Oscillation

Periodic Motion:
(i) The motion of earth around the sun and motion of moon around the earth
(ii) Motion of the pendulum of a clock
(iii) Motion of piston in the cylinder of an engine.

We find the same motion along the same path is repeated again and again after equal intervals of time - called periodic motion.

Motion in which the body describes the same path in the same way again and again in equal intervals of time - periodic motion

Circular periodic motion - If the body moves in a circular path.

Linear periodic motion - If the body repeatedly moves along the line.

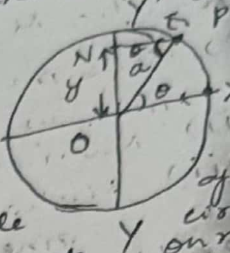
(SHM) Simple Harmonic Motion: It is a particular and fundamental type of periodic motion having single period. A SHM can be (a) linear and (b) angular depending upon the path described.

Simple harmonic motion can be defined as a motion in which the acceleration of the body is directly proportional to its displacement from a fixed point and is always directed towards the fixed point.

Characteristics of SHM:
(i) The motion should be periodic
(ii) when displaced from mean position, a restoring force, tending to bring it to the mean position & directed towards the mean position must act on the body.
(iii) The restoring force ∝ displacement from the mean position

SHM can also be defined as the projection of a uniform circular motion on a diameter of the circle (or) any other line in the plane of the O$^{le}$.

Consider a particle moving along a circle of radius 'a' with a constant speed 'v' as in fig (1). Draw a perpendicular from point P on YY' and let N be the projection of P. The circle is known as reference circle and the particle P is known as reference particle.




Projection of a uniform circular motion on reference circle.

When the particle is at X, the projection is at 'O'. When particle moves from X to Y' the projection moves from O to Y'. When particle reaches X' the projection comes back from Y' to O. When the particle reaches Y and when particle comes to X, the projection also reaches Y and the projection comes back to O. Thus as the particle moves along the reference circle, the projection moves along the diameter YY'. If particle move as the projection moves under the influence of some force, the particle is said to perform a linear SHM along YOY'.

## Characteristics of SHM:

(i) Displacement: It is the distance of the particle measured along the path of the motion from its mean position, at a given instant. In fig(i), the displacement of the particle N = y

$$ON = OP \sin\theta \quad — (1)$$

$$y = a \sin\theta$$

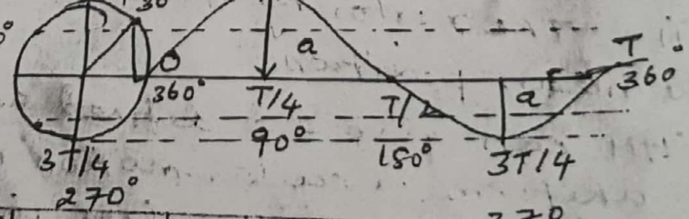
If angular velocity 'ω' describes this displacement in 't' second, we have

$$\frac{\theta}{t} = \omega \quad (or) \quad \theta = \omega t \quad — (2)$$

Sub (2) in (1) $\boxed{y = A \sin\omega t}$ — (3)

The position of the particle N at any instant can be obtained with the help of displacement curve. A displacement curve is a graph b/w time that elapses since the particle was at mean position O. Table 1 shows the displacement with time.

$T \to$ period of vibration

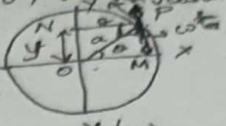



| Angle | 0 | 90° | 180° | 270° | 360° |
|---|---|---|---|---|---|
| Time | 0 | T/4 | T/2 | 3T/4 | T |
| Displacement | 0 | a max | 0 | −a max | 0 |

Table 1

(2) Amplitude: The maximum distance covered by the body on either side of the mean position.

(3) Velocity: Velocity of particle at N = component of the velocity of particle P along YOY'. If V is the velocity of reference particle:



Velocity of N = $v \cos \omega t$

Velocity of N = $a\omega \cos \omega t$ $(\because v = a\omega)$

velocity of N

$= a\omega \sqrt{(1-\sin^2 \omega t)}$ $\left(\because \sin \omega t = \frac{ON}{OP} = \frac{y}{a}\right)$

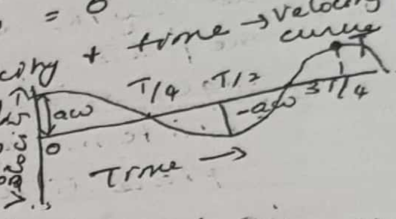
$= a\omega \sqrt{\left\{1 - \left(\frac{OM}{OP}\right)^2\right\}}$.

$= a\omega \sqrt{\left(1 - \frac{y^2}{a^2}\right)} = a\omega \sqrt{\left(\frac{a^2-y^2}{a^2}\right)}$

Hence $\boxed{\text{Velocity of } N = \omega\sqrt{a^2-y^2}}$ — (4)

In eqn (4) velocity of N depends on displacement only. when $y = 0$, velocity of N = $a\omega$ (maxm)
" " " = 0 (minm)
$y = a$)

Graph b/w instantaneous velocity + time → velocity curve



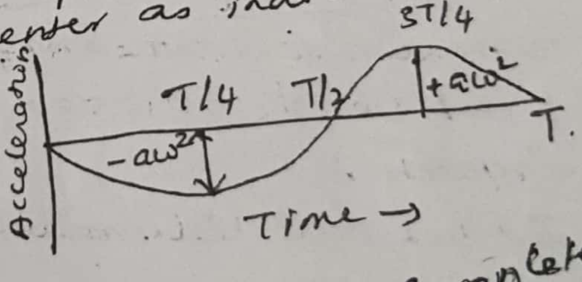
From the curve it is understood the velocity of the particle is maxm when it passes thro' mean position and = 0 when displacement is equal to amplitude.

3. Acceleration: Acceleration of a particle executing SHM = component of the acceln of the reference particle along YOY'. Since the particle P is moving with a velocity v in a circle of radius a, its acceleration will be $v^2/a$ (or) $\omega^2 a$. directed towards the center of the O$^{le}$. Resolving the acceleration along PN and PM. The component along NO will be $\omega^2 a \sin\theta$. Thus acceleration of N = $-\omega^2 a \sin\theta$.

(or) $\boxed{\text{Acceleration of } N = -\omega^2 y}$ — (5)

Acceleration of the particle $\propto y$ and always directed towards the center as indicated by '-' sign.

Acceleration curve.



From eqn (5) acceln is maxm. at the end pts when $y = a$ and minm when $y = 0$

4. Time period: The time required to complete one vibration and is denoted by 'T'. The time period of particle P = time taken by the particle to move from

O to y, y to y' & y' to O

Angular velocity $\omega$ = $\frac{\text{Angle described in one revolution}}{\text{Time taken to complete one revolution}}$

$$\boxed{\omega = \frac{2\pi}{T} \text{ (or) } T = \frac{2\pi}{\omega}} \quad \text{--- (6)}$$

$$T = \frac{2\pi}{\sqrt{\text{Accn}/y}} \text{ (or) } T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Accn.}}} \quad \text{--- (7)}$$

5) **Frequency**: The number of vibrations made by the particle in one second.

$$\boxed{n = \frac{1}{T}} \quad \text{--- (8)}$$

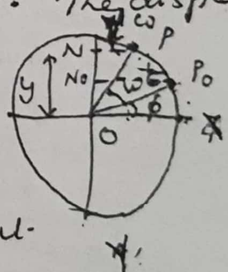
6) **Phase and phase difference**: The phase of a vibrating body gives us an idea of the position of the body at any instant of time.

The phase of particle is measured either interms of angle described by the reference particle as well as in terms of fraction of the time period

Phase difference between two SHMs indicates how much the two motions are out of step with each other (or) how much time one is ahead of the other.

From fig (1), let a point P start its motion from x axis at $t=0$ and trace an angle $\theta$ in time 't' with angular velocity $\omega$'. The displacement

$y = ON = a\sin\theta = a\sin\omega t$




Time taken by the particle to move from P to Po, where angle $P_0OB = \phi$. Now the displacement becomes $y = a\sin(\omega t + \phi)$

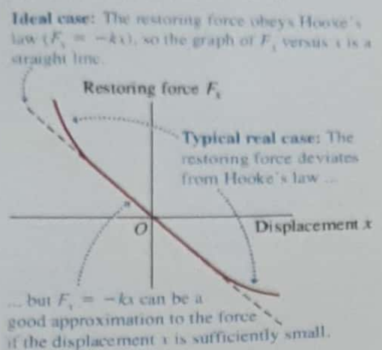
where $\omega t$ is the angle traced in time 't' from the position $P_0$. At $t=0$, $\phi$ is called as initial phase (or) 'epoch'.

If the particle makes $\phi$ below x axis, then the displacement $y = a\sin(\omega t - \phi)$

Hence the general equation of SHM, displacement

$$y = a\sin(\omega t \pm \phi)$$

**14.4** In most real oscillations Hooke's law applies provided the body doesn't move too far from equilibrium. In such a case small-amplitude oscillations are approximately simple harmonic.




Why is simple harmonic motion important? Keep in mind that not all periodic motions are simple harmonic; in periodic motion in general, the restoring force depends on displacement in a more complicated way than in Eq. (14.3). But in many systems the restoring force is *approximately* proportional to displacement if the displacement is sufficiently small (Fig. 14.4). That is, if the amplitude is small enough, the oscillations of such systems are approximately simple harmonic and therefore approximately described by Eq. (14.4). Thus we can use SHM as an approximate model for many different periodic motions, such as the vibration of the quartz crystal in a watch, the motion of a tuning fork, the electric current in an alternating-current circuit, and the oscillations of atoms in molecules and solids.

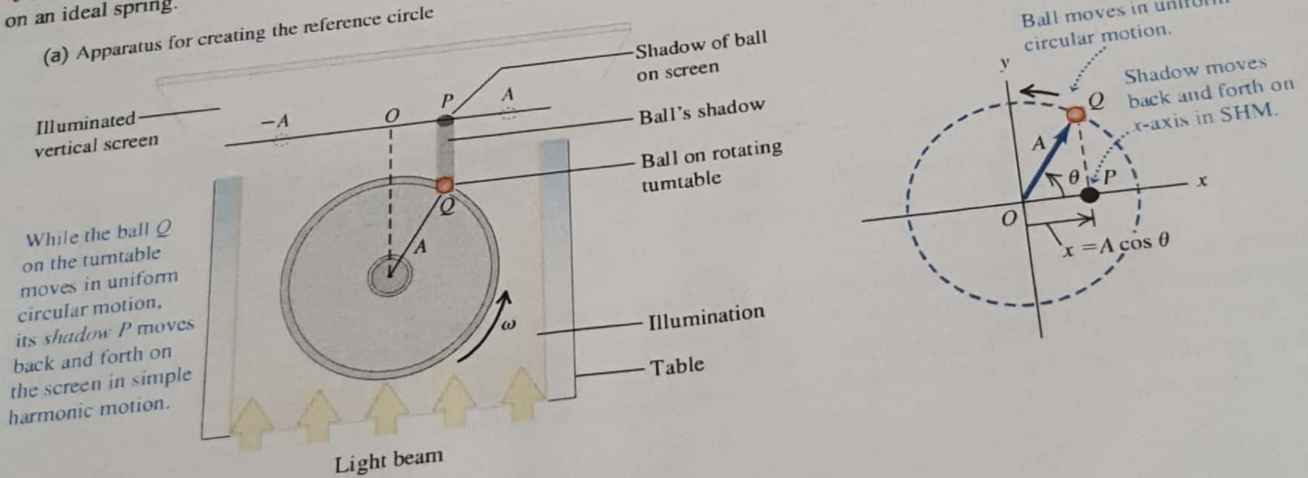
## Circular Motion and the Equations of SHM

To explore the properties of simple harmonic motion, we must express the displacement $x$ of the oscillating body as a function of time, $x(t)$. The second derivative of this function, $d^2x/dt^2$, must be equal to $(-k/m)$ times the function itself, as required by Eq. (14.4). As we mentioned, the formulas for constant acceleration from Section 2.4 are no help because the acceleration changes constantly as the displacement $x$ changes. Instead, we'll find $x(t)$ by noticing a striking similarity between SHM and another form of motion that we've already studied.

