

## Contributions of a Venus Swing-by Maneuver in Earth-Mars Transfers

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# Introduction

## Our Goal:

To investigate *the effect of Jupiter* gravitational field in *Earth-Mars transfers with a Venus Swing-by*

## Motivation:

To get trajectories for *exploration and colonization of Mars* with distinct profiles aiming different purposes

## Challenge:

*interplanetary transfer design involves many-body effects, primaries orbit geometry, elevated cost, suitable time window for departure and return, transfer time, & others*

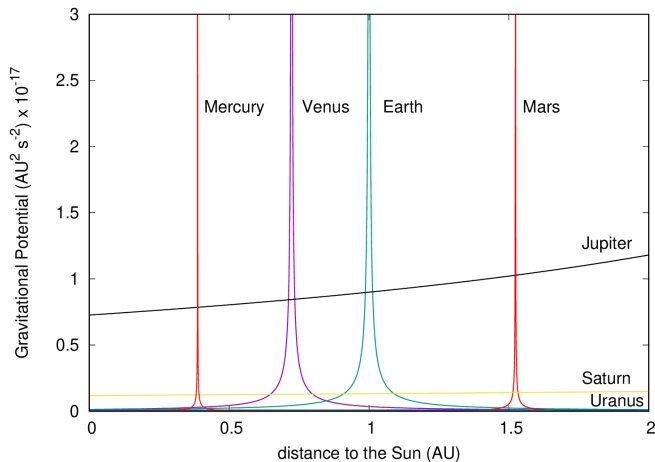
## Our Approach:

To build both *Direct Earth-Mars transfers* and *Earth-Mars transfers with a Venus Flyby* in the context of the *Sun-Jupiter System of the PCR3BP*.

# Introduction

## Why to consider the Sun-Jupiter system in Earth-Mars transfers?

*Inspecting the gravitational potential in the interplanetary region*

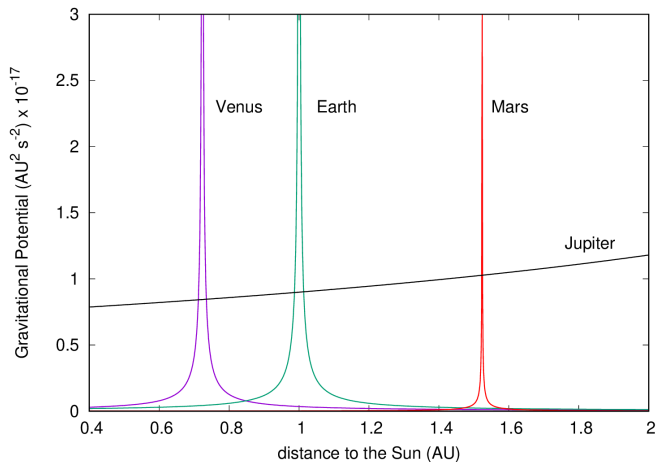


Inspired by Ren, Masdemont, Gomez, Fantino, CNSNS **17** (2012) 844.

# Introduction

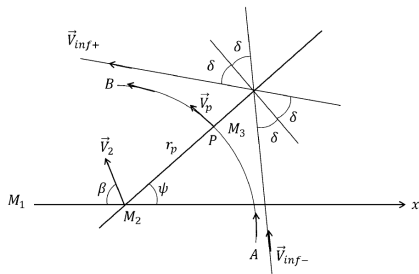
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## Earth Mars transfer with a Flyby Maneuver by Venus



Variation of SC velocity wrt the Sun:

$$\Delta \vec{V} \equiv \vec{V}_o - \vec{V}_i,$$

$$\Delta V = |\Delta \vec{V}| = 2V_{inf} \sin \delta$$

with  $V_{inf}$  defined wrt Venus

$$|\vec{V}_{inf+}| = |\vec{V}_{inf-}| = V_{inf}.$$

$$\sin \delta = \frac{1}{1 + \frac{r_p V_{inf}}{\mu_{venus}}}$$

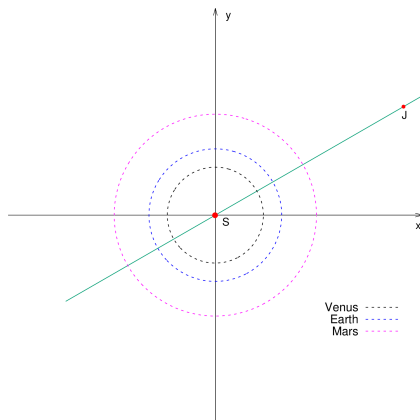
Periapsis altitude  $r_p$  as control parameter

Besides the opportunity to observe Venus, the flyby maneuver also is exploited to change the velocity of the spacecraft w.r.t the Sun at the input and output of the SOI of Venus.

