sa1 part II

Elisha Sophia Borromeo

2025-10-08

SUMMATIVE ASSESSMENT 1PART II

1. Find the (a) first, (b) second, (c) third, and (d) fourth moments of the set 2, 3, 7, 8, 10.
Given Set: 2, 3, 7, 8, 10. Recall the formula for reference:

$$m'_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$m'_2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2$$

$$m'_3 = \frac{1}{n}\sum_{i=1}^{n} x_i^3$$

$$m'_4 = \frac{1}{n}\sum_{i=1}^{n} x_i^4$$

```r
given <- c(2, 3, 7, 8, 10)
n <- length(given)

m1p <- mean(given)
m2p <- mean(given^2)
m3p <- mean(given^3)
m4p <- mean(given^4)

cat("$$",
    "\\begin{aligned}",
    "\\text{Moments of the set:} \\\\",
    "m_1' &= ", m1p, " \\\\",
    "m_2' &= ", m2p, " \\\\",
    "m_3' &= ", m3p, " \\\\",
    "m_4' &= ", m4p,
    "\\end{aligned}",
    "$$")
```

$$\begin{aligned}
\text{Moments of the set:} \\
m_1' &= 6 \\
m_2' &= 45.2 \\
m_3' &= 378 \\
m_4' &= 3318.8
\end{aligned}$$

2. Find the (a) first, (b) second, (c) third, and (d) fourth moments about the mean of the set 2, 3, 7, 8, 10. Now, we will be looking for the moments about the mean of the same set. Recall the formulas for reference:

$$m_1 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})$$

$$m_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$m_3 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3$$

$$m_4 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4$$

```r
given <- c(2, 3, 7, 8, 10)
n <- length(given)
meanV <- mean(given)

m1 <- mean(given - meanV)
m2 <- mean((given - meanV)^2)
m3 <- mean((given - meanV)^3)
m4 <- mean((given - meanV)^4)

cat("$$",
    "\\begin{aligned}",
    "\\text{Central Moments of the set:} \\\\[6pt]",
    "m_1 &= ", m1, " \\\\[6pt]",
    "m_2 &= ", m2, " \\\\[6pt]",
    "m_3 &= ", m3, " \\\\[6pt]",
    "m_4 &= ", m4,
    "\\end{aligned}",
    "$$")
```

$$\begin{aligned}
\text{Central Moments of the set:} \\[6pt]
m_1 &= 0 \\[6pt]
m_2 &= 9.2 \\[6pt]
m_3 &= -3.6 \\[6pt]
m_4 &= 122
\end{aligned}$$

3. Verify:

$$m_4 = m_4' - 4m_1'm_3' + 6m_1'^2m_2' - 3m_1'^4$$

We will still be using the same given set. For purposes of readability, let the right equation be u.

```r
given <- c(2, 3, 7, 8, 10)
n <- length(given)

m1p <- mean(given)
m2p <- mean(given^2)
m3p <- mean(given^3)
m4p <- mean(given^4)

meanV <- mean(given)
m4 <- mean((given - meanV)^4)
u <- m4p - 4*m1p*m3p + 6*(m1p^2)*m2p - 3*(m1p^4)

cat("$$",
    "\\begin{aligned}",
    "\\text{We get:} \\quad & m_4 = ", m4, " \\\\[6pt]",
    "\\text{Likewise:} \\quad & u = ", u, " \\\\[6pt]",
    "\\text{Hence,} \\quad & m_4 = u \\\\[6pt]",
    "\\text{Therefore,} \\quad & m_4 = m_4' - 4m_1'm_3' + 6(m_1')^2 m_2' - 3(m_1')^4",
    "\\end{aligned}",
    "$$")
```

$$\begin{aligned}\text{We get:} \quad & m_4 = 122 \\[6pt]\text{Likewise:} \quad & u = 122 \\[6pt]\text{Hence,} \quad & m_4 = u \\[6pt]\text{Therefore,} \quad & m_4 = m_4' - 4m_1'm_3' + 6(m_1')^2 m_2' - 3(m_1')^4\end{aligned}$$