Figure 14.5a shows a top view of a horizontal disk of radius $A$ with a ball attached to its rim at point $Q$. The disk rotates with constant angular speed $\omega$ (measured in rad/s), so the ball moves in uniform circular motion. A horizontal light beam shines on the rotating disk and casts a shadow of the ball on a screen. The shadow at point $P$ oscillates back and forth as the ball moves in a circle. We then arrange a body attached to an ideal spring, like the combination shown in Figs. 14.1 and 14.2, so that the body oscillates parallel to the shadow. We will prove that the motion of the body and the motion of the ball's shadow are *identical* if the amplitude of the body's oscillation is equal to the disk radius $A$, and if the angular frequency $2\pi f$ of the oscillating body is equal to the angular speed $\omega$ of the rotating disk. That is, *simple harmonic motion is the projection of uniform circular motion onto a diameter.*

We can verify this remarkable statement by finding the acceleration of the shadow at $P$ and comparing it to the acceleration of a body undergoing SHM, given by Eq. (14.4). The circle in which the ball moves so that its projection matches the motion of the oscillating body is called the **reference circle;** we will call the point $Q$ the *reference point.* We take the reference circle to lie in the

**14.5** (a) Relating uniform circular motion and simple harmonic motion. (b) The ball's shadow moves exactly like a body oscillating on an ideal spring.



xy-plane, with the origin $O$ at the center of the circle (Fig. 14.5b). At time $t$ the vector $OQ$ from the origin to the reference point $Q$ makes an angle $\theta$ with the positive $x$-axis. As the point $Q$ moves around the reference circle with constant angular speed $\omega$, the vector $OQ$ rotates with the same angular speed. Such a rotating vector is called a **phasor.** (This term was in use long before the invention of the Star Trek stun gun with a similar name. The phasor method for analyzing oscillations is useful in many areas of physics. We'll use phasors when we study alternating-current circuits in Chapter 31 and the interference of light in Chapters 35 and 36.)

The $x$-component of the phasor at time $t$ is just the $x$-coordinate of the point $Q$:

$$x = A\cos\theta \tag{14.5}$$

This is also the $x$-coordinate of the shadow $P$, which is the *projection* of $Q$ onto the $x$-axis. Hence the $x$-velocity of the shadow $P$ along the $x$-axis is equal to the $x$-component of the velocity vector of point $Q$ (Fig. 14.6a), and the $x$-acceleration of $P$ is equal to the $x$-component of the acceleration vector of $Q$ (Fig. 14.6b). Since point $Q$ is in uniform circular motion, its acceleration vector $\vec{a}_Q$ is always directed toward $O$. Furthermore, the magnitude of $\vec{a}_Q$ is constant and given by the angular speed squared times the radius of the circle (see Section 9.3):

$$a_Q = \omega^2 A \tag{14.6}$$

Figure 14.6b shows that the $x$-component of $\vec{a}_Q$ is $a_x = -a_Q\cos\theta$. Combining this with Eqs. (14.5) and (14.6), we get that the acceleration of point $P$ is

$$a_x = -a_Q\cos\theta = -\omega^2 A\cos\theta \quad \text{or} \tag{14.7}$$

$$a_x = -\omega^2 x \tag{14.8}$$

The acceleration of point $P$ is directly proportional to the displacement $x$ and always has the opposite sign. These are precisely the hallmarks of simple harmonic motion.

Equation (14.8) is *exactly* the same as Eq. (14.4) for the acceleration of a harmonic oscillator, provided that the angular speed $\omega$ of the reference point $Q$ is related to the force constant $k$ and mass $m$ of the oscillating body by

$$\omega^2 = \frac{k}{m} \quad \text{or} \quad \omega = \sqrt{\frac{k}{m}} \tag{14.9}$$

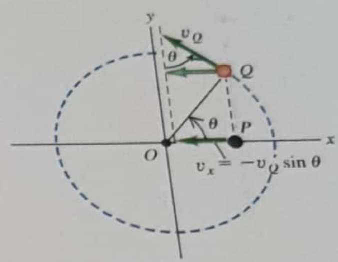
We have been using the same symbol $\omega$ for the angular *speed* of the reference point $Q$ and the angular *frequency* of the oscillating point $P$. The reason is that these quantities are equal! If point $Q$ makes one complete revolution in time $T$, then point $P$ goes through one complete cycle of oscillation in the same time; hence $T$ is the period of the oscillation. During time $T$ the point $Q$ moves through $2\pi$ radians, so its angular speed is $\omega = 2\pi/T$. But this is just the same as Eq. (14.2) for the angular frequency of the point $P$, which verifies our statement about the two interpretations of $\omega$. This is why we introduced angular frequency in Section 14.1; this quantity makes the connection between oscillation and circular motion. So we reinterpret Eq. (14.9) as an expression for the angular frequency of simple harmonic motion for a body of mass $m$, acted on by a restoring force with force constant $k$:

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(simple harmonic motion)} \tag{14.10}$$

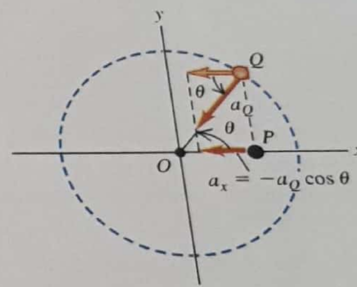
When you start a body oscillating in SHM, the value of $\omega$ is not yours to choose; it is predetermined by the values of $k$ and $m$. The units of $k$ are N/m or kg/s$^2$, so $k/m$ is in $(\text{kg/s}^2)/\text{kg} = \text{s}^{-2}$. When we take the square root in Eq. (14.10), we get s$^{-1}$, or more properly rad/s because this is an *angular* frequency (recall that a radian is not a true unit).

**14.6** The (a) $x$-velocity and (b) $x$-acceleration of the ball's shadow $P$ (see Fig. 14.5) are the $x$-components of the velocity and acceleration vectors, respectively, of the ball $Q$.

(a) Using the reference circle to determine the $x$-velocity of point $P$




(b) Using the reference circle to determine the $x$-acceleration of point $P$



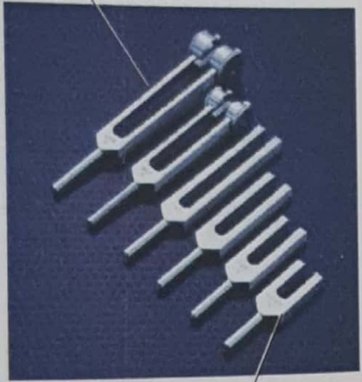
According to Eqs. (14.1) and (14.2), the frequency $f$ and period $T$ are

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{(simple harmonic motion)} \tag{14.11}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad \text{(simple harmonic motion)} \tag{14.12}$$

**14.7** The greater the mass $m$ in a tuning fork's tines, the lower the frequency of oscillation $f = (1/2\pi)\sqrt{k/m}$ and the lower the pitch of the sound that the tuning fork produces.

Tines with large mass $m$: low frequency $f = 128$ Hz



Tines with small mass $m$: high frequency $f = 4096$ Hz

We see from Eq. (14.12) that a larger mass $m$, with its greater inertia, will have less acceleration, move more slowly, and take a longer time for a complete cycle (Fig. 14.7). In contrast, a stiffer spring (one with a larger force constant $k$) exerts a greater force at a given deformation $x$, causing greater acceleration, higher speeds, and a shorter time $T$ per cycle.

**CAUTION** **Don't confuse frequency and angular frequency** You can run into trouble if you don't make the distinction between frequency $f$ and angular frequency $\omega = 2\pi f$. Frequency tells you how many cycles of oscillation occur per second, while angular frequency tells you how many radians per second this corresponds to on the reference circle. In solving problems, pay careful attention to whether the goal is to find $f$ or $\omega$.

## Period and Amplitude in SHM

Equations (14.11) and (14.12) show that the period and frequency of simple harmonic motion are completely determined by the mass $m$ and the force constant $k$. *In simple harmonic motion the period and frequency do not depend on the amplitude A.* For given values of $m$ and $k$, the time of one complete oscillation is the same whether the amplitude is large or small. Equation (14.3) shows why we should expect this. Larger $A$ means that the body reaches larger values of $|x|$ and is subjected to larger restoring forces. This increases the average speed of the body over a complete cycle; this exactly compensates for having to travel a larger distance, so the same total time is involved.

The oscillations of a tuning fork are essentially simple harmonic motion, which means that it always vibrates with the same frequency, independent of amplitude. This is why a tuning fork can be used as a standard for musical pitch. If it were not for this characteristic of simple harmonic motion, it would be impossible to make familiar types of mechanical and electronic clocks run accurately or to play most musical instruments in tune. If you encounter an oscillating body with a period that *does* depend on the amplitude, the oscillation is *not* simple harmonic motion.

### Example 14.2 Angular frequency, frequency, and period in SHM

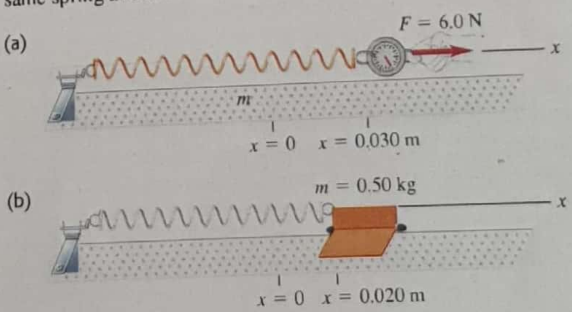
A spring is mounted horizontally, with its left end fixed. A spring balance attached to the free end and pulled toward the right (Fig. 14.8a) indicates that the stretching force is proportional to the displacement, and a force of 6.0 N causes a displacement of 0.030 m. We replace the spring balance with a 0.50-kg glider, pull it 0.020 m to the right along a frictionless air track, and release it from rest (Fig. 14.8b). (a) Find the force constant $k$ of the spring. (b) Find the angular frequency $\omega$, frequency $f$, and period $T$ of the resulting oscillation.

F = -kx

**SOLUTION**

**IDENTIFY and SET UP:** Because the spring force (equal in magnitude to the stretching force) is proportional to the displacement, the motion is simple harmonic. We find $k$ using Hooke's law, Eq. (14.3), and $\omega$, $f$, and $T$ using Eqs. (14.10), (14.11), and (14.12), respectively.

**14.8** (a) The force exerted *on* the spring (shown by the vector $\vec{F}$) has $x$-component $F_x = +6.0$ N. The force exerted *by* the spring has $x$-component $F_x = -6.0$ N. (b) A glider is attached to the same spring and allowed to oscillate.




F = -kx
6 = -k × 0.030
-k = 6/0.030
k = +200
ω = √(k/m) = ... 0.50

**EXECUTE:** (a) When $x = 0.030$ m, the force the spring exerts on the spring balance is $F_x = -6.0$ N. From Eq. (14.3),

$$k = -\frac{F_x}{x} = -\frac{-6.0\text{ N}}{0.030\text{ m}} = 200\text{ N/m} = 200\text{ kg/s}^2$$

(b) From Eq. (14.10), with $m = 0.50$ kg.

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200\text{ kg/s}^2}{0.50\text{ kg}}} = 20\text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{20\text{ rad/s}}{2\pi\text{ rad/cycle}} = 3.2\text{ cycle/s} = 3.2\text{ Hz}$$

$$T = \frac{1}{f} = \frac{1}{3.2\text{ cycle/s}} = 0.31\text{ s}$$

**EVALUATE:** The amplitude of the oscillation is 0.020 m, the distance that we pulled the glider before releasing it. In SHM the angular frequency, frequency, and period are all independent of the amplitude. Note that a period is usually stated in "seconds" rather than "seconds per cycle."

## Displacement, Velocity, and Acceleration in SHM

We still need to find the displacement $x$ as a function of time for a harmonic oscillator. Equation (14.4) for a body in simple harmonic motion along the $x$-axis is identical to Eq. (14.8) for the $x$-coordinate of the reference point in uniform circular motion with constant angular speed $\omega = \sqrt{k/m}$. Hence Eq. (14.5), $x = A\cos\theta$, describes the $x$-coordinate for both of these situations. If at $t = 0$ the phasor $OQ$ makes an angle $\phi$ (the Greek letter phi) with the positive $x$-axis, then at any later time $t$ this angle is $\theta = \omega t + \phi$. We substitute this into Eq. (14.5) to obtain

$$x = A\cos(\omega t + \phi) \quad \text{(displacement in SHM)} \tag{14.13}$$

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where $\omega = \sqrt{k/m}$. Figure 14.9 shows a graph of Eq. (14.13) for the particular case $\phi = 0$. The displacement $x$ is a periodic function of time, as expected for SHM. We could also have written Eq. (14.13) in terms of a sine function rather than a cosine by using the identity $\cos\alpha = \sin(\alpha + \pi/2)$. *In simple harmonic motion the position is a periodic, sinusoidal function of time.* There are many other periodic functions, but none so simple as a sine or cosine function.

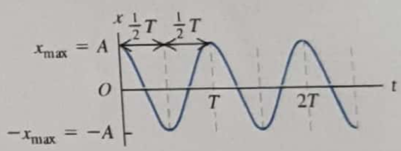
The value of the cosine function is always between $-1$ and 1, so in Eq. (14.13), $x$ is always between $-A$ and $A$. This confirms that $A$ is the amplitude of the motion.