# Mathematical Approach

## Assumptions of our preliminary analysis

1. Planets in coplanar circular orbits
2. Dynamics of the SC defined by Sun-Jupiter grav.potential
3. Gravitational effects of Earth and Mars compensate each other
4. Dynamics of the interplanetary flight given by R3BP

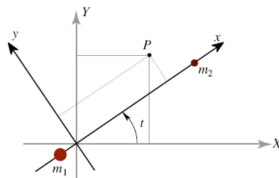


## Three reference frames:

1. Heliocentric inertial frame  
*Initial conditions*
2. Barycentric rotating frame  
*Interplanetary Flight*
3. Inertial frame centered at Venus  
*Inside the Sphere of Influence of Venus*

# The Restricted Three-body Problem: *Planar Version*

- 2 D.O.F. Hamiltonian system
- Particle  $P_3$  of negligible mass moving under the gravitational influence of  $P_1$  and  $P_2$  of masses  $m_1$  and  $m_2$ .
- The primaries describe circular coplanar orbits around the barycenter of  $P_1$ - $P_2$  and are fixed in the synodic reference frame (which rotates w.r.t. an inertial frame).
- Distance between the primaries, the sum of their masses and their angular velocity around the barycenter are normalized to one.
- $\mu = m_2/(m_1 + m_2)$ ,  $m_1 \geq m_2$  is the only parameter.



## Equations of Motion:

$$\ddot{x} - 2\dot{y} = \Omega_x, \quad \ddot{y} + 2\dot{x} = \Omega_y,$$
$$\Omega(x, y) = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{\mu(1-\mu)}{2},$$
$$r_1 = \sqrt{(x - \mu)^2 + y^2}, \quad r_2 = \sqrt{(x + 1 - \mu)^2 + y^2}$$

**Integral of motion:**  $J(x, y, \dot{x}, \dot{y}) = 2\Omega(x, y) - (\dot{x}^2 + \dot{y}^2) = C$ ,

3D manifold  $\mathcal{M}(\mu, C) = \{(x, y, \dot{x}, \dot{y}) \in \mathbb{R}^4 | J(x, y, \dot{x}, \dot{y}) = \text{const.}\}$

# Equilibrium points

## Hill regions:

- Projection of  $\mathcal{M}$  onto configuration space
- Areas accessible to trajectories for each  $C$
- Bounded by the Zero Velocity Surfaces

## Symmetries:

- $(x, y, \dot{x}, \dot{y}, t) \leftrightarrow (x, -y, -\dot{x}, \dot{y}, -t)$
- $(x, y, \dot{x}, \dot{y}, t) \leftrightarrow (x, y, \dot{x}, \dot{y}, t)$

## Collinear points: $L_{1,2,3}$ , on the $x$ -axis

- center-saddle
- 2D central manifold: horizontal Lyapunov orbits, invariant tori, other periodic orbits, chaotic regions.

## Triangular points: $L_{4,5}$ , at $x = \mu - \frac{1}{2}, y = \mp \frac{\sqrt{3}}{2}$

- Nonlinear stability (on a ball of small  $\mu$ -dependent radius around  $L_{4,5}$ ) for  $\mu \in (0, \mu_1) \setminus \{\mu_2, \mu_3\}$ ,  $\mu_1 = \frac{(9-\sqrt{69})}{18}$ ,  $\mu_2 = \frac{(45-\sqrt{1833})}{90}$ ,  $\mu_3 = \frac{(15-\sqrt{213})}{30}$ , except a set of initial conditions of small Lebesgue measure for fixed  $\mu$ .

