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UNIT-1

BOOLEAN ALGEBRA.

Basic concepts:

1. **set:**

Collection of well-defined objects.

2. **subset:**

Let A & B be any two sets. Then A is said to be a subset of B if each and every element of A is an element of B.

$A \subseteq B$ (A is the subset of B)

$A \subset B$ (proper subset)

A

Empty $\{\}$ — A (same set)

There two Empty & same sets are improper / Trivial subsets.

↳ that contains '0' elements / whose cardinality is '0'.

3. **cardinality:**

No. of elements in A

$|A| = 0 \Leftrightarrow A = \phi$

$A = \{a, e, i, o, u\}$

$|A| = 5$.

4. **Empty set:**

$|A| = 0 \Leftrightarrow A = \phi$

5. **single ton set:**

if its cardinality is one.

6. **Power set:**

Let A be a set. Then the power set of A

is denoted by P(A) or $\wp$(A) or 2^A and it is defined as the set of all the subsets of A.

Eg:, A = {a, b} → cardinality of A is 2

P(A) = {ϕ, {a}, {b}, {a, b}} → 4.

A = {a, b, c} → 3

P(A) = {ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}
{a, b, c}} → 8.

$|P(A)| = 2^{|A|}$ → $2^0 = 1$.

suppose A = ϕ → cardinality of A - 0.

P(A) = {ϕ} → |P(A)| = 1.

7. Union:

A & B (A∪B) A union B.

set of all x belongs to universal sets.
↳ it is subjective.

A∪B = {x ∈ E | x ∈ A or x ∈ B}

A = {1, 2, 3} ; B = {2, 4, 6}

A∪B = {1, 2, 3, 4, 6} Repeated no. should not be written.

8. Intersection:

A∩B = {x ∈ E | x ∈ A and x ∈ B}

A = {1, 2, 3} ; B = {2, 4, 6}

A∩B = {2}

A = {1, 3, 5} B = {2, 4, 6}

$A \cap B = \phi$ (They are dis

* Either $\underbrace{x \in A \text{ or, } x \in B}_{\text{exclusive arc.}}$ but not both.

9. Relative complement:-

Difference b/w A & B

$\Rightarrow A - B = \{ x \in E \mid x \in A \text{ and } x \notin B \}$

$\Rightarrow B - A = \{ x \in E \mid x \in B \text{ and } x \notin A \}$.

Relative complement of B w.r.t A.

Relative complement of A w.r.t B.

10. Symmetric difference:-

$\boxed{A \Delta B} = (A - B) \cup (B - A)$ (Definition)

Eg. $A = \{1, 2, 3, 4, 5\}$ $B = \{3, 4, 6\}$

$A - B = \{1, 2, 5\}$; $B - A = \{6\}$

set of elements in A

but not in B

$A \Delta B = \{1, 2, 5, 6\}$.

11. Complement:- $\bar{A}$ or A^c or A'.

A bar   A power   A prime.

$A' = \{ x \in E / x \notin A \}$

$E = \{1, 2, 3, \ldots, 25\}$

$A = \{1, 2, 3, \ldots, 15\}$

$A = \{16, \ldots, 25\}$.

12. <u>cartesian</u> <u>product</u>: $A \times B$

$A \times B = \{(a,b) \mid a \in A$ and $b \in B\}$

1st element & 2nd element
from 1st set from 2nd set.

<u>Eg</u>: $A = \{1,2,3\}$ $B = \{a,b\}$
cardinality - 3 car. 2.

$A \times B = \{(1,a)\ (1,b)\ (2,a)\ (2,b)\ (3,a)\ (3,b)\}$
car - 6

$B \times A = \{(a,1)\ (a,2)\ (a,3)\ (b,1)\ (b,2)\ (b,3)\}$

$A \times B \neq B \times A$?

Bcz $(1,a) \neq (a,1)$

* when $A \times B = B \times A$?

$A = B$ then $A \times B = B \times A$.

(X) $\boxed{|A \times B| = |A|\,|B|}$

<u>Relations</u>.

Any subset of $A \times B$ is the relation from A to B

$|A| = m$, $B|n| =$ How many relations are there b/w them?

$|A| = m$ $|B| = n$

2^{mn}

$2^{|A \times B|}$

$2^{|A|\,|B|}$ (with m above $|A|$ and n above $|B|$)

$\phi \subseteq A \times B$ — VOID

$A \times B \subseteq A \times B$ — UNIVERSAL

* Any subset of $A \times A$ is a relation on A

$A \times A \times A \times \ldots$ n times

$A = \{0, 1\}$ $A \times A = \{(0,0)(0,1)(1,0)(1,1)\}$

card - 2   card - 4

$A \times A \times A = \{(0,0,0)\ (0,0,1)\ (0,1,0)$

$(1,0,0)\ (0,1,1)\ (1,0,1)$

$(1,1,0)\ (1,1,1)\}$

card - 8.

$A \times A \times \ldots$ n times $= \{(0,0,\ldots,0),$

$(0,0,\ldots,1)\ (1,1,1\ldots,1)\}$.

* Any subset of $A \times A \times A \times \ldots$ n times is called as n-ary relation on A.

  ⇒ n = 2 → Binary

  ⇒ n = 3 → Ternary

Relations always refer binary relation. Whenever we see the relation it is binary relation.

Properties :-

Let R be a relation on A.

* Reflexive
* symmetric (A is related to B, B is related to A)
* Transitivity {(A, B) & (B, A) then A is related to C}
* Anti. symmetric (A is related to B, B is not related to A)
* Assymmetric.
* Irreflexive.

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Relation.

Any subset of $A \times B$ is a relation from A to B.

Any subset of $A \times A$ is a relation on A.

Void : $\{\}$ Universal : $A \times A$.

($\forall$ - for all).

Properties:

Let R be a relation on A. Then R is said to be;

(i) Reflexive : if $(a, a) \in R$, $\forall a \in A$.

(ii) symmetric : if $(a, b) \in R \Rightarrow (b, a) \in R$ $\forall\ a, b \in A$.

(iii) Transitive : if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$ $\forall\ a, b, c \in R$.

(iv) Anti-symmetric :- if $(a, b) \in R$ and $(b, a) \in R$ whenever $a = b$. (equal)

(v) Assymmetric :- if $(a, b) \in R$ then $(b, a) \notin R$ $\forall\ a, b \in A$.

(vi) Irreflexive :- if $(a, a) \notin R$ $\forall a \in A$.

Eg: A = set of all real no's.

(i) $R = \{(a, b) \mid a \leq b,\ a, b \in A\}$ (It is reflexive) $[\leq]$

$2.45 \leq 2.45$

$5 \leq 5$

$6 \leq 8$.

But it is not sym-

(iii) $a \preceq b$, $b \preceq c$. $\Rightarrow a \preceq c$. (It is transitive, reflexive, but it is not symmetric).

what is the relation b/w them? (a, c)

(iv) $a \preceq b$ and $b \preceq a$ $\Rightarrow$ only if $\boxed{a = b}$ (it is not irreflexive)
(It is antisymmetric, reflexive, transitive)

(v)
$\rightarrow$ Equal to relation ('=')

$R = \{(a,b) / a = b\}$ (It is sym, reflexive)

$\boxed{a = a}$ It is reflexive

$aRb \rightarrow a = b \Rightarrow b = a \Rightarrow bRA$ (sym).

aRb, bRC $\Rightarrow a = b$ & $b = c$

$\Rightarrow a = c$ (aRc) (it is transitive).

(iv) aRb and bRa

$a = b$ $\quad$ $b = a$ $\quad$ $\boxed{a = b}$

Suppose $R = \{(a,b) :$ a divides $b\}$ $\because a | b$.

a $\{$a does not divides $b\}$ $\because a \nmid b$.

Relation
- Equivilance
- Partial order relation.

Equivilance relation :- Reflexive, symmetric, Transitive that satifies.
Eg: ("=" relation) "equal to"

Partial Order relation :- That satifies reflexive, Anti-symmetric, transitive.

'Equal' relation:

Eg: $A = \{1, 2, 3, \ldots, 10\}$

$R = \{(a,b) / a \equiv b \pmod 4\}$ (concurent) meaning (a-b should be divisible by 4)

Or in other words;

when a is divided by 4, b will as 4 as remainder

$\boxed{4 / a-b}$

$a \equiv a \pmod 4 \rightarrow \frac{0}{4} = 0.$

↓

Reflexive.

$a \equiv b \pmod 4 \Rightarrow 4 \mid a-b$

$\Rightarrow 4 \mid -(a-b)$

$\Rightarrow 4 \mid (b-a)$

$\Rightarrow b \equiv a \pmod 4.$

∴ $a \equiv b \pmod 4 = b \equiv a \pmod 4$

∴ it is reflexive, symmetric, transitive.

if $a \equiv b \pmod 4$ and $b \equiv c \pmod 4$

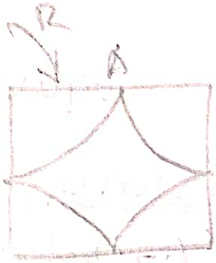
$\Rightarrow 4 \mid a-b$ and $4 \mid b-c$

$\Rightarrow a-b = 4K_1$ and $b-c = 4K_2$

$a-c = a-b+b-c.$

$= 4K_1 + 4K_2$

$= 4(K_1+K_2)$



$\Rightarrow 4 \mid a-c$

∴ $a \equiv c \pmod 4$.

