13/01/22

Unit - I.

Continuation

Probability distribution

Random experiment:

If in each trial of an expt conducted under identical conditions, the outcome is not unique, but may be any one of the possible outcomes, then such an expt. is called as random experiment.

sample space → set of all outcomes of a random experiment.

Event - a subset of a sample space.

Exhaustive events | Mutually exclusive events | Equally likely events.

Expt. rolling a dice.

A & B are Exhaustive events

A & B are Mutually exclusive events

A & B are equally likely events

A & C are not mutually exclusive

$S = \{1, 2, 3, 4, 5, 6\}$

A = set of all even no's.

$= \{2, 4, 6\}$

B = set of all odd no's

$= \{1, 3, 5\}$

$c = \{1, 2, 4, 6\}$

getting the no. 1 or 2 or 4 or 6.

Independent event:

Events are said to be independent if the happening of an event is not affected by the supplementary knowledge concery the occurance of any no. of the remaining events.

Ex: Expt:

Throwing a dice & picking a card from a pack of cards.

: Getting a no. ~~of~~ 3 doesn't have an influence in picking King in a pack of cards.

Pick a queen from a pack of cards do not have any influence in getting a no. of 6 while throwing a dice.

Probability:

If a random expt. which results in 'n' exhaustive mutually exlusive & equally likely outcomes, out of which m outcomes are favourable to an occurance of an event E, the probability of event E, denoted by $P(E)$ is defined as

$$P(E) = \frac{\text{no. of outcomes which are favouble}}{\text{Total outcomes}}$$

$$P(E) = \frac{m}{n}$$

$m \geq 0$ & $n > 0$ $E \subset S$

$$0 \leq P(E) \leq 1. \qquad \frac{1}{\infty} = 0$$

Axioms:-

any event

1) $0 \leq P(E) \leq 1$
2) $P(S) = 1$ sample space
3) If A & B are mutually exclusive events.

then $P(A \cup B) = P(A) + P(B)$

$$P(E) = \lim_{n \to \infty} \frac{m}{n}.$$

Russel's paradox:

set → collection of well defined objects.

A → set of all person's in a city who shave by themselves.

B → set of all persons in a city who get shaved by Barbers.

To which set will the Barber of the city be attached?

Independent events:-

Two events A & B are independent if

$$P(A \cap B) = P(A) \cdot P(B).$$

Let E be an event. $\bar{E}$ or E^c &

complement of event E

$$P(\bar{E}) = 1 - P(E)$$

sample space.

WKT; $P(S) = 1$

$$P(E \cup \bar{E}) = 1$$

$$P(E) + P(\bar{E}) = 1 \Rightarrow P(\bar{E}) = 1 - P(E)$$

Addition theorem:-

★ If A & B are any two events then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$$P(A\cup B) = \frac{n(A\cup B)}{n(S)}$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A\cap B)}{n(S)}.$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

★ If A & B are independent events P.T $\bar{A}$ & $\bar{B}$ are independent events

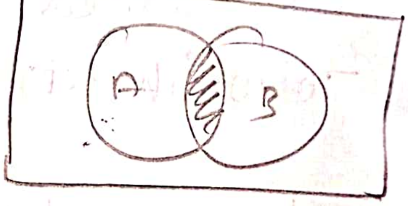
Soln:-

Given : $P(AB) = P(A)P(B)$ $\{A\cap B = AB\}$

To prove : $P(\bar{A}\bar{B}) = P(\bar{A})P(\bar{B})$ $\{\bar{A}\cap\bar{B} = \bar{A}\bar{B}\}$

S

$\bar{A}\cap\bar{B} = S - (A\cup B)$



$A\cap B$

$P(\bar{A})P(\bar{B}) = (1-P(A))(1-P(B))$

$= 1 - P(A) - P(B) + P(A)P(B)$

$= 1 - P(A) - P(B) + P(A\cap B)$ {A & B are independent}

$= 1 - \{P(A) + P(B) - P(A\cap B)\}$

$= 1 - P(A\cup B)$

$= P(S) - P(A\cup B)$

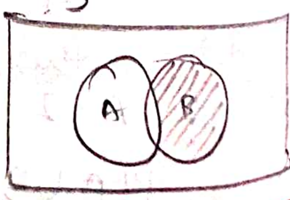
★ If A & B are independent, P.T $\bar{A}$ & B are independent.

Soln:

Given:- $P(AB) = P(A)P(B)$

T.P :- $P(\bar{A}B) = P(\bar{A})P(B)$

S



$\bar{A}\cap B = ?$

$B = (A \cap B) \cup (\bar{A} \cap B)$.

$A \cap B$ & $\bar{A} \cap B$ are mutually exclusive.

$P(B) = P(A \cap B) + P(\bar{A} \cap B)$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= P(B) - P(A)\, P(B)$ {A & B are indept}

$= P(B)\{1 - P(A)\}$

$= P(B)\, P(\bar{A})$.

$\bar{A}$ & B are independent.

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* If A & B are independent events then, P.T A & $\bar{B}$ are also independent.

17/01/22 Recall

Probability

sample space → set of all outcomes of an expt

Event → subset of a samplespace

Mutually exclusive events

Exhaustive events.

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{\text{no. of element in fav. event}}{\text{no. of}}$

Axioms of probability:-

1) $0 \le P(A) \le 1$

2) $P(S) = 1$ {Prob. of sample space is 1}

3) A & B are mutually exclusive events

$P(A \cup B) = P(A) + P(B)$

Addition theorem:

→ If A & B are ~~indep~~ any two events then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$

$\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$

Independent event:

Occurance of one event donot affect the occurance of the other.

$\boxed{P(A\cap B) = P(A) \cdot P(B)}$

If this condition satisfies then v can say that A & B are independent.

If A & B are independent then
$\bar{A}$ & $\bar{B}$ are " "
$\bar{A}$ & B are " " $\quad \bar{A} \rightarrow$ complement of A.
A & $\bar{B}$ are " "

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1. The probability of three students A, B, C solving a problem is $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. A problem is given to all the three students. Determine the probability that $\rightarrow \bar{A}\bar{B}\bar{C}$.

(i) No one will solve the problem
(ii) only one will solve the problem
(iii) Atleast one will solve the problem.

) $A\overline{BC} \cup \bar{A}B\bar{C} \cup \bar{A}\bar{B}C$.

$A\bar{B}\bar{C}$, $\bar{A}B\bar{C}$, $\bar{A}\bar{B}C$, $AB\bar{C}$,
$A\bar{B}C$, $\bar{A}BC$, ABC.

soln:

$P(A)$ = Prob. of A solving the problem $= \frac{1}{2}$ $\quad P(\bar{A}) = \frac{1}{2}$

$P(B)$ = Prob. of B " " " " $= \frac{1}{3}$ $\quad P(\bar{B}) = \frac{2}{3}$

$P(C)$ = " " C " " " " $= \frac{1}{4}$ $\quad P(\bar{C}) = \frac{1}{4}$

(i) Prob. of no one solving the problem

$= P(\bar{A}\bar{B}\bar{C})$. (A, B, C are independent events)

$= P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$

$= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$

$= \frac{1}{4}$ //

(ii) Exactly one solves the problem.

$= \cancel{P(\bar{A}\bar{B}C)}$

$= P(\bar{A}\bar{B}C \cup A\bar{B}\bar{C} \cup \bar{A}B\bar{C})$.

$= P(\bar{A}\bar{B}C) + P(A\bar{B}\bar{C}) + P(\bar{A}B\bar{C})$ {mutually exclusive events}

$= P(\bar{A}) P(\bar{B}) P(C) + P(A) P(\bar{B}) P(\bar{C}) + P(\bar{A}) P(B) P(\bar{C})$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}$

$= \frac{1}{12} + \frac{1}{4} + \frac{1}{8} = \frac{2+6+3}{24} = \frac{11}{24}$ //

(iii) Atleast one solving the problem is same as complement of none solving the problem.