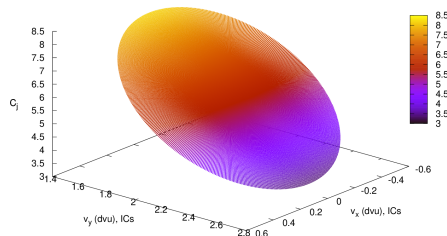


# Initial Conditions and Parameters of the Problem

## Five Parameters:

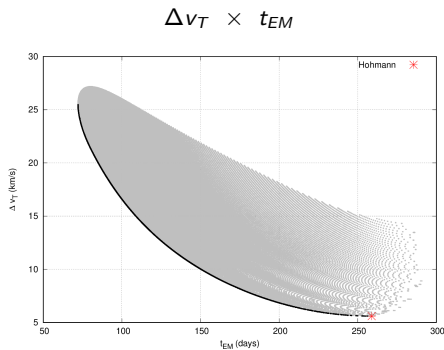
- At Earth departure: (4)
  - angle  $\alpha_{SJ}$   
(btw. Sun-Jupiter axis & x-axis of HIRF)
  - angle  $\theta$   
(Earth cir. orbit around the Sun,)
  - $\Delta v_E$  and  $\xi$ .  
(magnitude & phase of vel. increment)
- At Venus Swingby: (1)
  - $r_p$ , (due to maneuvers of negligible costs)  
(the altitude of the periapsis wrt Venus)

Initial Conditions for  $\alpha_{SJ} = \theta = 0$ :



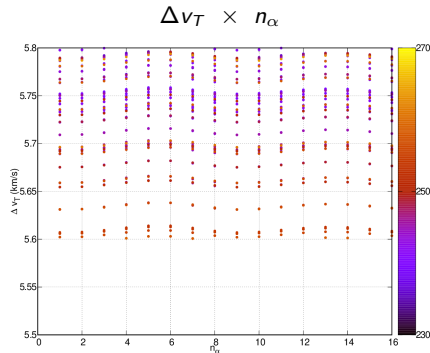
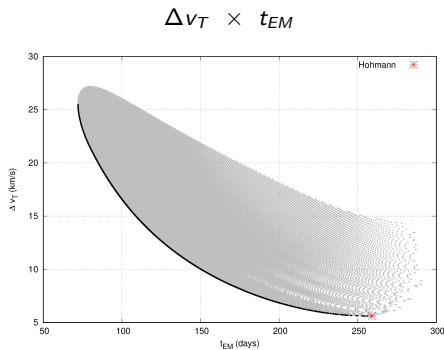
# Direct Earth-Mars transfers

16 equispaced values of  $\alpha_{SJ} = (n_\alpha - 1) \pi/8$  for  $\theta = 0$



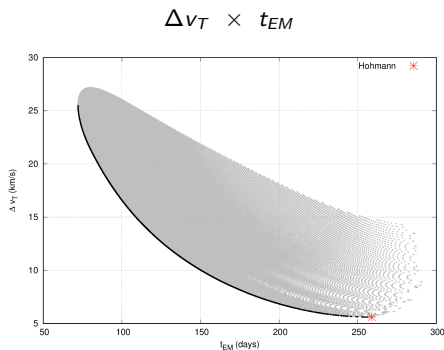
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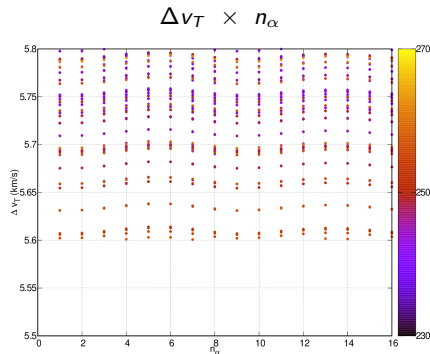


# Direct Earth-Mars transfers

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Gray dots: all solutions  
Black dots:  $\Delta v_T$  optimal solutions



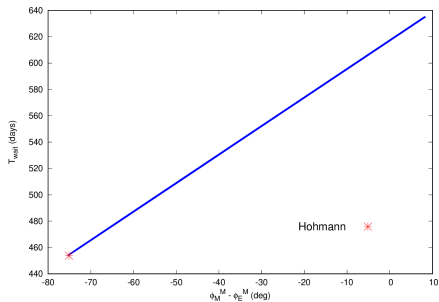
Color code:  $t_{EM}$

*Very tiny effect of Jupiter!!*

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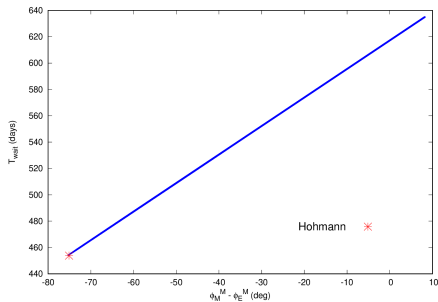
$$t_{wait} \propto \phi_M^M - \phi_E^M$$



