right) + q_{s,0}$$

$$q_s = -\frac{S_{y_{cis}}}{I_{xx}} . \left(\sum_{r=1}^{n} B_r y_r \right) + q_{s,0} = -\frac{S_y}{I_{xx}} . \left(\sum_{r=1}^{n} B_r y_r \right) + q_{s,0}$$

Passo 3: Análise do Carregamento

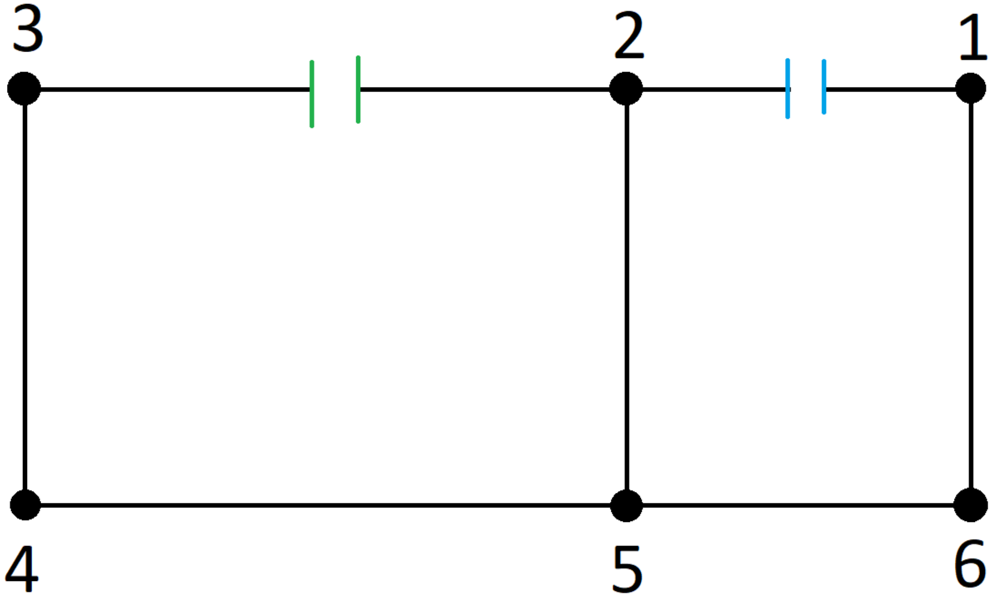
$$\begin{cases} S_y(z) = S_y \\ M_x(z) = -S_y(2000 - z) \end{cases} \qquad \Delta = \int \left\{ \left[\int \frac{q \delta q}{Gt} ds \right] + \frac{M_x \delta M_x}{E I_{xx}} \right\} dz$$



Passo 3.2: Contribuição do Cisalhamento

a) Cálculo do Fluxo

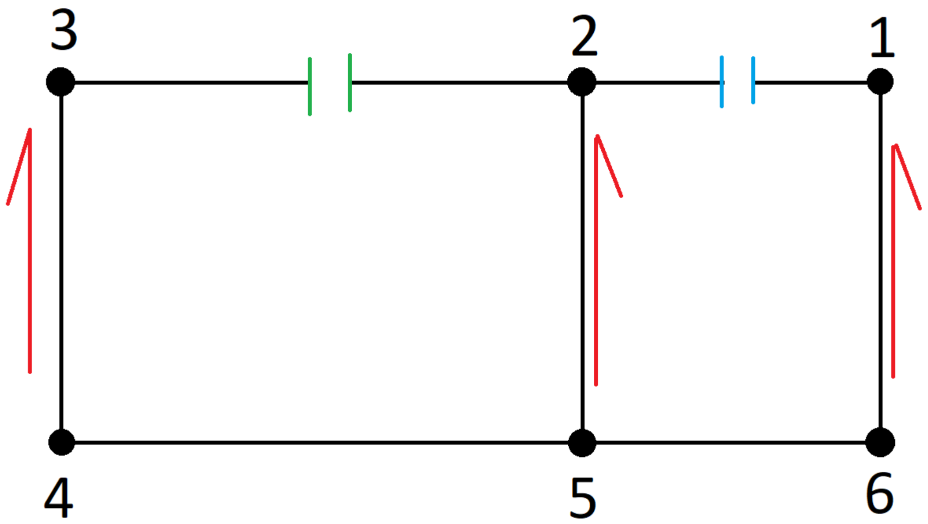
$$q_s = -\frac{S_{y_{cis}}}{I_{xx}} \cdot \left(\sum_{r=1}^{n} B_r y_r \right) + q_{s,0} = -\frac{S_y}{I_{xx}} \cdot \left(\sum_{r=1}^{n} B_r y_r \right) + q_{s,0}$$



Segmento	$q_b = -\frac{S_y}{81.25e6} \cdot \left(\sum_{r=1}^{n} B_r y_r \right)$	$q_{s,0}$
$1-2$	0	
$2-3$	0	
$2-5$	$-0.002 \times Sy$	
$3-4$	$-0.001 \times Sy$	
$4-5$	0	
$5-6$	0	
$6-1$	$0.001 \times S_y$	

Passo 3.2: Contribuição do Cisalhamento

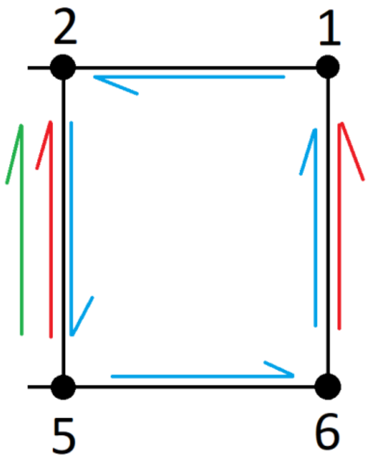
a) Cálculo do Fluxo



$\boldsymbol{Segmento}$	$q_b = -\dfrac{S_y}{81.25e6} \cdot \left(\sum_{r=1}^{n} B_r y_r\right)$	$q_{s,0}$
$1-2$	0	
$2-3$	0	
$2-5$	$-0.002 \times Sy$	
$3-4$	$-0.001 \times Sy$	
$4-5$	0	
$5-6$	0	
$6-1$	$0.001 \times S_y$	

Cálculo dos $q_{s,0,i}$

Análise da Célula I



$$\frac{d\theta}{dz} = \frac{1}{2A_r}\oint q\frac{ds}{Gt} = \frac{1}{2A_r Gt}\oint qds =$$

