Example 10.2

The cantilever beam shown in Fig. 10.6 is uniformly tapered along its length in both x and y directions and carries a load of 100 kN at its free end. Calculate the forces in the

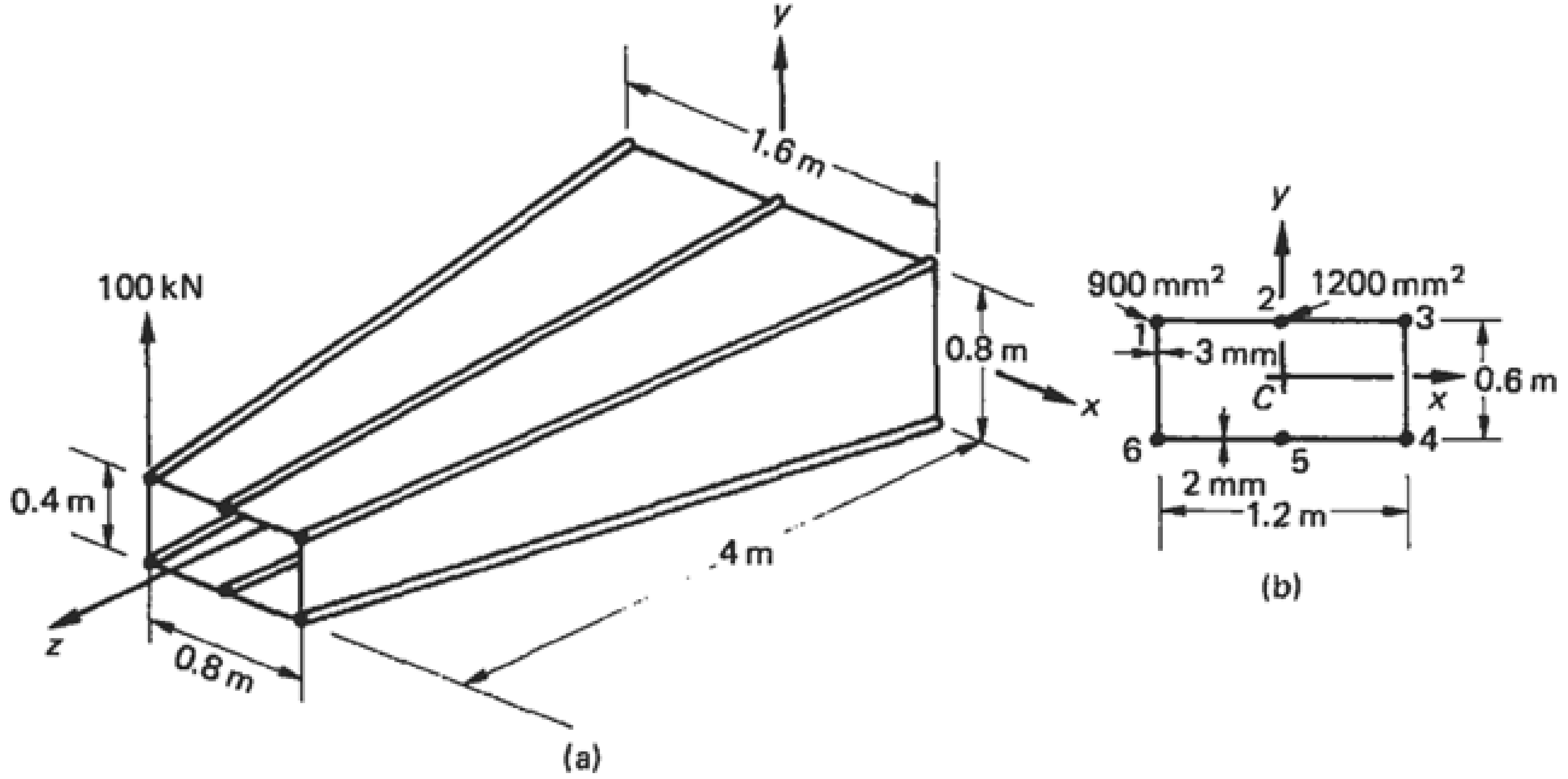


Fig. 10.6 (a) Beam of Example 10.2; (b) section 2 m from built-in end.

booms and the shear flow distribution in the walls at a section 2 m from the built-in end if the booms resist all the direct stresses while the walls are effective only in shear. Each corner boom has a cross-sectional area of $900\,\text{mm}^2$ while both central booms have cross-sectional areas of $1200\,\text{mm}^2$.