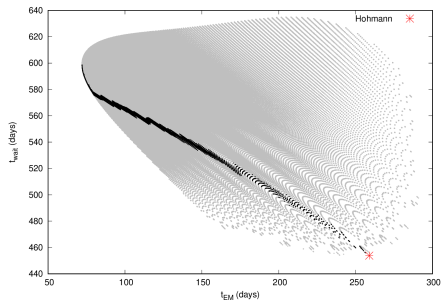
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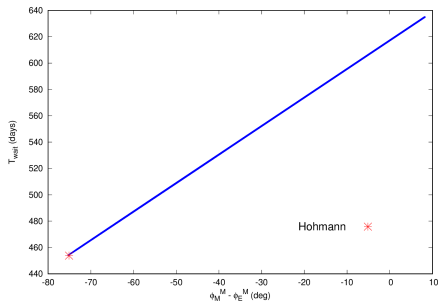
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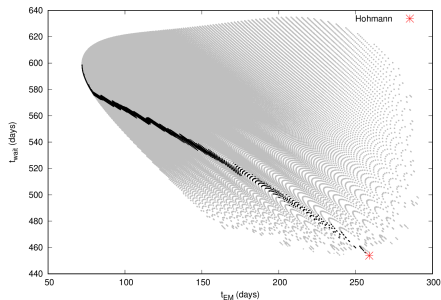
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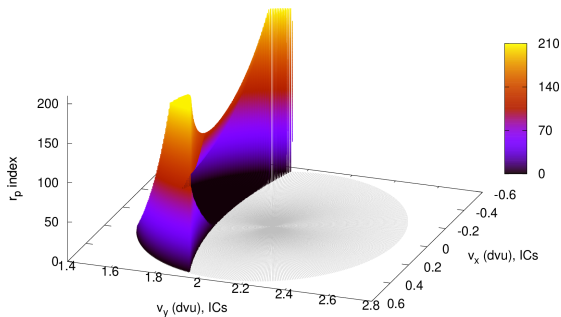


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*Long waiting time for a Earth return by Hohmann transfer!*

Inspecting Venus flyby process at first (for  $\alpha_{SJ} = \theta = 0$ )



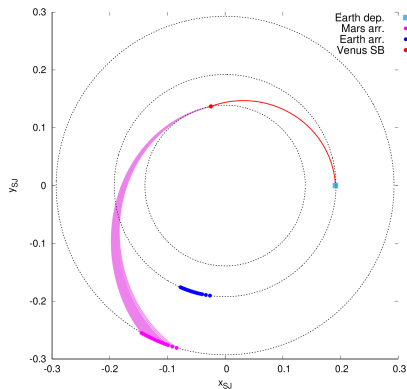
Venus pericenter altitude  $\times$  initial velocities



# Earth-Mars transfers with Venus Flyby

Inspecting Venus flyby process at first (for  $\alpha_{SJ} = \theta = 0$ )

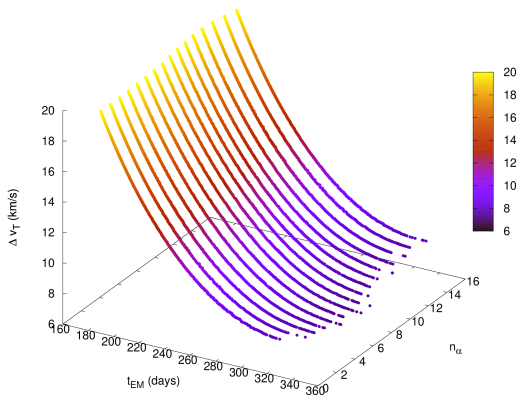
Illustration of EM transfers generated by one IC



# Earth-Mars transfers with Venus Flyby

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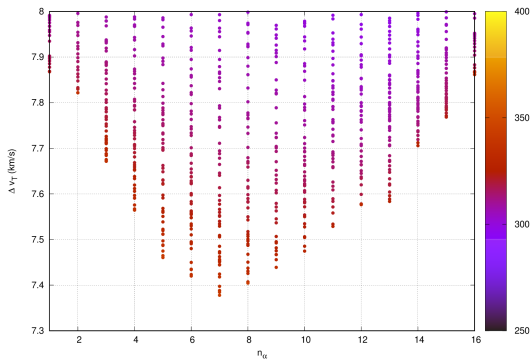
Pareto Optimal Solutions: minimum  $\Delta v_T$  as a function of  $t_{EM}$ , for each  $\alpha_{SJ}$



