

$$\pi(R) = 0.6$$

$$\pi(S) = 0.4$$

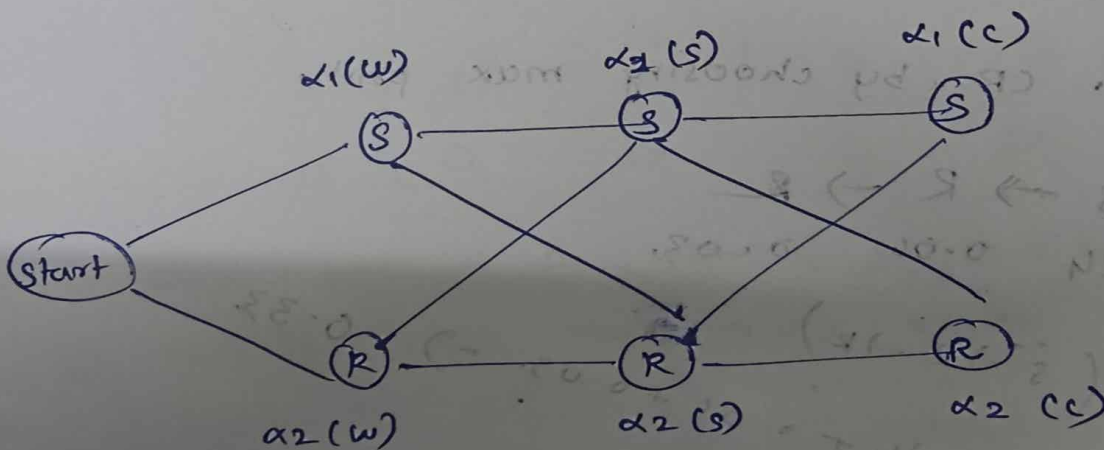
A → Transition

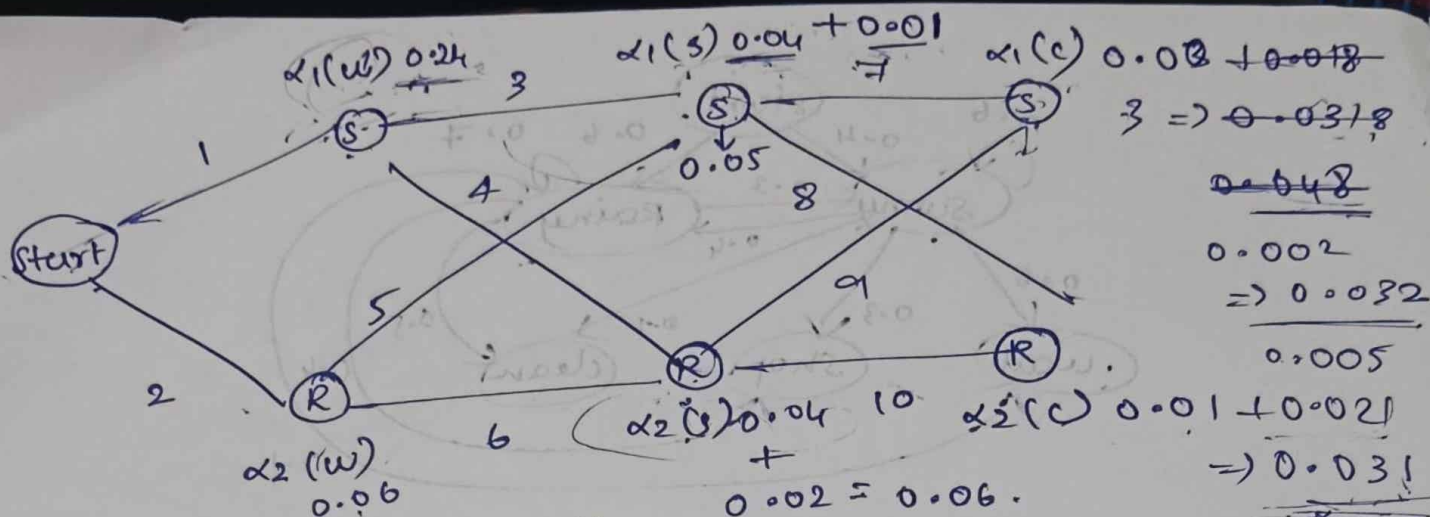
B → Emission

$$A \rightarrow \begin{matrix} & R & S \\ \begin{matrix} R \\ S \end{matrix} & \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \end{matrix}$$

$$B \rightarrow \begin{matrix} & w & s & c \\ \begin{matrix} R \\ S \end{matrix} & \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.6 & 0.3 & 0.1 \end{bmatrix} \end{matrix}$$

No. of observations : 3.