$= 1 - P(\bar{A}\bar{B}\bar{C})$

$= 1 - \frac{1}{4} = \frac{4-1}{4}$.

$= \frac{3}{4}$ //

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2. The odds are 9 to 5 against a person who is 50 years living till he is 70 & 8 to 8 against a person who is 60 years living till he is 80. Find the probablity that atleast one of them will ~~tear~~ live after 20 years.

Soln:

A -> event of 50 years man living till 70 yrs

$P(A) = \frac{5}{14} \rightarrow P(\bar{A}) = \frac{9}{14}$

B -> event of 60 yrs man living till 80 yrs

$P(B) = \frac{6}{14} \rightarrow P(\bar{B}) = \frac{8}{14}$

Prob of atleast one of them in alive =

$$= 1 - P(\bar{A}\,\bar{B})$$

$$= 1 - P(\bar{A})P(\bar{B})$$

$$= 1 - \frac{9}{14} \times \frac{8}{14}$$

$$= \frac{14^2 - 72}{14^2}$$

Conditional probability:

A & B are any two events, Event B has occured, the conditional probability of A given B is denoted as P(A/B) & is defined.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad (P(B) > 0)$$

Illy

$$P(B/A) = \frac{P(B \cap A)}{P(A)} \quad (P(A) > 0)$$

Note:-

If $P(A/B) = P(A)$ then A & B are independent events.

Multiplication theorem:-

Let A, B, C be any three events - then

$P(ABC) = P(A) \cdot P(B|A) \cdot P(C|AB)$.

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(19) If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$, $P(A|B) = \frac{1}{6}$. Find $P(B/A)$, $P(B/\bar{A})$.

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$\frac{1}{3}$

$1 - \frac{1}{3}$

$\frac{2}{3}$

$\frac{1}{2} \times \frac{2}{3}$

$\frac{2}{6}$

soln:

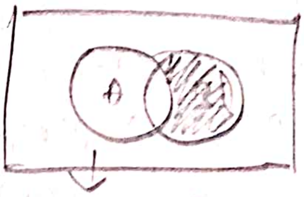
$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{1/12}{1/3}$$

$$= 1/4.$$

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{1}{6}$$

$$P(AB) = P(B) = \frac{1}{6} = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

$$P(B/\bar{A}) = \frac{P(B\bar{A})}{P(\bar{A})} \Rightarrow P(B\bar{A}) = ?$$



$B = (A \cap B) \cup (\bar{A} \cap B)$

$P(B) = P(AB) + P(\bar{A}B)$

$P(\bar{A}B) = P(B) - P(AB)$

$P(B) = \frac{1}{2}$, $P(AB) = \frac{1}{12}$

$$P(B/\bar{A}) = \frac{P(B\bar{A})}{P(\bar{A})}$$

$$= \frac{5/12}{1 - P(A)}$$

$$= \frac{5/12}{1 - \frac{1}{3}} = \frac{\frac{5}{12}}{\frac{2}{3}}$$

$$= \frac{5}{\cancel{12}_4} \times \frac{\cancel{3}}{2} = \frac{5}{4} \times \frac{1}{2} = \frac{5}{8}$$

18/01/22

Conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{if } P(B) > 0.$$

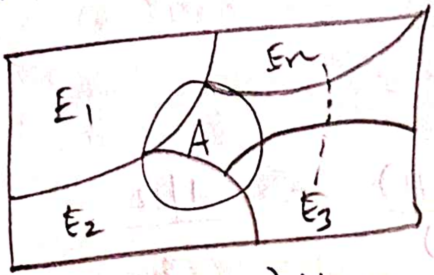
(Baye's theorem:-) I^st subdivision

If $E_1, E_2, E_3, \ldots, E_n$ are mutually disjoint events with $P(E_i) \neq 0$ $1 \le i \le n$, then for any arbitrary event A which is a subset of $\bigcup_{i=1} E_i$ such that $P(A) > 0$ then $P(E_i/A) = \dfrac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)}$

$E_i \cap E_j = \phi$

$\forall i \neq j$

$\& \bigcup_{i=1}^{n} E_i = S$



Proof. $A = (A \cap E_1) \cup (A \cap E_2) \cup \dots (A \cap E_n)$ [Bcz A is obtaind by inter-action of all events]

E_i s are mutually exclusive

$E_i \cap A$ is also " " " " "

$P(A) = P\{(A \cap E_1) \cup (A \cap E_2) \cup \dots \cup (A \cap E_n)\}$

$= P(A \cap E_1) + P(A \cap E_2) + \dots + P(A \cap E_n)$

$\left[\begin{array}{l} P(A/B) = \dfrac{P(A \cap B)}{P(B)} \\ P(A \cap B) = P(A/B)\,P(B) \end{array}\right]$ $= P(A/E_1)P(E_1) + P(A/E_2)P(E_2) + \dots + P(A/E_n)P(E_n)$

$= \sum_{i=1}^{n} P(A/E_i)\,P(E_i)$

$P(E_i/A) = \dfrac{P(E_i \cap A)}{P(A)}$ (By conditional prob)

$\boxed{= \dfrac{P(A/E_i)\,P(E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)}}$

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1. The prob. of X, Y, Z becaming managers are $\frac{4}{9}, \frac{2}{9}, \frac{1}{3}$ respectively, the prob that the bonus scheme will be introduced if X, Y, Z becames managers are $\frac{3}{10}, \frac{1}{2}, \frac{4}{5}$ respectively. If the bonus scheme is introduced what is the prob. that the manager appointed was X?. 2

Soln:

$E_1 \to X$ is appointed as manager

$E_2 \to Y$ " " " " "

$E_3 \to Z$ " " " " "

$B \to$ event that bonus scheme is introduced

To find prob. $\left(\dfrac{E_1}{B}\right)$

Given:

$P(E_1) = \frac{4}{9}$, $P(E_2) = \frac{2}{9}$, $P(E_3) = \frac{1}{3} = \frac{3}{9}$.

$P(B|E_1) = \frac{3}{10}$, $P(B|E_2) = \frac{1}{2}$, $P(B|E_3) = \frac{4}{5}$.

By Baye's theorem;

$$\boxed{P(E_1|B) = \frac{P(E_1)\,P(B|E_1)}{\sum_{i=1}^{n} P(E_i)\,P(B|E_i)}}$$

$$= \frac{\frac{4}{9} \times \frac{3}{10}}{\frac{4}{9} \times \frac{3}{10} + \frac{2}{9} \times \frac{1}{2} + \frac{3}{9} \times \frac{4}{5}} = \frac{12/90}{\frac{12}{90} + \frac{\cancel{2}^{1}}{\cancel{18}_{9}} + \frac{12}{45}}$$

$$= \frac{12/90}{(12+10+24)/90}$$

$$= \frac{\cancel{12}^{6}}{\cancel{46}_{23}}$$

$$= 6/23$$

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2. A & B are two students and their chances of solving a problem in statics are $\frac{1}{6}$, $\frac{1}{8}$ respectively. If the prob. of their making a common error is $\frac{1}{525}$ & they obtain the same answer, find the prob. that their answer is correct.

Soln:

$E_1 \to$ A & B solve the problem correctly

$E_2 \to$ Exactly one of them solves the problem correctly

$E_3 \to$ Neither of them solves the problem correctly.

$E \to$ They get the same answer.

Given:

$P(E_1) = \frac{1}{6} \times \frac{1}{8} = \frac{1}{48}$.

