MATLAB is a scripting language without types. That means a few things.

1. There is no need to declare variables before you assign values to them. This, for example, is something that is necessary in JAVA.

2. A variable initially defined to be a scalar than then be assigned to be an array and then assigned to be a function. This can be convenient but also makes it tougher to catch bugs.

3. Unless a specific environment is loaded, MATLAB will do all of its calculations in floating point arithmetic. In a language like PYTHON an expression like `1/4` will return `0` because `0` is the closest integer and `1` and `4` are integers. MATLAB will return `0.25`.

MATLAB code is stored in `.m`-files, or m-files for short. These are also referred to as MATLAB scripts. It is good practice to include the following at the top of every script that your write:

```
clear all;  close all;
```

This will help ensure that

1. The ability of your code to run is not affected by other previously set variables.

2. If you close MATLAB, your script will behave the same when the next time you open it.

MATLAB will also print the output of any line if you do not append a semicolon `;` at the end of the line. For example, executing the following produces output to the Command Window

```
x = 10
```

```
x =

    10
```

while

```
x = 10;
```

produces no output. In order to debug your code you will want to have nearly every line end with a semicolon. And then if the script does not run as expected you can remove one or two semicolons at a time to monitor the output.

The following code uses a *for loop* to add up all the positive integers that are less than or equal to n

```
n = 10;
SUM = 0; % using capital letters because sum() is a built-in function
for i = 1:n
    SUM = SUM + i;
end
SUM
n*(n+1)/2  % known answer to check
```

```
SUM =

    55


ans =

    55
```

Another type of loop is the *while loop*. Here is an example of performing the same sum as above using a while loop.

```
n = 10;
SUM = 0;
i = 0;
while i < n
    i = i + 1;
    SUM = SUM + i;
end
SUM
```

```
SUM =

    55
```

0.1 ▪ Using Matlab with some examples from calculus

Because Part I of this text concerns numerical linear algebra, there are few opportunities introduce the reader to the plotting functionality in MATLAB. So, we take some time to review some theorems from calculus and illustrate them using MATLAB.

0.1.1 ▪ Demonstrations from differential calculus

Definition 0.1. *A function f defined on a set X has limit L at x_0*

$$\lim_{x\to x_0} f(x) = L$$

if, given any real number $\epsilon > 0$, there exists a real number $\delta > 0$ such that

$$|f(x) - L| < \epsilon, \quad \textit{whenever} \quad x \in X \quad \textit{and} \quad 0 < |x - x_0| < \delta.$$

Definition 0.2. *Let f be a function defined on a set X of real numbers and $x_0 \in X$. Then f is continuous at x_0 if*

$$\lim_{x\to x_0} f(x) = f(x_0).$$

The function

$$f(x) = \begin{cases} \cos(\pi/x), & x \neq 0, \\ 1 & x = 0, \end{cases}$$

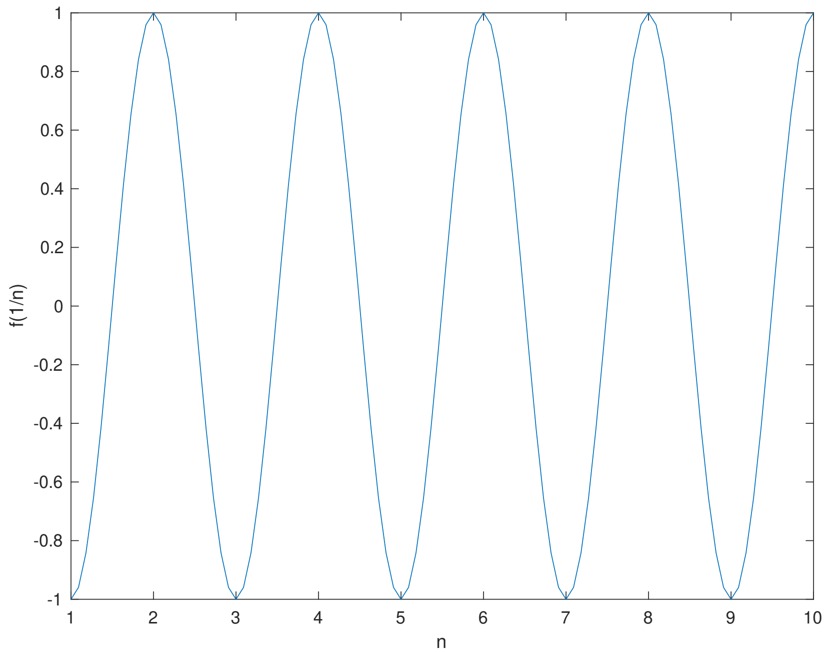

Figure 1. *A plot of the function $f(1/n)$*

is not continuous at $x = 0$. To show this, let $x_n = 1/n$. If f is continuous then $\lim_{n\to\infty} f(1/n) = 1$

```
f = @(x) cos(pi./x);
ns = linspace(1,10,100);
plot(ns,f(1./ns))
xlabel('n'); ylabel('f(1/n)') %label axes
```

The plot that results is shown in Figure 1

Definition 0.3. *Let $C^n[a, b] = \{f : [a, b] \to \mathbb{C} \mid f^{(n)}$ exists and is continuous$\}$.*

We use the notation $C[a, b] := C^0[a, b]$.