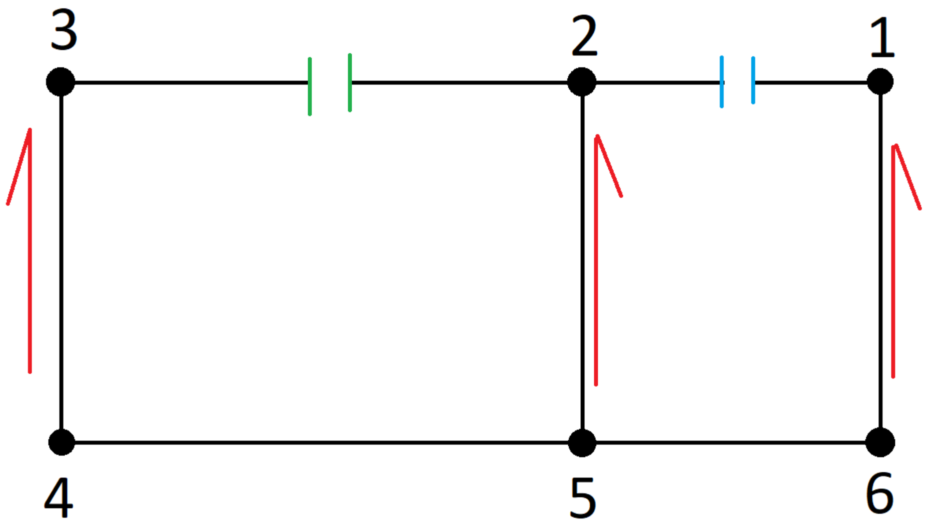
$$= \frac{1}{2A_r Gt}\left[q_{s,0,1} + \left(q_{s,0,1} - 0.002 \times Sy - q_{s,0,2}\right) + q_{s,0,1} + q_{s,0,1} + 0.001 \times S_y\right] \times 250 =$$

$$= \frac{1}{2 \times 250 \times 25900 \times 2}\left[4q_{s,0,1} - q_{s,0,2} - 0.001 \times S_y\right] =$$

$$\frac{d\theta}{dz} = 3.86 \times 10^{-8}\left(4q_{s,0,1} - q_{s,0,2} - 0.001 \times S_y\right)$$

Passo 3.2: Contribuição do Cisalhamento

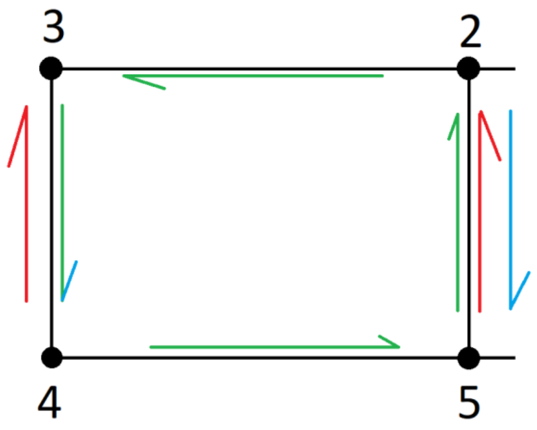
a) Cálculo do Fluxo



Segmento	$q_b = -\frac{S_y}{81.25e6} \cdot \left(\sum_{r=1}^{n} B_r y_r\right)$	$q_{s,0}$
$1-2$	0	
$2-3$	0	
$2-5$	$-0.002 \times Sy$	
$3-4$	$-0.001 \times Sy$	
$4-5$	0	
$5-6$	0	
$6-1$	$0.001 \times S_y$	

Cálculo dos $q_{s,0,i}$

Análise da Célula II



$$\frac{d\theta}{dz} = \frac{1}{2A_r}\oint q\frac{ds}{Gt} = \frac{1}{2A_rGt}\oint qds =$$

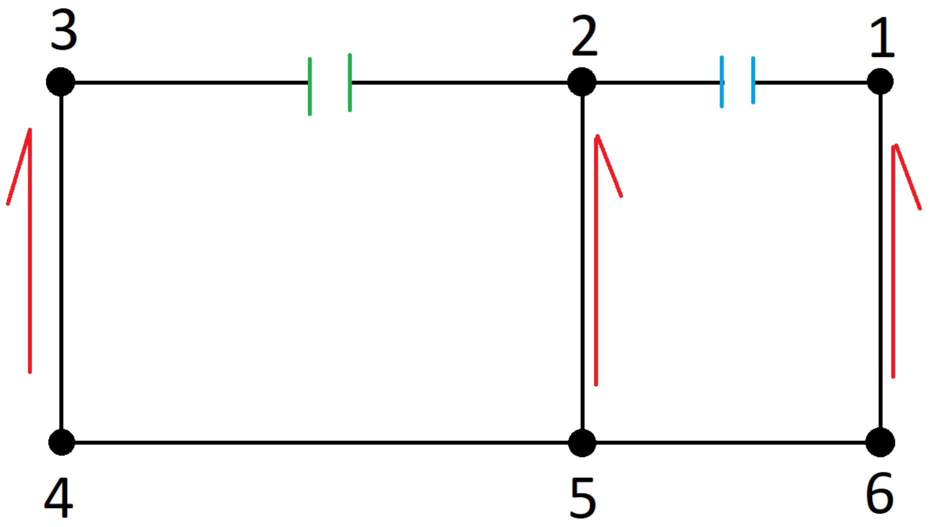
$$= \frac{1}{2A_rGt}\left[2q_{s,0,2} + \left(q_{s,0,2} - 0.001 \times Sy\right) + 2q_{s,0,2} + \left(q_{s,0,2} + 0.002 \times Sy - q_{s,0,1}\right)\right] \times 250 =$$

$$= \frac{1}{2 \times 500 \times 25900 \times 2}\left[6q_{s,0,2} - q_{s,0,1} + 0.001 \times S_y\right] =$$

$$\frac{d\theta}{dz} = 1.93 \times 10^{-8}\left(6q_{s,0,2} - q_{s,0,1} + 0.001 \times S_y\right)$$

Passo 3.2: Contribuição do Cisalhamento

a) Cálculo do Fluxo



$Segmento$	$q_b = -\frac{S_y}{81.25e6} \cdot \left(\sum_{r=1}^{n} B_r y_r\right)$	$q_{s,0}$
$1-2$	0	
$2-3$	0	
$2-5$	$-0.002 \times Sy$	
$3-4$	$-0.001 \times Sy$	
$4-5$	0	
$5-6$	0	
$6-1$	$0.001 \times S_y$	