$P(E_2) = \frac{1}{6} \times \frac{7}{8} + \frac{5}{6} \times \frac{1}{8} = \frac{12}{48}$; $P(E_3) = \frac{5}{6} \times \frac{7}{8} = \frac{35}{48}$

$P(E|E_1) = 1$, $P(E|E_2) = 0$, $P(E|E_3) = \frac{1}{525}$.

To find $P(E_1/E)$

$$P(E_1/E) = \frac{P(E_1)P(E/E_1)}{\sum_{i=1}^{3} P(E_i)P(E/E_i)}$$

$$= \frac{\frac{1}{48}\times 1}{\frac{1}{48}\times 1 + \frac{12}{48}\times 0 + \frac{\cancel{35}^{7}}{48}\times\frac{1}{\cancel{525}_{\cancel{105}_{15}}}} = \frac{\frac{1}{48}}{\frac{1}{48}\left\{1+\frac{1}{15}\right\}}$$

$$= \frac{15}{16}$$

Aliter:

E_1 : A & B get the same correct answer

E_2 : A & B " " " wrong " " "

E_3 : A & B " " " answer

To find: $P(E_1/E)$

$$P(E_1/E) = \frac{P(E_1\cap E)}{P(E)} \quad \{\text{conditional prob}\}$$

$$P(E) = P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

$$= \frac{1}{48} + \left(1-\frac{1}{6}\right)\left(1-\frac{1}{8}\right)\frac{1}{525}$$

$$= \frac{1}{48} + \frac{\cancel{5}}{6}\times\frac{\cancel{7}}{8}\times\frac{1}{\cancel{525}_{15}} = \frac{1}{48}\left(\frac{16}{15}\right) = \frac{1}{45}$$

$$P(E_1/E) = \frac{P(E_1\cap E)}{P(E)}$$

$$= \frac{P(E_1)}{P(E)} = \frac{\frac{1}{6}\times\frac{1}{8}}{1/45} = \frac{1/48}{1/45} = \frac{45}{48} = \frac{15}{16}$$

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3. The contents of Bags I, II, III are as follows:-

BI → 1W, 2B, 3R marbles.

BII → 2W, 1B, 1R marbles

BIII → 4W, 5B, 3R marbles.

A bag is choosen & two marbles are drawn from it. They happen to be white & Red. What is the prob. that they come from Bag I.

Soln:

$B_i \to$ Bag B_i is selected.

$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$

A = event that two marbles are selected from the bag (1W & 1R).

$$P(A|B_1) = \frac{1 \times 3}{6C_2} = \frac{3 \times 2}{6 \times 5} = \frac{1}{5}$$

$$P(A|B_2) = \frac{2 \times 1}{4C_2} = \frac{2 \times 2}{4 \times 3} = \frac{1}{3}$$

$$P(A|B_3) = \frac{4 \times 3}{12C_2} = \frac{12 \times 2}{12 \times 11} = \frac{2}{11}$$

$$P(B_1|A) = \frac{P(B_1)\,P(A|B_1)}{\sum_{i=1}^{3} P(B_i)\,P(A|B_i)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{5}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{11}} = \frac{1/5}{\frac{1}{5} + \frac{1}{3} + \frac{2}{11}}$$

$$= \frac{1/5}{\frac{33 + 55 + 30}{165}} = \frac{\cancel{165}^{33}}{\cancel{5} \times 118} = \frac{33}{118}$$

4. A speaks truth 4 out of 5 times. A dice is tossed. He reports that there is a six, what is the chance actually there was six?

Soln.

E_1: A speaks the truth $\qquad P(E_1) = 4/5$

E_2: A " " false $\qquad P(E_2) = 1/5$

E_3: A reports six.

$$P(E|E_1) = \frac{1}{6}, \quad P(E|E_2) = 5/6.$$

To find; $P(E_1/E)$.

$$P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E_1)P(E/E_1) + P(E_2)P(E/E_2)} = \frac{\frac{1}{6} \times \frac{4}{5}}{\frac{1}{6} \times \frac{4}{5} + \frac{1}{5} \times \frac{1}{5}}$$

$$= \frac{4}{9}$$

5. There are two bags A & B. A contains n white & 2 black balls & bag B contains 2 W & n black balls. One of the two bags is selected at random & two balls are drawn from it without replacement. If both the balls drawn are white & the prob. that bag A was used to draw the balls is $\frac{6}{7}$, find the value of n.

Soln:

$E_1 \to$ Event that Bag A is selected
$E_2 \to$ " " " B " " "
$P(E_1) = P(E_2) = \frac{1}{2}$.

$E \to$ " " 2 balls drawn are white

Given: $\boxed{P(E_1/E) = \frac{6}{7}}$.

$$P(E/E_1) = \frac{nC_2}{(n+2)C_2} = \frac{n(n-1)}{(n+2)(n+1)}$$

$$P(E/E_2) = \frac{2C_2}{(n+2)C_2} = \frac{2}{(n+2)(n+1)}$$

Given; $P(E_1/E) = 6/7$.

$$\frac{P(E_1)P(E/E_1)}{P(E_1)P(E/E_1) + P(E_2)P(E/E_2)} = \frac{6}{7}$$

$$\frac{1/2 \times \frac{n(n-1)}{(n+2)(n+1)}}{\frac{1}{2} \times \frac{n(n-1)}{(n+2)(n+1)} + \frac{1}{2} \times \frac{2}{(n+2)(n+1)}} = \frac{6}{7}$$

$$\frac{n(n-1)}{n(n-1)+2} = \frac{6}{7}$$

$$7n^2 - 7n = 6n^2 - 6n + 12$$

$$n^2 - n - 12 = 0.$$

$$(n-4)(n+3) = 0$$

$$\Rightarrow n = 4, -3.$$

$n \neq -3$ & $\therefore n = 4.$

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6. If A & B are two mutually exclusive events

ST: $P(A|\bar{B}) = \dfrac{P(A)}{1-P(B)}$

Soln:

$$P(A|\bar{B}) = \frac{P(A\cap\bar{B})}{P(\bar{B})}$$

$$= \frac{P(A)}{1-P(B)}$$

$\boxed{P(\bar{B}) = 1-P(B)}$

$A = (A\cap B) \cup (A\cap\bar{B})$

$P(A) = \underbrace{\cancel{P(A\cap B)}}_{0} + P(A\cap\bar{B})$

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7. If $P(A) > 0$, $P(B) > 0$ & $P(A|B) = P(B|A)$, PT: $P(A) = P(B)$.

Soln:

Given:

$$P(A|B) = P(B|A)$$

$$\frac{P(A\cap B)}{P(B)} = \frac{P(B\cap A)}{P(A)}$$

$$P(A).\cancel{P(A\cap B)} = \cancel{P(B\cap A)}.P(B)$$

$$P(A) = P(B).$$

Prove or disprove:-

If $P(B|\bar{A}) = P(B|A)$, then A & B are independent

{ A & B are independent if
$P(A\cap B) = P(A)P(B)$ }

Given:

$$P(B|\bar{A}) = P(B/A)$$

$$\frac{P(B\cap\bar{A})}{P(\bar{A})} = \frac{P(B\cap A)}{P(A)}$$

$$P(B\cap\bar{A})P(A) = P(\bar{A})\,P(A\cap B).$$

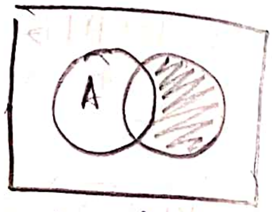
$$P(B\cap\bar{A})\,P(A) = P(A\cap B)\,P(\bar{A})$$

$$\{P(B) - P(A\cap B)\}\,P(A) = P(A\cap B)\,\{1 - P(A)\}.$$

$$P(B)P(A) - \cancel{P(A\cap B)P(A)} = P(A\cap B) - \cancel{P(A\cap B)P(A)}$$

$$P(A\cap B) = P(A)P(B).$$

$\Rightarrow$ A & B are independent.