Example 10.2

The cantilever beam shown in Fig. 10.6 is uniformly tapered along its length in both x and y directions and carries a load of 100 kN at its free end. Calculate the forces in the

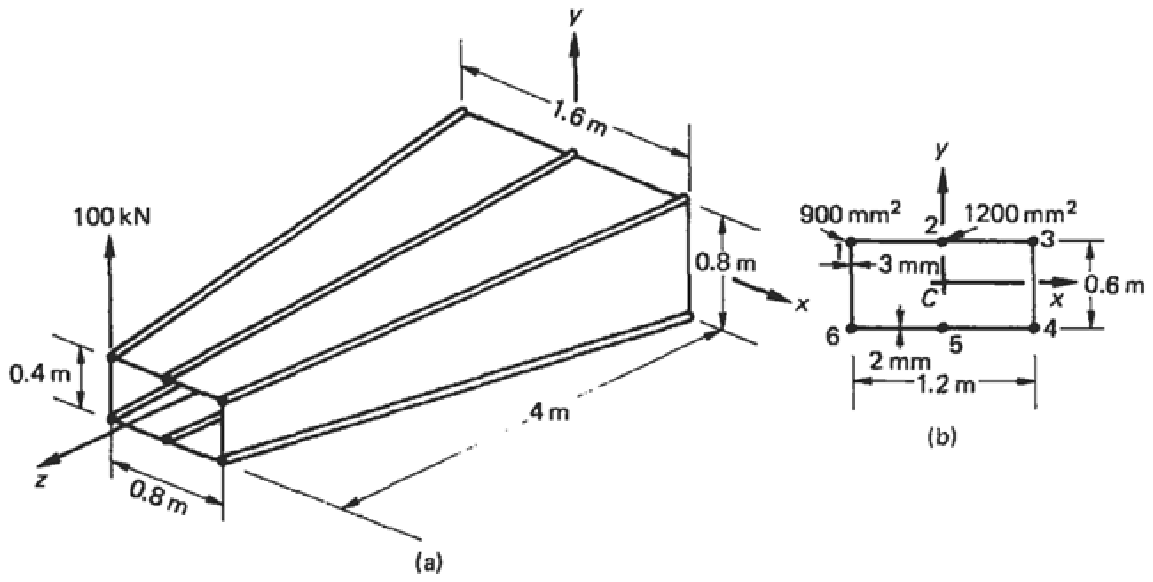
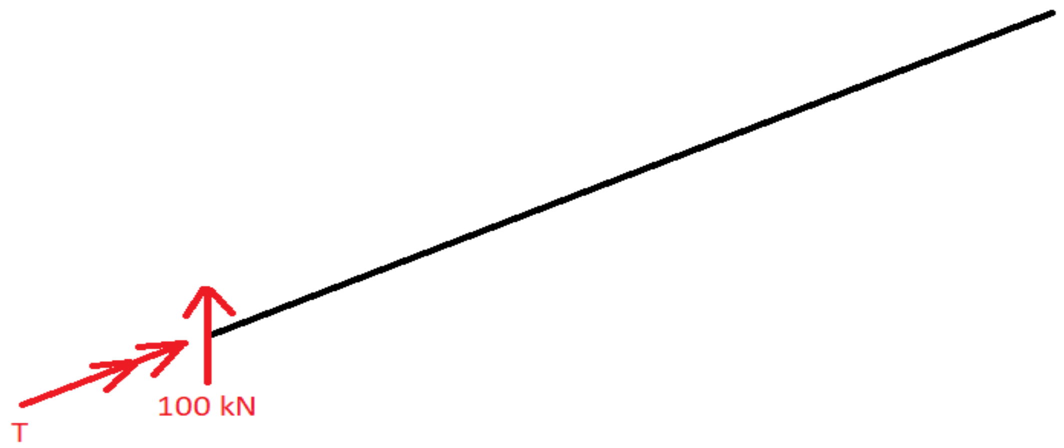


Fig. 10.6 (a) Beam of Example 10.2; (b) section 2 m from built-in end.

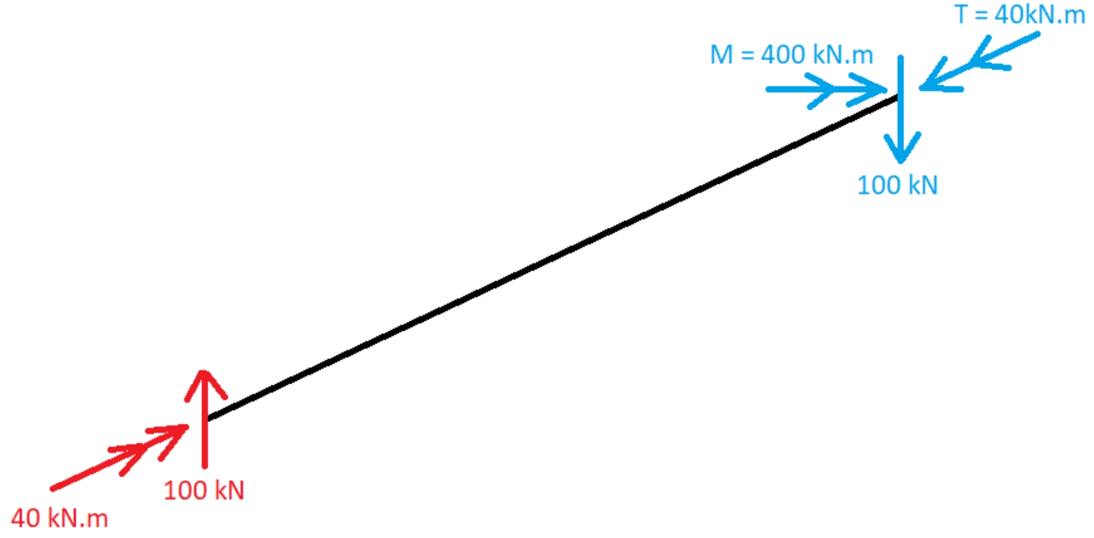
booms and the shear flow distribution in the walls at a section 2 m from the built-in end if the booms resist all the direct stresses while the walls are effective only in shear. Each corner boom has a cross-sectional area of 900 mm^2 while both central booms have cross-sectional areas of 1200 mm^2.