- ①  $\Rightarrow 0.4 \times 0.6 = 0.24$  ✓
- ②  $\Rightarrow 0.6 \times 0.1 = 0.06$  ✓
- ③  $\Rightarrow 0.24 \times 0.6 \times 0.3 = 0.04$  ✓
- ④  $\Rightarrow 0.24 \times 0.4 \times 0.4 = 0.04$
- ⑤  $\Rightarrow 0.06 \times 0.3 \times 0.3 = 0.01$
- ⑥  $\Rightarrow 0.06 \times 0.7 \times 0.4 = 0.02$
- ⑦  $\Rightarrow 0.05 \times 0.6 \times 0.1 = 0.003$
- ⑧  $\Rightarrow 0.05 \times 0.4 \times 0.5 = 0.01$
- ⑨  $\Rightarrow 0.06 \times 0.3 \times 0.1 = 0.0018 \approx 0.002$
- ⑩  $\Rightarrow 0.06 \times 0.7 \times 0.5 = 0.021$

Hidden seq. CR by choosing max prob.

$$S \Rightarrow R \rightarrow R$$

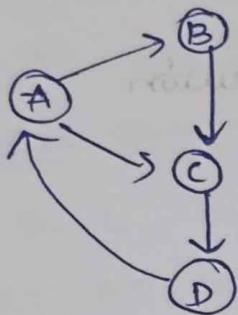
$0.24 \quad 0.06 \quad 0.03$

$$P(S \rightarrow R \rightarrow R)$$

$0.24 + 0.06 + 0.03 = 0.33$

## ②. Page Rank

To calculate the page rank for the directed graph with nodes A, B, C, D & the gr. edges  
 $A \rightarrow B$ ,  $A \rightarrow C$ ,  $B \rightarrow C$ ,  $C \rightarrow D$ ,  $D \rightarrow A$



a) Represent the graph as a transition matrix ie, probability of transitions from one-to-other node.

$$M = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1/2 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

col. wise sum  
 always one  
 col. sum = 1  
 one transition

Max. prob is 1, No. of edges = 2, so 1/2

b) Initialize the page rank vector.

$$P^0 = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

total node  $\rightarrow 4 \Rightarrow 1/4$

c) Applying damping factor ( $d$ ) for the prob. a user randomly jump to any page.  
 $d$  (or)  $d = 0.85$  (constant)

Page rank formula becomes

$$P^{(k+1)} = d \cdot M \cdot P^{(k)} + (1-d) \cdot v$$

$v \rightarrow$  vector that is uniform distribution

$$v = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}$$

$$P^{(k+1)} = 0.85 \times \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix} + (1-0.85) \times \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

$4 \times 4 \qquad 4 \times 1$

$$P^{(k+1)} = 0.85 \times \begin{bmatrix} 0.25 \\ 0.25/2 \\ 0.25/2 + 0.25 \\ 0.25 \end{bmatrix} + 0.15 \times \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2125 \\ 0.10625 \\ 0.31875 \\ 0.2125 \end{bmatrix} + \begin{bmatrix} 0.0375 \\ 0.0375 \\ 0.0375 \\ 0.0375 \end{bmatrix}$$

$$P^{(k+1)} \Rightarrow \begin{bmatrix} 0.25 \\ 0.14375 \\ 0.35625 \\ 0.25 \end{bmatrix} \Rightarrow P(1)$$



$$P(2) = 0.85 \times \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1/2 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.14 \\ 0.35 \\ 0.25 \end{bmatrix} + 0.15 \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.25 \\ 0.14 \\ 0.26 \\ 0.34 \end{bmatrix}$$

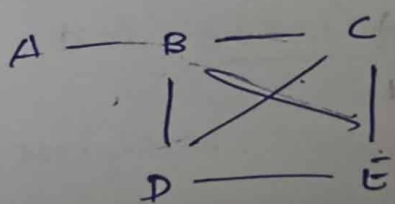
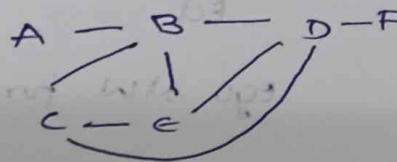
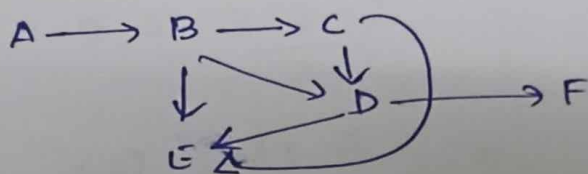
upto do for  $P(5) \hat{u}$ ;  $K=4$ .