$B = (A\cap B)\cup(\bar{A}\cap B)$

$P(\bar{A}\cap B) = P(B) - P(A\cap B)$

<u>Prove or disprove:-</u>

If $P(A|B) \geq P(A)$ then $P(B|A) \geq P(B)$.

<u>Given:</u>

$$P(A|B) \geq P(A)$$

$$\frac{P(A\cap B)}{P(B)} \geq P(A).$$

$$\frac{P(A\cap B)}{P(A)} \geq P(B)$$

$$\Rightarrow P(B|A) \geq P(B).$$

---

8. A & B are two independent events such that $P(A\cap\bar{B}) = \frac{3}{25}$ & $P(\bar{A}\cap B) = \frac{8}{25}$, If $P(A) < P(B)$, determine $P(A)$

<u>Soln:-</u>

$$P(\bar{A}\cap B) = 8/25$$

$$P(B) - P(A\cap B) = \frac{8}{25}$$

$$P(A\cap B) = P(B) - \frac{8}{25} \quad - \text{①}$$

$$P(A\cap B) = \frac{3}{25}$$

$$P(A) - P(A\cap B) = \frac{3}{25} \Rightarrow P(A\cap B) = P(A) - \frac{3}{25} \quad - \text{②}$$

$$\text{①} \rightarrow P(A\cap B) = P(B) - \frac{8}{25}.$$

$$P(A)P(B) - P(B) = -\frac{8}{25}.$$

$$-P(B)\{1 - P(A)\} = -\frac{8}{25}$$

$$P(B)\,P(\bar{A}) = \frac{8}{25} \quad - \text{③}$$

$$\text{②} \rightarrow P(A\cap B) = P(A) - \frac{3}{25}$$

$$P(A)P(B) = P(A) - \frac{3}{25}.$$

$$\frac{3}{25} = P(A) - P(A)P(B)$$

$$= P(A)(1 - P(B))$$

$$\boxed{P(A) < P(B)}$$

$$\text{③} \rightarrow P(\bar{B})P(\bar{A}) = \frac{8}{25} \qquad P(A)\,P(\bar{B}) = \frac{3}{25} \quad - \text{③}$$

$$P(A)\{1 - P(B)\} = \frac{3}{25}.$$

$$P(B)P(\bar{A}) = \frac{8}{25}; \quad P(A)P(\bar{B}) = \frac{3}{25}$$

$$P(B)\{1 - P(A)\} = \frac{8}{25}; \quad P(A)\{1 - P(B)\} = \frac{3}{25}.$$

$$P(B) - P(A)P(B) = \frac{8}{25}; \quad P(A) - P(A)P(B) = \frac{3}{25}$$

$$P(B) - P(A) = \frac{8}{25} - \frac{3}{25} = \frac{1}{5}$$

19/01/22.

Binomial distribution:-

Let a random experiment be performed repeatedly and in this each repetition is called a trial. & let the occurance of a trial be called as success and non-occurance is called as failure

1) Expection
2) variance
3) MGF.
4) Getting mean ~~and~~ from MGF.
5) Problems.

trials $\to n$; prob. of success is p; failure is q

$$q = 1-p.$$

SSSFSSFFSS
SFSFFFSFSFS
- - - .

success and failure can be in any pattern, so we represent it as a binomial distribution.

SSFF
SFSF
FFSS
FSSF

A random variable x is said to follow binomial distribution if it assumes only non-negative values and its probability mass function (Pdf) is gn. by;

$$P(X=x) = \begin{cases} nC_x\, p^x q^{n-x} & x = 0,1,2,\dots n \ \& \ q = 1-p \\ 0 & \text{Otherwise.} \end{cases}$$

Note:

$$\sum_x P(X=x) = 1.$$

$$\sum_{x=0}^{n} nC_x\, p^x q^x = 1 \longrightarrow (q+p)^n = 1^n = 1.$$

Expection of bionomial distribution:

$$E(X) = \sum_x x P(X=x)$$

$$= \sum_{x=0}^{n} x \cdot nC_x \, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{\lfloor n}{\lfloor x \, \lfloor n-x} p^x q^{n-x}.$$

$$= \sum_{x=1}^{n} \frac{\lfloor n}{\lfloor x-1 \, \lfloor n-x} p^x q^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{\lfloor n-1}{\lfloor x-1 \, \lfloor (n-1)-(x-1)} p^{x-1} q^{(n-1)-(x-1)}$$

$$= np(q+p)^{n-1}.$$

$$= np$$

variance: $(x) = E(x^2) - (E(x))^2$

$$E(x^2) = \sum_{x=0}^{n} x^2 \cdot nC_x \, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \{x(x-1) + x\} nC_x \, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) nC_x \, p^x q^{n-x} + \overset{np}{\cancel{\sum_{x=0}^{n} x \cdot p^x q^{n-x}}}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{\lfloor n}{\lfloor x \, \lfloor n-x} p^x q^{n-x} + np.$$

$\lfloor x = x(x-1) \lfloor x-2$

$$= \sum_{x=2}^{n} \frac{\lfloor n}{\lfloor x-2 \, \lfloor n-x} p^x \cdot q^{n-x} + np.$$

$$= n(n-1)p^x \sum \frac{\lfloor n-2}{\lfloor x-2 \, \lfloor (n-2)-(x-2)} p^{x-2} q^{(n-2)-(x-2)} + np$$

$= n(n-1)p^2 \cancel{(p+q)}^{n-2} + np$

$= n(n-1)p^2 + np$

$E(x^2) = n^2p^2 - n p^2 + np$

$$\boxed{Var(x) = E(x)^2 - (E(x))^2}$$

$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$

$= -np^2 + np$

$= np - np^2$

$= np(1-p)$

$= npq$

<u>Moment generating fncs</u>:

$M_x(t) = E(e^{tx})$

$M_x(t) = E(e^{tx}) = \sum_{x=0}^{n} e^{tx}\, nC_x p^x q^{n-x}$

$= \sum_{x=0}^{n} nC_x (pe^t)^x q^{n-x}$

$$\boxed{M_x(t) = (q + pe^t)^n}$$

$M_x(t) = (q + pe^t)^n$ (mgf)

$M_x'(t) = n(q+pe^t)^{n-1} \cdot pe^t$ (Diff w.r.t t)

$M_x''(t) = np\{e^t(q+pe^t)^{n-1} + e^t(n-1)(q+pe^t)^{n-2} \cdot pe^t\}$

$M_x'(0) = n(q+p)^{n-1} p = np = \mu_1' = \mu$

$M_x''(0) = np\{1(q+p)^{n-1} + (n-1)(q+p)^{n-2} \cdot p\}$

$= np\{1 + (n-1)p\} = np + n(n-1)p^2 \; (= \mu_2')$

$\mu_2 = \mu_2' - \cdots$

$\mu_2 = \mu_2' - (\mu_1')^2 = np + n(n-1)p^2 - n^2p^2$

$\cancel{n^2p^2} - np^2$

$= npq$

$= np + \cancel{n^2p^2} - np^2 - \cancel{n^2p^2}$

$= npq$

1. With usual notations, determine 'p' for a binomial distribution X if $n=6$ & $9P(X=4) = P(X=2)$.

soln:

~~Given~~: $9P(X=4) = P(X=2)$ $(n=6)$

$$9 \times 6C_4\, p^4 q^{6-4} = 6C_2\, p^2 q^{6-2}.$$

$$\Rightarrow 9 \times \frac{6 \times 5}{1 \times 2} p^4 q^2 = \frac{6 \times 5}{1 \times 2} p^2 q^4.$$

$$\Rightarrow 9 p^{\cancel{4}\,2} \cancel{q^2} = \cancel{p^2} q^{\cancel{4}\,2}$$

$$\Rightarrow 9p^2 = q^2.$$

$$\Rightarrow 9p^2 = q^2$$

$$9p^2 = (1-p)^2$$

$$= 1 - 2p + p^2$$

$$8p^2 + 2p - 1 = 0.$$

$$p = -\frac{1}{2} \text{ (or) } \frac{1}{4}.$$

p is always greater than 0 $\Rightarrow p = \frac{1}{4}$

---

2. comment on the following:

The mean & variance of a binomial distribution are 3 & 4 respectively.

soln:

Given:

mean of a binomial distribution is 3

$$np = 3$$

Variance of a binomial distribution is 4.

ie; $npq = 4$.

$$\frac{npq}{np} = \frac{4}{3} \Rightarrow q = \frac{4}{3} > 1.$$

which is impossible

such a binomial distribution do not exist

---

3. The prob. of a person hitting a target is $\frac{1}{4}$.
(i) If he fires 7 times what is the prob. of hitting the target atleast twice?
(ii) How many times he has to fire so that the prob. of his hitting the target at least once is $> \frac{2}{3}$.