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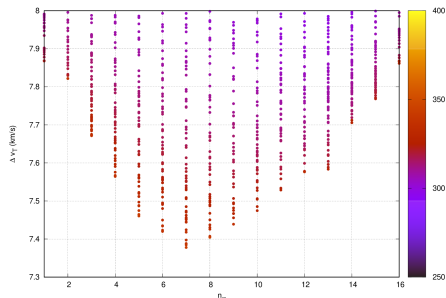
Optimal solutions: minimum  $\Delta v_T$  versus  $\alpha_{SJ}$  (i.e.,  $n_\alpha$ ) (colors depicts  $t_{EM}$ )



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$\Delta v_T$	$\alpha_{SJ}$
7.378 km/s	$3\pi/4$ ( $n_\alpha = 7$ )
7.868 km/s	0 ( $n_\alpha = 1$ )

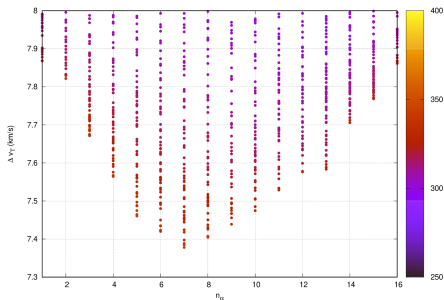
A variation of 0.490 km/s.

A save of 6.2% is possible

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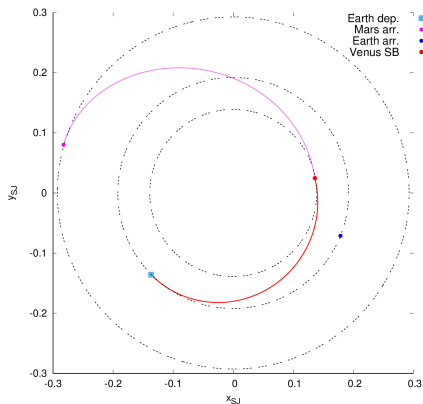
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*Jupiter grav. field plays an important role for transfers with Venus flyby*

# Earth-Mars transfers with Venus Flyby

Optimal transfer ( $\alpha_{SJ} = 3\pi/4$ )



*Sun-Jupiter Rotating Frame*

$$t_{EV} = 132 \text{ days}$$

$$t_{EM} = 343 \text{ days}$$

$$\Delta v_E = 2.60 \text{ km/s}$$

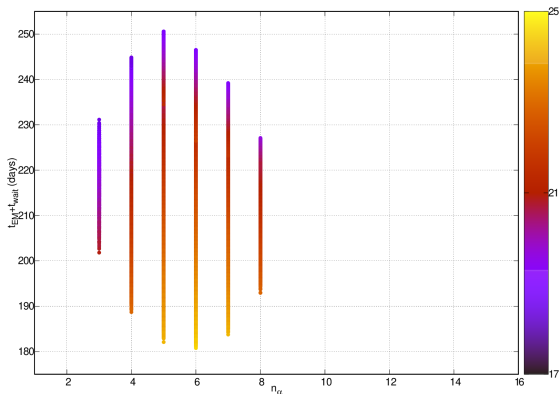
$$\Delta v_M = 4.478 \text{ km/s}$$

$$\text{Swing-by } \Delta v = 4.83 \text{ km/s}$$

$$t_{wait} = 594 \text{ days}$$

# Earth-Mars transfers with Venus Flyby

Non-optimal solutions with low waiting time: from 0 to 24 days  
with higher cost (color depicts  $\Delta v_T$ )



# Conclusions and Future Works

- Earth-Mars transfers with Venus swingby were obtained and *the role played by Jupiter* investigated.

- Possible applications:

- the judicious choice of the relative phase of Earth and Jupiter at the departure can lead to a significant fuel save.

*For the simplified model we employed and before a full optimization procedure, we found a save of 6.2 %.*

- combination of  $\alpha_{SJ}$  and  $r_p$  may leads to solutions with short waiting time for low cost Earth return.



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- combination of  $\alpha_{SJ}$  and  $r_p$  may leads to solutions with short waiting time for low cost Earth return.
- Further work: Inclusion of a multi-objective optimization strategy to allow the inclusion of more realistic geatures to the dynamical model, such as, the 3D orbits of Mars and Earth.

Besides that, an impulsive thrust must be included at different points in the swing-by by Venus aiming to decrease the value of  $\Delta v_M$  or even to enable future purposes after Mars passage.

**Thanks for your attention!**



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