※ It decomposes the set to equally disjoint subsets.

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

The elements that are related to one (1).

$[1]_R = \{1, 5, 9\}$

These 3 elements are related to one.

$[2]_R = \{2, 6, 10\}$

$[3]_R = \{3, 7\}$

$[4]_R = \{4, 8\}$

$[5]_R = \{5, 9, 1\}$

$[6]_R = \{2, 6, 10\}$

$[7]_R = \{3, 7\}$

$[8]_R = \{4, 8\}$

$[9]_R = \{1, 5, 9\}$

$[10]_R = \{2, 6, 10\}$.

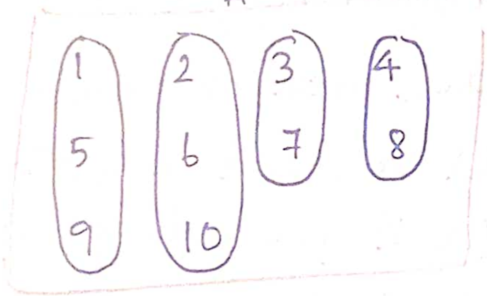
We have only 4 different classes.

$\{1, 5, 9\} = 3$
$\{2, 6, 10\} = 3$
$\{3, 7\} = 2$
$\{4, 8\} = 2$.

Equivalence classes (4)

~~$\{5, 9, 1\} = 2$~~
~~$\{2, 6$~~

A



$R = \{(a, b) / a \equiv b \pmod 4\}$

[Each subset is a equivalence class]

Note:

if it is having a huge data set we want to analyse each and every element but it is not possible. so in that time we want to define some equivalence relation that decomposes the set into the mutually disjoints subsets. from each subset take one representative and anylise it.

Boolean Algebra is a special type of latice. and latice it is a special type of poset or partially ordered set.

Partially ordered set is defined as a non-empty set together with a partial order relation.

Boolean algebra → lattice
↓
A non-empty set together with a partial order relation is called poset. ← Poset or partially ordered set.

Eg: $A = \{1, 2, 3, \ldots, 10\}$

$R = \{(a, b) /$ ~~$/$~~ → such that ~~that~~ $R = \{(a, b) : a \mid b\}$ → such that.

$(A, \preceq)$ → Poset / partially ordered set.

$\preceq$ → (Precedence symbol)

For eg: $(a \preceq b)$ It should not be read as a is lesser than or equal to b. It should be read as a is related to b. (a is precedence to b)

Relation is a subset of $A \times A$ (A cross A).

$A \times A$ has 100 elements.

$R = \{(1,1)\ (1,2)\ (1,3), \ldots (1,10)$
$(2,2)\ (2,4)\ (2,6)\ (2,8), (2,10)$
$(3,3)\ (3,6)\ (3,9)$
$(4,4)\ (4,8)\ (5,5)\ (5,10)\ (6,6)\ (7,7)$
$(8,8)\ (9,9)\ (10,10)\}$.

Eg: $A = \{1, 2, 3, 4\}$

$R = \{(1,1)\ (1,3)\ (1,4)\ (2,1)\ (2,2)\ (3,1)$
$(3,2)\ (3,3)\ (4,4)\}$.

∴ It is a subset of $A \times A$ ∴ R is the

1 -> 1, 2, 3, 4, 6, 8, 12, 24

2 -> 2, 4, 6, 8, 12, 24

3 -> 6, 12, 24, 3

4 -> 4, 8, 12, 24

6 -> 6, 12, 24

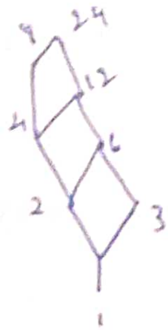
8 -> 8, 24

12 -> 12, 24

24 -> 24.

if $a \leq b$ means a must be placed below b..

→ Immidiate succer of 1 is (2, 3)
2 is (4, 6), 3 is (6), 4 is (8, 12), 6 is (12),
8 is (24), 12 is (24), 24 is nothing.



Importance of Hasse.
⇒ In Hasse diagram it says every element is related to itself.

it is Anti-symmetric, transitive,

H|W:

1) Draw Hasse diagram for the following posets.

(i) $(S_{36}, /)$

(ii) $(S_{32}, /)$

(iii) $(S_{48}, /)$

join → l u b
least upper bound

meet → great upper bound
g u b

Answers:

(i) $(S_{36}, /) = \{1, 2, 3, 4, 6, 9, 12, 36, 18\}$.

Prime factor

$36 = 2 \times 18$

$= 2 \times 2 \times 9$

$= 2^2 \times 3^2$

$= (2+1) \times (2+1)$

$= 3 \times 3 = 9$

$1 \to 1, 2, 3, \cancel{4}, \cancel{6}, \cancel{9}, \cancel{12}, \cancel{18}, \cancel{36}$.

$2 \to 2, 4, 6, \cancel{12}, \cancel{18}, \cancel{36}$.

$3 \to 3, 6, 9, 12, 18, 36$.

$4 \to 4, 12, 36$.

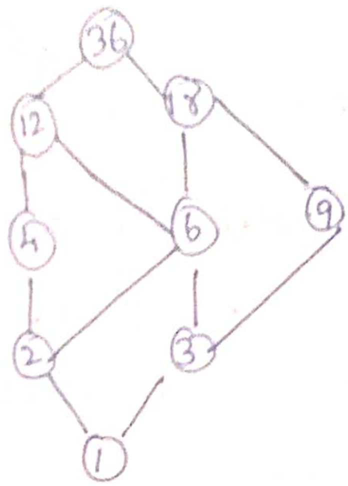
$6 \to 6, 12, 18, 36$.

$9 \to 9, 18, 36$.

$12 \to 12, 36$.

$18 \to 18, 36$.

$36 \to 36$.



(ii) $(S_{32}, /) = \{1, 2, 4, 8, 16, 32\}$

$1 \to 1, 2, 4, 8, 16, 32$

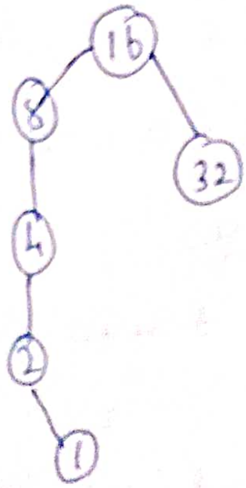
$2 \to 2, 4, 8, 16, 32$

$4 \to 4, 8, 16, 32$

$8 \to 8, 16, 32$

$16 \to 16, 32$.

$32 \to 32$



(iii) $(S_{48}, /) = \{1, 2, 4, 6, 3, 8, 12, 16, 24, 48\}$.
↳ division (or) (S_{36}, D)

$1 \rightarrow 1, 2, 3, 4, 6, 8, 12, 16, 24, 48$.

$2 \rightarrow 2, 4, 6, 8, 12, 16, 24, 48$

$3 \rightarrow 3, 6, 12, 24, 48$

$4 \rightarrow 4, 8, 12, 16, 24, 48$

$6 \rightarrow 6, 12, 24, 48$

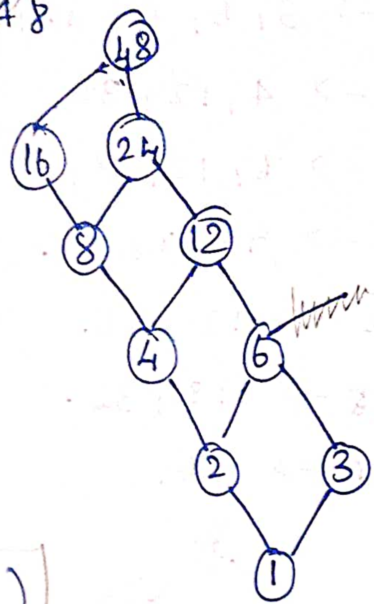
$8 \rightarrow 8, 16, 24, 48$

$12 \rightarrow 12, 24, 48$

$16 \rightarrow 16, 48$

$24 \rightarrow 24, 48$

$48 \rightarrow 48$



$\boxed{(S_n, /), (S_n, D), (D_n, /)}$

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Least element: (An element which is related to all element)

Let $(X, \leq)$ be a poset. Then an element $a \in X$ is said to be a least element if $a \leq$ (related) x, $\forall x \in X$.
↳ for all:

$\forall \rightarrow$ for all
$\leq \rightarrow$ related.

Greatest element:

Let $(X, \leq)$ be a poset. Then an element $a \in X$ is said to be a greatest element if $x \leq a$, $\forall x \in X$.

Note:

* Least element and the greatest element are unique.
* Least element is denoted by zero '0'.
* Greatest element is denoted by one '1'.

Subset of

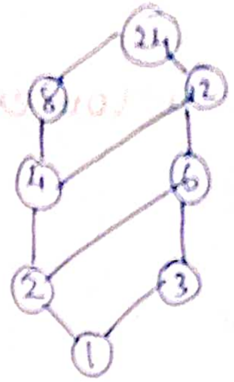
Lower bound:-

Let $(X, \leq)$ be a poset and $A \subseteq X$. Then an element $x \in X$ is said to be a lower bound of A if $x \leq a, \forall a \in A$.
↳ related.

Upper bound:-

Let $(X, \leq)$ be a poset and $A \subseteq X$. Then an element $x \in X$ is said to be an upper bound for A if $a \leq x, \forall a \in A$.