Soln:

(i) $P = \frac{1}{4}$ (success is hitting the target)

$q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$.

$n = 7$. To find $P(X \geq 2)$

$$P(X = x) = 7C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{7-x}.$$

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$

$$= 1 - 7C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{7-0} - 7C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^6.$$

(ii) Prob of hitting the target atleast one $= P(X \geq 1)$

$$= 1 - P(X=0)$$

$$= 1 - nC_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{n-0}.$$

$$= 1 - \left(\frac{3}{4}\right)^n$$

$$1 - \left(\frac{3}{4}\right)^n > \frac{2}{3} \Rightarrow 1 - \frac{2}{3} > \left(\frac{3}{4}\right)^n.$$

$$1 - \frac{2}{3} > \left(\frac{3}{4}\right)^n$$

$$\frac{1}{3} > \left(\frac{3}{4}\right)^n$$

$$\log\left(\frac{1}{3}\right) > n \log\left(\frac{3}{4}\right)$$

$$\log 1 - \log 3 > n\{\log 3 - \log 4\}$$

$$0 - 0.4771 > n\{0.4771 - 0.6021\}$$

$$n > 3.8.$$

=> person has to fire atleast 4 times in order to hit the target with atleast one with prob. $\frac{2}{3}$.

21/01/22.

Pattern for today
1. Defn of distn
2. Mean, variance, MGF, problems

Poisson distribution:

In binomial distribution, if $n \to \infty$ (no. of trials $= n$) p is small (Prob. of success is small) & np is finite then Binomial distribution tends to Poisson distribution.

Definition:

A r.v x is said to be a poisson distribution if it assumes only non-negative values & its probability mass function is given by

$$P(X=x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{\lfloor x} & x = 0, 1, 2, \ldots \quad \lambda > 0 \\ 0 & \text{otherwise.} \end{cases}$$

$\lambda \to$ parameter of the distribution.

Eg: (1)

- No. of accidents in a city (road traffic)

(2) No. of faculty HP computers in a production of 100 HP computers.

③ No. of deaths from disease like heart attack

Note:-

$$\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{\lfloor x}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{\lfloor x}$$

$$= e^{-\lambda}\left\{1 + \frac{\lambda}{\lfloor 1} + \frac{\lambda^2}{\lfloor 2} + \cdots\right\}$$

$$= e^{-\lambda} e^{\lambda} = 1 \qquad \left\{e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \cdots\right\}$$

Expection of poisson distribution:-

$$E(X) = \sum_x x\,P(X=x).$$

$$= \sum_{x=0}^{\infty} \frac{x \cdot e^{-\lambda}\lambda^x}{\lfloor x \lfloor x-1}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{\lfloor x-1}$$

$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{\lfloor x-1} = \lambda e^{-\lambda}\left\{1 + \frac{\lambda}{\lfloor 1} + \frac{\lambda^2}{\lfloor 2} + \cdots\right.$$

$$= \lambda \cancel{e^{-\lambda}} \cancel{e^{\lambda}} = \lambda$$

Variance:- of Poisson distributions

$$Var(X) = E(X^2) - (E(X))^2.$$

$$E(X^2) = \sum_x x^2 P(X=x)$$

$$= \sum_x x^2 \frac{e^{-\lambda}\lambda^x}{\lfloor x}$$

$$= \sum_{x=0}^{\infty} \{x(x-1) + x\} \frac{e^{-\lambda}\lambda^x}{\lfloor x}$$

$$= \sum_{x=0}^{\infty} \frac{x(x-1)e^{-\lambda}\lambda^x}{\lfloor x-2} + \sum_{x=0}^{\infty} \frac{x e^{-\lambda}\lambda^x}{\lfloor x}$$

$$= \lambda^2 e^{-\lambda} \cancel{e^{\lambda}} \cancel{\sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{\lfloor x-2}} + \lambda$$

$$= \lambda^2 + \lambda.$$

$$Var(x) = E(x^2) - (E(x))^2 = \cancel{\lambda^2} + \lambda - \cancel{\lambda^2} = \lambda.$$

$$\boxed{\text{Mean} = \text{variance} = \lambda}$$

Moment generating function:

$$M_x(t) = E(e^{tx})$$

$$= \sum_x \left\{ \frac{e^{tx} e^{-\lambda} \lambda^x}{\lfloor x} \right\}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{\lfloor x}$$

$$= e^{-\lambda} \left\{ 1 + \frac{\lambda e^t}{\lfloor 1} + \frac{(\lambda e^t)^2}{\lfloor 2} + \cdots \right\}$$

$$= e^{-\lambda} e^{\lambda e^t} = e^{-\lambda(1-e^t)}$$

$$M_x(t) = e^{-\lambda(1-e^t)}$$

$$M_x'(t) = e^{-\lambda(1-e^t)} \{-\lambda(-e^t)\}$$

$$= \lambda e^t e^{-\lambda(1-e^t)}$$

$$\boxed{M_x'(0) = \lambda e^0 e^{-\lambda(1-1)} = \lambda}$$

$$M_x''(t) = \lambda \left\{ e^t e^{-\lambda(1-e^t)} + e^t e^{-\lambda(1-e^t)} (\lambda e^t) \right\}$$

$$M_x''(0) = \lambda\{1+\lambda\} = \lambda^2 + \lambda.$$

$$var = M_x''(0) - M_x'(0))$$

$$= \cancel{\lambda^2} + \lambda - \cancel{\lambda^2}$$

$$= \lambda$$

1. A manufacturer of a product knows that 5% of his product is defence. If he sells his product in boxes of 100 & guarantees that not more than 10 products will be defective, what is the propability that a box will fail to meet the guaranteed quality?

Soln:

$$n=100;\ p=\frac{5}{100}=0.05.$$

{p → small when compared with n}

we use poisson distribution.

$$\text{Mean } \lambda = np = 100 \times \frac{5}{100} = 5$$

$$P(X>10) = 1-P(X\le 10)$$

$$= 1-\sum_{x=0}^{10} \frac{e^{-\lambda}\lambda^{x}}{\lfloor x}$$

$$= 1-\sum_{x=0}^{10} \frac{e^{-5}\lambda^{x}}{\lfloor 2}$$

$$= 1-e^{-5}\left\{1+\frac{5}{\lfloor 1}+\frac{5^2}{\lfloor 2}+\cdots+\frac{5^{10}}{\lfloor 10}\right\}$$

---

2. A car hire firm has two cars, which it hires out day by day. The no. of demands for a car on each day is distributed as a poisson distribution with mean (λ) 1.5. Calculate the propotion of days on which (i) neither car is used & (ii) the propotion of days on which some demand is refused.