$$\begin{cases} S_{y_0} = 100\text{kN} \\ M_{x_0} = -400\ \text{kN.m} \\ T_0 = -40\ \text{kN.m} \end{cases}$$

Passo 1: Diagrama de Corpo Livre



$$T = 100\text{kN} \times 0.4\text{m} = 40\text{kN.m}$$



Example 10.2

The cantilever beam shown in Fig. 10.6 is uniformly tapered along its length in both x and y directions and carries a load of 100 kN at its free end. Calculate the forces in the

Passo 2: Análise da Seção

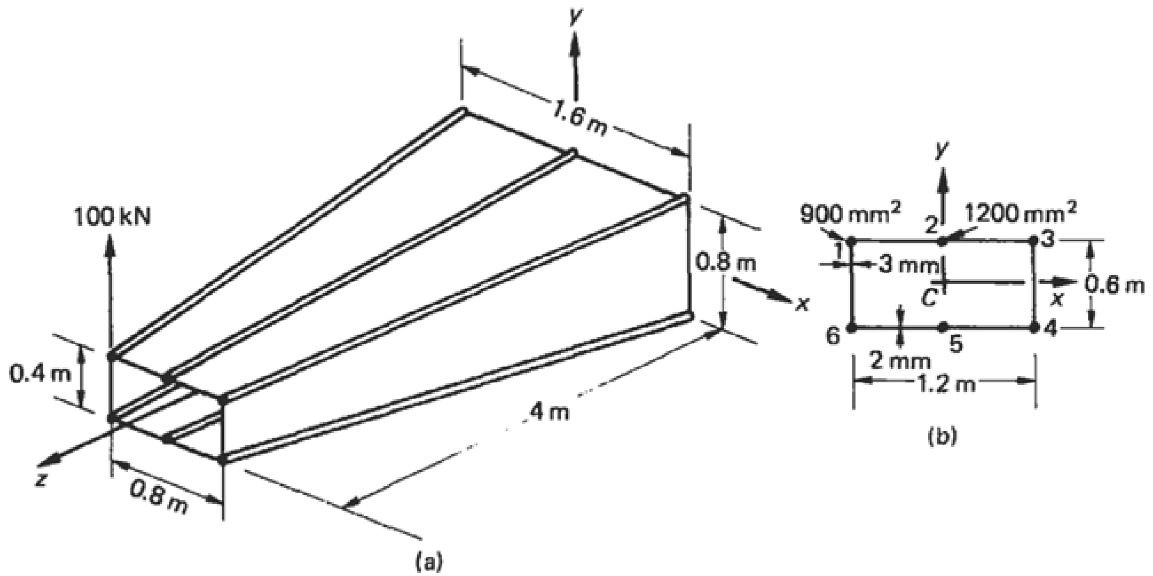
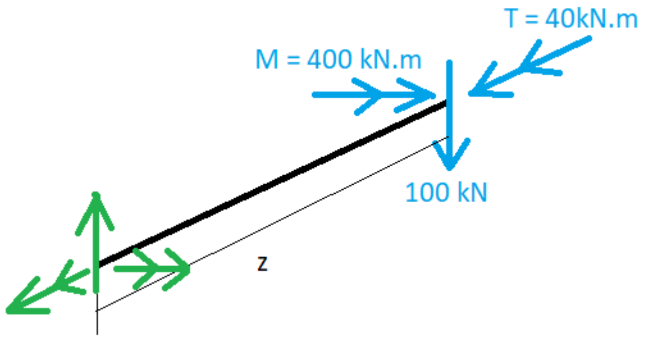


Fig. 10.6 (a) Beam of Example 10.2; (b) section 2 m from built-in end.



booms and the shear flow distribution in the walls at a section 2 m from the built-in end if the booms resist all the direct stresses while the walls are effective only in shear. Each corner boom has a cross-sectional area of 900 mm^2 while both central booms have cross-sectional areas of 1200 mm^2.

$$\begin{cases} S_y(z) = 100 \\ T(z) = -40 \\ M_x(z) = -400 + 100 \times z \end{cases} \rightarrow \begin{cases} S_y = 100 \text{ kN} \\ T = -40 \text{ kN.m} \\ M_x = -200 \text{ kN.m} \end{cases}$$

$$\begin{cases} S_{y_0} = 100\text{kN} \\ M_{x_0} = -400 \text{ kN.m} \\ T_0 = -40 \text{ kN.m} \end{cases}$$

Example 10.2

The cantilever beam shown in Fig. 10.6 is uniformly tapered along its length in both x and y directions and carries a load of 100 kN at its free end. Calculate the forces in the