Cálculo dos $q_{s,0,i}$

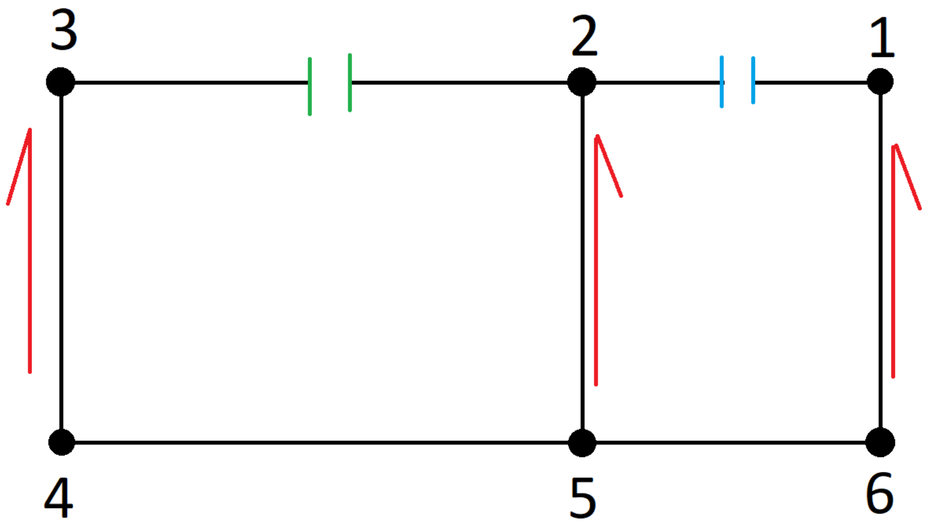
$$\frac{d\theta}{dz} = 1.93 \times 10^{-8}\left(6q_{s,0,2} - q_{s,0,1} + 0.001 \times S_y\right) = 0$$

$$\frac{d\theta}{dz} = 3.86 \times 10^{-8}\left(4q_{s,0,1} - q_{s,0,2} - 0.001 \times S_y\right) = 0$$

$$\begin{cases} -q_{s,0,1} + 6q_{s,0,2} = -0.001 \times S_y \\ 4q_{s,0,1} - q_{s,0,2} = 0.001 \times S_y \end{cases} \rightarrow \begin{cases} q_{s,0,1} = 0.00022 \times S_y \\ q_{s,0,2} = -0.00013 \times S_y \end{cases}$$

Passo 3.2: Contribuição do Cisalhamento

a) Cálculo do Fluxo



Segmento	$q_b = -\frac{S_y}{81.25e6} \cdot \left(\sum_{r=1}^{n} B_r y_r\right)$	$q_{s,0}$
$1-2$	0	
$2-3$	0	
$2-5$	$-0.002 \times Sy$	
$3-4$	$-0.001 \times Sy$	
$4-5$	0	
$5-6$	0	
$6-1$	$0.001 \times S_y$	

Cálculo dos $q_{s,0,i}$

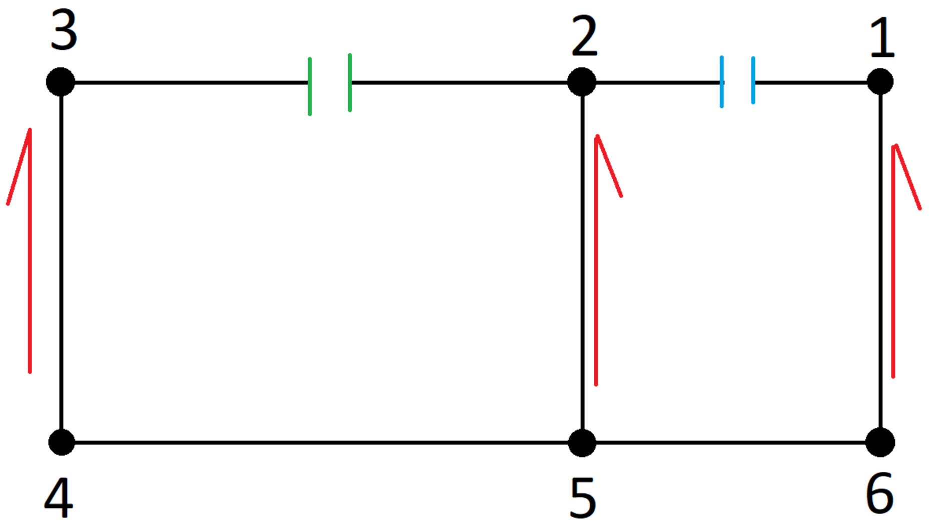
$$\frac{d\theta}{dz} = 1.93 \times 10^{-8}\left(6q_{s,0,2} - q_{s,0,1} + 0.001 \times S_y\right) = 0$$

$$\frac{d\theta}{dz} = 3.86 \times 10^{-8}\left(4q_{s,0,1} - q_{s,0,2} - 0.001 \times S_y\right) = 0$$

$$\begin{cases} -q_{s,0,1} + 6q_{s,0,2} = -0.001 \times S_y \\ 4q_{s,0,1} - q_{s,0,2} = 0.001 \times S_y \end{cases} \rightarrow \begin{cases} q_{s,0,1} = 0.00022 \times S_y \\ q_{s,0,2} = -0.00013 \times S_y \end{cases}$$

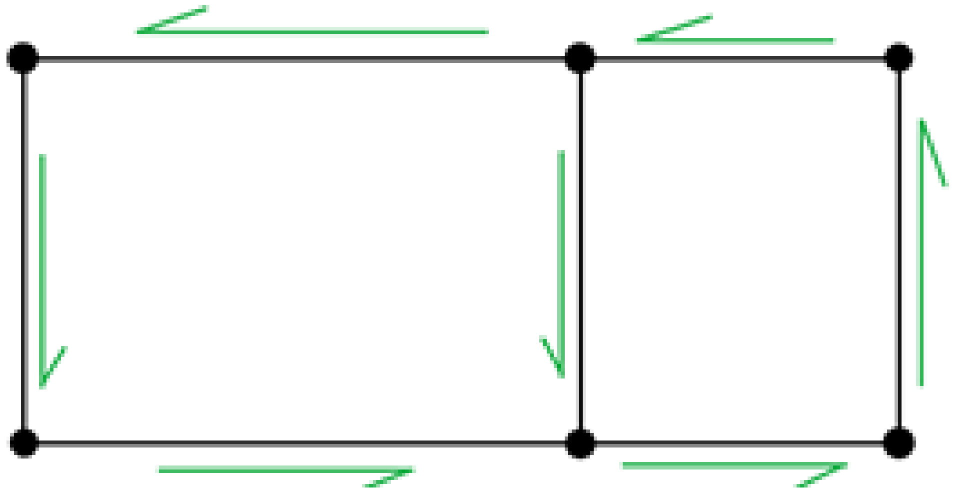
Passo 3.2: Contribuição do Cisalhamento

b) Cálculo do Fluxo Total



Segmento	q_b
$1-2$	0
$2-3$	0
$2-5$	$-0.002 \times Sy$
$3-4$	$-0.001 \times Sy$
$4-5$	0
$5-6$	0
$6-1$	$0.001 \times S_y$