Eg: (S_{24}, D)



$X = S_{24}$

$A = \{2, 4, 6\}$

X

1 3 8 24

2 4 6 12

lower bounds:-

$2 \geq 1, 2$

$4 \geq 1, 2, 4$

$6 \geq 1, 2, 3, 6.$

Lower bound for A are $\{1, 2\}$

Upper bounds:-

$2 \leq 2, 4, 6, 8, 12, 24$

$4 \leq 4, 8, 12, 24$

$6 \leq 6, 12, 24.$

Upper bound for A are $\{12, 24\}$

Least upper bound:- (lub)

Let $(X, \leq)$ be a poset and $A \subseteq X$. Then an element $x \in X$ is said to be a least upper bound for A if;

(i) x is an upper bound for A

(ii) $x \le y$ for all upper bounds y of A.

Among all upper bounds which is the least

?

Greatest lower bound:

Let $(X, \le)$ be a poset and $A \subseteq X$. Then an element $x \in X$ is said to be a glb of A. if;

(i) x is a lower bound for A.

(ii) x $y \le x$ for all lower bounds y of A.

Eg: (i) (S_{30}, D)

Let $X = S_{30}$

$S_{30} = X = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$A = \{3, 5, 6\}$

Least upper bound: (lub)

↓

upper bound $3 \le 3, 6, 15, 30$

$5 \le 5, 10, 15, 30$

$6 \le 6, 30.$

since; there is only one upper bound the lub is 30.

Greatest lower bound: (glb)

$3 \ge 1, 3$

$5 \ge 1, 5$

$6 \ge 1, 2, 3, 6.$

glb = 1.

(ii) $X = S_{30}$

$A = \{2, 5, 10)$

Least upper bound:

upper bounds

$2 \leq 2, 6, 10, 30)$

$5 \leq 5, 10, 15, 30)$

$10 \leq (10, 30)$

$UB - \{10, 30\}$

$LUB = 10.$

GLB:

$2 \geq 1, 2$

$5 \geq 1, 5$

$10 \geq 1, \cancel{10}\ 2, 5$

$LB = \{1\}$

$GLB = 1.$

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(iii) $(S_{72}, D) = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$

Let $X = S_{72}$

$A = \{4, 6, 8, 12\}$

Ub:

$4 \leq 4, 8, 12, 24, 36, 72$

$6 \leq 6, 12, 18, 24, 36, 72$

$8 \leq 8, 24, 72$ $\qquad$ $ub: \{24, 72\}$

$12 \leq 12, 24, 36, 72$

$lub(A) = \{24\}$

GLB:

$4 \succeq$ 1, 2, 4

$6 \succeq$ 1, 2, 3, 6

$8 \succeq$ 1, 2, 4, 8

$12 \succeq$ 1, 2, 3, 4, 12, 6.

GLB = {2}

join ← LUB - $\vee, \sqcup, +, \oplus$

meet ← GLB - $\wedge, \sqcap, \cdot, \odot$

LATICE:- (Both LUB and GLB both shld exist)

A poset $(X, \preceq)$ (precedence) is said to be a lattice if both $a \vee b$ (join) and $a \wedge b$ (meet) exist for all $a, b \in X$.

Eg: (i) $X = \{2, 3, 5, 6, 10, 15\}$.

$R = \{(a, b) : a \mid b\}$.

(X, R) is a poset.

$\boxed{\text{bcz R is a partial order relation (why X, R is a poset).}}$

Let $a = 2$, $b = 3$

LUB: $2 \preceq$ 2, 6, 10

$3 \preceq$ 3, 6, 15

$2 \vee 3 = 6$.

GLB: $2 \succeq 2$

$3 \succeq 3$.

GLB of 2 & 3 doesn't exist.

$2 \wedge 3$ doesn't exist

$\therefore$ (X, R) is not a lattice.

GLB {3, 10} = 1

LUB {3, 10} = Does not exist.

$\therefore$ (X, R) is not a lattice.

Bounded lattice: (Both least element and greatest element).

A lattice $(L, \preceq)$ (precedence) is said to be a bounded lattice if it has both least element and greatest element.

0 - denotes L element
1 - denotes G element

Complement:

Let $(L, \leq)$ be a bounded lattice (→ both 0 & 1) then an element $a \in L$ is said to have complement if such that $a \wedge b = 0$ (GLB, meet) and $a \vee b = 1$ (LUB, join).

$\{\{1,2,3,4,5\}, \leq\}$

$1 \leq 1, 2, 3, 4, 5$
$2 \leq 2, 3, 4, 5$
$3 \leq 3, 4, 5$
$4 \leq 4, 5$
$5 \leq 5$

usual less than or equal to relation

Hasse diagram: 5, 4, 3, 2, 1

Eg: (S_{24}, D).

$S_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$ (1 = LE, 24 = GE).

An element can have any no. of complements.

$2'$ (2 prime)

$2 \wedge 3 = 1$ (0)

$2 \vee 3 = 6$.

$2'$ → doesn't have any complement.

$3'$ → 8 is the complement of 3.

∵ 8 is the complement of 3, 3 is also the complement of 8.

$1'$ → 24
$2'$ → doesn't have any complement
$3'$ → 8 complement.
$4'$ → - (Doesn't have complement).
$6'$ → - (" " " " ").
$8'$ → 3
$12'$ → -
$24'$ → 1

$x \wedge y$ (and) → 0, $x \vee y$ (or) → 1

least element is the greatest element and greatest element is the least element.

Complemented lattice:

A bounded lattice $(L, \leq)$ is said to be a complemented lattice if each and every element of L has atleast one complement.

Boolean Algebra:

A complemented distributive lattice is known as Boolean algebra.

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c).$$

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c).$$

Note:

In Boolean algebra the complement is unique.

Algebric system or Alegebric structure or Algebra:

$X \rightarrow$ One are more n-ary operations.

HW:

Find the complements of all the elements of the following lattice.

(i) (S_{12}, D) (ii) (S_{30}, D) (iii) (S_{48}, D) (iv) (S_{18}, D)

(v) (S_{36}, D)

that element can't be the another element

Answers:

(i) $(S_{12}, D) = \{\overset{LE}{\underset{0}{1}}, 2, 3, 4, \cancel{6}\}\ 6, \overset{GE}{\underset{1}{12}}\}$

$1' \rightarrow \cancel{6}\,12$

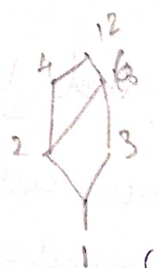
$2' \rightarrow -$

$3' \rightarrow 4$

$4' \rightarrow 3$

$6' \rightarrow \cancel{\phi}\ -$

$12' - 1.$



$\boxed{\begin{array}{l} 0' = 1 \\ 1' = 0. \end{array}}$

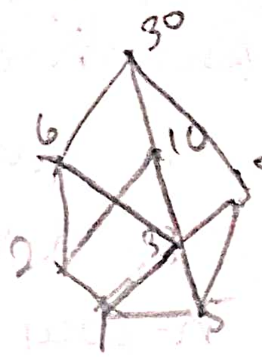
$a \mid b\ (3, 4)$

$3 \wedge 4 = 1$

(lub) $3 \vee 4 = 12$

(ii) $(S_{30}, D) = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$1' - 30$	$1 \to 1, 2, 3, 5, 6, 10, 15, 30$
$2' - 15$	$2 \to 2, 6, 10, 30$
$3' - 10$	$3 \to 3, 6, 15, 30$
$5' - 6$	$5 \to 5, 10, 15, 30$
$6' - 5$	$6 \to 6, 30$
$10' - 3$	$10 \to 10, 30$
$15' - 2$	$15 \to 15, 30$
$30' - 1$	$30 \to 30$



(iii) $(S_{48}, D) = \{1, 2,$

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Boolean algebra:-

A complemented distributive lattice is known as boolean algebra.

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

n-ary operations:

Any function from $A \times A \times A \times \dots$ n times is called as n-ary operation.

$\boxed{A^n}$

$A = \{0, 1\}$ $\quad A \times A = \{(0,0)\ (0,1)\ (1,0)\ (1,1)\}$

$A \times A \times \dots n \text{ times} = \{(0,0,0\dots,0)(0,0,\dots 1)(0,0-1,0)$
$\dots\ (1,1,1,\dots 1)\}$

?

$n=1 \rightarrow$ Unary operation.

$n=2 \rightarrow$ binary operation

$n=3 \rightarrow$ ternary operation.

Algebra or Algebric system or Alegebric structure :-

$(X, *, \oplus, \odot, \dots)$

$(B, \overset{lub}{+}, \overset{glb}{\cdot}, \overset{complement}{'})$ - ?

$\underbrace{+, \cdot}_{\text{Binary operation.}}$

Boolean algebra:-

Boolean algebra is an algebric structure defined on a set of elements B together with two binary operators + and $\cdot$ provided the following postulates are satisfied;

(1) (a) closure with respective the operator $+$

(b) closure w.r.t the operator $\cdot$

(2) (a) An identity element w.r.t. $(+)$ designated by zero (0) ie;

B
x y
$x+y = z \in B$
$x \cdot y = z \in B$

$$x+0=0+x=x$$

(b) An identity element w.r.t. $(\cdot)$ designated by one (1) ie;

$$x \cdot 1 = 1 \cdot x = x$$

(3) (a) commutative w.r.t $(+)$ ie;

$$x+y=y+x$$

(b) commutative w.r.t $(\cdot)$ ie;

$$x \cdot y = y \cdot x.$$

(4) (a) The operator $(\cdot)$ is distributive over $(+)$

$$x \cdot (y+z) = x \cdot y + x \cdot z.$$

(b) The operator $(+)$ is distributive over $(\cdot)$

$$x+(y \cdot z) = (x+y) \cdot (x+z).$$

(5) (a) For every element $x \in B$ there exists
$\exists$ symbol for there exist
($\exists$) an element $x' \in B$ such that $x+x'=1$ and $x \cdot x' = 0$.