③ K-core graph.

A, B, C, D, E, F

Edges: AB, BC, BE, BD, CD, CE, DE, DF

a) Compute the K-core collapse sequence for  $K=1, 2, 3, 4$ .



$K=1$

$K=2$

$K=3$

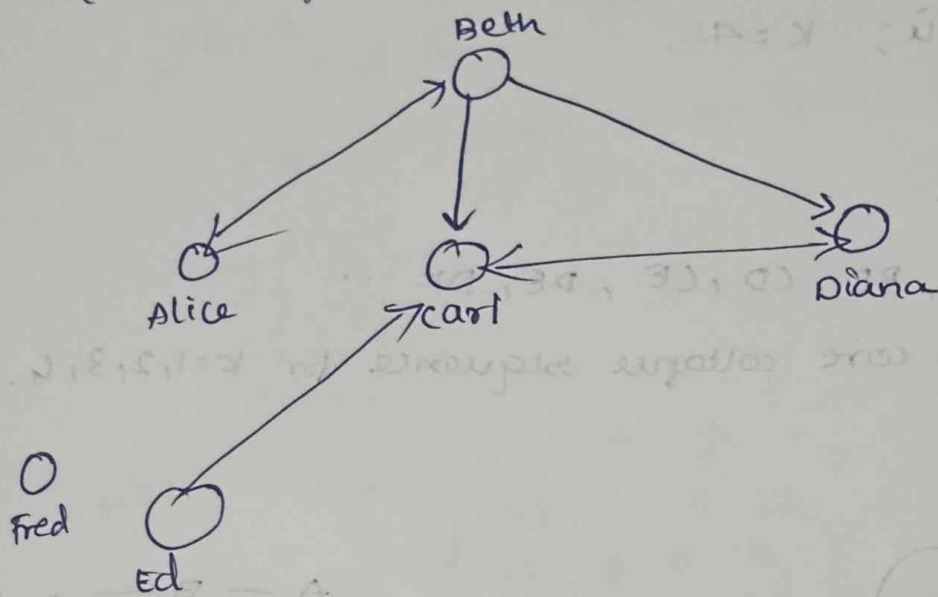
$K=4$

	Removed nodes	remaining nodes
$K=1$	A, F	A, B, C, D, E, F
$K=2$	A, F	B, C, D, E
$K=3$	C, E	B, D
$K=4$	B, D	

# ④ NW Analysis

Ego analysis.

Extract ego NW and analyze as independent (Directed graph)



Ego NW for Alice

	Alice	Beth	Row sum
Alice	0	1	1
Beth	1	0	1

Ego NW for Beth

<del>Beth</del>	Carl	Alice	Diana	Row sum	Beth
Beth	1	0	1	2	0
Carl	0	1	0	1	0
Alice	0	0	0	0	1
Diana	1	0	0	1	0

Fred has no connections  $\rightarrow$  so independent.

### ⑤ Whole NW Analysis

consider a NW with 4 nodes. The neighbors of each node are connected as follow.

Node A - 2 neighbors with 1 connection b/w them  
 Node B - 3 " " 3 " "  
 Node C - 1 " " 0 " "  
 Node D - 3 " " 2 " "

Total possible  $\Rightarrow \frac{n(n-1)}{2}$   $\rightarrow n \rightarrow$  neighbors

$$A \Rightarrow \frac{2(2-1)}{2} = \frac{2(1)}{2} = 1$$

$$B \Rightarrow \frac{3(3-1)}{2} = \frac{6}{2} = 3$$

$$C \Rightarrow \frac{1(1-1)}{2} = 0$$

$$D \Rightarrow \frac{3(3-1)}{2} = 3$$

Max possible connections.

an: Neighbors: conn - among neighbors.