Theorem 0.4 (Intermediate Value Theorem). *Let $f \in C[a, b]$. Assume $f(a) \neq f(b)$. For every real number y, $f(a) \leq y \leq f(b)$, there exists $c \in [a, b]$ such that $f(c) = y$.*

```
x = linspace(-3,3,100);
f = @(x) sin(x);
c = @(x) 0*x+.1;
plot(x,f(x),'k')
hold on
plot(x,c(x)) %every value between f(-3) and f(3) is attained at least once
```

The plot that results is shown in Figure 2

Definition 0.5. *Let f be a function defined on an open interval containing x_0. The*

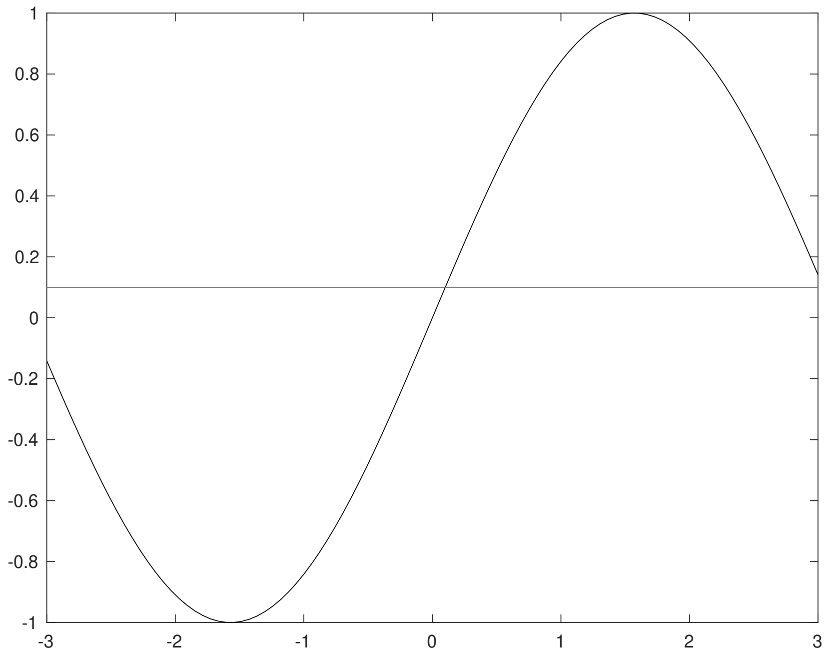

Figure 2. *A demonstration of the intermediate value theorem for $f(x) = \sin(x)$.*

function f is differentiable at x_0 if

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

exists. In which case, $f'(x_0)$ is the derivative of $f(x)$ at x_0. If f has a derivative at each point in a set X then f is said to be differentiable on X.

```
format long %to see more digits
f = @(x) sin(x); df = @(x) cos(x);
x = .0001; x0 = 0;
(sin(x)-sin(x0))/(x-x0)-cos(x0)
```

```
ans =

   -1.666666582522680e-09
```

Here are some of the most important theorems from single-variable calculus:

Theorem 0.6. *If a function f is differentiable at x_0, it is continuous at x_0.*

Theorem 0.7 (Rolle's Theorem). *Suppose $f \in C[a, b]$ and f is differentiable on $[a, b]$. If $f(a) = f(b)$, the a number c in (a, b) exists with $f'(c) = 0$.*

Theorem 0.8 (Mean Value Theorem). *Suppose $f \in C[a, b]$ and f is differentiable*

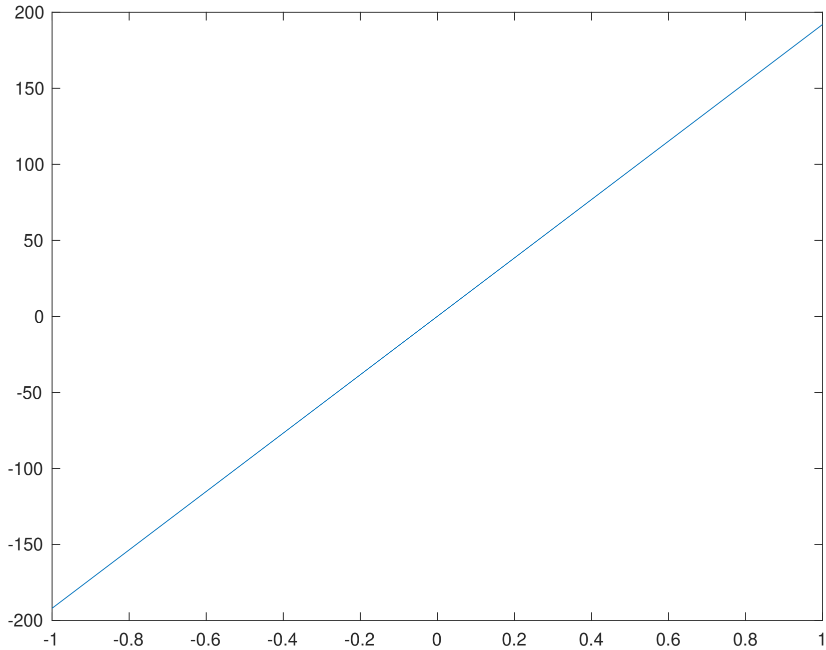

Figure 3. *A demonstration of Theorem 0.10 with* $f(x) = 8x^4 - 8x^2 + 1$.