(6) ($\exists$) atleast 2 elements $x, y \in B$ such that $x \neq y$. (Every boolean algebra must have 2 element)

(X) These postulates are HUNTINGTON POSTULATES.

B×B   B

(0,0) → 0
(0,1) → 1
(1,0) → 1
(1,1) → 1

$0 \vee 0 = 0$ (lub) join
$0 \vee 1 = 1$

(+)

(+) → denotes lub
(·) → denotes glb.

B   B

(0,0) → 0
(0,1) → 1
(1,0) → 1
(1,1)

$0 \wedge 0 = 0$
$0 \wedge 1 = 0$ meet (glb)
$1 \wedge 0 = 0$
$1 \wedge 1 = 0$

(·)

(+) operator (OR)

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

(·) operator (AND)

x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

LUB → $0 \le 0$, $\cancel{0} \le 1$

GLB → $0 \ge 0$, $1 \ge 0, 1$.

GLB → 0

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[cardinality of boolean algebra is 2^n].

Two-valued Boolean algebra.

$B = \{0, 1\}$

$B = \{0, 1\}$

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

(OR)

x	y	$x \cdot y$
0	0	0
0	1	0
1	1	1
1	0	0

(AND)

x	x'
0	1
1	0

complement.

$x+y+z$

2^n

n· no· of variable

$2^3 = 8$

$8 \div 2 = 4$

$4 \div 2 = 2$

$2 \div 2 = 1$

x	y	z	$x+y+z$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

(i) $x \cdot (y+z) = (x \cdot y) + (x \cdot z)$

(ii) $x + (y \cdot z) = (x+y) \cdot (x+z)$.

(i)

x	y	z	$y+z$	$x \cdot (y+z)$	$x \cdot y$	$x \cdot z$	$(x \cdot y)+(x \cdot z)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

(III) $x + (y \cdot z) = (x+y) \cdot (x+z)$

x	y	z	$y \cdot z$	$x+(y\cdot z)$ ~~$x+y$~~	$x + y$	$(x+z)$	$(x+y)\cdot(x+z)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

1) closure 2) identity 3) commutative 4) distributive
5) complement 6) $x \neq y$.

(I). Associative. $\longrightarrow$ $a+(b+c) = (a+b)+c$
$a \cdot (b \cdot c) = (a \cdot b) \cdot c$.

(II) Idempotent. $\longrightarrow$ $a+a=a$; $a \cdot a = a$.

(III) ABSORPTION $\longrightarrow$ $a \cdot (a+b) = a$; $a+(a \cdot b) = a$.

Duality principle:

Every algebric expression deducible from the postulate of Boolean algebra remains valid if the operators and the identity elements are interchanged.

$x+y = y+x \rightarrow$ comm $\qquad x \cdot (y \cdot z) = (x \cdot y) \cdot z$

$x \cdot y = y \cdot x \qquad x \cdot (y+z) = (x \cdot y) + (x \cdot z)$

$x+(y+z) = (x+y)+z \qquad x+(y \cdot z) = (x+y)\cdot(x+z)$

$x \cdot x' = 0$

$x + x' = 1$.

Theorem 1:- (Idepotent property).

(a) $x + x = x$

(b) $x \cdot x = x$

$x + 0 = x$

$x \cdot 1 = x$.

Proof:-

b) $x = x \cdot 1$ [$\because x = x \cdot 1$]

$= x \cdot (x + x')$ [$\because x + x' = 1$].

$= (x \cdot x) + (x \cdot x')$ [By distributive law)

$\therefore x \cdot x = x$ // [$\because x \cdot x' = 0$].

a) $x = x + 0$ (By identity property.

$= x + (x \cdot x')$ [$\because x \cdot x' = 0$]

$= (x + x) \cdot (x + x')$ [By distributive property

$= (x + x) \cdot 1$ [$\because x + x' = 1$]

$\therefore = x + x$. [$\because x \cdot 1 = x$].

$\therefore x + x = x$ //

Hence proved//

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Theorem:2: (Dominance property)

(a) $x + 1 = 1$

(b) $x \cdot 0 = 0$.

Proof:

$(x + 1) = x + (x + x')$

$= (x + x) + x'$ (Associative law)

$= x + x'$ (Idepotent law)

$= 1$.

(or)

$x+1 = 1\cdot(x+1)$ — P2 (Postulate 2) [$\because x \cdot 1 = x$]

$= (x+x')\cdot(x+1)$ (Postulate 5) [$x+x'=1$]

$= x + (x' \cdot 1)$

$\Rightarrow x + x' = 1$ (By complement law)

$= 1$ //

Rough

$a = x$

$b = x'$

$c = 1.$

$a\cdot(b+c) = a\cdot b + a\cdot c$

$a+(b\cdot c) = (a+b)\cdot(a+c)$

(b) $x\cdot 0 = 0 + (x\cdot 0)$ [By $x \cdot 0 = 0$]

$= (x\cdot x') + (x\cdot 0)$ [By complement law]

$= x\cdot(x'+0)$. [By distributive law]

$= x\cdot x'$ [By ~~complement~~ Identity law]

$\Rightarrow x\cdot x' = 0$ [By complement law]

$x \cdot 0 = 0$ //

Rough

$a = x$

$b = x'$

$c = 0$

Theorem 3: (Involution)

$(x')' = x$ [Involution].

Proof: $x + x' = 1 \quad x\cdot x' = 0$ → By complement law.

$(x')' = x.$

lub glb (+), (·), complement only these are having no other things

$a \cdot b = 0$

$a + b = 1$

$a' = b$, $b' = a$.

Theorem 4: (Associative)

(a) $x+y+z = (x+y)+z$

(b) $x\cdot y\cdot z = (x\cdot y)\cdot z$

Theorem 5:- (De-morgon's law)

(a) $(x+y)' = x' \cdot y'$

(b) $(x \cdot y)' = x' + y'$

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

$\cup \rightarrow +$

$\cap \rightarrow \cdot$

$- \rightarrow$ '

A B

x y

$\overset{I}{(1)}\ 2\ \overset{II}{(3)}$

$1=2$

$2=3$

$3=1.$

Proof:

(a) $\overset{I}{\underset{1}{(x+y)}} \underset{2}{+} \overset{II}{\underset{3}{(x' \cdot y')}}$

$= ((x+y)+x') \cdot ((x+y)+y')$. (By distributive property)

$= ((y+x)+x') \cdot ((x+y)+y')$ (By commutative property.

$= (y+(x+x')) \cdot (x+(y+y'))$ (By associative property)

$= (y+1) \cdot (x+1)$ (By complement law)

$= 1 \cdot 1$ $[\because x+1 = x]$

$= 1$ $[\because 1 \cdot 1 = 1]$

$\therefore (x+y) + (x' \cdot y') = 1$ — ①

Now, $(x+y) \cdot (x' \cdot y') = (x \cdot (x' \cdot y')) + (y \cdot (x' \cdot y'))$

(By distributive property)

$= [(x \cdot x') \cdot y'] + [y \cdot (x' \cdot y')]$

[By associative prop

$= (0 \cdot y') + (y \cdot (y' \cdot x'))$ (By commutative law & complement law)

$= 0 + ((y \cdot y') \cdot x')$ [By identity law) and (associative law)

$= (0 \cdot x')$ [By identity law & complement

$= 0$ $[\because x \cdot 0 = 0]$

$\therefore (x+y)\cdot(x'\cdot y') = 0$ — ②

From ① & ②, we infer that

$$(x+y)' = x'\cdot y'.$$

(b) Proof (H/W).

$$(x\cdot y)' = x'+y'$$

$(x\cdot y)\cdot(x'+y') = (x\cdot y)\cdot x' + (x\cdot y)\cdot y'$ (distributive)

$= (y\cdot x)\cdot x' + \cancel{(y\cdot x)\cdot y'}$ ~~(com~~ $x\cdot(y\cdot y')$ ((commutative & associative)

$= y\cdot(x\cdot x') + x\cdot 0$ (Associative & complement)

$= y\cdot 0 + x\cdot 0$ [complement & identity).

$= 0 + 0 = \cancel{0}$ (identity).

$= \cancel{0+0} = 0$ (idopotent)

$\therefore (x\cdot y)\cdot(x'+y') = 0$ — ③

Now; $(x\cdot y) + (x'+y') = x + (x'+y')\cdot(y+x'+y')$ (distributive)

$= ((x+x')+y')\cdot$ ~~(As~~ $(x'+y'+y)$ (Associative & commutative)

$= (1+y')\cdot(x'+(y'+y))$ (complement & associative)

$= 1\cdot(x'+1)$ (Dominace & complement)

$= 1\cdot 1$ [identity & dominance)

$= 1$ (idopotent)

$\therefore (x\cdot y) + (x'+y') = 1$ — ④

Fom ③ & ④ we infer that

$$(x\cdot y)' = x'+y'.$$

Theorem 6:- (Absorption law)

(a) $x \cdot (x+y) = x$

(b) $x + (x \cdot y) = x$.