Soln: Given: demand is poissnally distributed

X → demand.

$$\lambda = 1.5$$

(i) To find $P(X=0)$

$$P(X=x) = \frac{e^{-1.5}(1.5)^{x}}{\lfloor x}$$

$$P(X=0) = \frac{e^{-1.5}(1.5)^0}{\lfloor 0} = e^{-1.5}$$

(ii) To find $P(X>2)$

$$P(X>2) = 1 - P(X \leq 2)$$

$$= 1 - P(X=0) - P(X=1) - P(X=2)$$

$$= 1 - e^{-1.5} - e^{-1.5}(1.5) - e^{-1.5}\frac{(1.5)^2}{\lfloor 2}$$

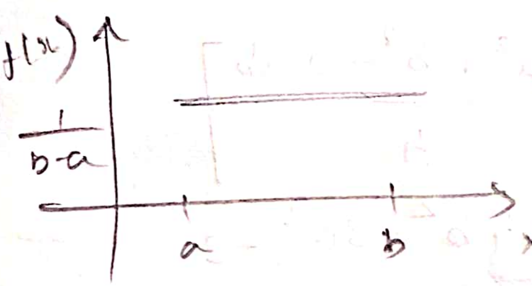
$$= \ast\ast\ast$$

---

continous distribution:-

Rectangular distribution / uniform distribution

Defn: A random variable x is said to have a rectangular distribution over (a, b) if pdf is
$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b. \\ 0 & \text{otherwise.} \end{cases}$$



→ ~~length~~ graph of uniform distribution

Expection of uniform distributions:

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_a^b x \cdot \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\left\{\frac{x^2}{2}\right\}_a^b$$

$$= \frac{b^2 - a^2}{2(b-a)} = \frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}}$$

$$= \frac{b+a}{2}$$

variance of uniform distribution:

$$\text{var}(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$= \int_a^b x^2 \cdot \frac{1}{b-a} dx = \frac{b^3 - a^3}{3(b-a)}$$

$$= \frac{\cancel{(b-a)}(b^2 + ab + a^2)}{3\cancel{(b-a)}}$$

$$\text{var}(x) = E(x^2) - [E(x)]^2$$

$$= \frac{b^2 + ab + b^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{b^2 + ab + b^2}{3} - \left[\frac{a^2 + b^2 + 2ab}{4}\right]$$

$$= \frac{4b^2 + 4ab + 4b^2 - 3a^2 - 3}{12}$$

$$= \frac{b^2 - 2ab + a^2}{12}$$

$$= \frac{(a-b)^2}{12}$$

MGF of uniform distribution:

$$M_x(t) = E(e^{tx})$$

$$= \int_a^b \frac{1}{b-a} e^{tx} \, dx$$

$$= \frac{1}{b-a} \left\{ \frac{e^{tx}}{t} \right\}_a^b$$

$$= \frac{1}{b-a} \left(\frac{e^{tb} - e^{ta}}{t} \right)$$

$$M_x(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}$$

$$\cancel{M'_x(t) = \frac{1}{b-a} \left\{ \frac{e^{tb} \cdot t - e^{ta} \cdot t}{t^2} \right\}}$$

$$\cancel{= \frac{1}{b-a} \left\{ \frac{e^{tb} - e^{ta}}{t} \right\}}$$

$$M'_x(t) = \frac{1}{b-a} \left\{ \frac{t(e^{tb} \cdot b - e^{ta} \cdot a) - (e^{tb} - e^{ta}) \cdot 1}{t^2} \right\}$$

$$= \frac{1}{b-a} \left\{ \frac{e^{tb}\{bt-1\} - e^{ta}(-at+1)}{t^2} \right\}.$$

$M'_x(0) =$

24/01/22

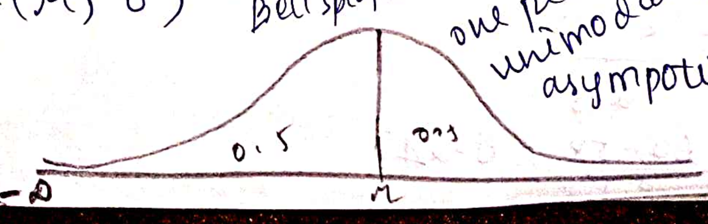
Normal distribution:-

Best continous distribution

Two parameters → mean μ
→ SD σ

$N(\mu, \sigma)$ Bell shaped

one peak
unimodal fnc.
asympotic



Distribution
1. uniform (or) Rectangular
2. Normal
3. Exponential
4. ~~Gamma~~

Large sample test

Compared the calculated value with the normal table values 2.58, 1.96 which represents the area of the normal curve.

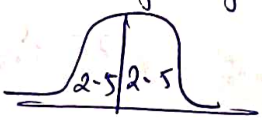
1. Fit a distribution to a data:
2. compare this with normal distribution.
3. we make conclusions on the data.

Standard normal distribution: $\varepsilon N(\mu, \sigma^2)$.

$$Z = \frac{x - \mu}{\sigma} \rightarrow N(0,1).$$

Testing of an hypothesis:-

Two - tailed test → 5% level of significance



single tailed test

Left tailed test ↓

$\bar{x}$

Right tailed test ↓

$\bar{x}$

Eg: the marks obtained by a large group of students in a examination have a mean 58 & SD 8.5. Assuming that these marks are normally distributed what is the % of students with marks in b/w 60 and 69 both inclusive.

Soln:

$x \rightarrow$ marks obtained by the students.

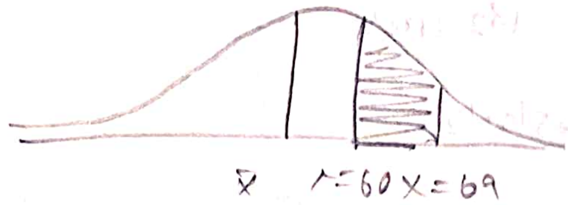
To find $P(60 \le x \le 69) \rightarrow P(0.24 \le Z \le 1.29)$

$$Z = \frac{x - \mu}{\sigma} = \frac{x - 58}{8.5}$$

when $x = 60$, $\Rightarrow \frac{60 - 58}{8.5} = 0.24$

when $x = 69$, $Z = \frac{69 - 58}{8.5}$

$Z = 1.29$.



$P(0.24 \le Z \le 1.29)$

$= P(0 \le Z \le 1.29) - P(0 \le Z \le 0.24)$.

$= 0.4015 - 0.0948$. → Normal table.

$= 0.3067$ //

(or)

⇒ 30.67% of students has obtained

marks in b/w 60 & 69.

(or)

$\{P(0.24 \le Z \le 1.29)\}$

$= P(-\infty \le Z \le 1.29) - P(-\infty \le Z \le 0.24)$.

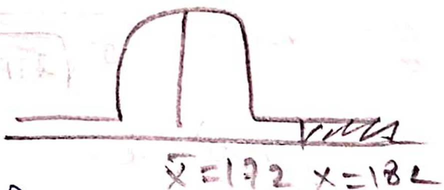
$= 0.90147 - 0.59483$.

$= 0.30664$.

$= 30.66\%$ (have obtained marks

b/w 60 & 69)

Eg: assume the mean height of a soldier to be 172 cms with SD 27 cm. The height of the soldiers are distributed normally. How many soldiers in a regiment of 1000 can be expected to be over 182 cms.



soln.

x → Height of the soldiers

To find $P(X \ge 182) \to P(Z \ge 0.37)$.

$Z = \frac{x - \mu}{\sigma} = \frac{182 - 172}{27} = \frac{10}{27} = 0.37$.

$P(X \ge 182) = P(Z \ge 0.37) = 0.5 - P(0 \le Z < 0.37)$.

$= 0.5 - 0.1443$

$= 0.3557$.

approximately 355 soldiers have an height above

182 cms.