The period $T$ is the time for one complete cycle of oscillation, as Fig. 14.9 shows. The cosine function repeats itself whenever the quantity in parentheses in Eq. (14.13) increases by $2\pi$ radians. Thus, if we start at time $t = 0$, the time $T$ to complete one cycle is given by

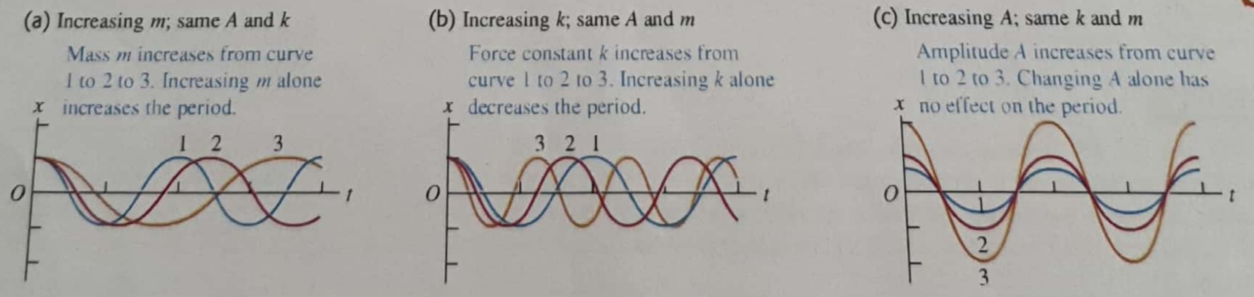
$$\omega T = \sqrt{\frac{k}{m}}T = 2\pi \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

which is just Eq. (14.12). Changing either $m$ or $k$ changes the period of oscillation, as shown in Figs. 14.10a and 14.10b. The period does not depend on the amplitude $A$ (Fig. 14.10c).

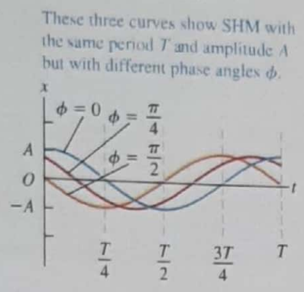
**14.9** Graph of $x$ versus $t$ [see Eq. (14.13)] for simple harmonic motion. The case shown has $\phi = 0$.




**14.10** Variations of simple harmonic motion. All cases shown have $\phi = 0$ [see Eq. (14.13)].



**14.11** Variations of SHM: displacement versus time for the same harmonic oscillator with different phase angles $\phi$.




The constant $\phi$ in Eq. (14.13) is called the **phase angle.** It tells us at what point in the cycle the motion was at $t = 0$ (equivalent to where around the circle the point $Q$ was at $t = 0$). We denote the position at $t = 0$ by $x_0$. Putting $t = 0$ and $x = x_0$ in Eq. (14.13), we get

$$x_0 = A\cos\phi \tag{14.14}$$

If $\phi = 0$, then $x_0 = A\cos 0 = A$, and the body starts at its maximum positive displacement. If $\phi = \pi$, then $x_0 = A\cos\pi = -A$, and the particle starts at its maximum *negative* displacement. If $\phi = \pi/2$, then $x_0 = A\cos(\pi/2) = 0$, and the particle is initially at the origin. Figure 14.11 shows the displacement $x$ versus time for three different phase angles.

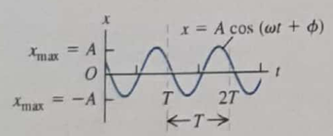
We find the velocity $v_x$ and acceleration $a_x$ as functions of time for a harmonic oscillator by taking derivatives of Eq. (14.13) with respect to time:

$$v_x = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi) \qquad \text{(velocity in SHM)} \tag{14.15}$$

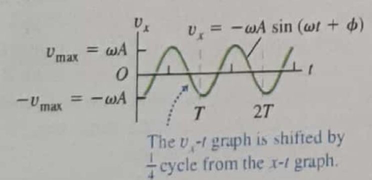
$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi) \qquad \text{(acceleration in SHM)} \tag{14.16}$$

**14.12** Graphs of (a) $x$ versus $t$, (b) $v_x$ versus $t$, and (c) $a_x$ versus $t$ for a body in SHM. For the motion depicted in these graphs, $\phi = \pi/3$.

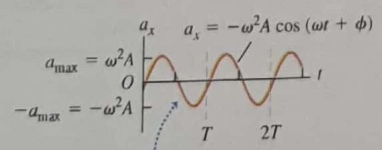
(a) Displacement $x$ as a function of time $t$




(b) Velocity $v_x$ as a function of time $t$




(c) Acceleration $a_x$ as a function of time $t$




The $a_x$-$t$ graph is shifted by $\frac{1}{4}$ cycle from the $v_x$-$t$ graph and by $\frac{1}{2}$ cycle from the $x$-$t$ graph.

The velocity $v_x$ oscillates between $v_{max} = +\omega A$ and $-v_{max} = -\omega A$, and the acceleration $a_x$ oscillates between $a_{max} = +\omega^2 A$ and $-a_{max} = -\omega^2 A$ (Fig. 14.12). Comparing Eq. (14.16) with Eq. (14.13) and recalling that $\omega^2 = k/m$ from Eq. (14.9), we see that

$$a_x = -\omega^2 x = -\frac{k}{m}x$$

which is just Eq. (14.4) for simple harmonic motion. This confirms that Eq. (14.13) for $x$ as a function of time is correct.

We actually derived Eq. (14.16) earlier in a geometrical way by taking the $x$-component of the acceleration vector of the reference point $Q$. This was done in Fig. 14.6b and Eq. (14.7) (recall that $\theta = \omega t + \phi$). In the same way, we could have derived Eq. (14.15) by taking the $x$-component of the velocity vector of $Q$, as shown in Fig. 14.6b. We'll leave the details for you to work out.

Note that the sinusoidal graph of displacement versus time (Fig. 14.12a) is shifted by one-quarter period from the graph of velocity versus time (Fig. 14.12b) and by one-half period from the graph of acceleration versus time (Fig. 14.12c). Figure 14.13 shows why this is so. When the body is passing through the equilibrium position so that the displacement is zero, the velocity equals either $v_{max}$ or $-v_{max}$ (depending on which way the body is moving) and the acceleration is zero. When the body is at either its maximum positive displacement, $x = +A$, or its maximum negative displacement, $x = -A$, the velocity is zero and the body is instantaneously at rest. At these points, the restoring force $F_x = -kx$ and the acceleration of the body have their maximum magnitudes. At $x = +A$ the acceleration is negative and equal to $-a_{max}$. At $x = -A$ the acceleration is positive: $a_x = +a_{max}$.

If we are given the initial position $x_0$ and initial velocity $v_{0x}$ for the oscillating body, we can determine the amplitude $A$ and the phase angle $\phi$. Here's how to do it. The initial velocity $v_{0x}$ is the velocity at time $t = 0$; putting $v_x = v_{0x}$ and $t = 0$ in Eq. (14.15), we find

$$v_{0x} = -\omega A\sin\phi \tag{14.17}$$

To find $\phi$, we divide Eq. (14.17) by Eq. (14.14). This eliminates $A$ and gives an equation that we can solve for $\phi$:

$$\frac{v_{0x}}{x_0} = \frac{-\omega A \sin\phi}{A\cos\phi} = -\omega \tan\phi$$

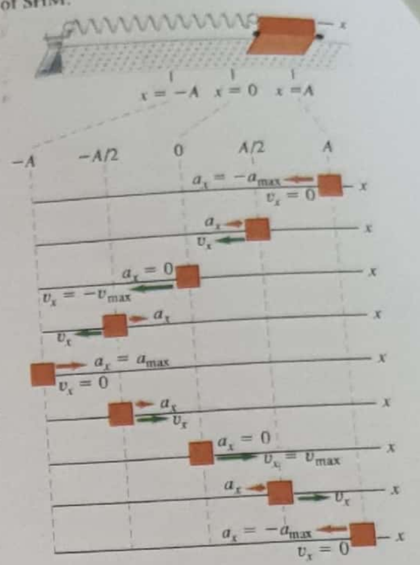
$$\boxed{\phi = \arctan\left(-\frac{v_{0x}}{\omega x_0}\right)} \quad \text{(phase angle in SHM)} \qquad (14.18)$$

It is also easy to find the amplitude $A$ if we are given $x_0$ and $v_{0x}$. We'll sketch the derivation, and you can fill in the details. Square Eq. (14.14); then divide Eq. (14.17) by $\omega$, square it, and add to the square of Eq. (14.14). The right side will be $A^2(\sin^2\phi + \cos^2\phi)$, which is equal to $A^2$. The final result is

$$\boxed{A = \sqrt{x_0^2 + \frac{v_{0x}{}^2}{\omega^2}}} \quad \text{(amplitude in SHM)} \qquad (14.19)$$

Note that when the body has both an initial displacement $x_0$ and a nonzero initial velocity $v_{0x}$, the amplitude $A$ is *not* equal to the initial displacement. That's reasonable; if you start the body at a positive $x_0$ but give it a positive velocity $v_{0x}$, it will go *farther* than $x_0$ before it turns and comes back.

**14.13** How $x$-velocity $v_x$ and $x$-acceleration $a_x$ vary during one cycle of SHM.



## Problem-Solving Strategy 14.1 Simple Harmonic Motion I: Describing Motion

**IDENTIFY** *the relevant concepts:* An oscillating system undergoes simple harmonic motion (SHM) *only* if the restoring force is directly proportional to the displacement.

**SET UP** *the problem* using the following steps:
1. Identify the known and unknown quantities, and determine which are the target variables.
2. Distinguish between two kinds of quantities. *Properties of the system* include the mass $m$, the force constant $k$, and quantities derived from $m$ and $k$, such as the period $T$, frequency $f$, and angular frequency $\omega$. These are independent of *properties of the motion*, which describe how the system behaves when it is set into motion in a particular way; they include the amplitude $A$, maximum velocity $v_{max}$, and phase angle $\phi$, and values of $x$, $v_x$, and $a_x$ at particular times.
3. If necessary, define an $x$-axis as in Fig. 14.13, with the equilibrium position at $x = 0$.

**EXECUTE** *the solution* as follows:
1. Use the equations given in Sections 14.1 and 14.2 to solve for the target variables.
2. To find the values of $x$, $v_x$, and $a_x$ at particular times, use Eqs. (14.13), (14.15), and (14.16), respectively. If the initial position $x_0$ and initial velocity $v_{0x}$ are both given, determine $\phi$ and $A$ from Eqs. (14.18) and (14.19). If the body has an initial positive displacement $x_0$ but zero initial velocity ($v_{0x} = 0$), then the amplitude is $A = x_0$ and the phase angle is $\phi = 0$. If it has an initial positive velocity $v_{0x}$ but no initial displacement ($x_0 = 0$), the amplitude is $A = v_{0x}/\omega$ and the phase angle is $\phi = -\pi/2$. Express all phase angles in *radians*.

**EVALUATE** *your answer:* Make sure that your results are consistent. For example, suppose you used $x_0$ and $v_{0x}$ to find general expressions for $x$ and $v_x$ at time $t$. If you substitute $t = 0$ into these expressions, you should get back the given values of $x_0$ and $v_{0x}$.

## Example 14.3 Describing SHM

We give the glider of Example 14.2 an initial displacement $x_0 = +0.015$ m and an initial velocity $v_{0x} = +0.40$ m/s. (a) Find the period, amplitude, and phase angle of the resulting motion. (b) Write equations for the displacement, velocity, and acceleration as functions of time.

**SOLUTION**

**IDENTIFY and SET UP:** As in Example 14.2, the oscillations are SHM. We use equations from this section and the given values $k = 200$ N/m, $m = 0.50$ kg, $x_0$, and $v_{0x}$ to calculate the target variables $A$ and $\phi$ and to obtain expressions for $x$, $v_x$, and $a_x$.

*Continued*

k = 200 N/m
y = 0.015 m
V = 0.40 m/s
T = ?
a = ?
σ = ?

amplitude
V = aω
V = ω√(a² − y²)

① ω = √(k/m) ⇒ 200/0.50 ⇒ 20.
② V = aω
0.40 = a × 20
a = 0.40/20 ⇒ 0.02

**EXECUTE:** (a) In SHM the period and angular frequency are *properties of the system* that depend only on $k$ and $m$, not on the amplitude, and so are the same as in Example 14.2 ($T = 0.31$ s and $\omega = 20$ rad/s). From Eq. (14.19), the amplitude is

$$A = \sqrt{x_0^2 + \frac{v_{0x}^2}{\omega^2}} = \sqrt{(0.015\ \text{m})^2 + \frac{(0.40\ \text{m/s})^2}{(20\ \text{rad/s})^2}} = 0.025\ \text{m}$$

We use Eq. (14.18) to find the phase angle:

$$\phi = \arctan\left(-\frac{v_{0x}}{\omega x_0}\right)$$
$$= \arctan\left(-\frac{0.40\ \text{m/s}}{(20\ \text{rad/s})(0.015\ \text{m})}\right) = -53° = -0.93\ \text{rad}$$

(b) The displacement, velocity, and acceleration at any time are given by Eqs. (14.13), (14.15), and (14.16), respectively. We substitute the values of $A$, $\omega$, and $\phi$ into these equations:

$$x = (0.025\ \text{m})\cos[(20\ \text{rad/s})t - 0.93\ \text{rad}]$$
$$v_x = -(0.50\ \text{m/s})\sin[(20\ \text{rad/s})t - 0.93\ \text{rad}]$$
$$a_x = -(10\ \text{m/s}^2)\cos[(20\ \text{rad/s})t - 0.93\ \text{rad}]$$

**EVALUATE:** You can check the expressions for $x$ and $v_x$ by confirming that if you substitute $t = 0$, they yield $x = x_0 = 0.015$ m and $v_x = v_{0x} = 0.40$ m/s.