Proof:-

(a) $x \cdot (x+y) = x$.

$x \cdot (x+y) = x \cdot x + x \cdot y$. (By distributive law)

$= x + x \cdot y$ (By idempotent law)

$= x \cdot 1 + x \cdot y$ (By identity law)

$= x \cdot (1+y)$ (By distributive law)

$= \cancel{x \cdot (+)}\ x \cdot 1$ [$\because x + 1 = x$]

$= x \cdot 1 \rightarrow x$. (By identity law)

$\therefore x \cdot (x+y) = x$.

(b) $x + (x \cdot y) = x$.

$x + (x \cdot y) = x \cdot 1 + x \cdot y$ (By identity law)

$= x \cdot (1+y)$ (By distributive law)

$= x \cdot (1)$ [By $x + 1 = x$] theo-2

$= x \cdot 1 \rightarrow x$ (By identity law)

$\therefore x + (x \cdot y) = x$.

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Generalised De-morgan's law:-

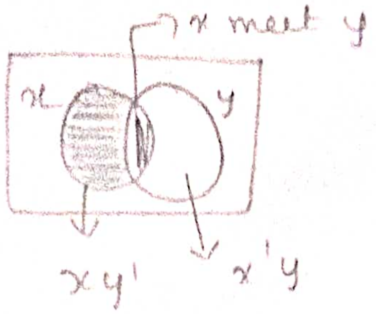
$(x_1 + x_2 + \cdots + x_n)' = x_1' \cdot x_2' \cdot x_3' \cdots x_n'$.

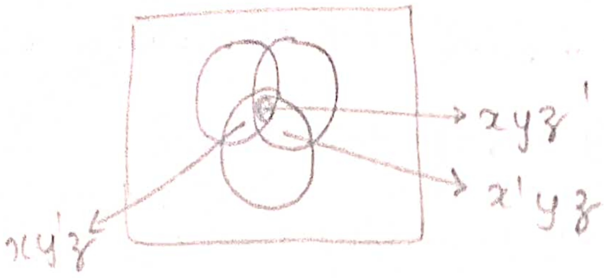
$(x_1 \cdot x_2 \cdots x_n)' = x_1' + x_2' + \cdots + x_n'$

Venn diagram:

we don't have the (- minus), (∩ intersection)

we have only three lub, glb, complement $(+), (\cdot), (')$.





$(x+y)'+xy$.

(i) ( ) [Paranthesis]

(ii) complement

(iii) and, or.

Boolean functions:-

Binary variable -?

A variable can take only two values ie; 0 (or) 1.

It is an expression formed by binary variables, the two binary operators And ~~(or)~~ and OR and uniary operator NOT, paranthesis and an equal sign (=).

Eg: (i) $F = xyz'$ } For boolean functs.

(ii) $F = xy + xz'$.

(i)

x	y	z	z'	xy	xyz'
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	1	0	0
0	1	1	0	0	0
1	0	0	1	0	0
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	1	1

(ii) $xy + xz'$.

x	y	z'	xy	xz'	$xy + xz'$
0	0	1	0	0	0
0	0	0	0	0	0
0	1	1	0	0	0
0	1	0	0	0	0
1	0	1	0	1	1
1	0	0	0	0	0
1	1	1	1	1	1
1	1	0	1	0	~~0~~ 1

Literal:-

A literal is a primed or unprimed variable.

Primed variable:

complement

$x + x'y$. How to simplify this?

$$x + x'y = x + (x' \cdot y) \text{ (Distributive)}$$

$$\boxed{\begin{array}{c} x + (y \cdot z) \\ (x+y) \\ (x+z) \end{array}}$$

$$= (x + x') \cdot (x + y)$$

$$= 1 \cdot (x + y)$$

$$= x + y \text{ (identity)}$$

(2) $x \cdot (x' + y) = (x \cdot x') + x \cdot y$

$= 0 + x \cdot y$

$= x \cdot y.$

(3) $x'y'z + x'yz + xy'.$

$= x'z\,(y' + y) + xy'$

$= x'z \cdot 1 + xy'$

$= x'z + xy'.$

(4) $xy + x'z + yz \neq \cancel{xy + (x' + y) \cdot z}$

$= xy + x'z + y \cdot z \cdot 1$

$= xy + x'z + yz\,(x + x')$

$= xy + x'z + yzx + yzx'$ (Distributive

$= xy + x'z + 1 + yzx + yzx'$

$= x'z \cdot 1 + xy \cdot 1$

$= x'z + xy$

(5) $(x + y)(x' + z)(y + z)$

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Complement of a function :- (function mean boolean)

$(x+y+z')' = (x'+y'+z')$

(i) $(x+y+z)' = x' \cdot y' \cdot z'$ (Generalised De-morgon's law).

x	y	z	x'	y'	z'	$x+y+z$	$(x+y+z)'$	$x' \cdot y' \cdot z'$
0	0	0	1	1	1	0	1	1
0	0	1	1	1	0	1	0	0
0	1	0	1	0	1	1	0	0
0	1	1	1	0	0	1	0	0
1	0	0	0	1	1	1	0	0
1	0	1	0	1	0	1	0	0
1	1	0	0	0	1	1	0	0
1	1	1	0	0	0	1	0	0

Hence it is proved//.

(ii) Find the complement of $F = (x' \cdot y \cdot z') + (x' \cdot y' \cdot z)$

$(x+y'+z) \cdot (x+y+z')$. [De-morgon's law]

$F' = x'yz' + x'y'z$.

$F' = (x'yz')' \cdot (x'y'z)'$.

On applying generalised De-morgon's law.

$F' = (x+y'+z) \cdot (x+y+z')$

(iii) Find F' if $F = x(y+z') + y(x+z) + z(x'+y')$.

$F = x(y+z') + y(x+z) + z(x'+y')$

$F' = (x' + y'z) \cdot (y' + x'z') \cdot (z' + xy)$.

⇒ Let us consider two variable x and y.

* **AND operation:**

How many operations can it have?

$x \cdot y$, $x' \cdot y$, $x \cdot y'$, $x' \cdot y'$.

* **OR operation:**

$x+y$, $x'+y$, $x+y'$, $x'+y'$.

⇒ Let us consider 3 variables x, y, z.

* **AND operation:**

$x'y'z'$, $x'y'z$, $x'yz'$, $xy'z'$, $x'yz$, $xy'z$, xyz', xyz.

* **OR operation:**

$x'+y'+z'$, $x'+y'+z$, $x'+y+z'$, $x+y'+z'$, $x'+y+z$, $x+y'+z$, $x+y+z'$, $x+y+z$.

⇒ Let us consider n variables,

So it have 2^n terms.

AND Operator - minterms / standard products $(\cdot)$

OR Operator - max terms / standard sums $(+)$.

x	y	Minterm x/y	Desig	Maxterm	Desig
0	0	$x'y'$	m_0	$x+y$	M_0
0	1	$x'y$	m_1	$x+y'$	M_1
1	0	xy'	m_2	$x'+y$	M_2
1	1	xy	m_3	$x'+y'$	M_3

Minterm:
If the value of is 'zero' is v should write complement. If it is 'one' no complement. It is for minterms. For max in place of 0

Minterm denotes the m_j → decimal equivalent of no.

Maxterm denotes the M_j → " " " " " "

1 → 1 6 → 110

2 → 10 7 → 111

3 → 11 8 → 1000

4 → 100 9 → 1001.

5 → 101

x	y	z	Minterm (m_j) Term	Desig	(Max term) (M_j) Term	Desig.
0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
0	1	0	$x'yz'$	m_2	$x+y'+z$	M_2
0	1	1	$x'yz$	m_3	$x+y'+z'$	M_3
1	0	0	$xy'z'$	m_4	$x'+y+z$	M_4
1	0	1	$xy'z$	m_5	$x'+y+z'$	M_5
1	1	0	xyz'	m_6	$x'+y'+z$	M_6
1	1	1	xyz	m_7	$x'+y'+z'$	M_7

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Min terms: (m_j)

Maxterms: (M_j)

$x+y+z$, $x+y+z'$, $x+y'+z$, $x+y'+z'$,

$x'+y+z$ + $x'+y+z'$ + $x'+y'+z$, $x'+y'+z'$.

It is denoted by the M_j

where $j \rightarrow$ denotes the de

Boolean function can be expressed as a sum of minterms. (Sum of product canonical form)

Boolean function can be expressed as a product of maxterms. (Product of sum canonical form).

1. Obtain the sum of product canonical form for the following boolean functions:

(i) $F(x,y) = x + x'y$

(ii) $F = xy + xz + yz$

(iii) $F = xy + x'y + z$.

Soln:-

(i) $F = x + x'y$. (By using T.T).

F

x	y	x'	$x'y$	$x+x'y$	Minterms
0	0	1	0	0	$x'y'$ m_0
0	1	1	1	1	$x'y$ m_1
1	0	0	0	1	xy' m_2
1	1	0	0	1	xy m_3

In Formula coloumn select the '1'.

$\therefore$ The sum of product canonial form of F is

$x'y + xy' + xy$

(or) (or) (or)

$m_1 + m_2 + m_3$ $\Sigma(1,2,3)$

16, 8, 4, 2,

without using truth table (T.T):-

$F = x + x'y$.