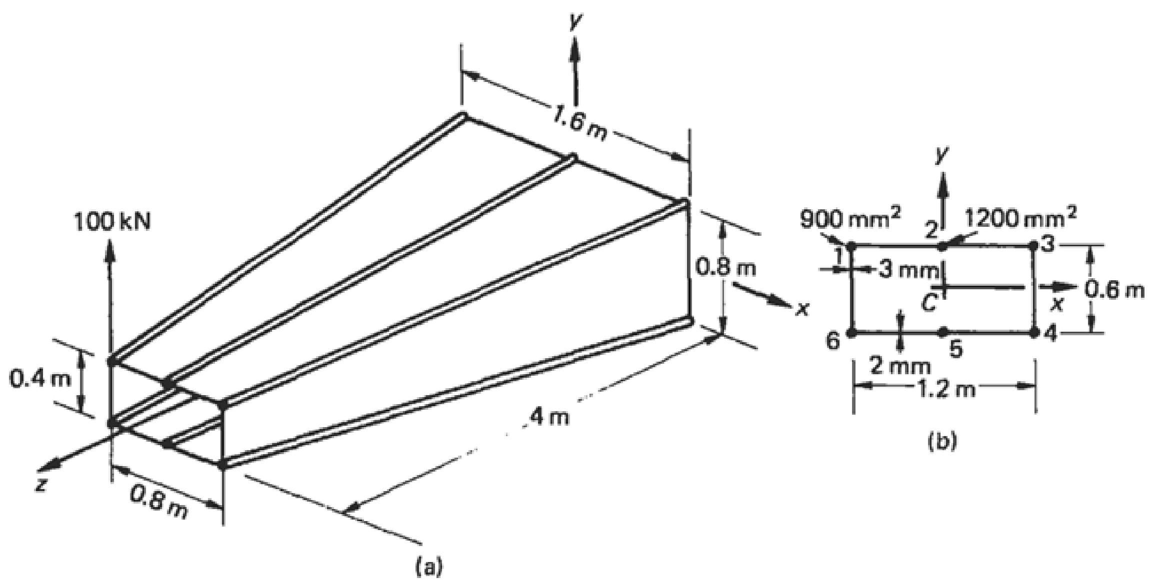
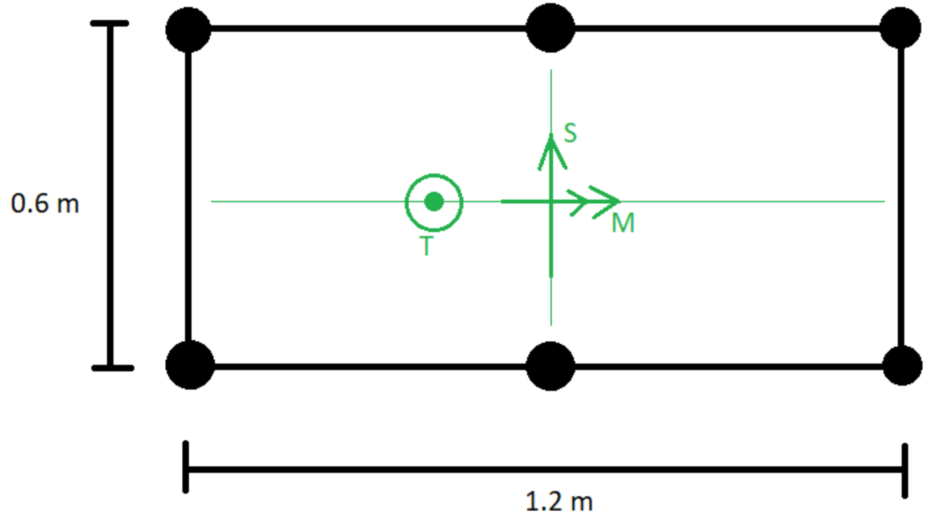


Fig. 10.6 (a) Beam of Example 10.2; (b) section 2 m from built-in end.

booms and the shear flow distribution in the walls at a section 2 m from the built-in end if the booms resist all the direct stresses while the walls are effective only in shear. Each corner boom has a cross-sectional area of 900 mm^2 while both central booms have cross-sectional areas of 1200 mm^2.

$$\begin{cases} S_y = 100 \text{ kN} \\ T = -40 \text{ kN.m} \\ M_x = -200 \text{ kN.m} \end{cases}$$

Passo 3: Análise das Cargas (Seção z = 2m)



Passo 3.1: Cálculo das tensões axiais

$$\sigma = \frac{M_y . I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} . x + \frac{M_x . I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} . y = \frac{M_x}{I_{xx}} . y$$

a. Cálculo das Propriedades Geométricas

$$I_{xx} = \sum B_i y_i^2 = 4 \times 900 \times 0.3^2 + 2 \times 1200 \times 0.3^2$$

$$I_{xx} = 540 \times 10^6 \text{ mm}^4$$

Example 10.2

The cantilever beam shown in Fig. 10.6 is uniformly tapered along its length in both x and y directions and carries a load of 100 kN at its free end. Calculate the forces in the