25/01/22.

Normal distribution:-

A r.v x is said to have a normal distri-bution with mean μ & SD 'σ' if the pdf

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < \infty \quad (\sigma > 0)$$

$$Z = \frac{x-\mu}{\sigma}$$

$$\& \; g(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad -\infty < z < \infty$$

table values

> standard normal variate

Moment generating function of a normal variate:-

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

substitude

$$\left.\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ dz &= \frac{dx}{\sigma} \end{aligned}\right\} \Rightarrow \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t(\mu+\sigma z)} e^{-\frac{z^2}{2}} \, \cancel{dx}\, dy$$

$$\Rightarrow \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\{z^2 - t2\sigma z + 2t\mu\}} dz.$$

M M

$$M_x(t) = \frac{1}{\sqrt{2\pi}} e^{t\mu} \int e^{-\frac{1}{2}(\sigma^2 - \cancel{t}2\sigma z)} dz$$

$$= \frac{1}{\sqrt{2\pi}} e^{t\mu} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\{(\sigma - tz)^2 + t^2 z^2\}} dz$$

$$= \frac{1}{\sqrt{2\pi}} e^{t\mu} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\{z^2 - 2t\sigma z\}} dz$$

$$= \frac{1}{\sqrt{2\pi}} e^{t\mu} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\{(z-\sigma t)^2 - \sigma^2 t^2\}} dz$$

$$= \frac{1}{\sqrt{2\pi}} e^{t\mu + \frac{\sigma^2 t^2}{2}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\underbrace{(z-\sigma t)^2}_{u^2}} \underbrace{dz}_{du}$$

$$\boxed{\begin{array}{l} u = z - \sigma t \\ du = dz \end{array}}$$

$$= e^{t\mu + \frac{\sigma^2 t^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{u^2}{2}} du.$$

$$\Rightarrow e^{t\mu + \frac{\sigma^2 t^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{u^2}{2}} du //$$

Moments of normal distribution:

$$\mu_{2n+1} = E(x-\mu)^{2n+1}$$

$$= \int_{-\infty}^{\infty} (x-\mu)^{2n+1} f(x)\, dx$$

$$= \int_{-\infty}^{\infty} (x-\mu)^{2n+1} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \int_{-\infty}^{\infty} (\sigma z)^{2n+1} \cdot \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

$$\boxed{\begin{array}{l} z = \frac{x-\mu}{\sigma} \\ dz = \frac{dx}{\sigma} \end{array}}$$

$$= 0$$

$g(z) = (\sigma z)$

$g(-z) = -(\sigma z)^{2n+1} e^{-z^2/2}$

$g(z)$ is odd fnc

$= -g(z)$.

$$\mu_{2n} = \int_{-\infty}^{\infty} (x-\mu)^{2n} f(x)\, dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^{2n} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z)^{2n} \cdot e^{-\frac{z^2}{2}} dz$$

$$= \frac{\sigma^{2n}}{\sqrt{\pi}} \times 2^{n} \int_{-\infty}^{\infty} u^{(n+\frac{1}{2})-1} \cdot e^{-u} du.$$

$$= \frac{\sigma^{2n}}{\sqrt{\pi}} \times 2^{n-\frac{1}{2}} \int_{-\infty}^{\infty} u^{(n+\frac{1}{2})-1} e^{-u} du$$

$$= \frac{\sigma^{2n} 2^{n-1} \times 2}{\sqrt{\pi}} \int_{0}^{\infty} u^{(n+\frac{1}{2})-1} e^{-u} du$$

$$= \frac{\sigma^{2n} 2^{n-1} \times 2}{\sqrt{\pi}} \Gamma\left(n+\frac{1}{2}\right) \left\{\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx\right\}$$

$$= \frac{\sigma^{2n} 2^{n}}{\sqrt{\pi}} \Gamma\left(n+\frac{1}{2}\right)$$

$z = \frac{x-\mu}{\sigma}$

$dz = \frac{dx}{\sigma}$

$\frac{z^2}{2} = u$

$\frac{\cancel{2} z\, dz}{\cancel{2}} = du$

$z^2 = 2u$

$z^{2n} = (z^2)^n$

$= (2u)^n$

$= 2^n u^n$

$z^{2n+1} = \frac{z^{2n}}{z}$

$\Rightarrow \frac{2^n u^n}{\sqrt{2u}}$

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1. If the ~~length~~ skulls are classified as A, B, C according to length-breadth index is under 75, b/w 75 & 80 & over 80, find approximately (assuming that the distribution is normal) the mean & SD of a series in which A are 58%, B are 38% & C are 4% respectively.

Soln:

$X \to$ length-breadth index of a skull.

Given $P(X < 75) = 0.58$; $P(75 < X < 80) = 0.38$

$P(X > 80) = 0.04$

To find mean (μ) & S.D (σ)

$$\boxed{Z = \frac{X-\mu}{\sigma}}$$

when $x = 75$, $z_1 = \frac{75-\mu}{\sigma}$ $P(x<75) = 0.58$.

when $x = 80$, $z_2 = \frac{80-\mu}{\sigma}$ $P\left(\frac{X-\mu}{\sigma} < \frac{75-\mu}{\sigma}\right) = 0.58$.

$P(x<75) = 0.58 \Rightarrow P(z<z_1) = 0.58$

$\Rightarrow$ from the normal table $P(z<z_1) = 0.58$

$\hookrightarrow$ $z_1 = 0.20$

$P(X>80) = 0.04 \Rightarrow P(z>z_2) = 0.04$

$\Rightarrow$ from the normal table $z_2 = 1.75$

$z_1 = \frac{75-\mu}{\sigma} \Rightarrow 0.2 = \frac{75-\mu}{\sigma}$

$$\boxed{\mu + 0.20\sigma = 75} \quad -① $$

$z_2 = \frac{80-\mu}{\sigma} \Rightarrow 1.75 = \frac{80-\mu}{\sigma}$

$$\boxed{\mu + 1.75\sigma = 80} \quad -② $$

solving ① & ②, we get

$(0.2 - 1.75)\sigma = -5$

$\sigma = \frac{5}{1.75 - 0.2} = \frac{5}{1.55}$

$\mu + 1.75\left(\frac{5}{1.55}\right) = 80$

$$\boxed{\mu = 80 - \frac{1.75 \times 5}{1.55}}$$

27/01/22

Exponential distribution:-

Definition:

A r.v X is said to have an

exponetial distribution with parameter $\theta > 0$, if its PDF is given by

$$f(x) = \begin{cases} \theta e^{-\theta x} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

<u>Note:</u>

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{\infty} \theta e^{-\theta x}\,dx = \theta \left\{ \frac{e^{-\theta x}}{-\theta} \right\}_0^{\infty}$$

$$= -\left(e^{-\infty} - e^{-0}\right) = 1.$$

<u>Find the mean and variance of exponential distribution:-</u>

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^{\infty} x \theta e^{-\theta x}\,dx$$

$$= \theta \int_0^{\infty} x\, d\left(\frac{e^{-\theta x}}{-\theta}\right)$$

Bernoullis formula

$\int u\,dv = uv - u'v_1 + u''v_2$

$u' \rightarrow$

$$E(X) = \theta \int_0^{\infty} x\, d\left(\frac{e^{-\theta x}}{-\theta}\right)$$

$\int u\,dv = uv - u'v_1 + u''v_2 + \dots$

$$= \theta \left\{ x \frac{e^{-\theta x}}{-\theta} - 1 \cdot \frac{e^{-\theta x}}{\theta^2} \right\}_0^{\infty}$$

$$= \theta \left\{ \frac{e^0}{\theta^2} \right\} \boxed{= \frac{1}{\theta}}$$

The mean of exponential distribution is $\frac{1}{\theta}$.