**Test Your Understanding of Section 14.2** A glider is attached to a spring as shown in Fig. 14.13. If the glider is moved to $x = 0.10$ m and released from rest at time $t = 0$, it will oscillate with amplitude $A = 0.10$ m and phase angle $\phi = 0$. (a) Suppose instead that at $t = 0$ the glider is at $x = 0.10$ m and is moving to the right in Fig. 14.13. In this situation is the amplitude greater than, less than, or equal to 0.10 m? Is the phase angle greater than, less than, or equal to zero? (b) Suppose instead that at $t = 0$ the glider is at $x = 0.10$ m and is moving to the left in Fig. 14.13. In this situation is the amplitude greater than, less than, or equal to 0.10 m? Is the phase angle greater than, less than, or equal to zero?

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**ActivPhysics 9.4:** Two Ways to Weigh Young Tarzan
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**ActivPhysics 9.7:** Releasing a Vibrating Skier II
**ActivPhysics 9.8:** One- and Two-Spring Vibrating Systems
**ActivPhysics 9.9:** Vibro-Ride

## 14.3 Energy in Simple Harmonic Motion

We can learn even more about simple harmonic motion by using energy considerations. Take another look at the body oscillating on the end of a spring in Figs. 14.2 and 14.13. We've already noted that the spring force is the only horizontal force on the body. The force exerted by an ideal spring is a conservative force, and the vertical forces do no work, so the total mechanical energy of the system is *conserved*. We also assume that the mass of the spring itself is negligible.

The kinetic energy of the body is $K = \frac{1}{2}mv^2$ and the potential energy of the spring is $U = \frac{1}{2}kx^2$, just as in Section 7.2. (You'll find it helpful to review that section.) There are no nonconservative forces that do work, so the total mechanical energy $E = K + U$ is conserved:

$$E = \tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2 = \text{constant} \qquad (14.20)$$

(Since the motion is one-dimensional, $v^2 = v_x^2$.)

The total mechanical energy $E$ is also directly related to the amplitude $A$ of the motion. When the body reaches the point $x = A$, its maximum displacement from equilibrium, it momentarily stops as it turns back toward the equilibrium position. That is, when $x = A$ (or $-A$), $v_x = 0$. At this point the energy is entirely potential, and $E = \frac{1}{2}kA^2$. Because $E$ is constant, it is equal to $\frac{1}{2}kA^2$ at any other point. Combining this expression with Eq. (14.20), we get

Derivation

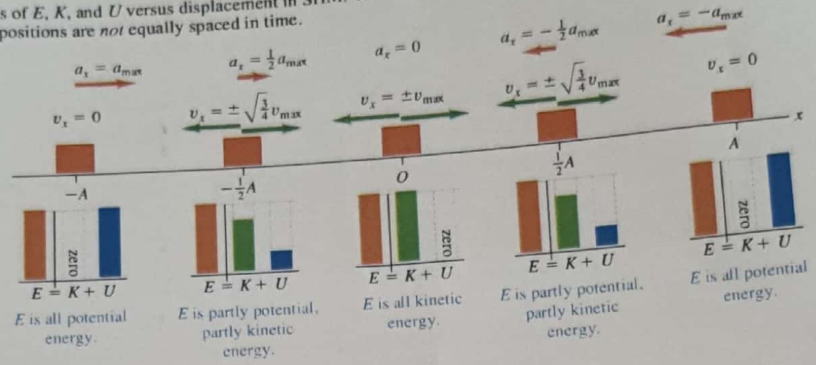
$$E = \tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 = \text{constant} \qquad \text{(total mechanical energy in SHM)} \qquad (14.21)$$

We can verify this equation by substituting $x$ and $v_x$ from Eqs. (14.13) and (14.15) and using $\omega^2 = k/m$ from Eq. (14.9):

$$E = \tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}m[-\omega A\sin(\omega t + \phi)]^2 + \tfrac{1}{2}k[A\cos(\omega t + \phi)]^2$$
$$= \tfrac{1}{2}kA^2\sin^2(\omega t + \phi) + \tfrac{1}{2}kA^2\cos^2(\omega t + \phi)$$
$$= \tfrac{1}{2}kA^2$$

amplitude

**14.14** Graphs of $E$, $K$, and $U$ versus displacement in SHM. The velocity of the body is *not* constant, so these images of the body at equally spaced positions are *not* equally spaced in time.




(Recall that $\sin^2\alpha + \cos^2\alpha = 1$.) Hence our expressions for displacement and velocity in SHM are consistent with energy conservation, as they must be.

We can use Eq. (14.21) to solve for the velocity $v_x$ of the body at a given displacement $x$:

$$v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2} \qquad (14.22)$$

The $\pm$ sign means that at a given value of $x$ the body can be moving in either direction. For example, when $x = \pm A/2$,

$$v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - \left(\pm\frac{A}{2}\right)^2} = \pm\sqrt{\frac{3}{4}}\sqrt{\frac{k}{m}}A$$

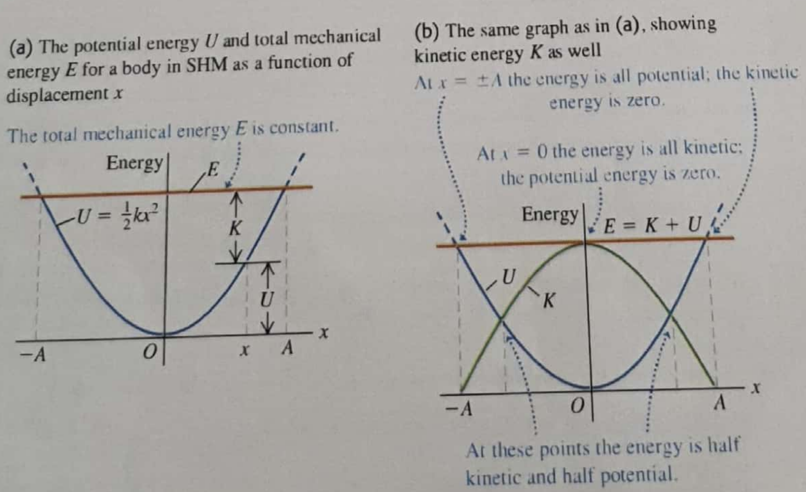
Equation (14.22) also shows that the *maximum* speed $v_{max}$ occurs at $x = 0$. Using Eq. (14.10), $\omega = \sqrt{k/m}$, we find that

$$v_{max} = \sqrt{\frac{k}{m}}A = \omega A \qquad (14.23)$$

This agrees with Eq. (14.15): $v_x$ oscillates between $-\omega A$ and $+\omega A$.

## Interpreting $E$, $K$, and $U$ in SHM

Figure 14.14 shows the energy quantities $E$, $K$, and $U$ at $x = 0$, $x = \pm A/2$, and $x = \pm A$. Figure 14.15 is a graphical display of Eq. (14.21); energy (kinetic, potential, and total) is plotted vertically and the coordinate $x$ is plotted horizontally.




**14.15** Kinetic energy $K$, potential energy $U$, and total mechanical energy $E$ as functions of position for SHM. At each value of $x$ the sum of the values of $K$ and $U$ equals the constant value of $E$. Can you show that the energy is half kinetic and half potential at $x = \pm\sqrt{\frac{1}{2}}A$?

The parabolic curve in Fig. 14.15a represents the potential energy $U = \frac{1}{2}kx^2$. The horizontal line represents the total mechanical energy $E$, which is constant and does not vary with $x$. At any value of $x$ between $-A$ and $A$, the vertical distance from the $x$-axis to the parabola is $U$; since $E = K + U$, the remaining vertical distance up to the horizontal line is $K$. Figure 14.15b shows both $K$ and $U$ as functions of $x$. The horizontal line for $E$ intersects the potential-energy curve at $x = -A$ and $x = A$, so at these points the energy is entirely potential, the kinetic energy is zero, and the body comes momentarily to rest before reversing direction. As the body oscillates between $-A$ and $A$, the energy is continuously transformed from potential to kinetic and back again.

Figure 14.15a shows the connection between the amplitude $A$ and the corresponding total mechanical energy $E = \frac{1}{2}kA^2$. If we tried to make $x$ greater than $A$ (or less than $-A$), $U$ would be greater than $E$, and $K$ would have to be negative. But $K$ can never be negative, so $x$ can't be greater than $A$ or less than $-A$.

## Problem-Solving Strategy 14.2 Simple Harmonic Motion II: Energy

MP

The SHM energy equation, Eq. (14.21), is a useful relationship among velocity, position, and total mechanical energy. If the problem requires you to relate position, velocity, and acceleration without reference to time, consider using Eq. (14.4) (from Newton's second law) or Eq. (14.21) (from energy conservation). Because Eq. (14.21) involves $x^2$ and $v_x^2$, you must infer the *signs* of $x$ and $v_x$ from the situation. For instance, if the body is moving from the equilibrium position toward the point of greatest positive displacement, then $x$ is positive and $v_x$ is positive.

## Example 14.4 Velocity, acceleration, and energy in SHM

Q/1

(a) Find the maximum and minimum velocities attained by the oscillating glider of Example 14.2. (b) Find the maximum and minimum accelerations. (c) Find the velocity $v_x$ and acceleration $a_x$ when the glider is halfway from its initial position to the equilibrium position $x = 0$. (d) Find the total energy, potential energy, and kinetic energy at this position.

**SOLUTION**

**IDENTIFY and SET UP:** The problem concerns properties of the motion at specified *positions*, not at specified *times*, so we can use the energy relationships of this section. Figure 14.13 shows our choice of $x$-axis. The maximum displacement from equilibrium is $A = 0.020$ m. We use Eqs. (14.22) and (14.4) to find $v_x$ and $a_x$ for a given $x$. We then use Eq. (14.21) for given $x$ and $v_x$ to find the total, potential, and kinetic energies $E$, $U$, and $K$.

**EXECUTE:** (a) From Eq. (14.22), the velocity $v_x$ at any displacement $x$ is

$$v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$$

The glider's maximum *speed* occurs when it is moving through $x = 0$:

$$v_{max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{200\text{ N/m}}{0.50\text{ kg}}}(0.020\text{ m}) = 0.40\text{ m/s}$$

Its maximum and minimum (most negative) *velocities* are $+0.40$ m/s and $-0.40$ m/s, which occur when it is moving through $x = 0$ to the right and left, respectively.

(b) From Eq. (14.4), $a_x = -(k/m)x$. The glider's maximum (most positive) acceleration occurs at the most negative value of $x$, $x = -A$:

$$a_{max} = -\frac{k}{m}(-A) = -\frac{200\text{ N/m}}{0.50\text{ kg}}(-0.020\text{ m}) = 8.0\text{ m/s}^2$$

The minimum (most negative) acceleration is $a_{min} = -8.0\text{ m/s}^2$, which occurs at $x = +A = +0.020$ m.

(c) The point halfway from $x = x_0 = A$ to $x = 0$ is $x = A/2 = 0.010$ m. From Eq. (14.22), at this point

$$v_x = -\sqrt{\frac{200\text{ N/m}}{0.50\text{ kg}}}\sqrt{(0.020\text{ m})^2 - (0.010\text{ m})^2} = -0.35\text{ m/s}$$

We choose the negative square root because the glider is moving from $x = A$ toward $x = 0$. From Eq. (14.4),

$$a_x = -\frac{200\text{ N/m}}{0.50\text{ kg}}(0.010\text{ m}) = -4.0\text{ m/s}^2$$

Figure 14.14 shows the conditions at $x = 0$, $\pm A/2$, and $\pm A$.

(d) The energies are

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(200\text{ N/m})(0.020\text{ m})^2 = 0.040\text{ J}$$
$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(200\text{ N/m})(0.010\text{ m})^2 = 0.010\text{ J}$$
$$K = \tfrac{1}{2}mv_x^2 = \tfrac{1}{2}(0.50\text{ kg})(-0.35\text{ m/s})^2 = 0.030\text{ J}$$

x=0 potential

**EVALUATE:** At $x = A/2$, the total energy is one-fourth potential energy and three-fourths kinetic energy. You can confirm this by inspecting Fig. 14.15b.

given $F = -6.0$ N
$F = -kx$
$x = 0.030$
$m = 0.50$ kg
$x = 0.020$ m

Q//

Sometimes we take amplitude = displacement. why?

## Example 14.5 Energy and momentum in SHM

A block of mass $M$ attached to a horizontal spring with force constant $k$ is moving in SHM with amplitude $A_1$. As the block passes through its equilibrium position, a lump of putty of mass $m$ is dropped from a small height and sticks to it. (a) Find the new amplitude and period of the motion. (b) Repeat part (a) if the putty is dropped onto the block when it is at one end of its path.