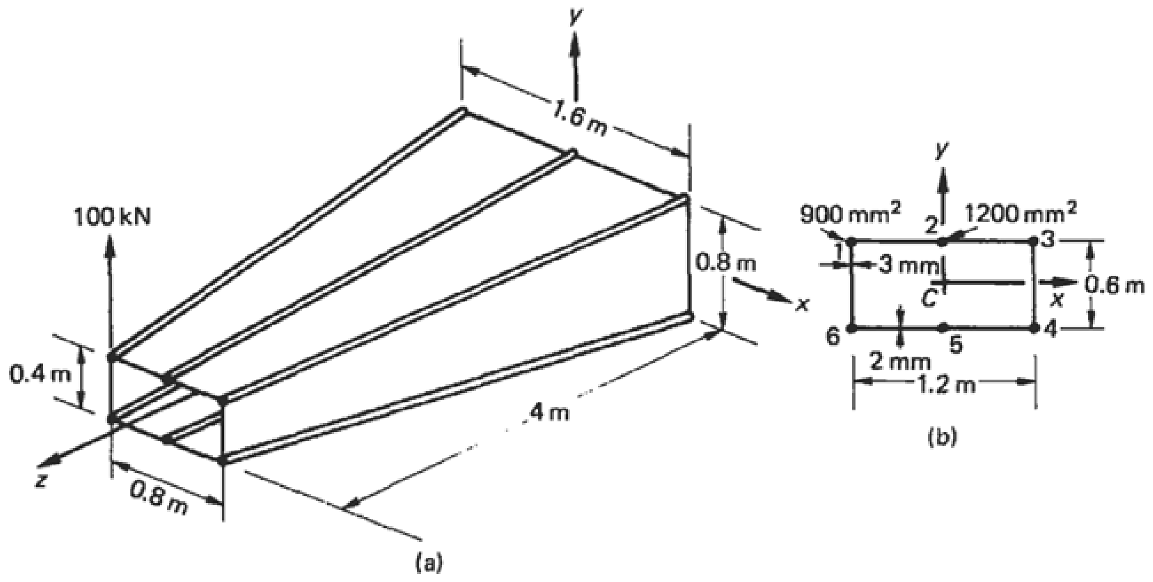


Fig. 10.6 (a) Beam of Example 10.2; (b) section 2 m from built-in end.

booms and the shear flow distribution in the walls at a section 2 m from the built-in end if the booms resist all the direct stresses while the walls are effective only in shear. Each corner boom has a cross-sectional area of 900 mm^2 while both central booms have cross-sectional areas of 1200 mm^2.

$$\begin{cases} S_y = 100\ \text{kN} \\ T = -40\ \text{kN.m} \\ M_x = -200\ \text{kN.m} \end{cases}$$

b. Cálculo das Tensões nos Booms

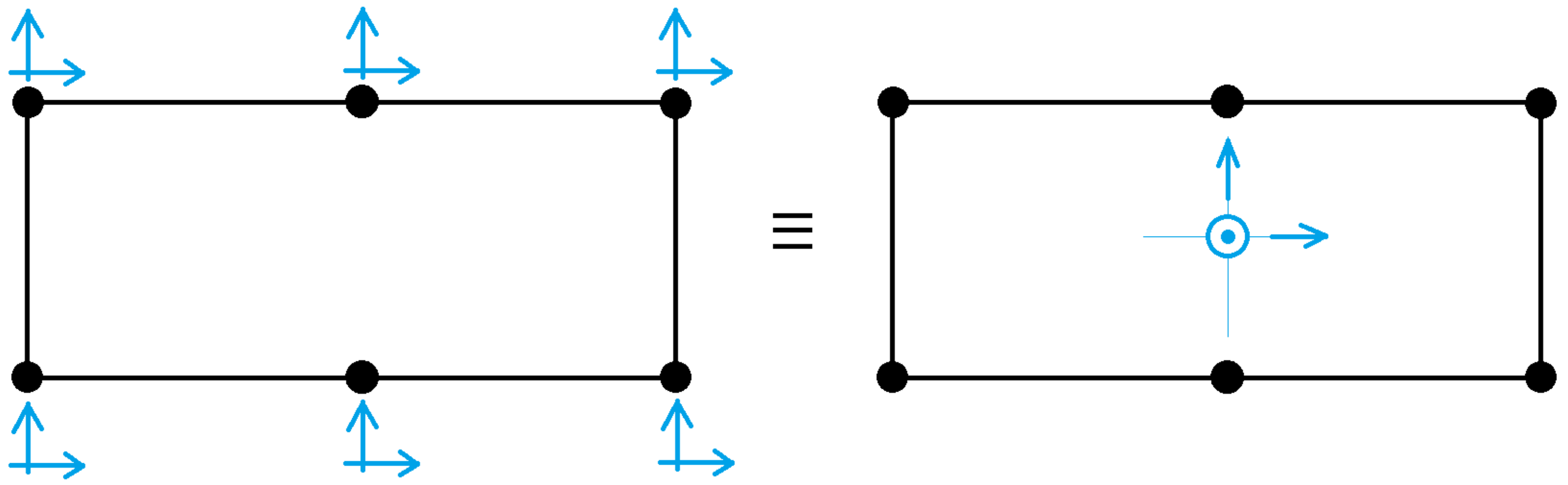
y_i(m)	σ_i(MPa)	$P_{z,i}$ (kN)
0.3	−111.1	−100
0.3	−111.1	−133.33
0.3	−111.1	−100
−0.3	111.1	100
−0.3	111.1	133.33
−0.3	111.1	100

$$\sigma = \frac{-200\ \text{kN.m}}{540e6\ \text{mm}^4} . y = \frac{-200\ \text{N.mm}}{540\ \text{mm}^4} . y$$