A  $\rightarrow$  2

B  $\rightarrow$  3

C  $\rightarrow$  1

D  $\rightarrow$  3

A  $\rightarrow$  1

B  $\rightarrow$  3

C  $\rightarrow$  0

D  $\rightarrow$  2

$$cc(A) = 1/1 = 1$$

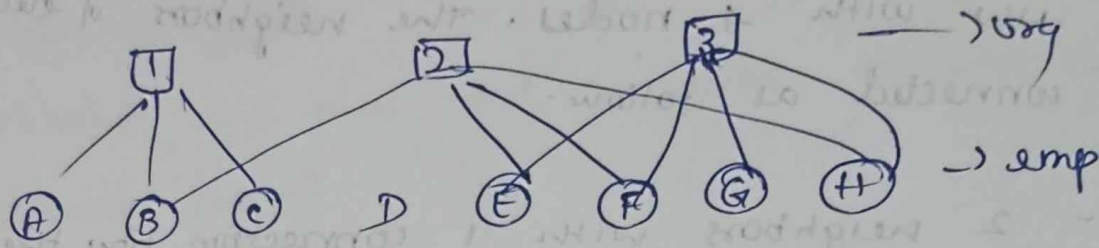
$$cc(B) = 3/3 = 1$$

$$cc(C) = 0/0 = 0$$

$$cc(D) = 2/3 = 0.67$$

$$\text{Avg. clustering} = \frac{1 + 1 + 0 + 0.67}{4} = 0.67$$

# ⑥ Two mode NW analysis



Adj-matrix

	<u>1</u>	<u>2</u>	<u>3</u>
A	1	0	0
B	1	1	0
C	1	0	0
D	0	0	0
E	0	1	1
F	0	1	1
G	0	0	1
H	0	0	1

density for 2 mode NW analysis

$$\Rightarrow \frac{L}{m \times n}$$

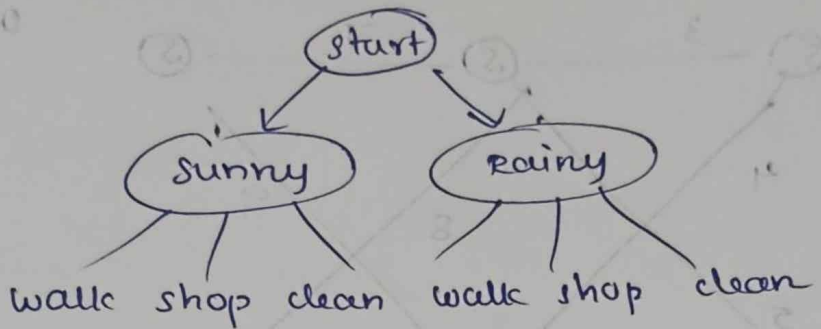
m → nod nodes

n → " " org

$$\Rightarrow \frac{11}{3 \times 8} = \frac{11}{24}$$



⑦ Viterbi



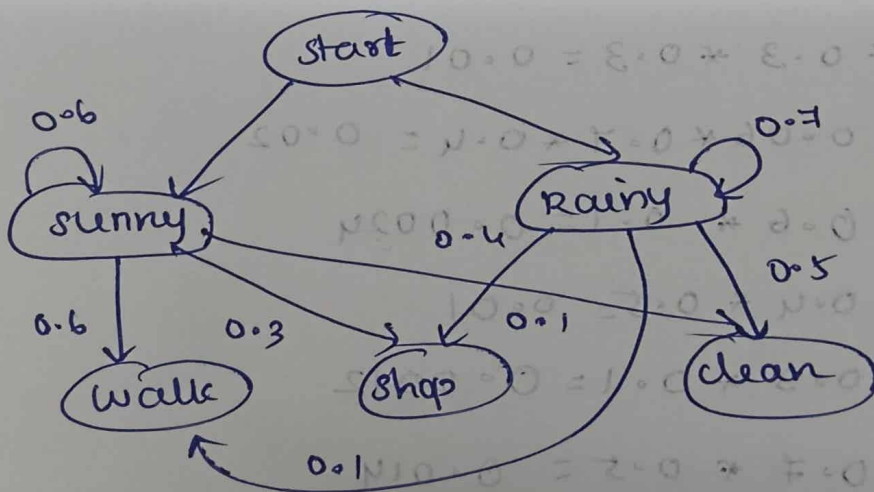
$$\begin{aligned} \pi(R) &= 0.6 \\ \pi(S) &= 0.4 \end{aligned} \quad \left. \begin{array}{l} \pi(R) \\ \pi(S) \end{array} \right\} \text{Initial prob (}\pi\text{)}$$

