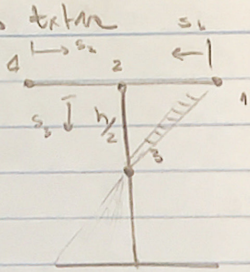
Exemplo extra



$$\Gamma_R = \int_C (2A_{R,0})^2 \cdot t \cdot ds - (2A'_R)^2 \cdot \int_C t \, ds$$

$$2A'_R = \frac{\int_C 2A_{R,0} \cdot t \cdot ds}{\int_C t \, ds}$$

$$\underset{1 \to 2}{2A_{R,0}} = \frac{s \cdot h}{2} \qquad 2A'_R = \frac{1}{t(b+h/2)} \left[\int_0^{b/2} \frac{s \cdot h}{2} \cdot t \, ds - \int_0^{b/2} -\frac{s h}{2} \cdot t \cdot ds_2 + \int_0^{h/2} 0 \cdot t \cdot ds_3 \right]$$

$$\underset{4 \to 2}{2A_{R,0}} = -\frac{s \cdot h}{2} \qquad 2A'_R = 0$$

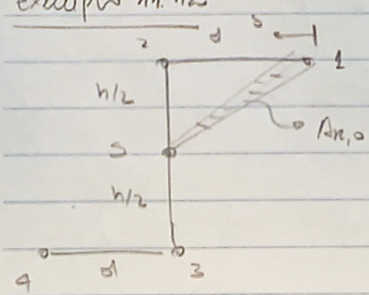
simetria

$$\underset{2 \to 3}{2A_{R,0}} = 0 \qquad \frac{1}{2}\Gamma_R = \int_0^{b/2} \left(\frac{s \cdot h}{2}\right)^2 \cdot t \, ds + \int_0^{b/2} \left(-\frac{s h}{2}\right)^2 \cdot t \, ds$$

$$\frac{1}{2}\Gamma_R = 2 \cdot \frac{th^2}{4} \cdot \frac{s^3}{3} \Big|_0^{b/2} \Rightarrow \Gamma_R = th^2 \cdot \frac{b^3}{24} \Rightarrow \frac{tb^3h^2}{24}$$

(I)

Exemplo 11.12



$$\Gamma_R = \int_C (2A_{R,0})^2 \cdot t \cdot ds - (2A'_R)^2 \int_C t \cdot ds$$

$$2A'_R = \frac{\int_C 2A_{R,0} \cdot t \cdot ds}{\int_C t \cdot ds}$$

$$\underset{1 \to 2}{2A_{R,0}} = \frac{s \cdot h}{2} \Rightarrow 2A'_R = \frac{1}{t \cdot (d+h/2)} \left[\int_0^{d} \frac{s \cdot h}{2} \cdot t \, ds + \int_0^{h/2} \frac{dh}{2} \cdot t \cdot ds \right]$$

$$\underset{2 \to S}{2A_{R,0}} = \frac{dh}{2} \Rightarrow 2A'_R = \frac{1}{t(d+h/2)} \left[\frac{h \cdot t}{2} \cdot \frac{s^2}{2} \Big|_0^{d} + \frac{d \cdot h \cdot t \cdot s}{2} \Big|_0^{h/2} \right]$$

$$2A'_R = \frac{1}{t(d+h/2)} \left[\frac{h \cdot t}{2} \cdot \frac{d^2}{2} + \frac{ht}{2} \cdot d \cdot \frac{h}{2} \right]$$

$$2A'_R = \frac{\cancel{2}^{1}}{\cancel{4}(2d+h)}\left[\frac{dh\cancel{t}(d+h)}{\cancel{4}2}\right] \therefore 2A'_R = \frac{hd(h+d)}{2(h+2d)}$$

$$\text{I} \quad \int_0^d \left(\frac{s \cdot h}{2}\right)^2 \cdot t \cdot ds + \int_0^{h/2} \left(\frac{dh}{2}\right)^2 \cdot t \cdot ds$$

$$= \frac{h^2 \cdot t}{4} \left.\frac{s^3}{3}\right|_0^d + \left.\frac{d^2h^2 \cdot t}{4} \cdot s\right|_0^{h/2}$$

$$= \frac{h^2t}{12} \cdot d^3 + \frac{d^2h^3 \cdot t}{8} \therefore \frac{h^2d^2t}{4}\left(\frac{d}{3}+\frac{h}{2}\right)$$