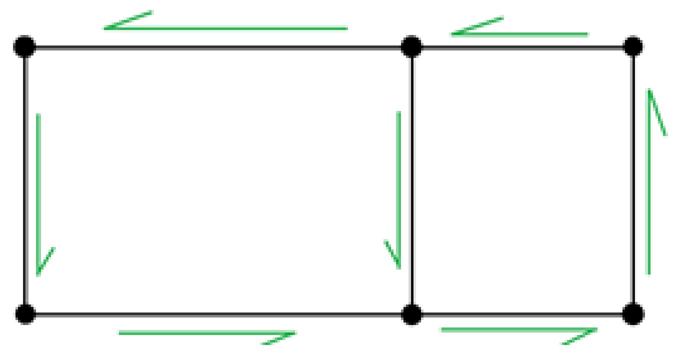
$$\begin{cases} q_{s,0,1} = 0.00022 \times S_y \\ q_{s,0,2} = -0.00013 \times S_y \end{cases}$$



Segmento	q
$1-2$	$0.00022 \times S_y$
$2-3$	$-0.00013 \times S_y$
$2-5$	$-0.00165 \times S_y$
$3-4$	$-0.00113 \times S_y$
$4-5$	$-0.00013 \times S_y$
$5-6$	$0.00022 \times S_y$
$6-1$	$0.00122 \times S_y$

Passo 3.2: Contribuição do Cisalhamento

c) Cálculo do Deslocamento



$Segmento$	q
$1-2$	$0.00022 \times S_y$
$2-3$	$-0.00013 \times S_y$
$2-5$	$-0.00165 \times S_y$
$3-4$	$-0.00113 \times S_y$
$4-5$	$-0.00013 \times S_y$
$5-6$	$0.00022 \times S_y$
$6-1$	$0.00122 \times S_y$

$$\Delta = \int \left[\int \frac{q\delta q}{Gt} ds \right] dz = 2000 \int \frac{q\delta q}{Gt} ds = \frac{2000}{2 \times 25900} \int q\delta q\, ds =$$

$$= \frac{2000}{2 \times 25900} [0.00022^2 + 0.00013^2 \times 2 + 0.00165^2 + 0.00113^2 + 0.00013^2 \times 2 + 0.00022^2 + 0.00122^2] \times S_y \times 250 =$$

$$= \frac{2000}{2 \times 25900} [2 \times 0.00022^2 + 0.00013^2 \times 4 + 0.00165^2 + 0.00113^2 + 0.00122^2] \times 44500 \times 250 =$$

$$\Delta_S = 2.43 \text{ mm}$$

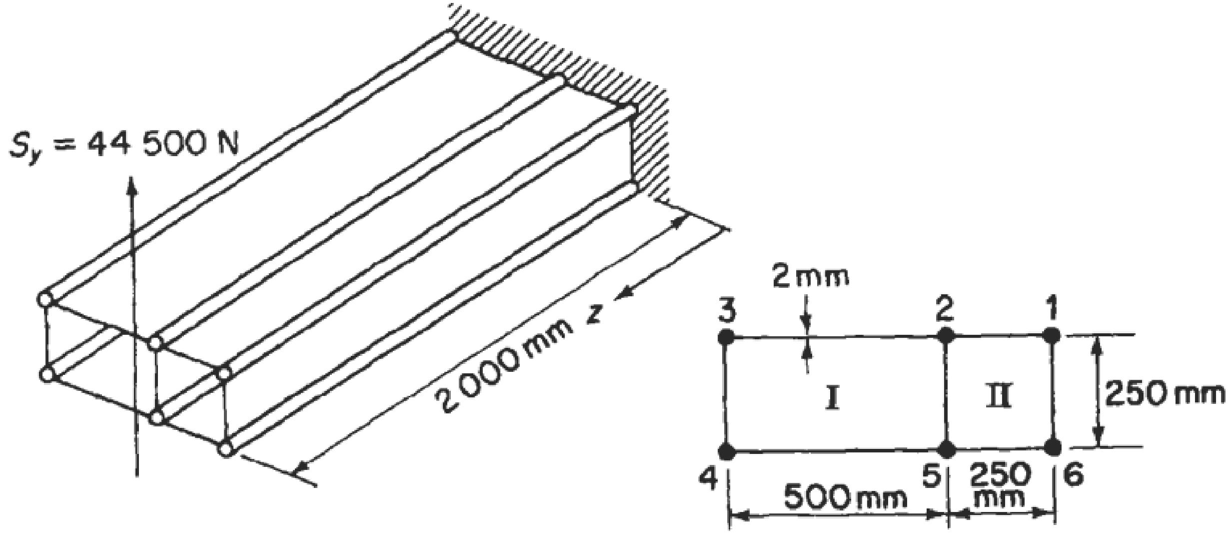
Example 10.12

Calculate the deflection at the free end of the two-cell beam shown in Fig. 10.39 allowing for both bending and shear effects. The booms carry all the direct stresses while the skin panels, of constant thickness throughout, are effective only in shear.

$$\text{Take } E = 69\,000\ \text{N/mm}^2 \text{ and } G = 25\,900\ \text{N/mm}^2$$

$$\text{Boom areas: } -B_1 = B_3 = B_4 = B_6 = 650\ \text{mm}^2,\ B_2 = B_5 = 1300\ \text{mm}^2$$

The beam cross-section is symmetrical about a horizontal axis and carries a vertical load at its free end through the shear centre.



Passo 4: Resultado Final

$$\Delta = 21.17 + 2.43 = 23.6\ \text{mm}$$

P.10.8 Determine the torsional stiffness of the four-cell wing section shown in Fig. P.10.8.

Data:

Wall	12	23	34					
	78	67	56	45^{o}	45^{i}	36	27	18
Peripheral length (mm)	762	812	812	1525	356	406	356	254
Thickness (mm)	0.915	0.915	0.915	0.711	1.220	1.625	1.220	0.915

Cell areas (mm^2) $\quad A_{I} = 161\,500, \quad A_{II} = 291\,000$

$A_{III} = 291\,000, \quad A_{IV} = 226\,000$

Ans. $522.5 \times 10^6 G\,\mathrm{N\,mm^2/rad}$.