9/11

**SOLUTION**

**IDENTIFY and SET UP:** The problem involves the motion at a given position, not a given time, so we can use energy methods. Figure 14.16 shows our sketches. Before the putty falls, the mechanical energy of the block–spring system is constant. In part (a), the putty–block collision is completely inelastic: The horizontal component of momentum is conserved, kinetic energy decreases, and the amount of mass that's oscillating increases. After the collision, the mechanical energy remains constant at its new value. In part (b) the oscillating mass also increases, but the block isn't moving when the putty is added; there is effectively no collision at all, and no mechanical energy is lost. We find the amplitude $A_2$ after each collision from the final energy of the system using Eq. (14.21) and conservation of momentum. The period $T_2$ after the collision is a *property of the system*, so it is the same in both parts (a) and (b); we find it using Eq. (14.12).

**EXECUTE:** (a) Before the collision the total mechanical energy of the block and spring is $E_1 = \frac{1}{2}kA_1^2$. The block is at $x = 0$, so $U = 0$ and the energy is purely kinetic (Fig. 14.16a). If we let $v_1$ be the speed of the block at this point, then $E_1 = \frac{1}{2}kA_1^2 = \frac{1}{2}Mv_1^2$ and

$$v_1 = \sqrt{\frac{k}{M}}A_1$$

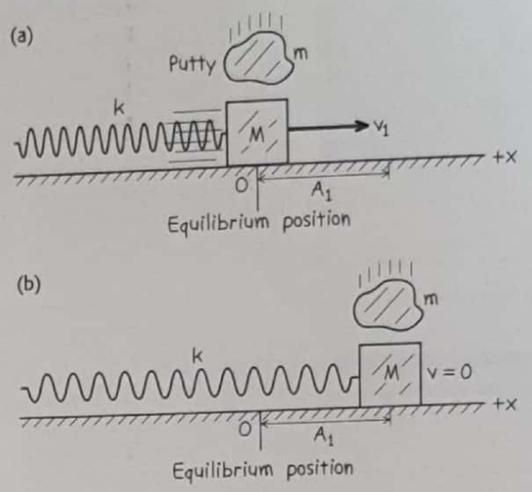
During the collision the $x$-component of momentum of the block–putty system is conserved. (Why?) Just before the collision this component is the sum of $Mv_1$ (for the block) and zero (for the putty). Just after the collision the block and putty move together with speed $v_2$, so their combined $x$-component of momentum is $(M + m)v_2$. From conservation of momentum,

$$Mv_1 + 0 = (M + m)v_2 \qquad \text{so} \qquad v_2 = \frac{M}{M + m}v_1$$

We assume that the collision lasts a very short time, so that the block and putty are still at the equilibrium position just after the collision. The energy is still purely kinetic but is *less* than before the collision:

$$E_2 = \tfrac{1}{2}(M + m)v_2^2 = \tfrac{1}{2}\frac{M^2}{M + m}v_1^2$$
$$= \frac{M}{M + m}\left(\tfrac{1}{2}Mv_1^2\right) = \left(\frac{M}{M + m}\right)E_1$$

**14.16** Our sketches for this problem.




Since $E_2 = \frac{1}{2}kA_2^2$, where $A_2$ is the amplitude after the collision, we have

$$\tfrac{1}{2}kA_2^2 = \left(\frac{M}{M + m}\right)\tfrac{1}{2}kA_1^2$$
$$A_2 = A_1\sqrt{\frac{M}{M + m}}$$

From Eq. (14.12), the period of oscillation after the collision is

$$T_2 = 2\pi\sqrt{\frac{M + m}{k}}$$

(b) When the putty falls, the block is instantaneously at rest (Fig. 14.16b). The $x$-component of momentum is zero both before and after the collision. The block and putty have zero kinetic energy just before and just after the collision. The energy is all potential energy stored in the spring, so adding the putty has *no effect* on the mechanical energy. That is, $E_2 = E_1 = \frac{1}{2}kA_1^2$, and the amplitude is unchanged: $A_2 = A_1$. The period is again $T_2 = 2\pi\sqrt{(M + m)/k}$.

**EVALUATE:** Energy is lost in part (a) because the putty slides against the moving block during the collision, and energy is dissipated by kinetic friction. No energy is lost in part (b), because there is no sliding during the collision.

**Test Your Understanding of Section 14.3** (a) To double the total energy for a mass-spring system oscillating in SHM, by what factor must the amplitude increase? (i) 4; (ii) 2; (iii) $\sqrt{2} = 1.414$; (iv) $\sqrt[4]{2} = 1.189$. (b) By what factor will the frequency change due to this amplitude increase? (i) 4; (ii) 2; (iii) $\sqrt{2} = 1.414$; (iv) $\sqrt[4]{2} = 1.189$; (v) it does not change.

MP

## 14.4 Applications of Simple Harmonic Motion

So far, we've looked at a grand total of *one* situation in which simple harmonic motion (SHM) occurs: a body attached to an ideal horizontal spring. But SHM can occur in any system in which there is a restoring force that is directly proportional to the displacement from equilibrium, as given by Eq. (14.3), $F_x = -kx$. The restoring force will originate in different ways in different situations, so the force constant $k$ has to be found for each case by examining the net force on the system. Once this is done, it's straightforward to find the angular frequency $\omega$, frequency $f$, and period $T$; we just substitute the value of $k$ into Eqs. (14.10), (14.11), and (14.12), respectively. Let's use these ideas to examine several examples of simple harmonic motion.

### Vertical SHM

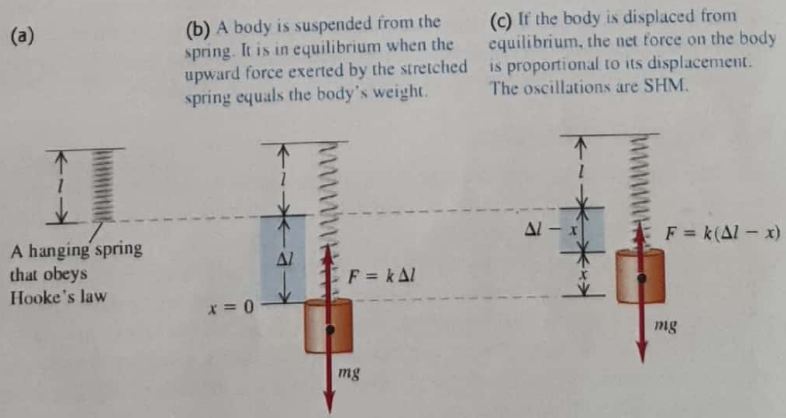
Suppose we hang a spring with force constant $k$ (Fig. 14.17a) and suspend from it a body with mass $m$. Oscillations will now be vertical; will they still be SHM? In Fig. 14.17b the body hangs at rest, in equilibrium. In this position the spring is stretched an amount $\Delta l$ just great enough that the spring's upward vertical force $k\,\Delta l$ on the body balances its weight $mg$:

$$k\,\Delta l = mg$$

Take $x = 0$ to be this equilibrium position and take the positive $x$-direction to be upward. When the body is a distance $x$ *above* its equilibrium position (Fig. 14.17c), the extension of the spring is $\Delta l - x$. The upward force it exerts on the body is then $k(\Delta l - x)$, and the net $x$-component of force on the body is

$$F_{\text{net}} = k(\Delta l - x) + (-mg) = -kx$$

that is, a net downward force of magnitude $kx$. Similarly, when the body is *below* the equilibrium position, there is a net upward force with magnitude $kx$. In either case there is a restoring force with magnitude $kx$. If the body is set in vertical motion, it oscillates in SHM with the same angular frequency as though it were horizontal, $\omega = \sqrt{k/m}$. So vertical SHM doesn't differ in any essential way from horizontal SHM. The only real change is that the equilibrium position $x = 0$ no longer corresponds to the point at which the spring is unstretched. The same ideas hold if a body with weight $mg$ is placed atop a compressible spring (Fig. 14.18) and compresses it a distance $\Delta l$.




**14.17** A body attached to a hanging spring.

given: $m = 1000$ kg
$F = 980$ N
$x = 2.8$ cm $2.8 \times 10^{-2}$ m
$T = ?$ $f = ?$

$F = -kx$
$T = 2\pi\sqrt{\frac{m}{k}}$



θ//

**Example 14.6** **Vertical SHM in an old car**

The shock absorbers in an old car with mass 1000 kg are completely worn out. When a 980-N person climbs slowly into the car at its center of gravity, the car sinks 2.8 cm. The car (with the person aboard) hits a bump, and the car starts oscillating up and down in SHM. Model the car and person as a single body on a single spring, and find the period and frequency of the oscillation.

**SOLUTION**

**IDENTIFY and SET UP:** The situation is like that shown in Fig. 14.18. The compression of the spring when the person's weight is added tells us the force constant, which we can use to find the period and frequency (the target variables).

**EXECUTE:** When the force increases by 980 N, the spring compresses an additional 0.028 m, and the $x$-coordinate of the car changes by $-0.028$ m. Hence the effective force constant (including the effect of the entire suspension) is

$$k = -\frac{F_x}{x} = -\frac{980\ \text{N}}{-0.028\ \text{m}} = 3.5 \times 10^4\ \text{kg/s}^2$$

The person's mass is $w/g = (980\ \text{N})/(9.8\ \text{m/s}^2) = 100$ kg. The *total* oscillating mass is $m = 1000\ \text{kg} + 100\ \text{kg} = 1100$ kg. The period $T$ is

$F = mg \Rightarrow m = F/g$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1100\ \text{kg}}{3.5 \times 10^4\ \text{kg/s}^2}} = 1.11\ \text{s}$$

The frequency is $f = 1/T = 1/(1.11\ \text{s}) = 0.90$ Hz.

**EVALUATE:** A persistent oscillation with a period of about 1 second makes for a very unpleasant ride. The purpose of shock absorbers is to make such oscillations die out (see Section 14.7).

## Angular SHM

A mechanical watch keeps time based on the oscillations of a balance wheel (Fig. 14.19). The wheel has a moment of inertia $I$ about its axis. A coil spring exerts a restoring torque $\tau_z$ that is proportional to the angular displacement $\theta$ from the equilibrium position. We write $\tau_z = -\kappa\theta$, where $\kappa$ (the Greek letter kappa) is a constant called the *torsion constant*. Using the rotational analog of Newton's second law for a rigid body, $\Sigma\tau_z = I\alpha_z = I\,d^2\theta/dt^2$, we can find the equation of motion:

$$-\kappa\theta = I\alpha \quad \text{or} \quad \frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta$$

The form of this equation is exactly the same as Eq. (14.4) for the acceleration in simple harmonic motion, with $x$ replaced by $\theta$ and $k/m$ replaced by $\kappa/I$. So we are dealing with a form of *angular* simple harmonic motion. The angular frequency $\omega$ and frequency $f$ are given by Eqs. (14.10) and (14.11), respectively, with the same replacement:

$$\omega = \sqrt{\frac{\kappa}{I}} \quad \text{and} \quad f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}} \quad \text{(angular SHM)} \qquad (14.24)$$

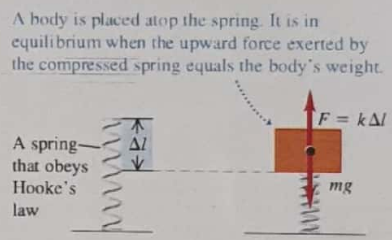
The motion is described by the function

$$\theta = \Theta\cos(\omega t + \phi)$$

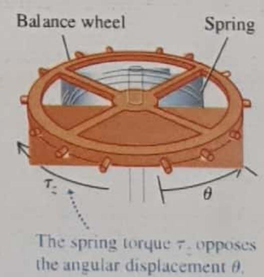
where $\Theta$ (the Greek letter theta) plays the role of an angular amplitude.

It's a good thing that the motion of a balance wheel *is* simple harmonic. If it weren't, the frequency might depend on the amplitude, and the watch would run too fast or too slow as the spring ran down.

**14.18** If the weight $mg$ compresses the spring a distance $\Delta l$, the force constant is $k = mg/\Delta l$ and the angular frequency for vertical SHM is $\omega = \sqrt{k/m}$—the same as if the body were suspended from the spring (see Fig. 14.17).




**14.19** The balance wheel of a mechanical watch. The spring exerts a restoring torque that is proportional to the angular displacement $\theta$, so the motion is angular SHM.




## Vibrations of Molecules

The following discussion of the vibrations of molecules uses the binomial theorem. If you aren't familiar with this theorem, you should read about it in the appropriate section of a math textbook.

When two atoms are separated from each other by a few atomic diameters, they can exert attractive forces on each other. But if the atoms are so close to each other that their electron shells overlap, the forces between the atoms are repulsive. Between these limits, there can be an equilibrium separation distance at which two atoms form a *molecule*. If these atoms are displaced slightly from equilibrium, they will oscillate.

amplitude → max displacement

Periodic motion:

→ In physics, motion repeated in equal intervals of time.