simetria

$$\text{Então: } \frac{1}{2}\Gamma_R = \frac{h^2d^2t}{4}\left(\frac{d}{3}+\frac{h}{2}\right) - \left[\frac{hd(h+d)}{2(h+2d)}\right]^2 \cdot t\,(d+h/2)$$

$$\frac{1}{2}\Gamma_R = \frac{h^2d^2t}{4}\left(\frac{d}{3}+\frac{h}{2}\right) - \frac{(hd(h+d))^2}{\cancel{4}(h+2d)^{\cancel{2}}} \cdot \frac{t\,\cancel{(h+2d)}}{2}$$

$$\frac{1}{\cancel{2}}\Gamma_R = \frac{h^2d^2t}{2\,\cancel{4}}\left(\frac{d}{3/2}+\frac{h}{\cancel{2}/3}\right) - \frac{h^2d^2(h+d)^2 \cdot t}{4\,\cancel{8}\,(h+2d)}$$

$$\Gamma_R = \frac{h^2d^2t}{12}\left[\frac{(2d+3h)(h+2d) - 3(h+d)^2}{h+2d}\right]$$

$$\Gamma_R = \frac{h^2d^2t}{12(h+2d)}\left[2dh + 4d^2 + \cancel{3h^2} + \cancel{6dh} - \cancel{3h^2} - \cancel{6hd} - 3d^2\right]$$

$$\Gamma_R = \frac{h^2d^2t}{12}\left(\frac{d^2+2dh}{(h+2d)}\right) \Rightarrow \underline{\frac{td^3h^2}{12}\,\frac{(2h+d)}{(h+2d)}}$$

$T = T_S + T_r$ $\qquad \cosh x = \frac{e^x + e^{-x}}{2}$; $\sinh x = \frac{e^x - e^{-x}}{2}$

$$T = GJ \frac{d\theta}{dz} - E\Gamma_F \frac{d}{dz}\left(\frac{d^2\theta}{dz^2}\right)$$

$$\frac{d^3\theta}{dz^3} - \frac{GJ}{E\Gamma_F}\frac{d\theta}{dz} = -\frac{1}{E\Gamma_F} T$$, mas $\mu^2 = \frac{GJ}{E\Gamma_F}$

$$\frac{d^3\theta}{dz^3} - \mu^2 \frac{d\theta}{dz} = -\mu^2 \cdot \frac{T}{GJ}$$

resolvendo para $\frac{d\theta}{dz} = A.\cosh(\mu z) + B.\sinh(\mu z) + \frac{T}{GJ}$

Condições de contorno:

i) no suporte o empenamento é zero. $w = -2A_n \frac{d\theta}{dz} = 0$

$\Rightarrow \frac{d\theta}{dz} = 0$ para $z = 0$.

$A.(1) + B.(0) + \frac{T}{GJ} = 0 \Rightarrow \underline{A = -\frac{T}{GJ}}$.

ii) na face livre tem-se que $\sigma_r = 0$; $z = L$, mas

$\sigma_r = -2A_n \cdot E \cdot \frac{d^2\theta}{dz^2} \Rightarrow \frac{d^2\theta}{dz^2} = 0$.

$$\frac{d^2\theta}{dz^2} = A.\mu.\sinh(\mu z) + B\mu.\cosh(\mu z)$$

$$\left.\frac{d^2\theta}{dz^2}\right|_{z=L} = A\cancel{\mu}.\sinh(\mu L) + B\cancel{\mu}\cosh(\mu L) = 0$$

$$\Rightarrow B = \frac{T}{GJ} \cdot tgh(\mu L)$$

$$\therefore \frac{d\theta}{dz} = -\frac{T}{GJ}\left[\cosh(\mu z) - tgh(\mu L).\sinh(\mu z) - 1\right]$$

$$\frac{d\theta}{dz} = \frac{T}{GJ}\left[1 - \cosh(\mu z) + tgh(\mu L).\sinh(\mu z)\right]$$

integrando: $\theta(z=0)=0$

$$\theta = \int_0^z \frac{T}{GJ}\left(1-\cosh(\mu z)+tgh(\mu L)\cdot \operatorname{senh}(\mu z)\right)\cdot dz$$

$$\theta(z) = \frac{T}{GJ}\left[z-\frac{\operatorname{senh}(\mu z)}{\mu}+\frac{tgh(\mu L)\cdot\cosh(\mu z)}{\mu}-\frac{tgh(\mu L)}{\mu}\right]$$

para $z=L$.