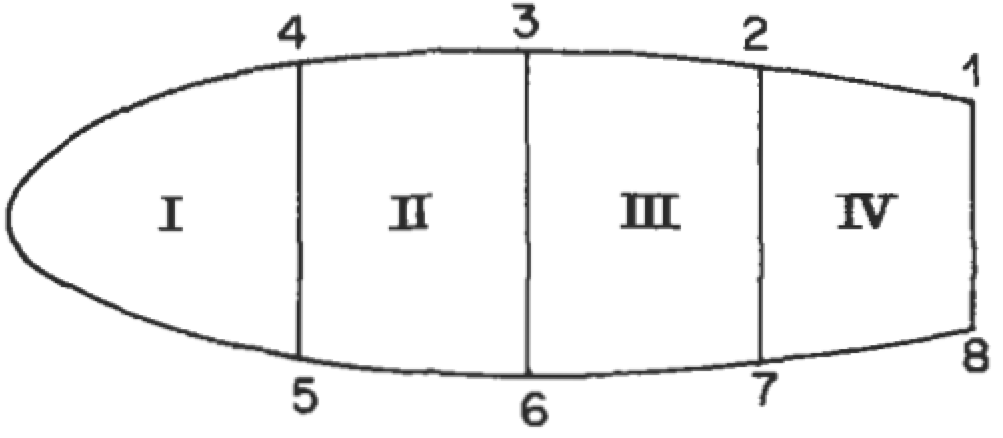


Fig. P.10.8

P.10.8 Determine the torsional stiffness of the four-cell wing section shown in Fig. P.10.8.

Data:

Wall	12	23	34					
	78	67	56	45^{o}	45^{i}	36	27	18
Peripheral length (mm)	762	812	812	1525	356	406	356	254
Thickness (mm)	0.915	0.915	0.915	0.711	1.220	1.625	1.220	0.915

Cell areas (mm^2) $A_{I} = 161\,500$, $A_{II} = 291\,000$

$A_{III} = 291\,000$, $A_{IV} = 226\,000$

Ans. $522.5 \times 10^6 G\,\text{N}\,\text{mm}^2/\text{rad}$.

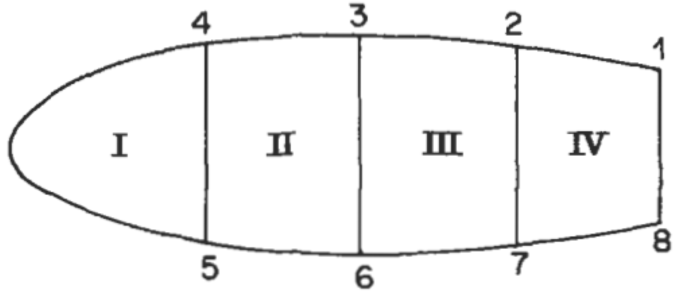


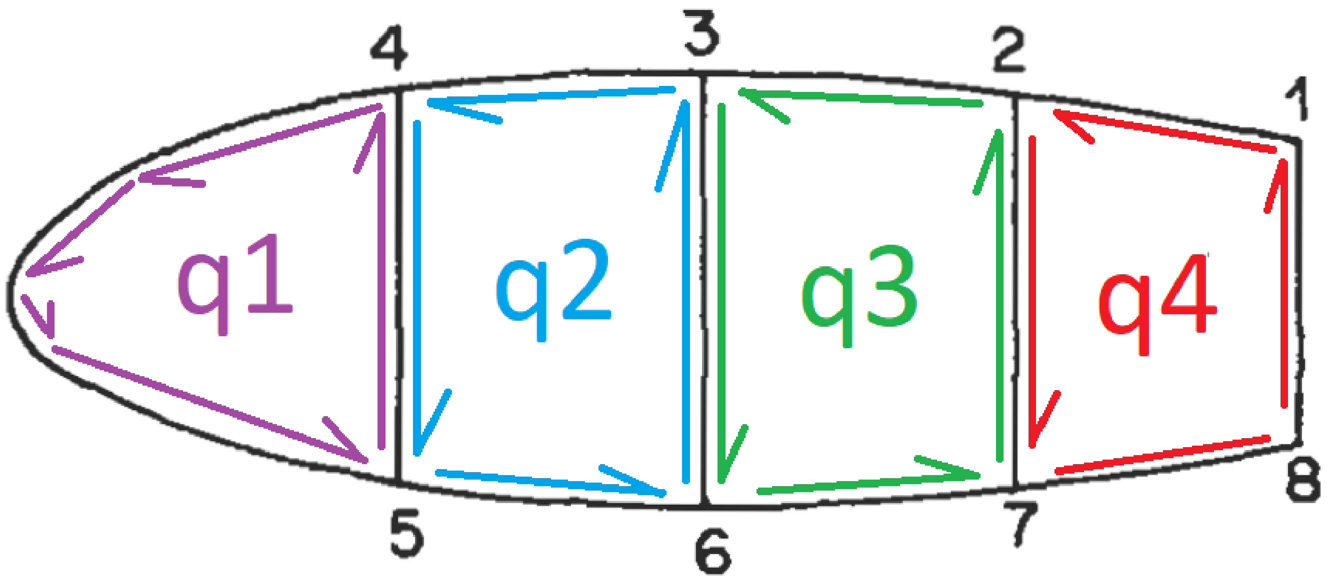
Fig. P.10.8

$$T = GJ.\frac{d\theta}{dz} \rightarrow \frac{T}{\frac{d\theta}{dz}} = GJ \text{ (rigidez torsional)}$$

Passo 1: Análise da Carga

$$T = T_1 + T_2 + T_3 + T_4 =$$
$$= q_1.2A_1 + q_2.2A_2 + q_3.2A_3 + q_4.2A_4 \rightarrow$$