→ eg: rocking chair, a bouncing ball, a vibrating fork, a swing in motion, the Earth in its orbit around the sun, and a water wave.

→ In each case the interval of time for a repetation, a cycle of the motion is called period.

→ In physics, a motion that is regular and repeating is reffered to as a periodic motion.

→ motion in which the body describes the same path in the same way again and again in equal intervals of time is called the periodic motion.

→ The number of periods per unit time is called frequency.

Thus the period of the Earth's orbit is one year, and its frequency is one orbit per year. A tuning fork might have a frequency of 1,000 cycles per second and a period of 1 millisecond (1 thousand of a second)

* Types of Periodic motion:-

1) Circular motion:- If the body moves in circular path

2) Linear motion: If the body moves along the line

## Amplitude, Period, Frequency and Angular Frequency

Amplitude of the motion, denoted by A is the maximum magnitude of displacement $x$ from equilibrium - that is, the maximum value of $|x|$ is always positive. If the spring in figure is an ideal one, the total ~~average~~ overall range of the motion is $2A$. The SI unit of $x$ is meter.

Period:- (T) is the time period for one cycle. it is always positive. The SI unit is the second, but it is sometimes expressed as "second per cycle".

Frequency: is the number of cycles in a unit time. it is always positive. The SI unit of frequency is Hertz.

$$1 \text{ hertz} = 1 \text{ Hz} = 1 \text{ cycle/s} = 1 \text{ s}^{-1}$$

Angular frequency $\omega$, is $2\pi$ times the frequency.

$f = \frac{1}{T}$ $\quad T = \frac{1}{f}$ (relation frequency and time period)

$\omega = 2\pi f = \frac{2\pi}{T}$ (angular frequency)

Q) An ultrasonic transducer used for medical diagnosis oscillate at 6.7 MHz, How long does each oscillation take, and what angular frequency?

$T = \frac{1}{f} = \frac{1}{6.7 \times 10^6 \text{ Hz}} = 1.5 \times 10^{-7} \text{ s} = 0.15 \mu s$

$\omega = 2\pi f = 2\pi (6.7 \times 10^6 \text{ Hz})$

$= (2\pi \text{ rad/cycle})(6.7 \times 10^6 \text{ cycle/s})$

$= 4.2 \times 10^7 \text{ rad/s}$

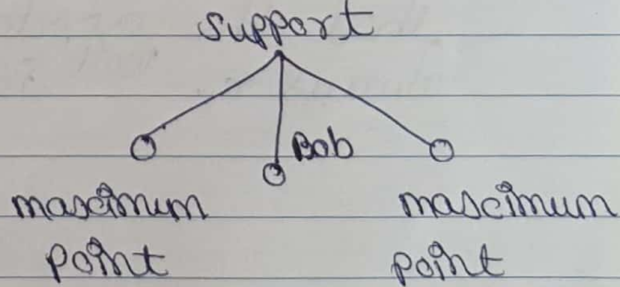
This is a very rapid vibration, with large $f$ and $\omega$ and small $T$. A slow vibration has small $f$ and $\omega$ and large $T$.

* Simple Harmonic motion (SHM)

It is a particular and common type of motion having single period. A system that oscillates with SHM is called a simple harmonic oscillator.

Simple harmonic motion is a repetitive movement back and forth through an equilibrium or central, position, so the maximum displacement on one side of the this position is equal to the maximum displacement of other side. The time interval of each complete vibration is the same.

The force responsible for the motion is always directed toward the equilibrium position and is directly proportional to the distance from it. That is $F = -kx$. where $F$ is the force, $x$ is the displacement, and $k$ is the constant.
The relation is called Hooke's law.




* characteristic of SHM:-

1) The motion should be periodic.
2) when ~~the~~ displaced from the ~~mean~~ (mean) position, a restoring force should act on the system to bring it back to the equilibrium.

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3) a restoring force should act on the body.
4) Body must have acceleration in a direction opposite to the displacement and the acceleration must directly proportional to displacement.
5) The system must have inertia (mass).

$F = -kx$
$F = ma$
or. $-kx = ma$

$$\boxed{\left(\frac{-k}{m}\right) x = a}$$

The force constant k is always + and has units N/m ($kg/s^2$). when the restoring force is directly propotional to displacement from equilibrium, as given by the above equation the oscillationes called SHM. The acceleration $a_x = d^2x/dt^2 = F_x/m$ of a body in SHM is given by

$$a_x = \frac{d^2x}{dt^2} = -\frac{k}{m} x$$

less mass high frequency

The minus sign indicates that the acceleration and displacement are always +. we'll see shortly how to solve this

Q) A particle execute SHM of period 10s and amplitude 1.5 m. Calculate the max acceleration and velocity.

~~$a = 1.5\,m$~~

~~$T = 10s$~~

~~$\boxed{V = \frac{a}{\omega}}$~~ ~~$\omega = 2\pi f = \frac{2\pi}{T}$~~

~~$\omega = \frac{2\pi}{10s} \Rightarrow \frac{\pi}{5} \Rightarrow 0.628$~~

~~$V = \frac{1.5}{0.628} \Rightarrow 2.38 \Rightarrow 3$~~

~~$A = 3\pi^2/50$~~

$T = 10s$ $\omega = ?$

$A = 1.5\,m$ $a = ?$

$v = ?$

$\omega = \frac{2\pi}{T} \Rightarrow \omega = \frac{2\pi}{10} \Rightarrow 0.628$ rad/sec

linear velocity $v = \omega\sqrt{a^2 - y^2}$

~~velocity~~ velocity is max only when displacement that is $(y) = 0$.

$v = \omega\sqrt{a^2 - y^2}$

$V = 0.628\sqrt{(1.5)^2 - 0}$

$= 0.628 \times 1.5 \Rightarrow 0.942$ m/s

acceleration $= \omega^2 y$

$\boxed{a = \omega^2 y}$

$a = (0.628)^2 \times 1.5 = 0.59\ m/s^2$

Q) velocity of the particle executing SHM = 16 cm/s when passing through the centre mean position. It goes 1 cm either sides of mean position. Calculate time period? given amplitude = 2 cm.

$v = 16$ cm/s $\rightarrow 16 \times 10^{-2}$ m

$y = 0$ $y = 1$ cm $a = 2$ cm

2

given:
amplitude $a = 2\,cm$
$v = 16\ cm/s$
$d = 1\,cm$.

$T = ?$

$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$

$\omega \rightarrow$ we have to find

$v = \omega\sqrt{a^2 - y^2}$

↳ passing through mean position so $y = 0$

$v = \omega\sqrt{a^2 - y^2}$

$16 = \omega\sqrt{2^2 - 0}$

$16 = \omega \times 2$

$\omega = 8\ rad/sec$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{8} \Rightarrow \frac{\pi}{4} \Rightarrow \cancel{0.25}\ 0.785\ s$ //

when the displacement is 1cm

$v = \omega\sqrt{a^2 - y^2}$

$16 = \omega\sqrt{2^2 - 1^2}$

$16 = \omega \times \sqrt{3}$

$\omega = \frac{16}{\sqrt{3}} \Rightarrow 9.24\ rad/s$

so, $T = \frac{2\pi}{\omega} = \frac{2\pi}{9.24} \Rightarrow 0.668\ s$ //

$m = 1.68 \times 10^{-27}$
$f = 10^{14}\ Hz$
$a = 10^{-10}\ m$
$F = ?$
$v = a\omega$

Q) A $H_2$ atom has a mass of $1.68 \times 10^{-27}$ kg. when attached to ~~mass~~ massive molecule it oscillate as a clastical oscillation with a frequency of $10^{14}$ Hz and amplitude $10^{-10}$ m. calculate force on $H_2$ atom.

givens

$m = 1.68 \times 10^{-27}$ kg

$f = 10^{14}$ Hz

$a = 10^{-10}$ m. amplitude & displacement

$F = ?$

$f = \frac{1}{T}$ & $T = \frac{1}{f} = \frac{1}{10^{14}}$

$T = \frac{2\pi}{\omega}$ so $\omega = \frac{2\pi}{T} \Rightarrow \frac{2\pi}{\frac{1}{10^{14}}} \Rightarrow 2\pi \times 10^{14}$

$\boxed{\text{acceleration} = \omega^2 y}$

$F = ma$

$= m \times \omega^2 y$

$= 1.68 \times 10^{-27} \times (2\pi \times 10^{14})^2 \times 10^{-10}$

$= 66.3 \times 10^{-9}$ newton N //

Q) A body executing SHM describe 120 vibration/min has a velocity 5 m/s. calculate length and velocity and position when half way through mean position.

$f = 120$ vib/min

$T = \frac{1}{\cancel{120}/\cancel{60}_2} \Rightarrow 0.5$ sec

$v = 5$ m/s

$T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T} \Rightarrow \frac{2\pi}{0.5}$

$v = \omega\sqrt{a^2 - y^2}$

$5 = \omega\sqrt{a^2 - 0}$ $y = 0$ [mean position]

$5 = \frac{2\pi}{0.5} \times a$

$\frac{5 \times 0.5}{2\pi} = a \Rightarrow 0.398$ m

now the total length is $2a \Rightarrow 2 \times 0.398$

$\Rightarrow 0.788$ m //

when the body is half way b/w its mean position

$y = a/2$

$v = \omega\sqrt{a^2 - y^2}$ $\Rightarrow \sqrt{0.398^2 - \left(\frac{0.398}{2}\right)^2}$

$= \frac{2\pi}{T}\sqrt{1 - \left(\frac{1}{2}\right)^2}$

$\Rightarrow 4.329$ m/s

— * —

Q) A body executes SHM with velocity = 1 m/s and accelerates at one of its extremity is 1.57 m/s$^2$. calculate time period of vibration!

$T = ?$ $T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T}$

$a = 1.57$ m/s$^2$ ($\omega$: angular frequency) ($T$: To find)

$v = 1$ m/s

$a = \omega^2 y$

acceleration

$v = \omega\sqrt{a^2 - y^2}$

$1 = \omega\sqrt{a^2 - 0}$ at mean position $y = 0$

$1 = \omega a \rightarrow (1)$

$a = \omega^2 y$

$1.57 = \omega^2 a \rightarrow (2)$ at extremity $y = a$

$\frac{(2)}{(1)} \Rightarrow \frac{1.57}{1} = \frac{\omega^{\cancel{2}} \cancel{a}}{\cancel{\omega} \cancel{a}} \Rightarrow \omega = 1.57$

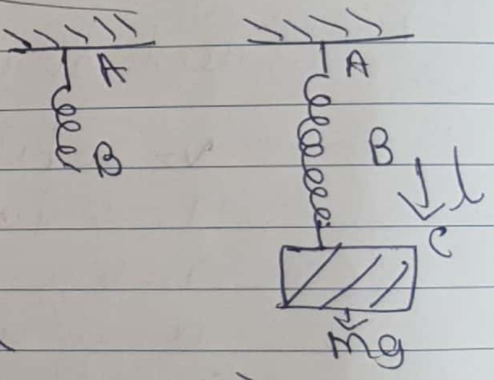
$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.57} \Rightarrow \frac{2 \times 3.14}{1.57} \Rightarrow 4$ sec

— * —

① vibration of simple spring mass system

i) vertical vibration: For a light spiral spring within the elastic limit the tention of the spring is $\propto$ to the exctension of the spring beyond its length where it obeys Hookes law.

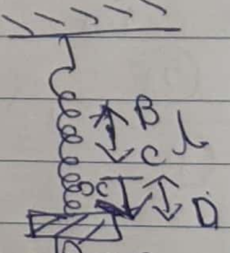
Fig(a) shows a spring of length 'l' suspended from a support A. If the mass m is attached ~~its~~ to its free end B. The spring will be streched down by an increased length 'l' $BC = l$, Due to the inc in the spring length the force executed by the spring on the mass, according to the Hookes Law will be $-kl$. where $k$ is the propotionality constant which depends on the size and the type of material.




Tension of the spring

$$T = mg = kl \rightarrow ①$$

If the load is displaced downward the distance $x$ ~~increases~~ makes the total length inc. $(x+l)$




$$T = k(x+l) \rightarrow ②$$ (virtical vibration of simple spring)

Resultant force acting on the mass would be

$$T - T' \text{ or } mg - T' = kl - k(l+x)$$

$$F = -kx$$

$F = ma$
$-kx = ma$

Acc of mass = $\frac{\text{Force}}{\text{mass}} = \frac{-kx}{m}$

$$\boxed{a = \frac{-kx}{m}}$$

acceleration

mass ∝ displacement since k is the mass constant. The mass will execute SHM with c as the equilibrium position.

$$\frac{k}{m} = \frac{a}{x}$$

Now time period

$$T = \frac{2\pi}{\omega} \Rightarrow \frac{2\pi}{\sqrt{\frac{\text{acceleration}}{x}}} \Rightarrow \frac{2\pi}{\sqrt{k/m}} = 2\pi\sqrt{\frac{m}{k}} \text{ sec}$$

since $mg = kl$ we have $k = mg/l$

$$T = 2\pi\sqrt{\frac{l}{g}} \rightarrow (5) \text{ is seconds}$$

~~Horizontal~~

The above equation shows that the time period of a spring with large k will be less and is ∝ proportional to the mass suspended. We assume mass of the spring < mass having. In case if mass is not negligible $\frac{m}{8}$ will be effective and has been added to the mass

$$T = 2\pi\sqrt{\frac{m + \frac{m_1}{3}}{k}} \rightarrow (6)$$

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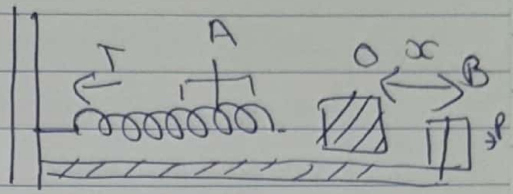
(2)

ii) Horizontal vibration

consider a spring of one end fixed to a rigid support and the other end to the restoring mass on a smooth horizontal surface.