$$\theta = \frac{T}{GJ}\left[L-\cancel{\frac{\operatorname{senh}(\mu L)}{\mu}}+\frac{\operatorname{senh}(\mu L)}{\cancel{\cosh(\mu L)}}\cdot\frac{\cancel{\cosh(\mu L)}}{\mu}-\frac{tgh(\mu L)}{\mu}\right]$$

$$\theta = \frac{TL}{GJ}\left(1-\frac{tgh(\mu L)}{\mu L}\right)$$

$$T_J = GJ\frac{d\theta}{dz}$$

$$T_J = T\left(1-\cosh(\mu z)+tgh(\mu L)\cdot\operatorname{senh}(\mu z)\right)$$

$$\frac{GJ}{T}\frac{d\theta}{dz} = \left(1-\cosh(\mu z)+\frac{\operatorname{senh}(\mu L)}{\cosh(\mu L)}\cdot\operatorname{senh}(\mu z)\right)$$

$$\frac{GJ}{T}\cdot\frac{d\theta}{dz} = \left(1+\frac{-\cosh(\mu z)\cdot\cosh(\mu L)+\operatorname{senh}(\mu L)\cdot\operatorname{senh}(\mu z)}{\cosh(\mu L)}\right)$$

revisão

$\cosh(a+b) = \cosh a\cosh b + \operatorname{senh} a + \operatorname{senh} b$

$\cosh(a-b) = \cosh a\cosh b - \operatorname{senh} a\operatorname{senh} b$

$$\frac{GJ}{T}\cdot\frac{d\theta}{dz} = \left(1-\frac{\cosh(\mu(L-z))}{\cosh(\mu L)}\right)$$

$$T_J = GJ\frac{d\theta}{dz} \Rightarrow T_J = \cancel{GJ}\frac{T}{\cancel{GJ}}\left(1 - \frac{\cosh(\mu(L-z))}{\cosh(\mu L)}\right)$$

$$\Rightarrow T_J = T\left(1 - \frac{\cosh(\mu(L-z))}{\cosh(\mu L)}\right)$$

$$T_\Gamma = -E\Gamma_R\frac{d^3\theta}{dz^3}, \text{ mas } T_\Gamma = T - GJ\frac{d\theta}{dz}$$

$$\Rightarrow T_\Gamma = \cancel{T} - \cancel{T} + \frac{T\cosh(\mu(L-z))}{\cosh(\mu L)}$$

$$\Rightarrow T_\Gamma = \frac{T\cosh(\mu(L-z))}{\cosh(\mu L)}$$

$$\sigma_\Gamma = -2A_R E\frac{d^2\theta}{dz^2} \Rightarrow \sigma_\Gamma = -2A_R E \cdot \frac{T}{GJ}\mu \cdot \frac{\sinh(\mu(L-z))}{\cosh(\mu L)}$$

$$\frac{d^2\theta}{dz^2} = \frac{T}{GJ}\left(+\mu \cdot \frac{\sinh(\mu(L-z))}{\cosh(\mu L)}\right); \quad \frac{d^3\theta}{dz^3} = \frac{-T}{GJ}\mu^2\frac{\cosh(\mu(L-z))}{\cosh(\mu L)}$$

$$q_\Gamma = E\frac{d^3\theta}{dz^3}\int_0^s 2A_R t \cdot ds \qquad \frac{hd^2(h+2d) - 2hd^2(h+d)}{4(h+2d)}$$

$$= \frac{-h^2d^2}{4(h+2d)}$$

mas $2A_R = 2A_{R,0} - 2A'_R$

Para o trecho 12

$$2A_R = \frac{hs_1}{2} - \frac{hd(h+d)}{2(h+2d)}$$

$$\int_0^s\left(\frac{hs_1}{2} - \frac{hd(h+d)}{2(h+2d)}\right)t\,ds_1 \Rightarrow t\left(\frac{hs_1^2}{4} - \frac{hd(h+d)s_1}{2(h+2d)}\right)\Bigg|_0^d$$

$$q_{\Gamma,1} = 0, \quad q_{\Gamma,2} = -E\frac{d^3\theta}{dz^3}\frac{h^2d^2t}{4(h+2d)}$$