# FA2

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**Github link: https://github.com/chieelo/STATS.git**

## 3.49 Prove

$$\sum_{i=1}^{n}(x_i - 1)^2 = \sum_{i=1}^{n} x_i^2 - 2\sum_{i=1}^{n} x_i + n.$$

### Proof

We know that

$$(a - b)^2 = a^2 - 2ab + b^2.$$

So, lets expand the left hand side of the equation.

$$(X_j - 1)^2 = X_j^2 - 2X_j + 1.$$

Now, sum over all $j = 1, 2, \ldots, N$:

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N}\left(X_j^2 - 2X_j + 1\right).$$

Then, we distribute the summation

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 \; - \; 2\sum_{j=1}^{N} X_j \; + \; \sum_{j=1}^{N} 1.$$

The third term is simply $N$, because we are adding 1 exactly $N$ times

So:

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N.$$

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Therefore, the right-hand side is just the expansion of the left-hand side.

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N.\blacksquare$$

### 3.51

Two variables, U and V, assume the values:

U1 = 3, U2 = 2, U3 = 5

V1 = 4, V2 = 1, V3 = 6

## Calculate:

```r
# Define the variables
U <- c(3, -2, 5)
V <- c(-4, -1, 6)


a <- sum(U * V)


b <- sum((U + 3) * (V - 4))


c <- sum(V^2)


d <- sum(U) * (sum(V))^2


e <- sum(U * V^2)


f <- sum(U^2 - 2*V^2 + 2)


g <- sum(U / V)
```

$(a)\ \sum UV$

= **20**

$(b)\ \sum(U+3)(V-4)$

= **-37**

$(c)\ \sum V^2$

$=$ **53**

$(d)\ (\sum U)(\sum V)^2$

$=$ **6**

$(e)\ \sum UV^2$

$=$ **226**

$(f)\ \sum (U^2 - 2V^2 + 2)$

$=$ **-62**

$(g)\ \sum (U/V)$

$=$ **2.08**

### 3.9

**Find the geometric mean of the sets:**

**(a)** $[3, 5, 8, 3, 7, 2]$

**(b)** $[28.5, 73.6, 47.2, 31.5, 64.8]$

**The geometric mean of a set of** $n$ numbers $x_1, x_2, \ldots, x_n$ is given by:

$$GM = \left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}}$$

```r
a <- c(3, 5, 8, 3, 7, 2)
geom_mean_a <- prod(a)^(1/length(a))
```

Set A: $[3, 5, 8, 3, 7, 2]$
Therefore, the geometric mean of set (a) is: **4.140681**

```r
b <- c(28.5, 73.6, 47.2, 31.5, 64.8)
geom_mean_b <- prod(b)^(1/length(b))
```

Set B: $[28.5, 73.6, 47.2, 31.5, 64.8]$
Therefore, the geometric mean of set (b) is: **45.8258**