<u>Variance:-</u>

$$E(X^2) = \int_0^{\infty} x^2 \theta e^{-\theta x}\,dx$$

$$= \theta \int_0^{\infty} x^2 \, d\left(\frac{e^{-\theta x}}{-\theta}\right)$$

$$= \theta \left\{ x^2 \frac{e^{-\theta x}}{-\theta} - 2x \cdot \frac{e^{-\theta x}}{\theta^2} + 2 \frac{e^{-\theta x}}{-\theta^3} \right\}_0^{\infty}$$

$$= \theta \left\{ 2 \cdot \frac{1}{\theta^3} \right\} = \frac{2}{\theta^2} //$$

$$E(x^2) = \frac{2}{\theta^2} //$$

$$Var(x) = E(x^2) - (E(x))^2$$

$$= \frac{2}{\theta^2} - \left(\frac{1}{\theta}\right)^2$$

$$= \frac{2}{\theta^2} - \frac{1}{\theta^2} = \frac{1}{\theta^2} //.$$

$$\boxed{Var(x) = \frac{1}{\theta^2} //}$$

<u>Note</u>:-

Mean of an exponential distribution is $\frac{1}{\theta}$, var of an " " " " is $\frac{1}{\theta^2}$.

$$Var(x) = \frac{1}{\theta^2} = \frac{1}{\theta} \times \frac{1}{\theta} = \frac{Mean}{\theta}.$$

$$\boxed{variance = \frac{mean}{\theta}}$$

$$\left\{ \begin{array}{l} \text{when, } \theta = 1 \Rightarrow \text{variance} = \text{mean} \\ \theta > 1 \Rightarrow \text{Variance} < \text{mean} \\ 0 < \theta < 1 \Rightarrow \text{Variance} > \text{mean.} \end{array} \right\} \boxed{\text{Simple note}}$$

<u>Moment generating function (MGF)</u>:- <u>of exponential distribution</u>:-

$$\boxed{M_x(t) = E(e^{tx})}$$

$$= \int_0^\infty e^{tx} \cdot \theta e^{-\theta x} \, dx$$

$$= \theta \int_0^\infty e^{-(\theta - t)x} \, dx$$

$$= \theta \left(\frac{e^{-(\theta - t)x}}{-(\theta - t)} \right)_0^\infty$$

$$\boxed{MGF = \frac{\theta}{\theta - t}}$$

$$M_x(t) = \frac{\theta}{\theta - t} = \frac{\theta}{\theta\left(1 - \frac{t}{\theta}\right)} = \left(1 - \frac{t}{\theta}\right)^{-1}$$

$$M_x(t) = 1 + \frac{t}{\theta} + \frac{t^2}{\theta^2} + \frac{t^3}{\theta^3} + \cdots \qquad \boxed{\mu_2' = \frac{1}{\theta^2}}$$

$$\mu_r' = E(x^r) = \text{coeff of } \frac{t^r}{\lfloor r} \text{ in } M_x(t)$$

$$\mu_1' = \frac{1}{\theta} = \mu, \quad \mu_2' = \frac{2}{\theta^2}$$

$$M_x(t) = \frac{\theta}{\theta - t} = \frac{\cancel{\theta}}{\cancel{\theta}\left(1 - \frac{t}{\theta}\right)} = \left(1 - \frac{t}{\theta}\right)^{-1}$$

$$M_x(t) = 1 + \frac{t}{\theta} + \frac{t^2}{\theta^2} + \frac{t^3}{\theta^3} + \cdots$$

$$\mu_r' = E(x^r) = \text{co-eff of } \frac{t^r}{\lfloor r} \text{ in } M_x(t).$$

$$= \frac{r!}{\theta^r} \qquad r = 1, 2, \ldots$$

$$M_x(t) = E(e^{tx})$$

$$= E\left\{1 + \frac{tx}{\lfloor 1} + \frac{t^2 x^2}{\lfloor 2} + \cdots + \frac{t^r x^r}{\lfloor r} + \cdots\right\}$$

$$= E(1) + \frac{t}{\lfloor 1} E(x) + \frac{t^2}{\lfloor 2} E(x^2) + \cdots + \frac{t^r}{\lfloor r} E(x^r) + \cdots$$

$= 1 + \frac{t}{\lfloor 1} \mu_1' + \frac{t^2}{\lfloor 2} \mu_2' + \frac{t^3}{\lfloor 3} \mu_3' + \cdots + \frac{t^r}{\lfloor r} \mu_r' + \cdots$

$\mu_r' = \text{coeff of } \frac{t^r}{\lfloor r} \text{ in } M_X(t)$

$\mu_1' = \frac{2}{\theta^2}$ $\quad \mu_2 = \mu_2' - \mu_1'^2 = \boxed{\frac{1}{\theta^2}}$

$\mu_2' = \text{coeff of } \left(\frac{t^2}{\lfloor 2}\right) \text{ in } M_X(t) = \boxed{\frac{2}{\theta^2}}$

$\boxed{\mu_3' = \frac{6}{\theta^3}}$ $\quad \frac{t^3}{\theta^3} = \frac{t^3}{\theta^3} \cdot \frac{\lfloor 3}{\lfloor 3}$

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d/02/22

1) X is uniformly distributed with mean 1 & variance 4/3. Find $P(x < 0)$.

Soln:- Mean $= \frac{b-a}{2} = 1 \Rightarrow b + a = 2$

variance $= \left(\frac{a-b}{1}\right)^2 \cdot \frac{1}{12} = \frac{4}{3}$ $\quad a < b$

$\Rightarrow (a-b)^2 = 16 \Rightarrow (b-a)^2 = 16.$

case (i):- $b - a = 4.$

$a + b = 2$

$-a + b = 4$

$2b = 6$

$\boxed{b = 3}$ $\boxed{a = -1}$

case (ii):- $b - a = -4.$

$a + b = 2$

$-a + b = -4$

$2b = -2$

$\boxed{b = -1}$ $\boxed{a = 3}$

case (i) is consider;

$a = -1, \; b = 3.$

$f(x) = \begin{cases} \frac{1}{4} & -1 \le x \le 3 \\ 0 & \text{othe} \end{cases}$

$P(X < 0) = \int_0^1 f(x)\,dx = \int_{-1}^{0} \frac{1}{4}\,dx = \frac{1}{4}(x)_{-1}^{0} = \frac{1}{4}$

2. Subway trains on a certain line run every half hour b/w midnight & 8 in the morning. What is prob that a man entering the station at random time during the period will have to wait atleast 20 minutes?

Soln:-

$X \to$ waiting time of the next train

Assume $\to$ uniformly distributed.

$$f(x) = \begin{cases} \frac{1}{30} & 0 < x < 30 \\ 0 & \text{otherwise} \end{cases}$$

$$P(X \geq 20) = \int_{20}^{\infty} f(x)\,dx = \int_{20}^{30} \frac{1}{30}\,dx = \frac{10}{30} = \frac{1}{3}$$

3. If x has uniform distribution is $[0,1]$, find PDF of $-2\log x$

$\boxed{\text{Pdf of } -2\log x}$

Soln: $G(y)$ = Distribution function of $Y = -2\log x$.

$= P(Y \leq y)$

$= P(-2\log x \leq y)$

$= P(\log x \geq -\frac{y}{2}) = P(x \geq e^{-y/2})$

$G(y) = 1 - P(x \leq e^{-y/3})$

$$= 1 - \int_0^{e^{-y/2}} f(x)\,dx = 1 - \int_0^{e^{-y/2}} dx$$

$G(y) = 1 - e^{-y/2}$

PDF of $Y = \frac{d}{dy}(G(y)) = e^{-y/2}$. $\quad x \to 0$ to 1

$Y \to$