If we pull the mass third a certain distance we find on releasing the mass vibrates about its position. The vibration are also SHM.

Let m be the mass of the body vibration b/w A and B. The force applied to pull the body ~~vibration b/tw~~ be P and the spring tention P is T the restoring force. T will be in the opposite direction of the applied force P.

If 'x' be the distance the body is pulled then the restoring force $\boxed{T = -kx} \rightarrow (7)$

$T \propto -x$ $\quad k \rightarrow \propto$ why cannot

$\boxed{T = -P} \rightarrow (8)$

From eq (7), it is clear that the restoring force makes the mass 'm' to vibrate about its mean position O. Hence,

Acceleration of the mass $= \frac{-kx}{m} \rightarrow (9)$

Since acc is $\propto$ displacement and directed toward the mean position O, the body will execute SHM

$\frac{Acc}{Displacement} = \frac{-kx/m}{x} \Rightarrow \frac{-k}{m}$

$T = 2\pi\sqrt{m/k} \rightarrow (11)$ $\quad T = 2\pi\sqrt{\frac{k}{m}}$ ) //

Q) A 1.75 kg particle moves as function of time as follows:

$x = 4\cos(1.33t + \pi/5)$ where distance is measured in meters and time in seconds. What is the amplitude, frequency, angular frequency and period of motion?

given:

$m = 1.75\,kg$

$x = 4\cos(1.33t + \pi/5)$

$x = A\cos(\omega t + \phi)$

$A = 4\,m$

$\omega = 1.33$ rad/s $\quad \omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$

$\phi = \pi/5$

$T = \frac{2\pi}{1.33} = \frac{6.28}{1.33} \Rightarrow 4.72\,s$ //

$f = \frac{1}{T} \Rightarrow \frac{1}{4.72} \Rightarrow 0.21\,Hz$ //

Q) An object oscillates with angular velocity $\omega = 8$ rad/s. At $t = 0$, it is at $x = 4$ cm with an initial velocity $v = -25$ cm/s. Find amplitude and initial phase constant for this motion. Write $x$ as a function of time $t$.

given:

$A = ?$ $\quad v = a\omega\cos\omega t$

$=$

$\omega = 8$ rad/s

$x = 4\,cm \Rightarrow 4\times10^{-2}\,m$

$v = -25\,cm/s \Rightarrow -25\times10^{-2}\,m/s$

Q) A body of mass 200 g executing SHM has a velocity of 3 cm/s when the displacement is 4 cm and a velocity of 4 cm/s when displacement is 3 cm. Calculate time period and amplitude of oscillation.

$m = 200\,g\,\cancel{\times 10} \Rightarrow 200 \times 10^{-2}$

$v_1 = 3\,cm/s \Rightarrow 3 \times 10^{-2}\,m/s$

$x_1 = 4\,cm \Rightarrow 4 \times 10^{-2}\,m$

$v_2 = 4\,cm/s \Rightarrow 4 \times 10^{-2}\,m/s$

$x_2 = 3\,cm \Rightarrow 3 \times 10^{-2}\,m$

$T = ?$ $\qquad v = \omega\sqrt{a^2 - y^2}$

$A = ?$

$v = A\omega \qquad v^2 = \omega^2[a^2 - y^2]$

$(3 \times 10^{-2})^2 = (\omega)^2 \sqrt{a^2 - 4^2}$

$9 \times 10^{-4} = \omega^2[A^2 - 16] \rightarrow (1)$

$16 = \omega^2[A^2 - 9] \rightarrow (2)$

$7 = -9\omega^2 + 16\omega^2$
$\omega^2 = 1\ rad/s$
$\omega = 1\ rad/s$

$T = \frac{2\pi}{\omega} \Rightarrow \frac{2\pi}{1} \Rightarrow 2\times3.14 \Rightarrow 6.28\,s$ //

$9 = \omega^2[A^2 - 16]$
$9 = A^2 - 16$
$A^2 = 9 + 16$
$A^2 = 25$
$A = 5\,cm$ //

Q) A block of mass m is attached with a spring constant K by means of a ~~spring~~ string going over a frictionless pulley. The block is held in position so that the spring is unstretched. The block is then released and allowed to oscillate with small amplitude. The max velocity of the block during oscillation, in terms of block mass (m) and the force constant k of the spring.

$v = A\omega$ $\quad\quad \omega = \sqrt{\frac{k}{m}}$
$v = A\sqrt{\frac{k}{m}}$

$v_{max} = A\omega$
$= A\sqrt{\frac{k}{m}}$ $\quad\quad F = -kx \Rightarrow -kA = mg$

$x = a$ $\quad\quad A = \frac{mg}{k}$
mass

$v = \frac{mg}{k} \times \sqrt{\frac{k}{m}}$

$v^2 = \frac{\cancel{m^2}\,g^2}{\cancel{k^2}} \times \frac{\cancel{k}}{\cancel{m}} \Rightarrow \frac{mg^2}{k}$ //

Q) A spring of stiffness factor 98 N/m is pulled from 20cm. Find the restore force compute mass ~~with~~ which should be attached to the spring by the same amount.

$x = 20cm$

$K = 98 \, N/m$ (stiffness factor ~~f~~ is the force constant)

$F = Kx = 20 \times 98 \times 10^{-2} \Rightarrow 19.6$

$F = mg$

$Kx = mg$

$19.6 = m \times 9.8$

$m = \frac{19.6}{9.8} = 2kg$ //

Q) A mass suspended from a spiral spring produces an extention of 79 cm. If mass displaced vertically perform 100 SHO in 57 sec. calculate g from the above experiment.

$l = x = 79cm \Rightarrow 79 \times 10^{-2} m$

$f = \frac{100 \text{ vib}}{57 \text{ sec}}$

$T = 2\pi \sqrt{\frac{l}{g}}$ $\boxed{g = \frac{4\pi^2 l}{T^2}}$

$g = 4 \times (3.14)^2 \times 79 \times 10^{-2} \cancel{\times \left(\frac{100}{57}\right)^2}$

$f = 1.754$ $T = \frac{1}{1.754}$

$g = 4 \times (3.14)^2 \times 79 \times 10^{-2} \times (0.57)^2$

$= 10.12 \, m/s^2$

Q) A 4kg mass is hung at the end of a helical spring and pulled down to vibrate. The mass complete 100 vibration in 55 sec. calculate stiffness of spring.

$m = 4kg$
$f = 100$ $t = 55$ seconds
$K = ?$ (stiffness)

$f = \frac{1}{T}$ $T = \frac{1}{f}$

$f = \cancel{\frac{1}{T}}$ $\frac{100}{55} \Rightarrow$ sec.

$T = \frac{1}{\frac{100}{55}}$

$T = 2\pi\sqrt{\frac{m}{K}}$

$\frac{55}{100} = 2\pi \times \sqrt{\frac{4}{K}}$ $\Rightarrow \left(\frac{55}{100}\right)^2 \times = \frac{\cancel{4}}{k}$

$\left(\frac{55}{100}\right)^2 = 2\times3.14 \times$ $\cancel{4}\pi^2$

$\cancel{0.3025 \times k = 2\times3.14\times2}$ $0.3025 \times k = \pi^2$

$K = 2\times3.14\times2 / 0.3025$

$k = \frac{3.14\times3.14}{0.3025}$

$\Rightarrow 47.72 N/m$

$= 9.85 / 0.3025$

Q) what mass should be hung on a spiral spring having ~~di~~ stiffness given $K = 89.2 N/m$
89.2 N/m ~~that~~ vibrates $T = 1$ sec
with periodic time 1 sec.

$T = 2\pi\sqrt{\frac{m}{K}}$

$m = ?$ (spring)
$K = 89.2 N/m$
$T = 1$ sec

$1 = 2\pi\sqrt{\frac{m}{89.2}}$

$(1)^2 = 2\pi \times \frac{m}{89.2}$

$\cancel{89.2 = 2\pi \times m}$
$\cancel{m = 89.2/2\pi}$

$\frac{1}{4\pi^2} = \frac{m}{89.2}$ $\Rightarrow m\times4\pi^2 = 89.2$

$m = \frac{89.2}{4\times3.14\times3.1}$

$m = 89.2/39.4$
$\Rightarrow 2.26 kg$

Q) A mass of 0.5 kg hangs from a spring. If the mass is pulled downward it execute SHM, calculate time period if spring stretched 16 cm by 0.4 kg mass.

$m = 0.4$ kg

$m = 0.5$ kg (spring)

$T = ?$

$x = \cancel{16}\ 16 \times 10^{-2}$

$mg = kx$

$\cancel{0.5\times}\ 4 \times 9.8 = k \times 16 \times 10^{-2}$

$k = 2.45$ N/m

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow 2\pi\sqrt{\frac{0.5}{2.45}}$

$T = 0.28$ s

~~when two springs~~

~~a) A & B each of length l has a force constant~~

~~b) K1 & K2 find the force constant.~~

~~a) Let x be the extention of the spring is 'm' in each spring~~

~~fig (a) $A = k_1 x$~~

~~$B = k_2 x$~~

~~$mg = (k_1 + k_2)x \Rightarrow$ when in series~~

~~so $k = k_1 + k_2$ be extention~~

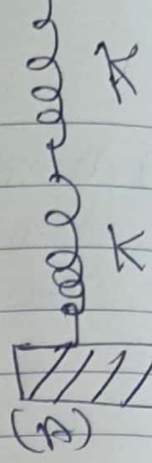
Derivation

* Serial spring system:-

(3) consider a serial spring system. When the mass is pulled down, both the springs experience different displacements but ~~diff~~ same force.

$F_{eq} = -k_{eq}(x_1 + x_2)$

But $F_1 = -kx_1$ and $F_2 = -kx_2$




$x_1 = \frac{F_1}{K_1}$ $x_2 = \frac{F_2}{K_2}$

$x_{eq} = \frac{F_{eq}}{K_{eq}} = x_1 + x_2$

$\frac{F_1}{K_1} \cancel{+} \frac{F_2}{K_2}$ $T = 2\pi\sqrt{\frac{m}{K_{eq}}} \Rightarrow 2\pi\sqrt{\frac{m(K_1+K_2)}{K_1K_2}}$

Therefore $\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} = \frac{K}{2}$ in this case.

$\hookrightarrow \boxed{K = \frac{K_1K_2}{K_1+K_2}}$

(4) * Parallel spring system:

consider a parallel spring system. when the mass point is pulled down then the restoring force is inupward direction since the displacement is same.

$F_1 \neq F_2$

$F_{eq} = F_1 + F_2$

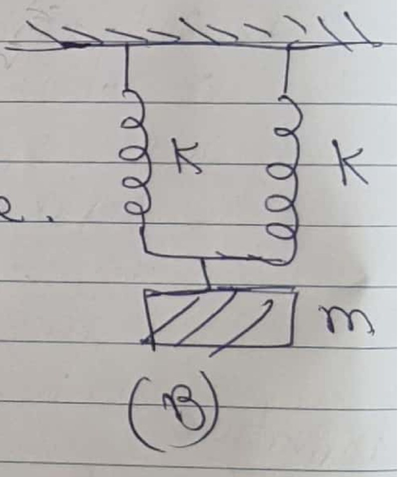
$= -K_1x + -K_2x$

$= -x(K_1+K_2)$

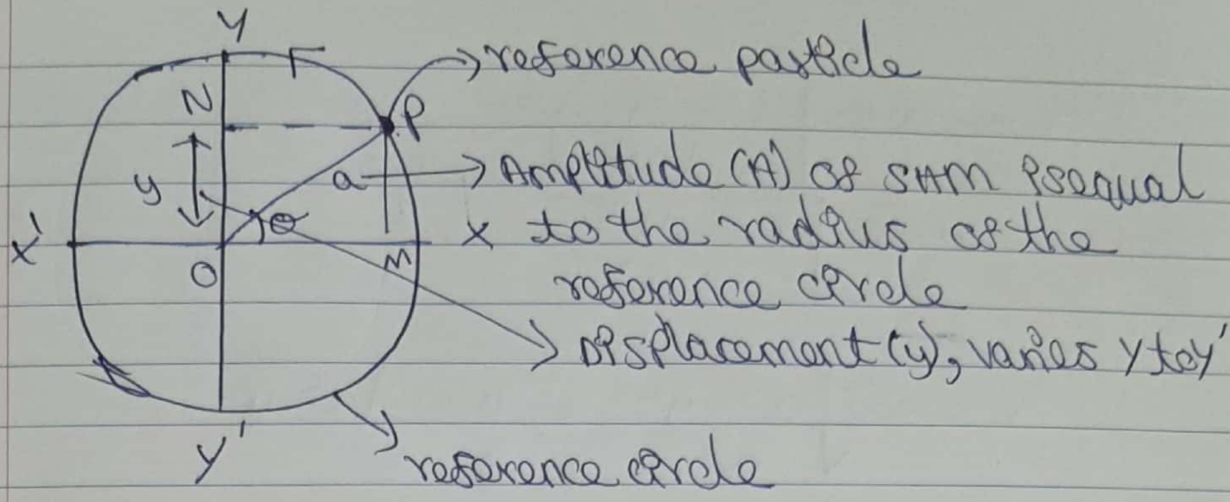
$K_{eq} = K_1 + K_2 = 2K$ in this case.

