

GPCM $d=1$ $\theta=90$

$$\begin{bmatrix} 1 & 4 & 5 & 10 & 10 \\ 2 & 4 & 3 & 11 & 10 \\ 3 & 4 & 2 & 10 & 11 \\ 4 & 4 & 4 & 11 & 11 \end{bmatrix} \Rightarrow 1, 2, 3, 4, 5, 10, 11$$

GPCM:

	1	2	3	4	5	10	11
1	0	0	0	0	0	0	0
2	1	0	1	0	0	0	0
3	0	1	0	0	1	0	0
4	0	1	1	3	0	0	0
5	0	0	0	0	0	0	1
10	0	0	0	0	0	1	1
11	0	0	0	0	0	3	1

Normalized GPCM

$$\frac{1}{15}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Sobel Operator

$$\text{Vertical Sobel} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{Shifting} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 4 & 5 & 10 & 10 \\ 2 & 4 & 3 & 11 & 10 \\ 3 & 4 & 2 & 10 & 11 \\ 4 & 4 & 4 & 11 & 11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= -12$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ -12 & 4 & 5 \\ 2 & 4 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= -35$$

$$\begin{bmatrix} 0 & 0 & 0 \\ -35 & 5 & 10 \\ 4 & 3 & 11 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= -97$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ -97 & 10 & 10 \\ 3 & 11 & 10 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= -221$$

$$\begin{bmatrix} 0 & 0 & 0 \\ -21 & 0 & 0 \\ 11 & 10 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= -431$$

$$\Rightarrow \begin{bmatrix} 0 & -12 & -35 \\ 0 & 2 & 4 \\ 0 & 3 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= 23$$

$$\Rightarrow \begin{bmatrix} -12 & -35 & -97 \\ 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= 126$$

$$\Rightarrow \begin{bmatrix} -35 & -97 & -229 \\ 12 & 3 & 11 \\ 4 & 2 & 10 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= -410$$

$$\Rightarrow \begin{bmatrix} -97 & -229 & -431 \\ 4 & 10 & 11 \\ 2 & 10 & 11 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= 1125$$

$$\Rightarrow \begin{bmatrix} 225 & -431 & 0 \\ 1125 & 10 & 0 \\ 0 & 11 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 225 & 4 & 2 \\ -431 & 4 & 4 \\ 1125 & 11 & 11 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= -667$$

$$\Rightarrow \begin{bmatrix} 126 & 2 & 11 \\ -667 & 4 & 11 \\ 4 & 11 & -2360 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 410 & 1125 & 2039 \\ -2360 & 10 & 11 \\ 2 & 11 & 11 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= -6378$$

$$\Rightarrow \begin{bmatrix} 1125 & 2039 & 0 \\ -6378 & 11 & 0 \\ 11 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= -11620$$

$$\Rightarrow \begin{bmatrix} 0 & 23 & 126 \\ 0 & -138 & -667 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= 1208$$

$$\Rightarrow \begin{bmatrix} 23 & 126 & 410 \\ 1208 & -667 & -2360 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= 67219$$



$$\Rightarrow \begin{bmatrix} 122 & 410 & 1125 \\ 6749 & -2366 & -6378 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= 25255$$

$$\Rightarrow \begin{bmatrix} 410 & 1125 & 2039 \\ 25255 & -6378 & -11620 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= 72070$$

$$\Rightarrow \begin{bmatrix} 1125 & 2039 & 0 \\ 72070 & -11620 & 0 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= 145265$$

$$\Rightarrow \begin{bmatrix} -12 & -35 & -97 & -221 & -431 \\ 23 & 126 & 410 & 1125 & 2039 \\ -138 & -667 & -2366 & -6378 & -11620 \\ 1208 & 6749 & 25255 & 72070 & 145265 \end{bmatrix}$$

$$\Rightarrow \text{Max filter} \begin{bmatrix} 1 & 4 & 5 & 10 & 10 \\ 2 & 4 & 3 & 11 & 10 \\ 3 & 4 & 2 & 10 & 11 \\ 4 & 4 & 2 & 11 & 11 \end{bmatrix}$$

Let's Assume 3x3 filter max

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 4 \end{bmatrix} \Rightarrow 4$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 4 & 4 & 5 \\ 2 & 4 & 3 \end{bmatrix} \Rightarrow 5$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 5 & 5 & 10 \\ 4 & 3 & 11 \end{bmatrix} \Rightarrow 11$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 11 & 10 & 10 \\ 3 & 11 & 10 \end{bmatrix} \Rightarrow 11$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 11 & 10 & 0 \\ 11 & 10 & 0 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 0 & 4 & 5 \\ 0 & 2 & 4 \\ 0 & 3 & 4 \end{bmatrix} = 5$$

$$\Rightarrow \begin{bmatrix} 4 & 5 & 11 \\ 5 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 5 & 11 & 11 \\ 11 & 3 & 11 \\ 4 & 2 & 10 \end{bmatrix} = 11$$

$$\begin{bmatrix} 11 & 11 & 11 \\ 11 & 11 & 10 \\ 2 & 10 & 11 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 11 & 11 & 0 \\ 11 & 10 & 0 \\ 10 & 11 & 0 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 0 & 5 & 11 \\ 0 & 3 & 4 \\ 0 & 4 & 4 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 5 & 11 & 11 \\ 5 & 4 & 2 \\ 11 & 4 & 4 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 11 & 11 & 11 \\ 11 & 2 & 10 \\ 4 & 4 & 11 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 11 & 11 & 11 \\ 11 & 10 & 11 \\ 4 & 11 & 11 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 11 & 11 & 0 \\ 11 & 11 & 0 \\ 11 & 11 & 0 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 0 & 11 & 11 \\ 4 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 11 & 11 & 11 \\ 11 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 11 & 11 & 11 \\ 11 & 4 & 11 \\ 0 & 0 & 0 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 11 & 11 & 0 \\ 11 & 11 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 11$$