Conforme observado, P_z é apenas uma componente do carregamento atuante sobre o Boom, sendo necessário o cálculo das demais componentes

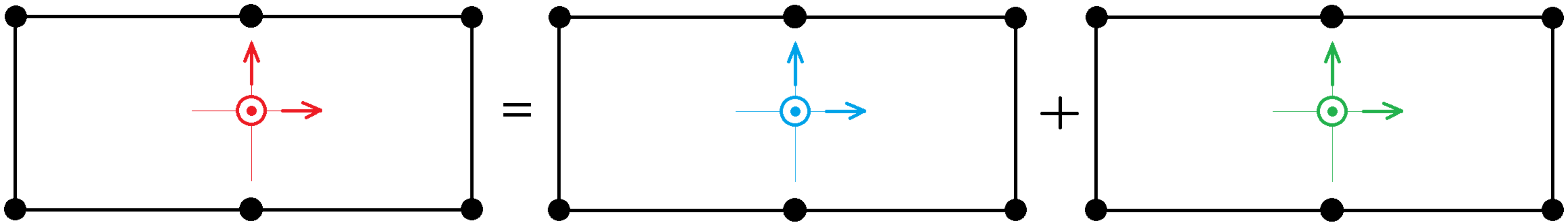
b. Cálculo das Tensões nos Booms

y_i(m)	σ_i(MPa)	$P_{z,i}$ (kN)	δ_x/δ_z	δ_y/δ_z	$P_{x,i}$ (kN)	$P_{y,i}$ (kN)	P_i(kN)	$\sigma_{t,i}$(MPa)	ξ (m)	η (m)	T_x(kN.m)	T_y(kN.m)
0.3	−111.1	−100	0.1	−0.05	−10	5	−100.62	−111.8	0.3	−0.6	−3	−3
0.3	−111.1	−133.33	0	−0.05	0	6.66	−133.5	−111.25	0.3	0	0	0
0.3	−111.1	−100	−0.1	−0.05	10	5	−100.62	−111.8	0.3	0.6	3	3
−0.3	111.1	100	−0.1	0.05	−10	5	100.62	111.8	−0.3	0.6	3	3
−0.3	111.1	133.33	0	0.05	0	6.66	133.5	111.25	−0.3	0	0	0
−0.3	111.1	100	0.1	0.05	10	5	100.62	111.25	−0.3	−0.6	−3	−3
Total					0	33.32					0	0



Passo 3.2: Cálculo dos Fluxos de Cisalhamento

a. Recalculo das cargas de cisalhamento atuantes apenas nas cascas



$$\begin{cases} S_y = 100 \text{ kN} \\ T = -40 \text{ kN.m} \\ M_x = -200 \text{ kN.m} \end{cases} \rightarrow \begin{cases} S_{x_{cis}} = S_x - \sum P_{x,i} \\ S_{y_{cis}} = S_y - \sum P_{y,i} \\ T_{cis} = T - \sum T_{y,i} + \sum T_{x,i} \end{cases} \rightarrow \begin{cases} S_{x_{cis}} = 0 \text{ kN} \\ S_{y_{cis}} = 66.68 \text{ kN} \\ T_{cis} = -40 \text{ kN.m} \end{cases}$$

Example 10.2

The cantilever beam shown in Fig. 10.6 is uniformly tapered along its length in both x and y directions and carries a load of 100 kN at its free end. Calculate the forces in the

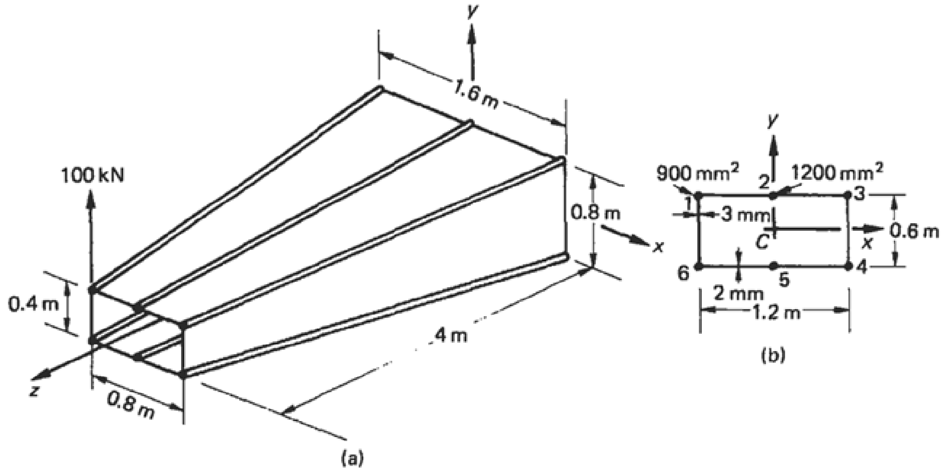


Fig. 10.6 (a) Beam of Example 10.2; (b) section 2 m from built-in end.

booms and the shear flow distribution in the walls at a section 2 m from the built-in end if the booms resist all the direct stresses while the walls are effective only in shear. Each corner boom has a cross-sectional area of 900 mm^2 while both central booms have cross-sectional areas of 1200 mm^2.