$$\rightarrow T = 2 \times (q_1.A_1 + q_2.A_2 + q_3.A_3 + q_4.A_4)$$



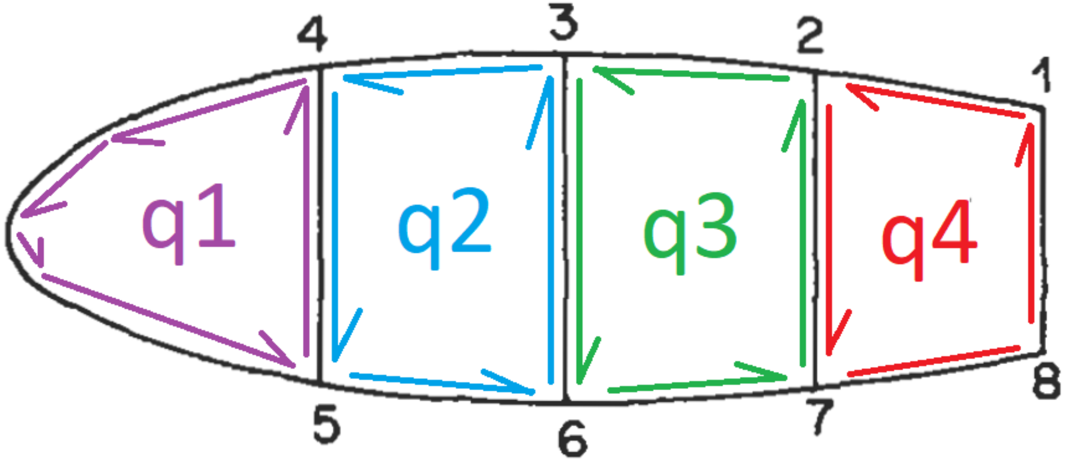
Passo 2: Análise de compatibilidade de deslocamento

Célula I

$$\frac{d\theta}{dz} = \frac{1}{2A_r}\oint q\,\frac{ds}{Gt} = \frac{1}{2A_rG}\oint \frac{q}{t}\,ds = \frac{1}{2A_1G}\left[q_1 \times \frac{1525}{0.711} + (q_1 - q_2) \times \frac{356}{1.22}\right] =$$

$$= \frac{1}{2 \times 161500 \times G}\left[q_1 \times \left(\frac{1525}{0.711} + \frac{356}{1.22}\right) - q_2 \times \frac{356}{1.22}\right] =$$

$$\frac{d\theta}{dz} = [75.4\,q_1 - 9.03\,q_2] \times \frac{10^{-4}}{G}$$



Data:

Wall	12	23	34					
	78	67	56	45^o	45^i	36	27	18
Peripheral length (mm)	762	812	812	1525	356	406	356	254
Thickness (mm)	0.915	0.915	0.915	0.711	1.220	1.625	1.220	0.915

Cell areas (mm^2) $A_I = 161\,500$, $A_{II} = 291\,000$

$A_{III} = 291\,000$, $A_{IV} = 226\,000$

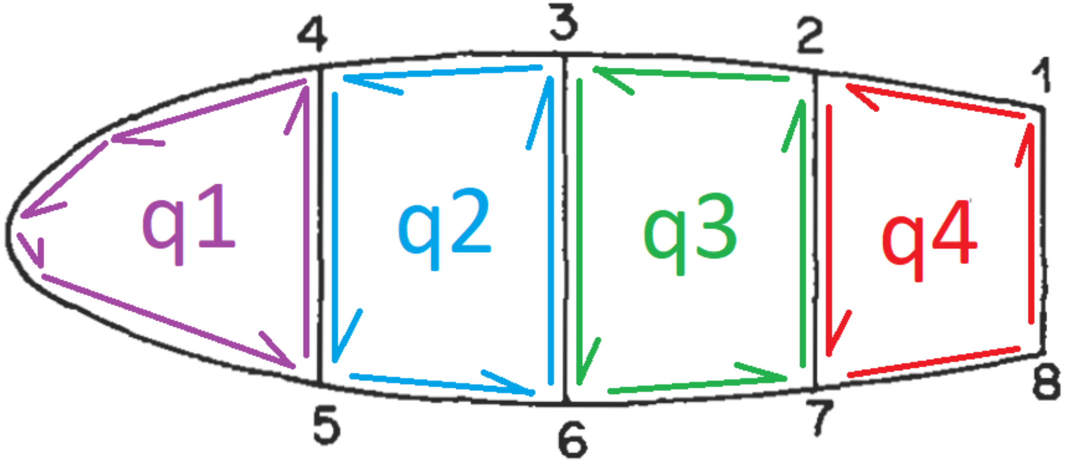
Passo 2: Análise de compatibilidade de deslocamento

Célula II

$$\frac{d\theta}{dz} = \frac{1}{2A_r}\oint q\frac{ds}{Gt} = \frac{1}{2A_rG}\oint \frac{q}{t}ds = \frac{1}{2A_2G}\left[q_2\left(2\times\frac{812}{0.915}+\frac{356}{1.22}+\frac{406}{1.625}\right)-q_1\frac{356}{1.22}-q_3\frac{406}{1.625}\right] =$$

$$= \frac{1}{2\times 291000\times G}\left[q_2\left(2\times\frac{812}{0.915}+\frac{356}{1.22}+\frac{406}{1.625}\right)-q_1\frac{356}{1.22}-q_3\frac{406}{1.625}\right] =$$

$$\frac{d\theta}{dz} = [-\,5.01\,q_1 + 39.8\,q_2 - 4.29\,q_3]\times\frac{10^{-4}}{G}$$



Data:

Wall	12	23	34					
	78	67	56	45°	45ⁱ	36	27	18
Peripheral length (mm)	762	812	812	1525	356	406	356	254
Thickness (mm)	0.915	0.915	0.915	0.711	1.220	1.625	1.220	0.915

Cell areas (mm^2) $A_I = 161\,500$, $A_{II} = 291\,000$

$A_{III} = 291\,000$, $A_{IV} = 226\,000$

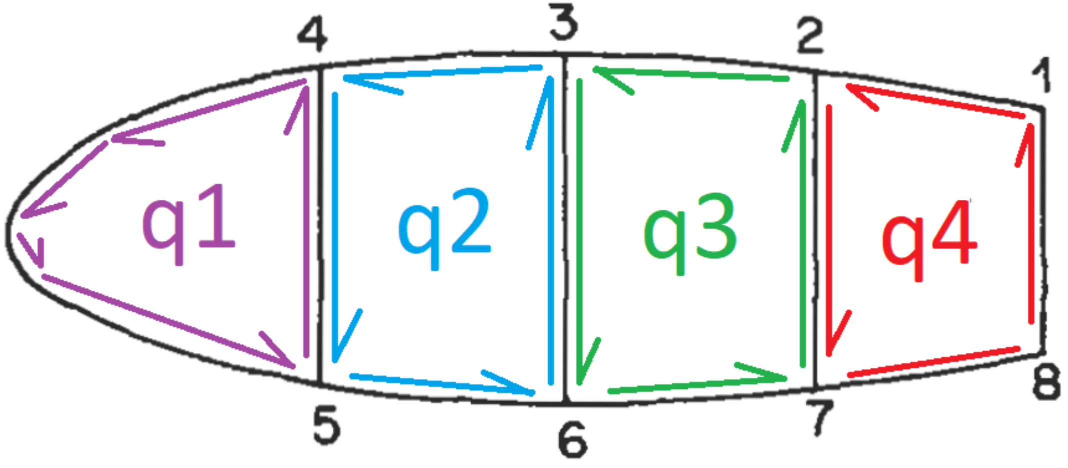
Passo 2: Análise de compatibilidade de deslocamento

