

# Formative Assessment 4

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github link:

The normal data are female height measurements, the skewed-right data are age at marriage for females, the skewed-left data are obituary data that give the age at death for females, and the uniform data are the amount of cola put into a 12 ounce container by a soft drinks machine.

```
library(knitr)

data <- read.csv("Data-1.csv")
kable(data)
```

| Normal | SK_right | SK_left | Uniform |
|--------|----------|---------|---------|
| 67     | 31       | 102     | 12.1    |
| 70     | 43       | 55      | 12.1    |
| 63     | 30       | 70      | 12.4    |
| 65     | 30       | 95      | 12.1    |
| 68     | 38       | 73      | 12.1    |
| 60     | 26       | 79      | 12.2    |
| 70     | 29       | 60      | 12.2    |
| 64     | 55       | 73      | 12.2    |
| 69     | 46       | 89      | 11.9    |
| 61     | 26       | 85      | 12.2    |
| 66     | 29       | 72      | 12.3    |
| 65     | 57       | 92      | 12.3    |
| 71     | 34       | 76      | 11.7    |
| 62     | 34       | 93      | 12.3    |
| 66     | 36       | 76      | 12.3    |
| 68     | 40       | 97      | 12.4    |
| 64     | 28       | 10      | 12.4    |
| 67     | 26       | 70      | 12.1    |
| 62     | 66       | 85      | 12.4    |
| 66     | 63       | 25      | 12.4    |
| 65     | 30       | 83      | 12.5    |
| 63     | 33       | 58      | 11.8    |
| 66     | 24       | 10      | 12.5    |
| 65     | 35       | 92      | 12.5    |
| 63     | 34       | 82      | 12.5    |

| Normal | SK_right | SK_left | Uniform |
|--------|----------|---------|---------|
| 69     | 40       | 87      | 11.6    |
| 62     | 24       | 104     | 11.6    |
| 67     | 29       | 75      | 12.0    |
| 59     | 24       | 80      | 11.6    |
| 66     | 27       | 66      | 11.6    |
| 65     | 35       | 93      | 11.7    |
| 63     | 33       | 90      | 12.3    |
| 65     | 75       | 84      | 11.7    |
| 60     | 38       | 73      | 11.7    |
| 67     | 34       | 98      | 11.7    |
| 64     | 85       | 79      | 11.8    |
| 68     | 29       | 35      | 12.5    |
| 61     | 40       | 71      | 11.8    |
| 69     | 41       | 90      | 11.8    |
| 65     | 35       | 71      | 11.8    |
| 62     | 26       | 63      | 11.9    |
| 67     | 34       | 58      | 11.9    |
| 70     | 19       | 82      | 11.9    |
| 64     | 23       | 72      | 12.2    |
| 63     | 28       | 93      | 11.9    |
| 68     | 26       | 44      | 12.0    |
| 64     | 31       | 65      | 11.9    |
| 65     | 25       | 77      | 12.0    |
| 61     | 22       | 81      | 12.0    |
| 66     | 28       | 77      | 12.0    |

Functions to use:

```
origin_moments <- function(x, k) mean(x^k)

mean_moments <- function(x, k) {
  mu <- mean(x)
  mean((x - mu)^k)
}

moment_about_mean <- function(x, k, c) mean((x - c)^k)
```

1. Find the (a) first, (b) second, (c) third, and (d ) fourth moments for each of the sets of data (normal, skewed-right, skewed-left, uniform).

```
origin_results <- sapply(1:4, function(k) c(
  Normal      = origin_moments(data$Normal, k),
  Skewed_Right = origin_moments(data$SK_right, k),
  Skewed_Left  = origin_moments(data$SK_left, k),
  Uniform      = origin_moments(data$Uniform, k)
))

origin_df <- as.data.frame(t(origin_results))
```

```
rownames(origin_df) <- c("1st", "2nd", "3rd", "4th")

kable(origin_df, digits = 3,
      caption = "Moments for each of the sets of data")
```