Passo 3.2: Cálculo dos Fluxos de Cisalhamento

a. Recalculo das cargas de cisalhamento atuantes apenas nas cascas

$$\begin{cases} S_{x_{cis}} = 0 \text{ kN} \\ S_{y_{cis}} = 66.68 \text{ kN} \\ T_{cis} = -40 \text{ kN.m} \end{cases}$$

b. Fluxo devido a torção em uma seção fechada

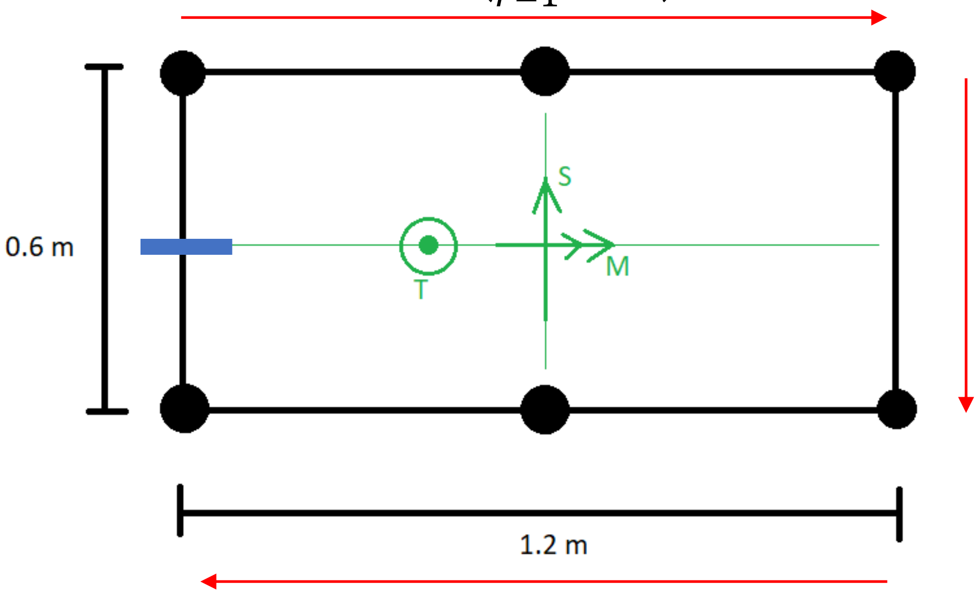
$$q_{0,t} = \frac{T_{cis}}{2A} = -\frac{40}{2 \times 0.6 \times 1.2} = -27{,}78 \text{ N/mm}$$

c. Fluxo devido ao cisalhamento

$$q_s = -\frac{S_{x_{cis}}.I_{xx} - S_{y_{cis}}I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}.\left(\int_0^s t_d x ds + \sum_{r=1}^{n} B_r x_r\right) - \frac{S_{y_{cis}}.I_{yy} - S_{x_{cis}}I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}.\left(\int_0^s t_d y ds + \sum_{r=1}^{n} B_r y_r\right) + q_{s,0}$$

c. Fluxo devido ao cisalhamento

$$q_s = -\frac{S_{y_{cis}}}{I_{xx}} \cdot \left(\sum_{r=1}^{n} B_r y_r \right) + q_{s,0}$$

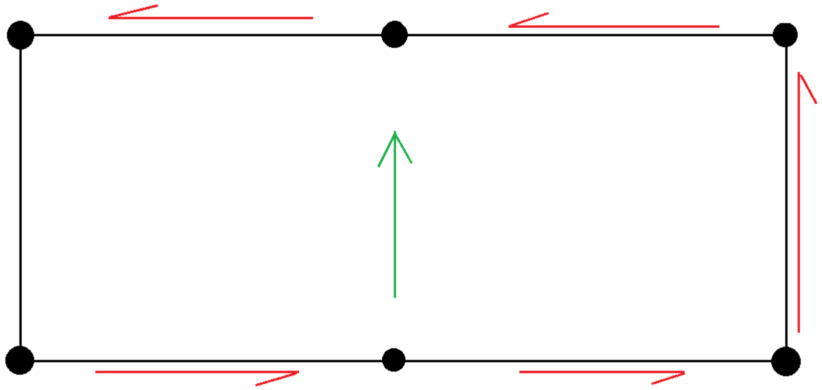


$$\begin{cases} S_{x_{cis}} = 0 \text{ kN} \\ S_{y_{cis}} = 66.68 \text{ kN} \\ T_{cis} = -40 \text{ kN.m} \end{cases}$$

Segmento	$q_b = -\frac{66.68}{540e6} \cdot \left(\sum_{r=1}^{n} B_r y_r \right)$	$q_{s,0}$
$0-1$	0	
$1-2$	-33.34 N/mm	
$2-3$	-77.82 N/mm	
$3-4$	-111.15 N/mm	
$4-5$	-77.82 N/mm	
$5-6$	-33.34 N/mm	
$6-0$	0	

c. Fluxo devido ao cisalhamento

$$q_s = -\frac{S_{y_{cis}}}{I_{xx}} \cdot \left(\sum_{r=1}^{n} B_r y_r\right) + q_{s,0}$$