Transition prob: A

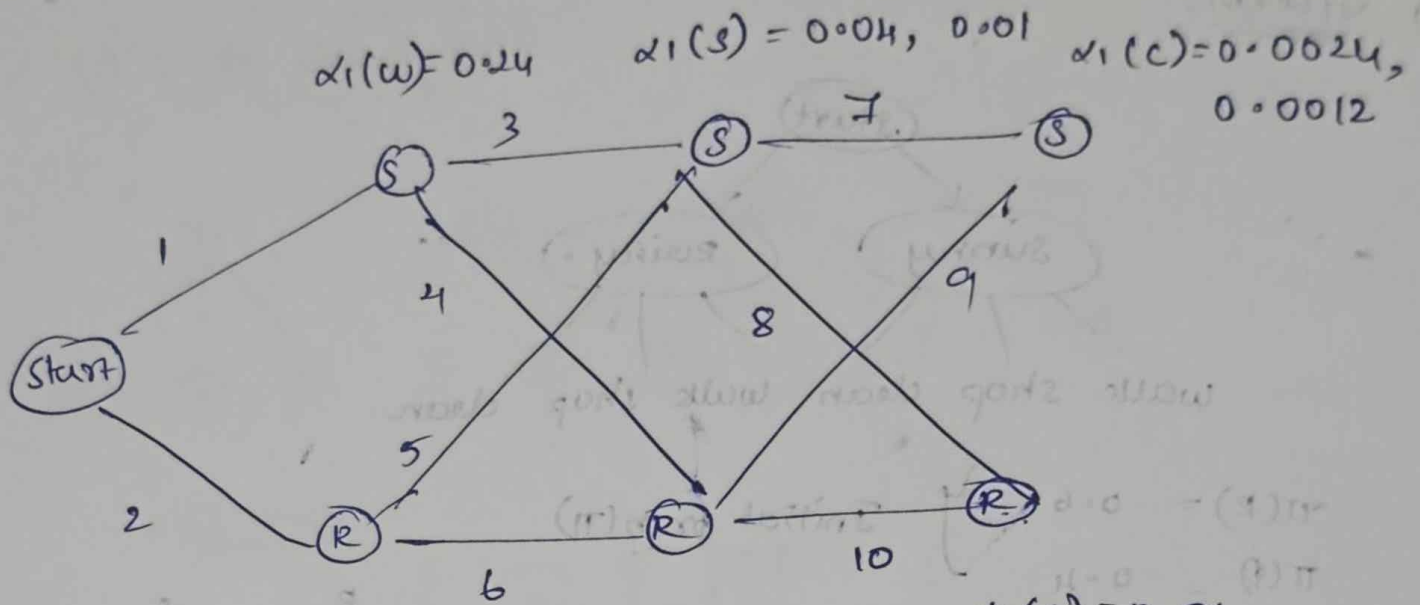
Emission prob: B

$$A: \begin{matrix} & R & S \\ \begin{matrix} R \\ S \end{matrix} & \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \end{matrix}$$

$$B: \begin{matrix} & R & S \\ \begin{matrix} w \\ s \\ c \end{matrix} & \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.6 & 0.3 & 0.1 \end{bmatrix} \end{matrix}$$



No. of observations: 3.



$$① \Rightarrow 0.4 * 0.6 = 0.24$$

$$② \Rightarrow 0.6 * 0.1 = 0.06$$

$$③ \Rightarrow 0.24 * 0.6 * 0.3 = 0.04$$

$$④ \Rightarrow 0.24 * 0.4 * 0.4 = 0.04$$

$$⑤ \Rightarrow 0.06 * 0.3 * 0.3 = 0.01$$

$$⑥ \Rightarrow 0.06 * 0.06 * 0.7 * 0.4 = 0.02$$

$$⑦ \Rightarrow 0.04 * 0.6 * 0.1 = 0.0024$$

$$⑧ \Rightarrow 0.04 * 0.4 * 0.5 = 0.01$$

$$⑨ \Rightarrow 0.04 * 0.3 * 0.1 = 0.0012$$

$$⑩ \Rightarrow 0.04 * 0.7 * 0.5 = 0.014$$

$$S \rightarrow R \rightarrow R$$

0.24    0.02    0.04