$= x \cdot 1 + x'y$ (identity law).

$= x \cdot (y+y') + x'y$ (complement law)

$= x \cdot y + x \cdot y' + x' \cdot y$ (Distributive property)//

(ii) $F(x,y) = xy + xz + yz$. (By using T.T).

x	y	z	xy	xz	yz	xy + xz + yz	Minterms
0	0	0	0	0	0	0	$x'y'$ m_0
0	0	1	0	0	0	0	$x'y'$ m_1
0	1	0	0	0	0	0	$x'y$ m_2
✓0	1	1	0	0	1	1✓	$x'y$ m_3
1	0	0	0	0	0	0	xy' m_4
✓1	0	1	0	1	0	1✓	xy' m_5
✓1	1	0	1	0	0	1✓	xy m_6
✓1	1	1	1	1	1	1✓	xy m_7

∴ the sum of product canonical form of F is

$x'yz + xy'z + xyz' + xyz$. / m_3, m_5, m_6, m_7 /

without using truth table (T.T):- $\Sigma(3,5,6,7)$

$F = xy + xz + yz$.

$= xy \cdot 1 + xz \cdot 1 + yz \cdot 1$ (identity law)

$= xy \cdot (z+z') + xz \cdot (y+y') + yz \cdot (x+x')$

(complement law)

$= xyz + xyz' + xyz + xy'z + xyz + x'yz$ (Distributive)

~~i.e.~~ $= xyz + xyz' + xy'z + x'yz$ -

$\therefore$ the sum of product minterms of F is

$$x'yz + xyz' + xy'z + xyz$$

(iii) $F = xy + x'y + z$. (H/w)

Using Truth table:

x	y	z	xy	x'	x'y	xy+x'y+z	minterms.
0	0	0	0	1	0	0	$x'y'$ m_0
0	0	1	0	1	0	1	$x'y'$ m_1
0	1	0	0	1	1	1	$x'y$ m_2
0	1	1	0	1	1	1	$x'y$ m_3
1	0	0	0	0	0	0	xy' m_4
1	0	1	0	0	0	1	xy' m_5
1	1	0	1	0	0	1	xy m_6
1	1	1	1	0	0	1	xy m_7

The sum of product canonical form of F is

$m_1, m_2, m_3, m_5, m_6, m_7$.

without using (TT):

$$F = xy + x'y + z$$

2. obtain product of sum canonical form for the foll. boolean function.

~~(i) x+~~ (i) $F(x,y) = x + x'y$

(ii) $F(x,y,z) = (x+y)\cdot(x+z)\cdot(y'+z)$

(iii) $F(x,y,z) = xy + yz + \cancel{xy}$.

Note:
maxterms-
1 → prime
minterms
0 → prime

Soln:-

(i) Using truth table:-

$x + x'y$

x	y	x'	$x'y$	$x+x'y$	Maxterms
0	0	1	0	0	$x+y\cancel{+z}$
0	1	1	1	1	$\cancel{x}+x+y'$
1	0	0	0	1	$x'+y$
1	1	0	0	1	$x'+y'$

∴ the product of sum of maxterms of F is
$(x+y)$ / ~~m~~ M_0 / $\Sigma'(1)$.

without using TT:-

$F = x + x'y$. (1+2·3)

$= (x+x')\cdot(x+y)$ (By distributive law)

$= 1\cdot(x+y)$ (By ~~iden~~ complement law)

$= x+y$ (By identity law)

∴ the product of sum of maxterms is $x+y$//

(ii) $F = (x+y)\cdot(x+z)\cdot(y'+z)$

without using truth table (TT).

$$\Rightarrow (x+y)\cdot(x+z)\cdot(y'+z)$$

$$= ((x+y)+0)\cdot((x+z)+0)\cdot((y'+z)+0) \quad \text{(Identity law)}$$

1 + 2·3

$$= ((x+y)+(z\cdot z'))\cdot((x+z)+(y\cdot y'))\cdot((y'+z)+(x\cdot x'))$$

(By complement law)

$$= ((x+y+z)\cdot(x+y+z'))\cdot((x+y+z)\cdot(x+y'+z))\cdot((x+y'+z)\cdot(x'+y'+z)).$$

$$= (x+y+z)\cdot(x+y+z')\cdot(x+y'+z)\cdot(x'+y'+z)$$

(iii) $F = xy + yz + xz$

x	y	z	xy	yz	xz	$xy+yz+xz$	maxterms
0	0	0	0	0	0	0	$x+y$
0	0	1	0	0	0	0	$x+y$
0	1	0	0	0	0	0	$x+y'$
0	1	1	0	1	0	1	$x+y'$
1	0	0	0	0	0	0	$x'+y$
1	0	1	0	0	1	1	$x'+y$
1	1	0	1	0	0	1	$x'+y'$
1	1	1	1	1	1	1	$x'+y'$

01/11/21:

Find the sum of maxterms.

$f(x,y,z) = xy + x'z$.

$= xy \cdot 1 + x'z \cdot 1$

$= xy \cdot (z + z') + x'z \cdot (y + y')$

$= xyz + xyz' + x'yz + x'y'z$.

$= \Sigma(1,3,6,7)$

$\therefore$ sum of minterms of F $\} = \Sigma(1,3,6,7)$.

$\therefore$ Product of maxterms of F $\} = \Pi.(0,2,4,5)$.

Standard forms.

Sum of products — Boolean expression contains 'AND' terms of one or more literals each.

product of sums — Boolean expression contains 'OR' items

Standard form

$(\cdot) + (\cdot) + (\cdot)$.

Every term is made up of only AND operator

canonical form.

sum of minterms

$(\cdot) + (\cdot) + (\cdot) + \dots$

Every term is made up of AND operator any containing all variables.

Disjunction → $\vee$ → OR

T or F

Conjunction → $\wedge$ → AND

Negation → $\neg$ or $\sim$ complement.

P	Q	$P \vee Q$	$P \wedge Q$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

T → Tautology

F → contradiction

SOPCF → PDNF

SOSCF → PCNF

Exclusive OR:- → $xy' + x'y$.

$x = 1, y = 1$ but not both

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

NAND:-

x	y	$x \uparrow y$
0	0	1
0	1	1
1	0	1
1	1	0

AND' $x \cdot y$

0

0

0

1

NOR:-

x	y	$x \downarrow y$
0	0	1
0	1	0
1	0	0
1	1	0

$x+y$

0

1

1

1

x	y	$F=0$	$F=1$	$F=x\cdot y$	$F=xy'$	$F=x'y$	$F=x+y$	$F=x$	$F=y$	$F=y'$	$F=x'$	$F=x'+y$	•	•	$F=x\uparrow y$	$F=x\oplus y$
0	0	0	1	0	0	0	1	0	0	1	1	0	1	1	1	0
0	1	0	1	0	0	1	0	0	1	0	1	1	0	1	1	1
1	0	0	1	0	1	0	0	1	0	1	0	1	1	0	1	1
1	1	0	1	1	0	0	0	1	1	0	0	1	1	1	0	0

•
1
0
0
1

$2 \to 16$
$3 \to 81$

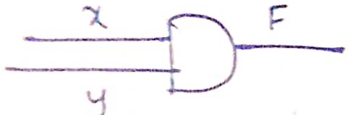
02/11/21.

Boolean function	Name	operator symbol
$F_0 = 0$	Null	.
$F_1 = xy$	AND	$x \cdot y$
$F_2 = xy'$	Inhibition	x/y
$F_3 = x$	Transfer	
$F_4 = x'y$	Inhibition	y/x
$F_5 = y$	Transfer	
$F_6 = xy' + x'y$	Exclusive OR	$x \oplus R$
$F_7 = x + y$	OR	$x + y$
$F_8 = (x+y)'$	NOR	$\downarrow$
$F_9 = xy + x'y'$	Equivalence	$x \odot y$
$F_{10} = y'$	complement	y'
$F_{11} = x + y'$	Implication	$x \subset y$
$F_{12} = x'$	complement	x'
$F_{13} = x' + y$	Implication	$y \subset x$
$F_{14} = (xy)'$	NAND	$\uparrow$
$F_{15} = 1.$	Identity	

x	y	y'	x'	$x \mid y$ (or) xy'	$y \mid x$	xy	$x'y'$	$x \cdot y = xy + x'y'$
0	0	1	1	0	0	0	1	1
0	1	0	1	0	1	0	0	0
1	0	1	0	1	0	0	0	0
1	1	0	0	0	0	1	0	1

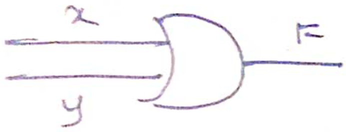
LOGIC GATES

AND



$F = x \cdot y$

OR



$F = x + y$

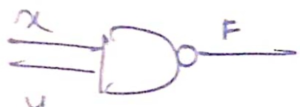
Inverter

$F = x'$

Buffer

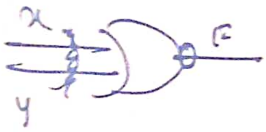
$F = x$

NAND



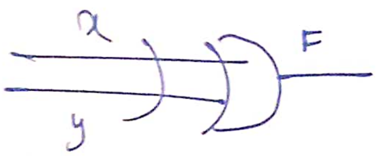
$F = (xy)'$

NOR



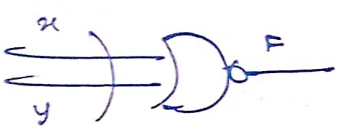
$F = (x+y)'$

XOR (Exclusive OR)