Segmento	$q_b = -\frac{66.68}{540e6} \cdot \left(\sum_{r=1}^{n} B_r y_r\right)$	$q_{s,0}$
$0-1$	0	
$1-2$	$33.34\ \mathrm{N/mm}$	
$2-3$	$77.82\ \mathrm{N/mm}$	
$3-4$	$111.15\ \mathrm{N/mm}$	
$4-5$	$77.82\ \mathrm{N/mm}$	
$5-6$	$33.34\ \mathrm{N/mm}$	
$6-0$	0	

$$\begin{cases} S_{x_{cis}} = 0\ \mathrm{kN} \\ S_{y_{cis}} = 66.68\ \mathrm{kN} \\ T_{cis} = -40\ \mathrm{kN.m} \end{cases}$$

Cálculo do $q_{s,0}$ (Boom 5)

$$M_{S_{x_{cis}}} + M_{S_{y_{cis}}} = \oint p q_b ds + 2Aq_{s,0} \rightarrow$$

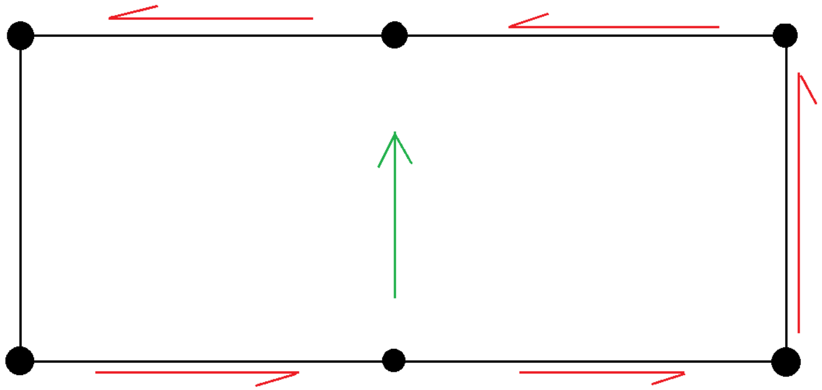
$$0 = 111.15 \times 600 \times 600 + (33.34 + 77.82) \times 600 \times 600 + 2Aq_{s,0} \rightarrow$$

$$0 = 80e4 + 2Aq_{s,0} \rightarrow$$

$$q_{s,0} = -\frac{80e6}{2 \times 600 \times 1200} = -55.6\ \mathrm{N/mm}$$

c. Fluxo devido ao cisalhamento

$$q_s = -\frac{S_{y_{cis}}}{I_{xx}} \cdot \left(\sum_{r=1}^{n} B_r y_r \right) + q_{s,0}$$



$$\begin{cases} S_{x_{cis}} = 0 \text{ kN} \\ S_{y_{cis}} = 66.68 \text{ kN} \\ T_{cis} = -40 \text{ kN.m} \end{cases}$$

Segmento	$q_b = -\frac{66.68}{540e6} \cdot \left(\sum_{r=1}^{n} B_r y_r \right)$	$q_{s,0}$	q_s
$1-2$	33.34 N/mm	-55.6 N/mm	-22.26 N/mm
$2-3$	77.82 N/mm	-55.6 N/mm	22.22 N/mm
$3-4$	111.15 N/mm	-55.6 N/mm	55.6 N/mm
$4-5$	77.82 N/mm	-55.6 N/mm	22.22 N/mm
$5-6$	33.34 N/mm	-55.6 N/mm	-22.26 N/mm
$6-1$	0	-55.6 N/mm	-55.6 N/mm

Passo 4: Resumo de Resultados

y_i(m)	σ_i(MPa)	$P_{z,i}$ (kN)	δ_x/δ_z	δ_y/δ_z	$P_{x,i}$ (kN)	$P_{y,i}$ (kN)	P_i(kN)	$\sigma_{t,i}$(MPa)
0.3	−111.1	−100	0.1	−0.05	−10	5	−100.62	−111.8
0.3	−111.1	−133.33	0	−0.05	0	6.66	−133.5	−111.25
0.3	−111.1	−100	−0.1	−0.05	10	5	−100.62	−111.8
−0.3	111.1	100	−0.1	0.05	−10	5	100.62	111.8
−0.3	111.1	133.33	0	0.05	0	6.66	133.5	111.25
−0.3	111.1	100	0.1	0.05	10	5	100.62	111.25

Segmento	$q_b = -\frac{66.68}{540e6} \cdot \left(\sum_{r=1}^{n} B_r y_r\right)$	$q_{s,0}$(N/mm)	$q_{t,0}$(N/mm)	q_{final} (N/mm)
1 − 2	33.34 N/mm	−55.6	−27,78	−50.04
2 − 3	77.82 N/mm	−55.6	−27,78	−5.56
3 − 4	111.15 N/mm	−55.6	−27,78	27.82
4 − 5	77.82 N/mm	−55.6	−27,78	−5.56
5 − 6	33.34 N/mm	−55.6	−27,78	−50.04
6 − 1	0	−55.6	−27,78	−83.38