on $[a, b]$. *There exists a point* $c \in (a, b)$ *such that*

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Theorem 0.9 (Extreme Value Theorem). *If* $f \in C[a, b]$, *then* $c_1, c_2 \in [a, b]$ *exist with* $f(c_1) \leq f(x) \leq f(c_2)$, *for all* $x \in [a, b]$. *Furthermore, if* f *is differentiable on* $[a, b]$ *then* c_1, c_2 *are either the endpoints (*a *or* b*) or at a point where* $f'(x) = 0$.

This theorem states that both the maximum and minimum values of $f(x)$ on a closed interval $[a, b]$ must be attained within the interval (at points c_2 and c_1). A more involved theorem is the following:

Theorem 0.10. *Suppose* $f \in C[a, b]$ *is* n*-times differentiable on* (a, b). *If* $f(x) = 0$ *at* $n + 1$ *distinct numbers* $a \leq x_0 < x_1 < \cdots < x_n \leq b$, *then a number* $c \in (x_0, x_n)$, *(and hence in* (a, b)*) exists with* $f^{(n)}(c) = 0$.

Consider the fourth degree polynomial $f(x) = 8x^4 - 8x^2 + 1$:

```
f = @(x) 8*x.^4-8*x.^2+1; % has 4 zeros on (-1,1)
dddf = @(x) 8*4*3*2*x; % must have 1 zero on (-1,1)
x = linspace(-1,1,100);
plot(x,dddf(x))
```

The plot that results is shown in Figure 3

Theorem 0.11 (Taylor's Theorem). *Suppose* $f \in C^n[a, b]$, *and that* $f^{(n+1)}$ *exists on*

$[a,b]$, and $x_0 \in [a,b]$. For every $x \in [a,b]$, there exists a number $\xi(x)$ between x_0 and x with

$$f(x) = P_n(x) + R_n(x),$$

where

$$P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n,$$

and

$$R_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x - x_0)^{n+1}.$$

0.1.2 ▪ Demonstrations from integral calculus

Definition 0.12. *The Riemann integral of the function f defined on the interval $[a,b]$ is the following limit (if it exists):*

$$\int_a^b f(x)dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(\bar{x}_i)\Delta x_i,$$

where the numbers $x_0, x_1, \ldots, x_n$ satisfy $a = x_0 \leq x_1 \leq \cdots \leq x_n = b$, $\Delta x_i = x_i - x_{i-1}$ for $i = 1, 2, \ldots, n$. And $\bar{x}_i$ is an arbitrary point in the interval $[x_{i-1}, x_i]$.

Let's choose the points x_i to be evenly spaced: $x_i = a + i\frac{b-a}{n}$ and $\bar{x}_i = x_i$. Then we have $\Delta x_i = \frac{b-a}{n}$ and

$$\int_a^b f(x)dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(x_i).$$

```
f = @(x) exp(x);
n = 10; a = -1; b = 1;
x = linspace(a,b,n+1); % create n + 1 points
x = x(2:end); % take the last n of these points
est = (b-a)/n*sum(f(x)) % evaluate f at these points and add them up
actual = exp(b)-exp(a) % the actual value
abs(est-actual)
```

```
est =

   2.593272082493666


actual =

   2.350402387287603
```

```
ans =

   0.242869695206063
```

Now choose and $\bar{x}_i = \dfrac{x_i + x_{i-1}}{2}$ to be the midpoint. We still have $\Delta x_i = \frac{b-a}{n}$ and

$$\int_a^b f(x)dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(\bar{x}_i).$$

```
f = @(x) exp(x);
n = 10; a = -1; b = 1;
x = linspace(a,b,n+1); % create n + 1 points
x = x(2:end); % take the last n of these points
x = x - (b-a)/(2*n); % shift to the midpoint
est = (b-a)/n*sum(f(x)) % evaluate f at these points and add them up
actual = exp(b)-exp(a) % the actual value
abs(est-actual)
```

```
est =

   2.346489615388305


actual =

   2.350402387287603


ans =

   0.003912771899298
```

Theorem 0.13 (Weighted Mean Value Theorem). *Suppose $f \in C[a,b]$, the Riemann integral of g exists on $[a,b]$, and $g(x)$ does not change sign on $[a,b]$. Then there exists a number c in (a,b) with*

$$\int_a^b f(x)g(x)dx = f(c)\int_a^b g(x)dx.$$