Célula III

$$\frac{d\theta}{dz} = \frac{1}{2A_r}\oint q\frac{ds}{Gt} = \frac{1}{2A_rG}\oint \frac{q}{t}ds = \frac{1}{2A_3G}\left[q_3\left(2\times\frac{812}{0.915}+\frac{406}{1.625}+\frac{326}{1.22}\right)-q_2\frac{406}{1.625}-q_4\frac{356}{1.22}\right] =$$

$$= \frac{1}{2\times 291000\times G}\left[q_3\left(2\times\frac{812}{0.915}+\frac{406}{1.625}+\frac{326}{1.22}\right)-q_2\frac{406}{1.625}-q_4\frac{356}{1.22}\right] =$$

$$\frac{d\theta}{dz} = [-\,4.29\,q_2 + 39.8\,q_3 - 5.01\,q_4]\times\frac{10^{-4}}{G}$$



Data:

Wall	12	23	34					
	78	67	56	45^o	45^i	36	27	18
Peripheral length (mm)	762	812	812	1525	356	406	356	254
Thickness (mm)	0.915	0.915	0.915	0.711	1.220	1.625	1.220	0.915

Cell areas (mm^2) $A_I = 161\,500$, $A_{II} = 291\,000$

$A_{III} = 291\,000$, $A_{IV} = 226\,000$

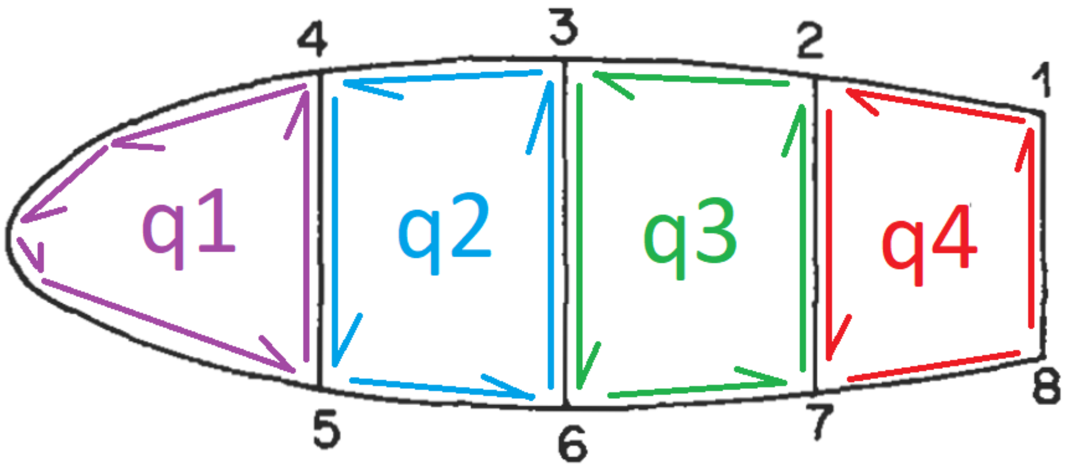
Passo 2: Análise de compatibilidade de deslocamento

Célula IV

$$\frac{d\theta}{dz} = \frac{1}{2A_r}\oint q\frac{ds}{Gt} = \frac{1}{2A_rG}\oint \frac{q}{t}ds = \frac{1}{2A_4G}\left[q_4\left(2\times\frac{762}{0.915}+\frac{254}{0.915}+\frac{326}{1.22}\right)-q_3\frac{356}{1.22}\right] =$$

$$= \frac{1}{2\times 226000\times G}\left[q_4\left(2\times\frac{762}{0.915}+\frac{254}{0.915}+\frac{326}{1.22}\right)-q_3\frac{356}{1.22}\right] =$$

$$\frac{d\theta}{dz} = [-6.45\,q_3 + 49.4\,q_4]\times\frac{10^{-4}}{G}$$



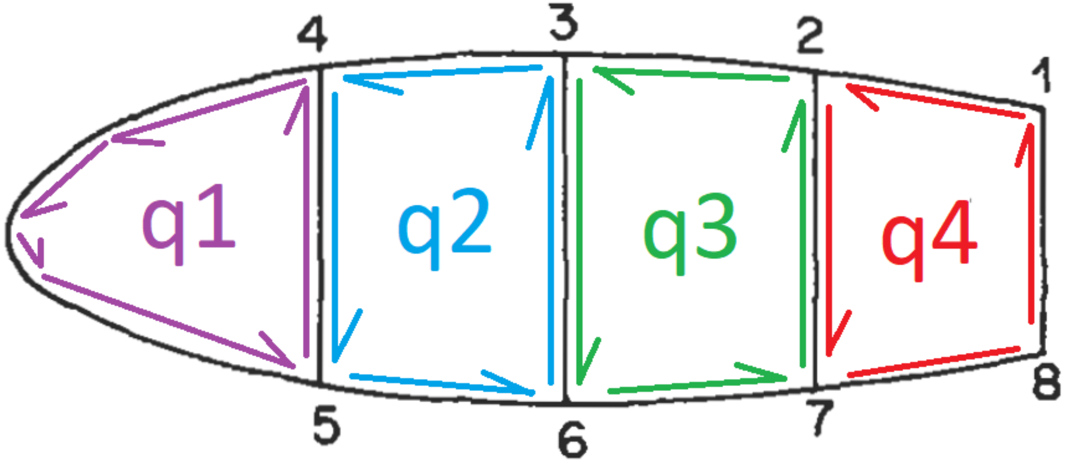
Data:

Wall	12	23	34					
	78	67	56	45^{o}	45^{i}	36	27	18
Peripheral length (mm)	762	812	812	1525	356	406	356	254
Thickness (mm)	0.915	0.915	0.915	0.711	1.220	1.625	1.220	0.915

Cell areas (mm^2) $A_{I} = 161\,500$, $A_{II} = 291\,000$, $A_{III} = 291\,000$, $A_{IV} = 226\,000$