$F = x'y + xy'$

Exclusive NOR



$F = x \odot y$

$= x'y + xy'$

1. Construct the logic circuit for the following Boolean functions.

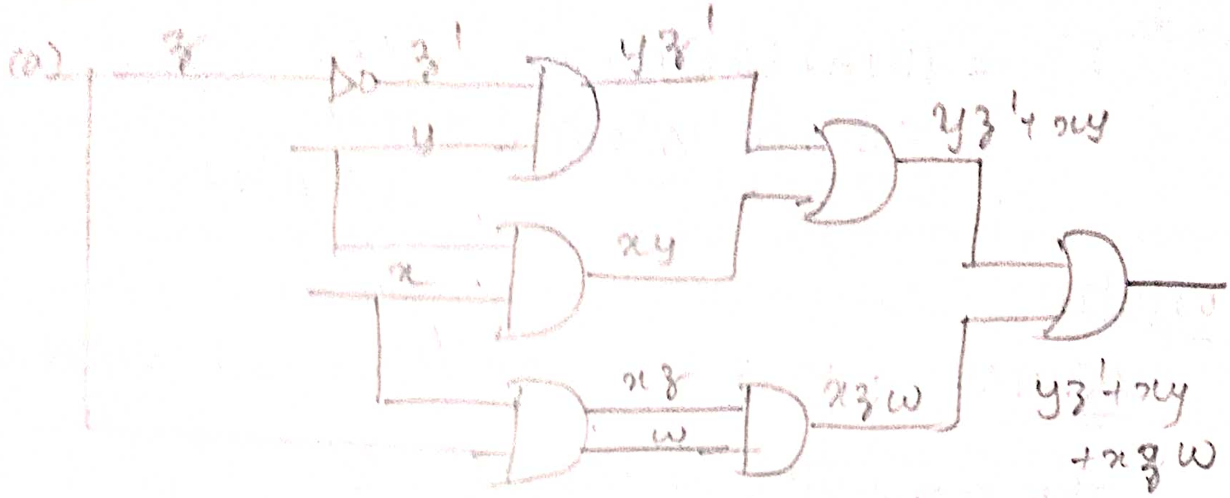
(a) $yz' + xy + xzw$

(b) $(A+B)(C+D)(A'+B+D)$

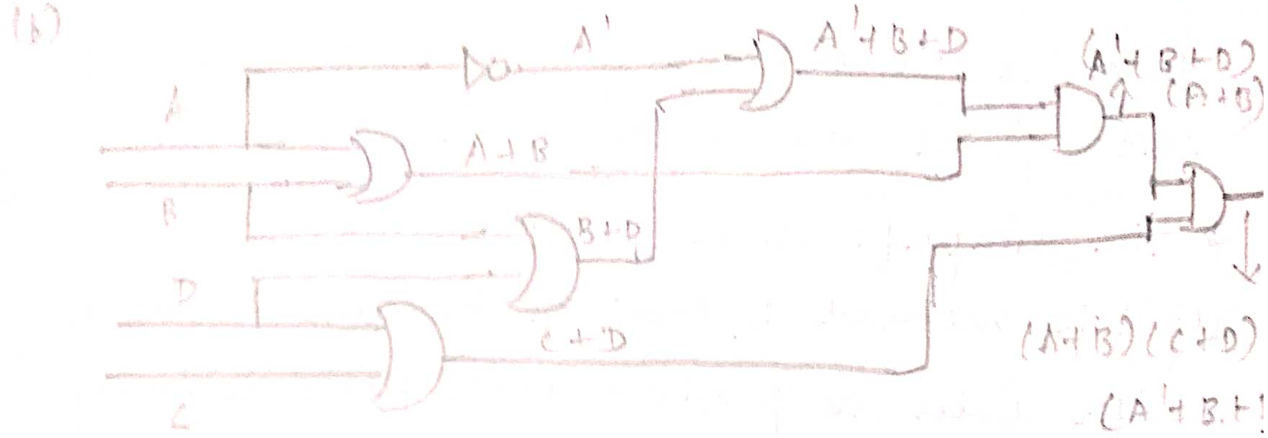
(c) $(AB + A'B')(CD' + C'D)$

Soln:

(a)



(b)



$(A+B)(C+D)(A'+B+D)$

$= (A+B)(A'+B+D)(C+D)$ (commutative)

$= ((A \cdot (A'+D))+B)(C+D)$ (distributive)

$= ((A \cdot A' + A \cdot D)+B)(C+D)$ (distributive)

$= ((0 + AD)+B)(C+D)$ (complement)

$= (AD+B)(C+D)$ (identity) $x+(x \cdot y) = x$

$AD + (AD \cdot C) = AD$

$= \boxed{ADC + AD + BC} + BD$ (distributive)

$= AD + BC + BD$ (Absoption) $A(D \cdot D)$

$= AD + B(C+D)$ (distributive)

$$(A+B)(A'+B+D)$$
$$= (A+B)((A'+D)+B)$$
$$= (B+A)(B+(A'+D))$$
$$= B+(A\cdot(A'+D)).$$

$$x+(y\cdot z)$$
$$= (x+y)\cdot(x+z)$$
$x = B$
$y = A$
$z = A'+D.$

05/11/21

KARNAUGH MAP (K-MAP):- / vietech diagram.

The complexity of the digital logic gates that implement a Boolean function is directly related to the complexity of the algebric expre-ssion from which the fnc. is implemented.

* The simplification of Boolean fnc. by algebric method is tedious because it lacks specific rules to predict each succeding step in the manipulative process.
* The K-map method provides a straight forward procedure for simplifying or minimizing the Boolean func.
* This method was 1st proposed by veitech and modified by karnaugh.
* It is also called as Veitech diagram
* this method provides a pictorial representa-tion of all posible ways a fnc. may be expressed in a standard form.
* The simplification of Boolean fnc. by algebric method is tedious because it lacks

specific rules to predict each succeeding step in the manipulative process.

- The K-map method provides a straight forward procedure for simplifying or minimizing the

$x'y', x'y, xy', xy$
$m_0 \quad m_1 \quad m_2 \quad m_3$ → AND operator.

- Each square represent one minterm
- Any two adjacent squares in the map differ by only one variable which is primed in one square and unprimed in other.

00, 11, 01, 10.
000, 111

- Minterms are not arranged in a binary sequence but in a sequence in which only one bit changes from 0 to 1 or 1 to 0.

__Two-variable map:-__ (It is made up of 4 squares).

$x \backslash y$	0	1
0	m_0	m_1
1	m_2	m_3

$m_0 - 00$
$m_1 - 01$
$m_2 - 10$
$m_3 - 11$

$2^n - 2^2 = 4$

Connects

__Three-Variable map:-__

$x \backslash yz$	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

row represents x
column represents y $2^3 = 8$
$2^4 = 16$

$m_0 - 000$
$m_1 - 001$ $\quad m_5 - 101$
$m_3 - 011$ $\quad m_7 - 111$
$m_2 - 010$ $\quad m_6 - 110$.
$m_4 - 100$

* what are the adjacent squares of m_0?

$\to x'y',z'$

$x'y'z \leftarrow m_1$ is adjacent to m_0.

* **Four-variable map:-**

wx\yz	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

row → wx
coloumn → yz

- 0000
- 0001
- 0011
- 0010
- 0100
- 0101
- 0111
- 0110
- 1000
- 1001
- 1011
- 1010
- 1100
- 1101
- 1111
- 1110

* what are the adjacent squares to m_0?

m_1, m_4, m_2, m_8.

* For m_4?

m_0, m_5, m_6, m_{12}.

* For m_{15}?

$m_{14}, m_{11}, m_7, m_{13}$

Problems: Two-Variable.

1. simplify $f(x,y) = \Sigma(0,1,2)$.

$m_0\ m_1\ m_2$

Soln:

x\y	0	1
0	1	1
1	1	

$x'y'\ x'y\ xy'\ y'$

00, 01, 10, 11.

if y-value is 0 should write '

y-value is 1 no need.

$f(x,y) = y' + x'$.

$f(x,y) = x'y' + x'y + xy'$

$$f(x,y) = x'y' + x'y + xy'$$

$$= x'\cdot(y'+y) + xy'$$

Verification:

$$= x'\cdot 1 + xy'$$

$$= x' + xy'$$

$$= (x'+x)\cdot(x'+y')$$

$$= x'+y'$$

Rough side

06/11/21

2. simplify $f(x,y) = \Sigma(0,2,3)$.
$m_0 \; m_2 \; m_3$

soln:

x|y	0	1
0	1	
1	1	1

$$f(x,y) = y' + x.$$

$$f(x,y) = x'y' + xy' + xy$$

Verification:

$$= \cancel{x'\cdot(y'+y') + xy'}$$

$$= y'(x'+x) + xy. \text{ (distributive)}$$

$$= 1\cdot y' + xy$$

$$= y' + xy$$

$$= (y'+x)\cdot(y'+y)$$

$$= y'+x$$

0	0	0	$x'y'$
1	0	1	$x'y$
2	1	0	xy'
3	1	1	xy

---

3. simplify $f(x,y,z) = \Sigma(2,3,4,5)$ three-variable.
$m_2 \; m_3 \; m_4 \; m_5$

soln:

x|yz	00	01	11	10
0			1	1
1	1	1		

011
010
100
101

011
010
$x'y$
+

$f(x,y,z) = x'y + xy'$

$f(x,y,z) = x'yz' + x'yz + xy'z' + xy'z$.