$$\Rightarrow \begin{bmatrix} 4 & 5 & 11 & 11 & 11 \\ 5 & 11 & 11 & 11 & 11 \\ 11 & 11 & 11 & 11 & 11 \\ 11 & 11 & 11 & 11 & 11 \end{bmatrix}$$

$$4) \begin{bmatrix} 1 & 1 & 5 & 11 & 10 \\ 2 & 3 & 3 & 11 & 10 \\ 3 & 5 & 2 & 11 & 10 \\ 4 & 6 & 4 & 11 & 10 \end{bmatrix} \quad SE = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Erosion: $(0,0)$
 $\Rightarrow (-1,-1)=0, ((-1,1)=0), (1,-1)=0$
 $(1,1)=3$
 $\min = 0.$

$(0,1)$:
 $SE = (-1,0)=0, (-1,2)=0, (1,0)=3$
 $(1,2)=3 \quad \min = 0.$

$$(0,2): SE: (-1,1)=0 \cdot (-1,3)=0, (1,1)=3 \\ (1,3)=11 \text{ min}=0$$

$$(0,3): SE: (-1,2)=0, (-1,4)=0, (1,2)=3 \\ (1,4)=10; \text{min}=0$$

$$(0,4): SE: (-1,3)=0, (-1,5)=0, (1,3)=11 \\ (1,5)=0; \text{min}=0$$

$$(1,0): SE: (0,-1)=0; (0,1)=1; (2,-1)=3 \\ (2,1)=5 \\ \text{min}=0$$

$$(1,1): SE: (0,0)=1; (0,2)=5 \\ (2,0)=3; (2,2)=2 \\ \text{min}=1$$

$$(1,2): SE: (0,1)=4; (0,3)=11; (2,1)=5 \\ (2,3)=4$$

$$(1,3): SE: (0,2)=5; (0,4)=10; (2,4)=2 \\ (2,4)=10 \\ \text{min}=2$$

$$(1,4): SE: (0,3)=11; (0,5)=0; (2,3)=11 \\ (2,5)=0; \\ \text{min}=0$$

$$(2,0): SE: (1,-1)=0; (1,1)=3; \\ (3,-1)=0; (3,1)=6 \\ \text{min}=0$$

$$(2,1): SE: (1,0)=2; (1,2)=3; \\ (3,0)=4; (3,2)=4 \\ \text{min}=2$$

$$(2,2): SE: (1,1)=3; (1,3)=11; (3,1)=6 \\ (3,3)=11 \\ \text{min}=3$$

$$(2,3): SE: (1,2)=3; (1,4)=10 \\ (3,2)=4; (3,4)=18 \\ \text{min}=3$$

$$(2,4): SE: (1,3)=11; (1,5)=0; \\ (3,3)=11; (3,5)=0; \\ \text{min}=0$$

$$(3,0): SE: (2,-1)=0; (2,1)=5; \\ (4,-1)=0; (4,1)=0 \\ \text{min}=0$$

$$(3,1):$$

$$SE = (2,0) = 3$$

$$(2,2) = 2$$

$$(4,0) = 0, (4,2) = 0$$

$$min = 0$$

$$(3,2):$$

$$SE = (2,1) = 5, (2,3) = 11,$$

$$(4,1) = 0, (4,3) = 0$$

$$min = 0$$

$$(3,3):$$

$$SE = (2,2) = 2$$

$$(2,4) = 10, (4,2) = 0,$$

$$(4,4) = 0$$

$$min = 0$$

$$(3,4):$$

$$SE = (2,3) = 11$$

$$(2,5) = 0, (4,3) = 0$$

$$min = 0$$

0	0	0	0	0
0	1	3	3	0
0	2	3	3	0
0	0	0	0	0

Dilation

$$(0,0) \max(0,0,0,1) = 1$$

$$(0,1) \max(0,0,0,1) = 1$$

$$(0,2) \max(0,0,1,2) = \max = 2$$

$$(0,3) \max(0,0,1,0) \max = 1$$

$$(0,4) \max(0,0,0,2,0) = \max = 2$$

$$(1,0) = \max(0,0,0,2) = 2$$

$$(1,1) = \max(0,0,0,3) = 3$$

$$(1,2) = \max(0,0,2,3) = 3$$

$$(1,3) = \max(0,0,3,0) = 3$$

$$(1,4) \max(0,0,3,0) = 3$$

$$(2,0) \max(0,1,0,0) = 1$$

$$(2,1) \max(0,1,0,0) = 1$$

$$(2,2) \max(1,2,0,0) = 2$$

$$(2,3) = \max(1,0,0,0) = 1$$

$$(2,4) = \max(2,0,0,0) = 2$$

$$(3,0) = \max(0,2,0,0) = 2$$

$$(3,1) = \max(0,3,0,0) = 3$$

$$(3,2) = \max(2,3,0,0) = 3$$

$$(3,3) = \max(3,0,0,0) = 3$$

$$(3,4) = \max(3,0,0,0) = 3$$

[Signature]

Final result

$$\left\{ \begin{array}{l} 2 \quad 2 \quad 2 \quad 2 \\ 1 \quad 2 \quad 1 \quad 2 \\ 2 \quad 2 \quad 2 \quad 2 \\ 1 \quad 2 \quad 1 \quad 2 \end{array} \right\}$$