Passo 3: Sistema de Equações

$$\begin{cases} T = 2 \times (q_1.A_1 + q_2.A_2 + q_3.A_3 + q_4.A_4) \\ \dfrac{d\theta}{dz} = [75.4\, q_1 - 9.03\, q_2] \times \dfrac{10^{-4}}{G} \\ \dfrac{d\theta}{dz} = [-5.01\, q_1 + 39.8\, q_2 - 4.29\, q_3] \times \dfrac{10^{-4}}{G} \\ \dfrac{d\theta}{dz} = [-4.29\, q_2 + 39.8\, q_3 - 5.01\, q_4] \times \dfrac{10^{-4}}{G} \\ \dfrac{d\theta}{dz} = [-6.45\, q_3 + 49.4\, q_4] \times \dfrac{10^{-4}}{G} \end{cases} \rightarrow \begin{cases} 75.4\, q_1 - 9.03\, q_2 = -5.01\, q_1 + 39.8\, q_2 - 4.29\, q_3 \\ 75.4\, q_1 - 9.03\, q_2 = -4.29\, q_2 + 39.8\, q_3 - 5.01\, q_4 \\ 75.4\, q_1 - 9.03\, q_2 = -6.45\, q_3 + 49.4\, q_4 \end{cases}$$



Data:

Wall	12	23	34					
	78	67	56	45^o	45^i	36	27	18
Peripheral length (mm)	762	812	812	1525	356	406	356	254
Thickness (mm)	0.915	0.915	0.915	0.711	1.220	1.625	1.220	0.915

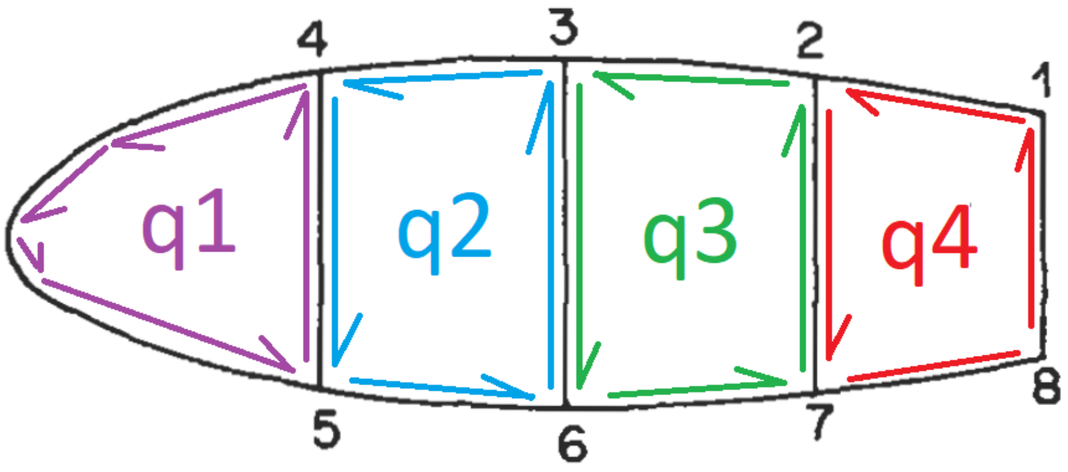
Cell areas (mm^2) $A_I = 161\,500$, $A_{II} = 291\,000$
$A_{III} = 291\,000$, $A_{IV} = 226\,000$

Passo 3: Sistema de Equações

$$\begin{cases} 75.4\, q_1 - 9.03\, q_2 = -\,5.01\, q_1 + 39.8\, q_2 - 4.29\, q_3 \\ 75.4\, q_1 - 9.03\, q_2 = -\,4.29\, q_2 + 39.8\, q_3 - 5.01\, q_4 \\ 75.4\, q_1 - 9.03\, q_2 = -6.45\, q_3 + 49.4\, q_4 \end{cases} \rightarrow \begin{cases} 80.41\, q_1 - 48.83\, q_2 + 4.29\, q_3 = 0 \\ 75.4\, q_1 - 4.74\, q_2 - 39.8\, q_3 = -5.01\, q_4 \\ 75.4\, q_1 - 9.03\, q_2 + 6.45\, q_3 = 49.4\, q_4 \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} 80.41\, q_1 - 48.83\, q_2 + 4.29\, q_3 = 0 \\ 75.4\, q_1 - 4.74\, q_2 - 39.8\, q_3 = -5.01\, q_4 \\ 75.4\, q_1 - 9.03\, q_2 + 6.45\, q_3 = 49.4\, q_4 \end{cases} \rightarrow \begin{cases} q_1 = 0.695\, q_4 \\ q_2 = 1.260\, q_4 \\ q_3 = 1.290 q_4 \end{cases}$$

$$\begin{cases} T = 2 \times (q_1 . A_1 + q_2 . A_2 + q_3 . A_3 + q_4 . A_4) \\ \dfrac{d\theta}{dz} = [-6.45\, q_3 + 49.4\, q_4] \times \dfrac{10^{-4}}{G} \end{cases} \rightarrow \begin{cases} T = 2 \times [0.695 \times 161500 + (1.26 + 1.29) \times 291000 + 226000] q_4 \\ \dfrac{d\theta}{dz} = \dfrac{41.07 \times 10^{-4}}{G} q_4 \end{cases}$$



Data:

Wall	12	23	34					
	78	67	56	45^o	45^i	36	27	18
Peripheral length (mm)	762	812	812	1525	356	406	356	254
Thickness (mm)	0.915	0.915	0.915	0.711	1.220	1.625	1.220	0.915

Cell areas (mm^2) $A_I = 161\,500$, $A_{II} = 291\,000$

$A_{III} = 291\,000$, $A_{IV} = 226\,000$

Passo 3: Sistema de Equações

$$\begin{cases} T = 2 \times (q_1.A_1 + q_2.A_2 + q_3.A_3 + q_4.A_4) \\ \dfrac{d\theta}{dz} = [-6.45\ q_3 + 49.4\ q_4] \times \dfrac{10^{-4}}{G} \end{cases} \rightarrow \begin{cases} T = 2 \times [0.695 \times 161500 + (1.26 + 1.29) \times 291000 + 226000] q_4 \\ \dfrac{d\theta}{dz} = \dfrac{41.07 \times 10^{-4}}{G} q_4 \end{cases}$$

$$\begin{cases} T = 2160585 q_4 \\ \dfrac{d\theta}{dz} = \dfrac{41.07 \times 10^{-4}}{G} q_4 \end{cases} \rightarrow \frac{T}{\frac{d\theta}{dz}} = \frac{2160585}{41.07 \times 10^{-4}} G = 526.08 \times 10^6\ G\ \mathrm{N}\frac{\mathrm{mm}^2}{\mathrm{rad}} \rightarrow$$

$$\rightarrow GJ = 526.08 \times 10^6\ G\ \mathrm{N}\frac{\mathrm{mm}^2}{\mathrm{rad}}$$