000
010 - 2
011 - 3
100 - 4
101 - 5

Verification:

$= x'y(z'+z) + xy'(z'+z)$

$= x'y + xy'$

000 m_0
001 m_1
010 - 2
011 - 3
100 - 4
101 - 5
110 - 6
111 - 7

4. Simplify: $f(a,b,c) = \Sigma(0,1,2,4,5,7)$

$m_0\ m_1\ m_2\ m_4\ m_5\ m_7$

Soln)

a \ bc	00	01	11	10
0	1	1		1
1	1	1	1	

⊃ ⊂ this means these two are combined.

$f(a,b,c) = \cancel{ab} + ac + a'c' + b'$.

$f(a,b,c) = ac + a'c' + b'$.

000
001
100
101
b'

111
101
ac

000
010
$a'c'$

Verification:

$f(a,b,c) = a'b'c' + a'b'c + a'bc'$
$+ ab'c' + ab'c + abc$.

5. Simplify: $f(x,y,z) = x'y'z' + xyz' + xy'z'$
$\Sigma(0,4,6)$.

x → row
y/z → coloumn

Soln:

x \ yz	00	01	11	10
0	1			
1	1			1

100
110
xyz

000
100
xyz

$f(x,y,z) = y'z' + xz'$.

6. Simplify :- $f(x,y,z) = \Sigma(0,1,2,3,5,7)$

x \ yz	00	01	11	10
0	1	1	1	1
1		1	1	

$f(x,y,z) = x' + \cancel{xy}z.$

000	101
001	111
011	001
010	011

Verification –

$f(x,y,z) = x'y'z' + x'y'z + x'yz' + x'yz + xy'z + xyz.$

$= x'y' + x'y + xz$

$= x' + (xz)$

$= \cancel{x' + xz}$ $= (x'+x)\cdot(x'+z)$

$\cancel{= y' + yz}$ $= x' + z.$

09/11/21

7. simplify : $f(x,y,z) = \Sigma(0,1,2,3,7).$

$x \backslash yz$	00	01	11	10
0	1	1	1	1
1			1	

011	000
111	001
xyz	011
	010
	x'

$f(x,y,z) = yz + x'.$

Verification :-

$f(x,y,z) = x'y'z' + x'y'z + x'yz' + x'yz + xyz.$

8. simplify : $f(x,y,z) = \Sigma(0,1,4,5)$

Simplify :- the boolean function whose sum of minterms form

$x \downarrow \backslash yz \rightarrow$	00	01	11	10
0	1	1		
1	1	1		

$f(x,y,z) = y'$

9. Simplify the Boolean function. Four variable

$F(x,y,z,w) = \Sigma(0,1,2,5,8,9,10)$.

Soln:

$wx \backslash yz$	00	01	11	10
00	1	1		1
01		1		
11				
10	1	1		1

0001
0101
$w'y'z$

1000
1001
$x'y'$

0000
0010
1000
1010
$wxyz$

1000
1001
$wx'y'$

$f(w,x,y,z) = w'z' + w'y'z + x'z'$
$+ wx'y'$.

Verification:

$w'x'y'z' + w'x'y'z + w'x'yz' +$
$w'xy'z + wx'y'z' + wx'y'z + wx'yz'$.

(0) 0000, (1) 0001, (2) 0010, (5) 0101, (8) 1000, (9) 1001, (10) 1010.

$= w'x'y'(z'+z) + wx'y'(z'+z) + x'yz'(w'+w)$
$+ w'xy'z$

$= w'x'y'\cdot 1 + wx'y'\cdot 1 + x'yz'\cdot 1 + w'xy'z$

$= w'x'y' + wx'y' + x'yz' + w'xy'z$

$= (w+w')x'y' + x'yz' + w'xy'$

$= x'y' + x'yz' + w'xy'$

$= x'(y' + yz') + wxy'$

$= x'((y'+y)\cdot(y'z')) + w'xy'z$

$= x'y'z + w'xy'z$

$= y'(x'z' + w'xz) \rightarrow (1\cdot 2)\cdot 3$

$= y'((x'+w'xz)\cdot(z'+w'xz)) \to 1+(2\cdot3)$

$= y'((x'+x)\cdot(x'+w'z))\cdot((z'+w'x)\cdot(z+z')$

$= y'\cdot(x'+w'z)\cdot(z'+w'x)$

$= (y'x'+w'y'z)(z'+w'x)$

$= x'y'z'+w'xy'z$

$w'y'z+wx'y'+x'z'$.

$= w'y'z\cdot(x+x')+wx'y'\cdot(z+z')+x'z'\cdot(w+w')$

$= w'xy'z+w'x'y'z+wx'y'z+wx'y'z'+wx'z'$
$+w'x'z'$.

$= w'xy'z+w'x'y'z+wx'y'z+wx'y'z'+$
$wx'z'(y+y')+w'x'z'(y+y')$

$= w'xy'z+w'x'y'z+wx'y'z+wx'y'z'+$
$wx'yz'+wx'y'z'+w'x'yz'+w'x'y'z'$

$= w'xy'z+w'x'y'z+wx'y'z+wx'y'z+$
$wx'yz'+w'x'yz'+w'x'y'z'$.

$= \underset{(5)}{0101}+\underset{(1)}{0001}+\underset{(9)}{1001}+\underset{(8)}{1000}+\underset{(10)}{1010}+\underset{(2)}{0010}$
$+\underset{(0)}{0000}$.

$= 5, 1, 9, 8, 10, 2, 0$.

$\therefore \Sigma(0,1,2,5,8,9,10)$.

12/11/21.

10. simplify the Boolean function

$F(w,x,y,z) = \Sigma(1,3,7,11,15)$.

soln:

wx \ yz	00	01	11	10
00		1	1	
01			1	
11			1	
10			1	

0001
0011
$w'x'z$

0011
0111
1111
1011
yz

$f(w,x,y,z) = w'x'z + yz.$

verification:

$f(w,x,y,z) = 0011 + 0001 + 1111 + 1011 + 0111$

$f(w,x,y,z) = w'x'yz + w'x'y'z + wxyz + wx'yz + w'xyz.$

$\Sigma(1,3,7,11,15).$

$w'x'(y+y')z + (w+w')yz.$

$= w'x'yz + w'x'y'z + wyz + w'yz.$

$w'x'yz + w'x'y'z + w(x+x')yz + w'(x+x')yz.$

$wxyz + wx'yz + w'xyz + w'x'yz.$

$x \cdot 1 = x$
$x + x' = 1$
$x \cdot (y+z) = x \cdot y + x \cdot z$

11. Simplify the Boolean function.

$F(w,x,y,z) = \Sigma(0,1,2,8,10,11,14,15).$

wx \ yz	00	01	11	10
00	1	1		1
01				
11			1	1
10	1		1	1

0000
1000
$x'y'z'$

1000
1010
$wx'z'$

1111
1110
1011
1010
wy

$f(w,x,y,z) = w'x'y' + w'x'z' + wx'z' + wy.$

12. Simplify the Boolean function:-

$$F(w,x,y,z) = \Sigma(0,1,4,5,8,9,12,13)$$

wx \ yz	00	01	11	10
00	1	1		
01	1	1		
11	1	1		
10	1	1		

0000
0001
0100
0101
1100
1101
1000
1001
$w x y' z$

0000
0001
$x'y'z'w$

$F(w,x,y,z) = y'.$

13/11/21.

Don't care conditions:- $F(x,y,z) = \Sigma(0,1,3)$

Eg:-

x \ yz	00	01	11	10
0	1	1	1	
1				

000
001
$x'y'z$

$x'y' + x'z$
001
011
$x'z'$

$\boxed{\Rightarrow x'y' + x'z.}$

$F(x,y,z) = x'y'z' + x'y'z + x'yz.$
000 001 011
(0)

$x'yz'$ doesn't occur.

x \ yz	00	01	11	10
0	1	1	1	0
1	0	0	0	0

1. Simplify $F(w,x,y,z) = \Sigma(1,3,7,11,15)$ that has dont' care conditions $\Sigma(0,2,5)$.

do not occur at all.

wx \ yz	00	01	11	10
00	X	1	1	X
01		X	1	
11			1	
10			1	

0011
0111
1111
1011
yz

1st (07)

0001
0101
0011
0111
$w'z$

$F(w,x,y,z) = yz + w\cancel{x'}z$

$F(w,x,y,z) = yz + w'z$.

2. Simplify $F(a,b,c,d) = \Sigma(0,2,4,6,8,12)$ that has don't care conditions $\Sigma(1,3,5,7)$.

wx / ab \ yz / cd	00	01	11	10
00	1	X	X	1
01	1	X	X	1
11	1			
10	1			

0000
0001
0011
0010
0100
0101
0111
0110
01

0000
0100
1100
1000
$c'd'$

$F(a,b,c,d) = c'd' + a'$

3. Simplify $F(x,y,z) = \Sigma(0,2,4,6)$ that has don't care conditions $\Sigma(1,5)$

x \ yz	00	01	11	10
0	1			1
1	1			1

000
010
100
110
z'

000
001
100
101
y'

010
110
yz