Table 2: Moments for each of the sets of data

|     | Normal      | Skewed_Right | Skewed_Left | Uniform   |
|-----|-------------|--------------|-------------|-----------|
| 1st | 65.12       | 35.48        | 74.2        | 12.056    |
| 2nd | 4248.92     | 1437.72      | 5925.4      | 145.426   |
| 3rd | 277770.92   | 68292.44     | 489458.8    | 1755.158  |
| 4th | 18194173.64 | 3797594.04   | 41396161.5  | 21194.593 |

2. Find the (a) first, (b) second, (c) third, and (d ) fourth moments about the mean for each of the sets of data (normal, skewed-right, skewed-left, uniform).

```
mean_result <- sapply(1:4, function(k) c(
  Normal      = mean_moments(data$Normal, k),
  Skewed_Right = mean_moments(data$SK_right, k),
  Skewed_Left  = mean_moments(data$SK_left, k),
  Uniform      = mean_moments(data$Uniform, k)
))

mean_df <- as.data.frame(t(mean_result))
rownames(mean_df) <- c("1st", "2nd", "3rd", "4th")

kable(mean_df, digits = 5,
      caption = "Moments about the mean for each of the sets of data ")
```

Table 3: Moments about the mean for each of the sets of data

|     | Normal    | Skewed_Right | Skewed_Left | Uniform |
|-----|-----------|--------------|-------------|---------|
| 1st | 0.00000   | 0.0000       | 0.00        | 0.00000 |
| 2nd | 8.30560   | 178.8896     | 419.76      | 0.07886 |
| 3rd | -0.47174  | 4588.1284    | -12498.26   | 0.00034 |
| 4th | 160.94863 | 210642.8834  | 927289.75   | 0.01125 |

3. Find the (a) first, (b) second, (c) third, and (d ) fourth moments about the number 75 for the set of female height measurements.

```
moments75 <- sapply(1:4, function(k) moment_about_mean(data$Normal, k, 75))

moments75_df <- data.frame(
  Moment = c("1st", "2nd", "3rd", "4th"),
  Value  = round(moments75, 2)
)

kable(moments75_df, caption = "Moments about the number 75 for the set of female height measurements")
```

Table 4: Moments about the number 75 for the set of female height measurements

| Moment | Value    |
|--------|----------|
| 1st    | -9.88    |
| 2nd    | 105.92   |
| 3rd    | -1211.08 |
| 4th    | 14572.64 |

#### 4. Using the results of items 2 and 3 for the set of female height measurements, verify the relations between the moments

Using the results of items 2 and 3 for the set of female height measurements, verify the relations between the moments

(a)

$$m_2 = m'_2 - (m'_1)^2$$

(b)

$$m_3 = m'_3 - 3m'_2m'_1 + 2(m'_1)^3$$

(c)

$$m_4 = m'_4 - 4m'_3m'_1 + 6m'_2(m'_1)^2 - 3(m'_1)^4$$

```
# about origin
m1p <- origin_moments(data$Normal, 1)
m2p <- origin_moments(data$Normal, 2)
m3p <- origin_moments(data$Normal, 3)
m4p <- origin_moments(data$Normal, 4)

# about mean
m2 <- mean_moments(data$Normal, 2)
m3 <- mean_moments(data$Normal, 3)
m4 <- mean_moments(data$Normal, 4)

m2_relation <- m2p - m1p^2
m3_relation <- m3p - 3*m2p*m1p + 2*(m1p^3)
m4_relation <- m4p - 4*m3p*m1p + 6*m2p*(m1p^2) - 3*(m1p^4)

moment_tab <- data.frame(
  Moment = c("m2", "m3", "m4"),
  Actual = c(m2, m3, m4),
  Relation = c(m2_relation, m3_relation, m4_relation)
)

kable(moment_tab, digits = 4,
  caption = "Relations between the moments")
```

Table 5: Relations between the moments

| Moment | Actual   | Relation |
|--------|----------|----------|
| m2     | 8.3056   | 8.3056   |
| m3     | -0.4717  | -0.4717  |
| m4     | 160.9486 | 160.9486 |