$T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow \sqrt{\frac{\cancel{2\pi} m}{K_1+K_2}}\ 2\pi$

$T = 2\pi\sqrt{\frac{m}{K_1+K_2}}$ //.



(5)




1) Displacement: The distance of the particle measured by the path of the motion in the mean position at the given instant.
Displacement of particle N = y    ON = OP sin θ

$y = a \sin \omega t \rightarrow ①$

Angular velocity $\omega$, with the time taken $t$ (second) and $\theta$

$\frac{\theta}{t} = \omega \Rightarrow \theta = \omega t \rightarrow ②$

$y = a \sin\theta \Rightarrow a \sin \omega t \rightarrow ③$

2) Amplitude: It is maximum distance covered by the particle in either sides of mean position. The total range is $2x$. Then the SI unit is m. Always positive.

3) Velocity: The velocity of the particle P will move along with YOY'. The velocity of the reference particle.

Velocity of particle N = $a\omega \cos \omega t$

$v = a\omega \cos \omega t$

$= a\omega\sqrt{1 - \sin^2 \omega t}$

$$v = a\omega\sqrt{1 - \left(\frac{ON}{OP}\right)^2}$$

$$= a\omega\sqrt{1 - \frac{y^2}{a^2}}$$

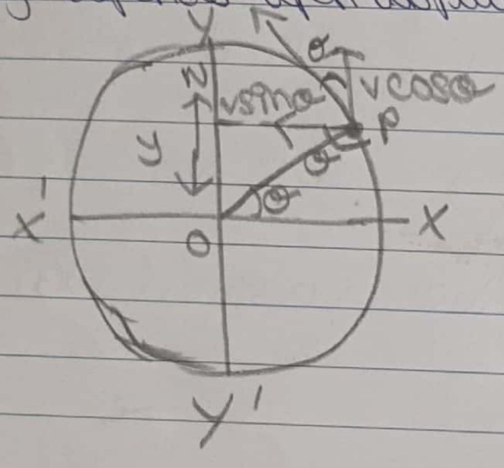
$$v = a\omega\sqrt{\frac{a^2 - y^2}{a^2}} \Rightarrow \frac{a\omega\sqrt{a^2-y^2}}{a}$$

$$v = \omega\sqrt{a^2 - y^2}$$

$\rightarrow v = a\omega$

it is max when $y = 0$ at mean position
it is min when $y = a \Rightarrow 0$

velocity depends upon displacement.




7) Acceleration: The particle P is travelling with velocity v in a circle with radius a, so the acceleration would be $-\omega^2 a$ directed toward the centre of circle. The acceleration is from PN and PM.

$N = \cancel{x} - \omega^2 a \sin\theta$

$a = -\omega^2 y$ $\quad\quad$ $x = -\omega^2 y \sin\theta$

it is max when $y = a$
" is min " $y = 0$

5) Time period: Time taken to complete one full vibration and denoted by T. always (+) It is known as seconds per cycle. SI unit is second.

$$T = 2\pi f \Rightarrow \frac{2\pi}{\omega}$$

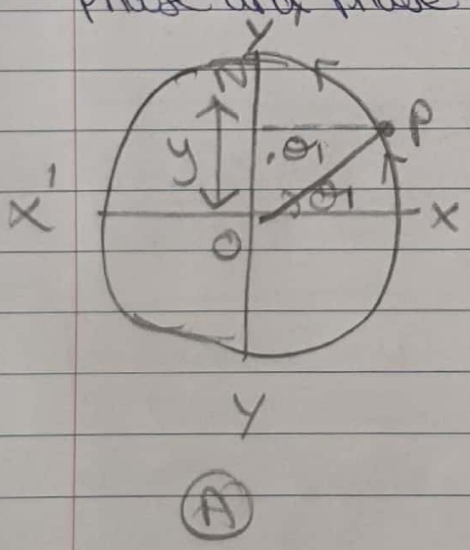
6) Angular velocity ($\omega$) = $\dfrac{\text{Angle described in one revolution}}{\text{Time taken to } \cancel{\text{cover}} \text{ complete one } \cancel{\text{vibration}} \text{ revolution}}$

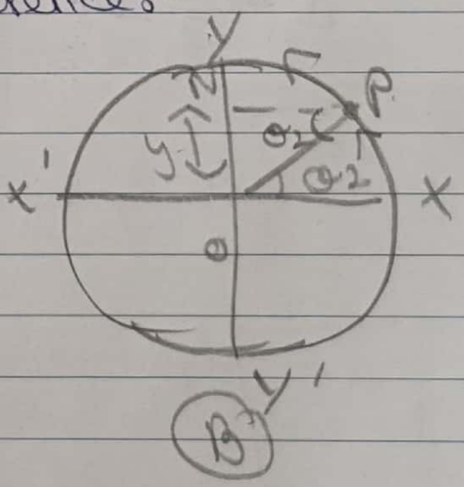
$$\boxed{\omega = \frac{2\pi}{T} \qquad T = \frac{2\pi}{\omega}}$$

7) frequency:- No of vibration of a particle in 1 second. SI unit Hz.

$$n = \frac{1}{T} \qquad f = \frac{1}{T}$$

8) phase and phase difference:-




Phase difference between 2 SHM A and B

$$\theta_1 - \theta_2 = \phi$$

Formulaes 8 -

1) $F = -kx$

↳ displacement

↳ proportionality constant

↓

Force

* when they say that the spring streches it becomes $F = -(-kx)$

2) $F = -kx$

↓

$mg = -kx$ → gravitational force take 9.8 or 10

↳ mass

↳ displacement

↳ proportionality constant

SI units

m (mass) = kg

$g = 9.8\ N/m^2$

$k = N/m$ check

$x = m$

$\omega = \sqrt{\frac{a}{x}}$ a → acceleration, x → displacement

3) $\boxed{\omega\ (\text{omega}) = \sqrt{\frac{k}{m}}}$ k → propotionality constant, m → mass in (kg)

4) $\boxed{v = a\omega}$ a → amplitude, ω → angular ~~freq~~ velocity

↳ velocity

$\boxed{v = \sqrt{\frac{k}{m}} \times a}$

when a not given take a = y or x.

5) $\boxed{v = \omega\sqrt{a^2 - y^2}}$ y → displacement, a → amplitude

↓ v → velocity, ω → angular velocity

$\boxed{v = \sqrt{\frac{k}{m}} \times \sqrt{a^2 - y^2}}$

when y=0 only at mean position that is at maximum

$v = a\omega$ (max) am

at extremity $y = a$ it becomes 0.

velocity = m/s
t = s

6) $v = a\omega \cos\omega t$

velocity $= v \cos\omega t$

$= a\omega \sqrt{1 - \sin^2 \omega t}$ ↳ time in seconds

$v = a\omega$

$\boxed{v = a \times \sqrt{k/m}}$ → amplitude

7) acceleration

$a = -\omega^2 y$ → displacement

8) $\boxed{\omega = 2\pi f}$

or $\frac{2\pi}{T}$ or $\boxed{\sqrt{\frac{k}{m}} = \omega}$ $\boxed{\omega = \frac{2\pi}{T}}$

omega ($\omega$)

9) frequency ⇒ $\boxed{f = \frac{1}{T}}$ ↳ time period

$\boxed{T = \frac{2\pi}{\omega}}$ $\boxed{T = \frac{1}{f}}$

$\omega = 2\pi f$

$\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega}$ $\omega \to \sqrt{k/m}$ ⇒ $\boxed{2\pi\sqrt{\frac{m}{k}} = T}$

10) $A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$

$v_0$ → initial velocity

$\omega$ → omega

$x_0$ → displacement (initial)

11) $\phi = \arctan\left(\frac{-v_0}{\omega \times x_0}\right)$ → initial velocity

phi

omega

→ displacement

displacement

$$x = A\cos(\omega t + \phi)$$

velocity

$$v_x = -\omega A \sin(\omega t + \phi)$$

$$a_x = - \quad \cos(\omega t + \phi)$$

$$x = A\sin(\omega t + \phi)$$

$$v_x = -\omega A\cos(\omega t + \phi)$$

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad T = 2\pi\sqrt{\frac{x}{a}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$y = A\sin\omega t$$

$$v = A\omega\cos\omega t$$

$$TE = \frac{1}{2}kA^2$$ Ampletude

$$PE = \frac{1}{2}kx^2$$ displacement

$$KE = \frac{1}{2}mv^2$$

(6)

Energy of SHM:-

$x = A\sin(\omega t + \phi)$

$v = \frac{dx}{dt}(A\sin(\omega t + \phi))$

$v = A\omega\cos(\omega t + \phi)$

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2} \times m \times (A^2\omega^2 \cos^2(\omega t + \phi))$

$= \frac{1}{2} m\omega^2 (A^2(1 - \sin^2(\omega t + \phi)))$

$= \frac{1}{2} m\omega^2 (A^2 - A^2\sin^2(\omega t + \phi))$

$= \frac{1}{2} m\omega^2 (A^2 - x^2)$

$= \frac{1}{2} \frac{\cancel{m} \times k}{\cancel{m}} (A^2 - x^2)$ $\left[\omega = \sqrt{\frac{k}{m}}\right]$

$E = \frac{1}{2} k(A^2 - x^2)$ when $x = A$ so it will Become $\frac{1}{2}kA^2$ //

(7)

* PE (Potential energy)

$F = kx$

$dW = F.dx$

$W = \int_0^x F.dx \Rightarrow \int_0^x kx.dx \Rightarrow k\int_0^x x.dx$

$\Rightarrow k\left[\frac{x^2}{2}\right]_0^x$

$U \Rightarrow \frac{1}{2}kx^2$ //

$PE \Rightarrow \frac{1}{2}kx^2$ //

## Total energy

$PE + KE = TE$

$TE = \frac{1}{2} k[A^2 - x^2] + \frac{1}{2} kx^2$

$= \frac{1}{2} kA^2 - \cancel{\frac{1}{2} kx^2} + \cancel{\frac{1}{2} kx^2}$

$TE = \frac{1}{2} kA^2$

$$\boxed{\begin{aligned} TE &= \tfrac{1}{2} kA^2 \\ PE &= \tfrac{1}{2} kx^2 \\ KE &= \tfrac{1}{2} mv^2 \end{aligned}}$$

At extreme position

$x = A$

$PE = \frac{1}{2} kA^2$

$KE = 0$

at centre

$x = 0$

$KE = \frac{1}{2} kA^2$

$PE = 0$

(8) Effective mass (spring-mass system)

$dm \rightarrow$ mass of single element

$v \Rightarrow$ velocity of mass element

$m \Rightarrow$ mass of entire spring

$dy \Rightarrow$ displacement produced by one circle

$y \Rightarrow$ entire spring displacement

$V \Rightarrow$ velocity of vibration in entire spring

$l \Rightarrow$ length.

$$T = \int \frac{1}{2} u^2 dm$$

$$dm = \left(\frac{dy}{L}\right) \times m$$

$$T = \int \frac{1}{2} u^2 \frac{dy}{L} \times m$$

$$= \frac{1}{2} \frac{m}{L} \int_0^L u^2 dy$$

velocity of mass element ~~d~~ of the spring
$\alpha$ length from the position

$$u = \frac{vy}{L} \qquad u \propto y$$

$$= \frac{1}{2} \frac{m}{L} \int_0^L \left(\frac{vy}{L}\right)^2 dy$$

$$= \frac{1}{2} \frac{m \times v^2}{L \times L^2} \int_0^L y^2 dy$$

$$= \frac{1}{2} \frac{mv^2}{L^3} \int_0^L \frac{y^3}{3}$$

$$= \frac{1}{2} \frac{mv^2}{\cancel{L^3}} \times \frac{\cancel{L^3}}{3}$$

$$= \frac{1}{2} \frac{mv^2}{3} //$$